Price Sensitivities for U.S. Frozen Dairy Products

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To be presented at the annual meetings of the American Agricultural Economics Association, Long Beach, Ca., July 28-31, 2002.

Abstract: Price elasticities and flexibilities for a system of frozen dessert products are estimated from scanner data. Simultaneity tests reject exogeneity of conditional expenditures, but not prices or quantities, at the weekly level. Inverting the elasticity matrix to obtain flexibilities, while theoretically appropriate, appears to be empirically unacceptable.

keywords: dairy products, price flexibility, scanner data, simultaneity

JEL: D12

Background and Objectives

In the dairy sector, both private and public decision makers require contemporary demand analysis. For example, in 1998 the International Dairy Foods Association commissioned a retail demand analysis of a wide range of dairy products, motivated by the industry's perception that dairy demand was becoming more elastic (Maynard, 1999). In 1999, one of the authors testified in a breach-of-contract civil case in which the own-price elasticity of demand for frozen novelties was a salient issue. Recently, the GAO sought contemporary dairy demand elasticities for use in preparing its analysis of Northeast Dairy Compact impacts (United States General Accounting Office, 2001).

One gap in the existing dairy demand literature is estimation of flexibilities (the percentage change in price given a one percent increase in quantity) from inverse demand systems. Although quantity is the individual consumer's choice variable, aggregate quantity of perishable dairy products at any given time may be predetermined, and price is a choice variable from the retailer's perspective. Cash and futures market analysts can use demand flexibilities to forecast price changes resulting from supply shocks. Demand flexibilities may be used in price transmission models and in market power studies of price distortion.

One might be tempted to substitute reciprocals of price elasticities of demand where flexibilities are needed. A reciprocal relationship would theoretically hold only for goods that had no substitutes or complements. One might next be tempted to substitute the inverse of the price elasticity matrix for the matrix of flexibilities. Inverting the elasticity matrix is theoretically appropriate, but empirically inappropriate because elasticities are stochastic estimates.

Huang (1994, 1996) and Eales (1995) debated about how potential simultaneity of prices and quantity should affect estimation of elasticities and flexibilities. Huang argued that flexibilities should always be estimated directly, while Eales countered that simultaneity tests should first determine whether ordinary or inverse demand models were appropriate. If prices were predetermined and quantities were endogenous, Eales argued that an ordinary demand system was appropriate, and that flexibilities should be obtained by inverting the elasticity matrix. Huang (1996) responded that inverting a matrix estimated from stochastic variables would produce inaccurate and possibly unstable flexibility estimates. Huang clarified that small (in absolute value) elasticity estimates do not necessarily imply large flexibility estimates.

The objective of this study was to estimate sensitivities (i.e., price elasticities and quantity flexibilities) of retail demand using weekly U.S. average scanner data for seven types of products within the frozen dessert category. Synthetic ordinary and inverse demand systems were estimated that allow flexibility in how expenditure shares affect parameter estimates. Hausman tests were used to test whether quantity-dependent (ordinary) or price-dependent (inverse) are appropriate at the weekly retail level.

Demand estimates for specific products such as frozen desserts are (perhaps understandably) rare. Boehm (1975) estimated price elasticities of demand for ice cream and frozen novelties, Huang (1993) estimated price elasticities for ice cream as part of a large-scale complete demand system for food using annual U.S. data from 1953 to 1990, and Maynard and Liu (1999) estimated price elasticities for ice cream, frozen yogurt, and frozen novelties. To the authors' knowledge, this is the most comprehensive demand analysis of frozen dairy products and their primary substitutes. While certain stakeholders require estimates at this level of product disaggregation, the results also motivate a general discussion about the role of a publicly available database of demand sensitivity estimates in private and public decision making.

Methods

Ordinary synthetic demand system

Elasticities were estimated directly from an ordinary (i.e., quantity-dependent) conditional demand system. A synthetic model developed by Barten (1993) aided model selection by parameterizing, rather than assuming, the influence of expenditure shares on marginal expenditure shares and Slutsky terms.

Lee, Brown, and Seale (1994) and Brown, Lee, and Seale (1994) provide details of the ordinary synthetic demand system. The synthetic system nests four differential demand systems: the Rotterdam, the linear approximate almost ideal demand system (LA/AIDS), the CBS system (named after the Dutch Central Bureau of Statistics), and the NBR system (named after the National Bureau of Research). Marginal budget shares and Slutsky terms are treated as constants in the Rotterdam model, but they are treated as functions of budget share levels in the LA/AIDS model. The CBS model has the LA/AIDS income coefficients and the Rotterdam price coefficients, while the NBR model has Rotterdam income coefficients and LA/AIDS price coefficients. One first estimates the following nonlinear model (using proc MODEL in SAS, for example) to identify which of the four specifications best describes the data:

$$w_i d\ln q_i = (d_i + {}_1 w_i) d\ln Q + \sum_j [e_{ij} - d_2 w_i (d_{ij} - w_j)] d\ln p_j \qquad ,$$

where q_i denotes the quantity demanded of the i^{th} good, $dlnQ$ denotes the Divisia volume index, p_i denotes the price of the i^{th} good, and $ä_{ij}$ denotes the Kronecker delta such that $ä_{ij}$=1 if i=j, and $ä_{ij}$=0 if $i…j$. The parameter d_i is a weighted average of the expenditure parameters $â_i$ and $è_i$ in the LA/AIDS and Rotterdam models, respectively. Likewise, the parameter e_{ij} is a weighted average of the compensated price parameters ã$_{ij}$ and ð$_{ij}$ in the LA/AIDS and Rotterdam models, respectively:

$$d_i = d_1 b_i + (1 - d_1) q_i ; \quad e_{ij} = d_2 g_{ij} + (1 - d_2) p_{ij} \quad .$$

Restricting the value of ä$_1$ and ä$_2$ yields the following demand systems:

Rotterdam	ä$_1$=ä$_2$=0
LA/AIDS	ä$_1$=ä$_2$=1
CBS	ä$_1$=1, ä$_2$=0
NBR	ä$_1$=0, ä$_2$=1.

Likelihood ratio tests evaluated at q=2 restrictions allow one to choose which set of restrictions best describes the data.

One may either impose restrictions on ä$_1$ and ä$_2$ and re-estimate a specific model, or obtain elasticity estimates directly from the nonlinear synthetic model (typically at the expenditure share means):

expenditure elasticity ç$_i$ = $(d_i$ + ä$_1 w_i)/w_i$

compensated price elasticity ç$_{ij}$ = $(e_{ij}$ - ä$_2 w_i($ä$_{ij} - w_j)/w_i$

uncompensated price elasticity ç$_{ij}$* = ç$_{ij}$ + w_j ç$_i$.

Theoretical demand restrictions in the synthetic model are as follows, where equations are indexed by i and price terms within an equation are indexed by j:

Adding-up Ó$_i d_i$ = 1 - ä$_1$, Ó$_i$ e_{ij} = 0 for all j

Homogeneity Ó$_j$ e_{ij} = 0 for all i

Symmetry $e_{ij} = e_{ji}$ for all i, j .

Inverse synthetic demand system

Flexibilities were estimated directly from an inverse (i.e., price-dependent) conditional demand system. Brown, Lee, and Seale (1995) developed a synthetic inverse demand system analogous to Barten's (1993) synthetic ordinary demand system, and applied it to orange varieties for which quantities were expected to be predetermined. Brown, Lee, and Seale (1995) provide details of the following summary.

The synthetic inverse system nests four differential demand systems: the inverse Rotterdam (RIDS), the almost ideal inverse demand system (AIIDS), the Laitinen-Theil system, and the RAIIDS system (a RIDS/AIIDS hybrid). The relationships between expenditure shares and compensated quantity and Antonelli coefficients are parameterized to relax the maintained assumptions of specific inverse demand systems. The Antonelli matrix is the generalized inverse of the Slutsky substitution matrix, with each element representing the compensated price impact of a unitary change in quantity (Deaton and Muellbauer, 1980, p. 57). The synthetic inverse demand system is:

$$w_i d \ln p_i = (d_i + d_1 w_i) d \ln Q + \sum_j [e_{ij} - d_2 w_i (d_{ij} - w_j)] d \ln q_j \quad ,$$

where $ð_i$ denotes p_i/x, x denotes total expenditure, and all other variables are defined as in the ordinary demand system. The parameter d_i is a weighted average of the scale parameters in the RIDS and AIIDS models, respectively. Likewise, the parameter e_{ij} is a weighted average of the compensated quantity parameters in the RIDS and AIIDS models.

Restricting the value of $ä_1$ and $ä_2$ yields the following inverse demand systems:

RIDS	$ä_1=ä_2=0$
AIIDS	$ä_1=ä_2=1$

Laitinen-Theil $\ddot{a}_1=1,\ \ddot{a}_2=0$

RAIIDS $\ddot{a}_1=0,\ \ddot{a}_2=1.$

Likelihood ratio tests evaluated at q=2 restrictions allow one to choose which set of restrictions best describes the data.

As with the ordinary synthetic model, one may either impose restrictions on $\ddot{a}_1$ and $\ddot{a}_2$ and re-estimate a specific model, or obtain flexibility estimates directly from the nonlinear synthetic model:

scale flexibility $f_i = (d_i + \ddot{a}_1 w_i)/w_i$

compensated price flexibility $f_{ij} = (e_{ij} - \ddot{a}_2 w_i(\ddot{a}_{ij} - w_j)/w_i$

uncompensated price flexibility $f_{ij}{}^* = f_{ij} + w_j f_i$.

Theoretical demand restrictions in the synthetic inverse model are as follows, where equations are indexed by i and price terms within an equation are indexed by j:

Adding-up $\acute{O}_i d_i = -1 + \ddot{a}_1$, $\acute{O}_i\, e_{ij} = 0$ for all j

Homogeneity $\acute{O}_j\, e_{ij} = 0$ for all i

Symmetry $e_{ij} = e_{ji}$ for all i, j .

Endogeneity testing

Quantity-dependent demand models produce consistent elasticity estimates when prices are predetermined or exogenous. Inverse demand models are appropriate when quantities are predetermined, and are commonly used when biological lags characterize food production. Incorrect assumptions about exogeneity produce biased and inconsistent estimates (see, e.g., Pindyck and Rubinfeld, 1991, ch. 11). If endogenous variables appear on both sides of an equation, consistent estimates may be obtained by replacing endogenous right-hand-side variables with exogenous or predetermined instruments.

Suppose prices are predetermined in a demand system, but one needs to obtain flexibility estimates (for example, to calculate a Lerner index of market power-induced price distortion). Should one invert the consistently estimated elasticity matrix, or should one estimate an inverse system via instrumental variables?

Eales (1995) argued that simultaneity tests should first determine whether ordinary or inverse demand models were appropriate. If prices were predetermined, Eales argued that flexibilities should be obtained by inverting the elasticity matrix. While agreeing that the flexibility matrix is theoretically equivalent to the inverted elasticity matrix, Huang (1996) used the Cauchy-Schwartz inequality to demonstrate that the inverse of a directly estimated elasticity matrix will not equal the flexibility matrix estimated from the same data. Furthermore, inverted statistical estimates may be unstable. Huang (1994, 1996) suggested direct flexibility estimation.

The debate clarified, but did not resolve, the analytical tradeoffs between simultaneity bias and parameter instability. The approach used in this study integrated both perspectives. Elasticities and flexibilities were estimated directly from quantity-dependent and price-dependent models, respectively. If Hausman tests rejected exogeneity of right-hand-side variables, instrumental variable (IV) estimators were used to obtain consistent estimates. IV estimators such as 3SLS are consistent but are generally biased (Davidson and MacKinnon, 1993, p. 217). The potential bias of the IV estimator was deemed less costly than the potential instability of inverting parameter matrices obtained from nonlinear models.

Hausman tests were performed by regressing potentially endogenous variables on a set of exogenous and predetermined instruments, and including the residuals as a regressor in the original demand model (Davidson and MacKinnon, 1993, p. 239). Statistical significance of the generated residual term indicates that parameter estimates in the demand system are significantly affected by endogeneity of right-hand-side variables. If endogeneity was detected, the affected demand system was estimated via 3SLS instead of SUR.

In the ordinary demand system, price terms (dlnp_i) were jointly tested for exogeneity, as was the Divisia volume index (dln$Q$). In the inverse demand system, quantity terms (dlnq_i) were jointly tested, as was dln$Q$. Instruments in all cases consisted of current and lagged seasonality and holiday variables, lagged price terms, lagged quantity terms, and lagged dln$Q$.

Eales and Unnevehr (1993) performed Hausman tests using livestock production costs as instruments in ordinary and inverse AIDS models of meat demand. The annual data suggested that only beef quantity was predetermined; all other prices and quantities were endogenous. Brown, Behr, and Lee (1994) performed Hausman tests, using current and lagged exogenous variables and lagged endogenous variables as instruments, on a conditional ordinary Rotterdam system for fruit juices using weekly scanner data. Neither prices nor conditional expenditures were found to be endogenous. The exogeneity of conditional expenditures was interpreted as support for rational random behavior. Lee, Brown, and Seale (1994) also determined that dln$Q$ was exogenous in a complete ordinary AIDS system using annual Taiwanese data for highly aggregated goods.

Data and Estimation

Demand for frozen dessert products was estimated using weekly national average retail scanner data provided by A.C. Nielsen via the International Dairy Foods Association for the weeks ending August 3, 1996 through November 21, 1998 (n = 121). The raw data consist of nominal prices and quantities for seven products: ice cream, frozen yogurt, sherbet, sorbet, branded frozen novelties, private label frozen novelties, and "other packaged frozen" products. The data reflect sales at retail grocery stores with over $2 million in annual sales, and are similar to the juice data used by Brown, Behr, and Lee (1994) in that they are highly aggregated across space but quite disaggregated across time and form. Table 1 provides descriptive statistics.

Prices were deflated by the Consumer Price Index. Seasonality was represented by a cosine transformation that fluctuated between one on July 1 and negative one on January 1. A holiday dummy variable equaled one during weeks containing Memorial Day, July 4th, and Labor Day. Prior experience with scanner data indicated that complete demand systems in which the products of interest have very small budget shares often produce unstable parameter estimates. Interpolating monthly personal consumer expenditure data to obtain weekly observations also has the potential to introduce intra-month measurement errors that would bias parameter estimates. Conditional demand systems were therefore estimated, with total expenditures defined as expenditures on the group of seven frozen dessert product types. The data add up by construction. F-tests failed to reject any of the homogeneity and symmetry restrictions at the .05 level, and all theoretical restrictions were imposed in subsequent estimation. All parameter estimates reflect correction for autocorrelation.

Results

Regarding choice of functional form, table 2 shows likelihood ratio tests for restrictions on $\ddot{a}_1$ and $\ddot{a}_2$ in both the ordinary and inverse synthetic demand systems. All four specific models were rejected in each system. Estimated values of $\ddot{a}_1$ were 1.29 in the ordinary system and 1.12 in the inverse system, while estimated values of $\ddot{a}_2$ were 2.53 in the ordinary system and 0.27 in the inverse system. Each estimated value of $\ddot{a}_1$ and $\ddot{a}_2$ was significantly different from zero at the .01 level, and the estimated values of ä_2 in each system were significantly different from one at the .01 level. Rather than estimate specific functional forms that had been rejected by the data, the nonlinear synthetic models themselves were used for subsequent estimation.

Table 3 contains the results of Hausman tests for exogeneity of right-hand-side variables. Neither prices in the ordinary system, nor quantities in the inverse system, were sufficiently endogenous to significantly affect the vector of contrasts between parameters estimated via nonlinear SUR versus nonlinear 3SLS. Eales and Unnevehr (1993) generally rejected exogeneity in annual data, while Brown, Behr, and Lee (1994) failed to reject exogeneity in weekly prices. The frozen dessert data suggest that market-clearing adjustments in both prices and quantities occur over durations exceeding one week. The implication is that ordinary and inverse demand systems may both be consistently estimated via SUR, without resorting to an IV estimator such as 3SLS.

Hausman tests for exogeneity of conditional expenditures were more ambiguous. The null hypothesis of exogeneity was not rejected in either system when all seven dln*Q* terms were tested jointly. However, in both systems, exogeneity was rejected at the .05 level in a joint test of dln*Q* involving the three products that accounted for 90 percent of frozen dessert expenditures: ice cream, frozen yogurt, and branded frozen novelties. Accordingly, subsequent estimation treated dln*Q* as endogenous, and nonlinear 3SLS replaced the nonlinear SUR estimator.

Table 4 contains the compensated price elasticity matrix estimated from the ordinary synthetic demand system. Adjusted R-squared statistics ranged from 0.42 in the sherbet equation to 0.91 in the ice cream equation. Except for sherbet (-0.71), all product types were price elastic, with branded frozen novelties being the most elastic (-2.39). The elasticity magnitudes were similar to those of other dairy products estimated from scanner data (Maynard and Liu, 1999). The dominant roles of ice cream and branded frozen novelties in the frozen dessert category are evident in the cross-price elasticities.

Table 5 contains the compensated price flexibility matrix. Explanatory power was higher in the price-dependent system, with adjusted R-squared statistics ranging from 0.82 in the “other packaged frozen” equation to 0.99 in the ice cream equation. All own-price flexibilities are less than one in absolute value (inflexible). While this is qualitatively consistent with the elastic values in table 4, quantities appear to be less elastic than the reciprocal or inverse of the flexibilities would suggest. Alternatively, prices are less flexible than the reciprocal or inverse of the elasticities would suggest. A similar pattern exists in the Eales and Unnevehr (1993) SUR estimates and in Huang (1994).

Discussion

The objective of this study was to provide public and private decision makers with accurate demand elasticity and flexibility estimates for frozen dairy products and their primary substitutes, with particular emphasis on functional form selection and treatment of endogeneity. The results supported Huang’s (1994, 1996) contention that inverting an elasticity matrix will produce substantially different outcomes than direct estimation of flexibilities from a price-dependent demand system.

The most interesting discussion point involves the debate between Huang (1994, 1996) and Eales (1996) regarding the propriety of obtaining flexibility estimates by direct estimation or inversion of the elasticity matrix. Consider the position of the analyst working for a government agency or a consulting firm, who may need price flexibility estimates for forecasting or policy analysis purposes. Frequently, time and data constraints prohibit direct estimation. Available information may consist only of previously estimated own-price elasticities. Given these constraints, the only feasible approach may be to calculate flexibilities as the reciprocal of available own-price elasticities.

Inverting the elasticity matrix is theoretically appropriate, and calculating own-price flexibilities as reciprocals of elasticities would be theoretically appropriate only for products with no substitutes or complements. However, if weak substitute/complement relationships exist, the reciprocals may not differ significantly from their inverted counterparts (Huang's 1994 results illustrate this). The bigger culprit is the difference between inverted elasticities and directly estimated flexibilities. In Eales and Unnevehr (1993), inverted elasticities differ from estimated flexibilities by 15 percent (pork) to 128 percent (chicken). In Huang (1994), inverted elasticities differ from estimated flexibilities by 37 percent (high-quality beef) to 1,071 percent (manufacturing-grade beef). In the present study, cross-price terms for ice cream in the branded and private label frozen novelties equations caused the inverted elasticity matrix to blow up, illustrating the sensitivity to numeric structure referred to by Huang (1996).

Earlier in the manuscript, Lerner's index of price distortion was used as an example where researchers have alternately used inverted elasticities (e.g., Schroeter, 1988) or advocated direct flexibility estimation (e.g., Sexton, 2000). The flexibility discrepancies described above could easily make the difference between attributing either modest or extreme price distortions to market power, with subsequent impacts on policy recommendations.

Theoretical rationale notwithstanding, it appears empirically inappropriate in most cases to use inverted elasticity matrices as demand flexibilities. Where does this leave the agency analyst or consultant who does not have the time or data to estimate flexibilities directly? In marketing courses, we tell our students that elasticities and flexibilities are useful because they can be inserted into many economic models without requiring that a full-blown demand study accompany every economic analysis. Analysts would be well-served by a publicly-accessible database of directly estimated demand elasticities and flexibilities for food products over a wide range of temporal, spatial, and product aggregation. A coordinated effort could exploit economies of scale in methods development, data collection, and estimation procedures, and would generate outputs of value both within the discipline and among our stakeholders.

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Table 1. Descriptive statistics

Quantities (000)	Mean	Std. Dev.	Min	Max
ice cream	10,599.43	1,530.14	7,939.96	14,295.07
frozen yogurt	862.19	180.16	525.39	1,322.48
sherbet	421.62	65.08	311.15	563.47
sorbet	68.47	15.25	43.48	96.81
branded novelties	7,414.34	2,572.68	3,670.27	12,793.66
private label novelties	2,631.32	909.71	1,135.57	4,586.91
other frozen	72.34	20.27	48.33	182.07

Nominal Prices	Mean	Std. Dev.	Min	Max
ice cream	$6.16	$0.27	$5.67	$7.13
frozen yogurt	$7.58	$0.24	$7.01	$8.25
sherbet	$5.54	$0.30	$5.01	$6.38
sorbet	$20.41	$0.73	$18.46	$21.79
branded novelties	$3.60	$0.18	$3.24	$4.02
private label novelties	$2.03	$0.11	$1.77	$2.32
other frozen	$25.58	$1.63	$20.91	$28.90

Expenditure Shares	Mean	Std. Dev.	Min	Max
ice cream	60.35%	3.25%	53.78%	66.67%
frozen yogurt	6.07%	1.11%	4.39%	8.15%
sherbet	2.16%	0.15%	1.92%	2.80%
sorbet	1.28%	0.11%	1.06%	1.51%
branded novelties	23.69%	3.08%	17.84%	29.08%
private label novelties	4.74%	0.63%	3.39%	5.93%
other frozen	1.72%	0.44%	1.24%	3.38%

Table 2. Likelihood ratio tests reject common functional forms

Ordinary Demand	L.R. Statistic	Inverse Demand	L.R. Statistic
Rotterdam (ä1=0, ä2=0)	58.34 ***	RIDS (ä1=0, ä2=0)	245.15 ***
AIDS (ä1=1, ä2=1)	11.70 ***	AIIDS (ä1=1, ä2=1)	191.64 ***
CBS (ä1=1, ä2=0)	29.78 ***	Laitinen-Theil (ä1=1, ä2=0)	28.49 ***
NBR (ä1=0, ä2=1)	41.83 ***	RAIIDS (ä1=0, ä2=1)	437.92 ***

*** denotes likelihood ratio statistic > critical ÷^{2} value for 2 d.f. at the .01 level

Table 3. Hausman tests suggest conditional expenditure terms may be endogenous

	Potentially endogenous regressors	L.R. Statistic
	dlnp_i, i=1-7	2.98
Ordinary Demand	dln$Q$ (all equations)	8.93
	dln$Q$ (ice cream, frozen yogurt, branded novelties)	8.03 **
	dlnq_i, i=1-7	1.10
Inverse Demand	dln$Q$ (all equations)	10.15
	dln$Q$ (ice cream, frozen yogurt, branded novelties)	9.39 **

** denotes likelihood ratio statistic > critical $\div^2$ value for 3 d.f. at the .05 level

Table 4. Compensated price elasticity matrix, estimated from ordinary demand system

	Ice cream	Frozen yogurt	Sherbet	Sorbet	Branded novelties	Private-label novelties	Other frozen
Ice cream	**-1.30**	0.13	0.06	0.05	0.91	0.20	-0.04
Frozen yogurt	1.24	**-1.72**	0.00	0.01	0.53	-0.07	0.01
Sherbet	1.78	-0.01	**-1.43**	-0.44	0.22	-0.16	0.04
Sorbet	2.28	0.05	-0.74	**-0.71**	-0.65	-0.09	-0.14
Branded novelties	2.31	0.14	0.02	-0.04	**-2.39**	-0.23	0.19
Private-label novelties	2.51	-0.09	-0.07	-0.03	-1.15	**-1.59**	0.43
Other frozen	-1.45	0.04	0.05	-0.10	2.65	1.17	**-2.37**

Table 5. Compensated price flexibility matrix, estimated from inverse demand system

	Ice cream	Frozen yogurt	Sherbet	Sorbet	Branded novelties	Private-label novelties	Other frozen
Ice cream	**-0.09**	0.03	0.00	0.00	0.04	0.00	0.01
Frozen yogurt	0.26	**-0.37**	-0.01	0.01	0.06	0.09	-0.03
Sherbet	0.12	-0.04	**-0.17**	0.00	0.11	-0.03	0.00
Sorbet	0.17	0.03	0.00	**-0.34**	0.18	-0.04	-0.01
Branded novelties	0.10	0.02	0.01	0.01	**-0.14**	0.02	-0.02
Private-label novelties	0.06	0.12	-0.01	-0.01	0.09	**-0.25**	0.01
Other frozen	0.44	-0.12	0.00	0.00	-0.26	0.02	**-0.08**