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A Nonparametric Approach to the von Liebig-Paris Technology

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Working Paper 94-03

April 1994

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Quirino Paris and several co-authors have argued in a series of papers that von Liebig's "law of the minimum" should guide formulation of nutrient-response models (Ackello-Ogutu, Paris, and Williams; Paris and Knapp; Paris). This "law" posits two essential characteristics governing nutrient use by crops: (1) non-substitution between nutrients, and (2) a yield plateau. Polynomial specifications of nutrient response, which are frequently used to make fertilizer recommendations, have neither of these features and thus generally result in higher recommended fertilizer application rates than would emerge from a von Liebig specification.

Ackello-Ogutu, Paris and Williams compared a linear von Liebig model of nutrient response for phosphorus and potassium on corn against quadratic and square-root models; non-nested hypothesis tests tended to reject the polynomial models but not the von Liebig. Frank, Beattie and Embleton compared a linear von Liebig model for potassium and nitrogen on corn with quadratic and Mitscherlich-Baule models using the Heady, Pesek and Brown data. Non-nested hypothesis tests tended to reject the quadratic and linear von Liebig formulations, but not the Mitscherlich-Baule, which exhibits a yield plateau but not non-substitution between nutrients. Paris re-examined these data, comparing quadratic, square-root, linear von Liebig and Mitscherlich-Baule models with a nonlinear von Liebig specification that allows diminishing marginal productivity but maintains the properties of non-substitution and a yield plateau. Non-nested hypothesis tests rejected the quadratic, square-root, and linear von Liebig models at a 1% significance level and the Mitscherlich-Baule at a 5% level, but did not reject the nonlinear von Liebig. In the agronomy literature, Cerrato and Blackmer compared quadratic, square-root, Mitscherlich-Baule, linear von Liebig and quadratic-with-plateau specifications for nitrogen on corn. They concluded that the quadratic-with-plateau model best described the observed yield responses in their study, thus supporting Paris' finding of diminishing marginal productivity combined with a yield plateau.

We take up this issue using a different approach. Rather than specify particular functional forms for nutrient response and use non-nested hypothesis tests to distinguish between them, we develop dual representations of a von Liebig technology and then show (1) how to use parametric and nonparametric methods to determine whether yield plateaus exist for differing bundles of fixed inputs and (2) how to investigate the degree of substitutability. We then apply the nonparametric yield-plateau methods to an experimental data set. Our findings support the nonlinear von Liebig specification for that data set.

Primal and Dual Representations of the von Liebig-Paris Technology

Our starting point is Paris' generalized von Liebig technology. Our notation is: $\mathbf{x} \in \mathbb{R}^n_+$ is a vector of inputs, $x_i \in \mathbb{R}_+$ represents the ith element of $\mathbf{x}$, $y \in \mathbb{R}_+$ represents output. The von Liebig-Paris (vLP) technology satisfies:

(1) $$y = \text{Min}\,\{f_1(x_1), ..., f_n(x_n)\}$$

where each f_i: $\mathbb{R}_+ \rightarrow \mathbb{R}_+$ is an arbitrary numeric function. For the sake of simplicity we shall assume that each f_i is strictly increasing (and hence invertible). As Paris demonstrates, the vLP technology can easily be made consistent with the law of diminishing marginal returns by choosing each f_i to be concave.

The input-requirement set for this technology, which gives all input combinations capable of producing a given output level, is defined by the correspondence V: $\mathbb{R}_+ \rightarrow \mathbb{R}^n_+$,

$$V(y) = \{\mathbf{x} : \text{Min}\,\{f_1(x_1), ..., f_n(x_n)\} \geq y\}$$

$$= \cap_i \{x_i : f_i(x_i) \geq y\}$$

$$= \cap_i \{x_i : x_i \geq g_i(y)\},$$

where $g_i(y) = f_i^{-1}(y)$. Hence, the vLP technology is a special case of what Chambers calls the 'Kohli-output (KO) nonjoint' or nonlinear Leontief production technology. The cost function associated with the vLP technology can therefore be written (Chambers, p.297):

$$(2) \qquad c(\mathbf{w}, y) = \mathrm{Min}_x\{\mathbf{w}\mathbf{x} : \mathbf{x} \in V(y)\} = \sum_{i=1}^{n} w_i g_i(y),$$

where $\mathbf{w} \in \mathbb{R}^n_{++}$ is a vector of strictly positive prices and w_i denotes the ith element of $\mathbf{w}$. Paris reports two special cases of (2).

Perhaps the most interesting aspect of (2) is that it offers a straightforward and simple way to test the vLP hypothesis given data on prices, output, and inputs or data on price, output, and total cost. That test would involve specifying a suitably flexible form for the cost function c(w, y), estimating that form subject only to the homogeneity and symmetry restrictions, and then testing parametrically to determine whether the parameters of the general form can be restricted to assume the linear (in $\mathbf{w}$) form in (2). For the generalized Leontief form, this would involve a simple parametric test involving only linear restrictions on parameters.

The form in (2) is the long-run or unrestricted cost function associated with the vLP technology. In many instances, in particular in our empirical applications below, one will be interested in short-run versions of the technology, i.e., ones involving some fixed factors. In formal terms, the distinguishing characteristic of the vLP technology is that there exist output levels at which any subvector of $\mathbf{x}$ acting as fixed factors will become strictly limitational. Therefore, versions of (2) consistent with the existence of fixed factors will be informative.

Obtaining the short-run, variable-cost function for the vLP technology is slightly less straightforward but still easy. For simplicity, and without any true loss of generality, suppose that the first k inputs (denoted by the subvector $\mathbf{x}^1$) are variable and that inputs k+1 to n are fixed (denoted by the subvector $\mathbf{x}^{-1}$). The maximum output obtainable with $\mathbf{x}^{-1}$ is:

$$y^*(\mathbf{x}^{-1}) = \text{Min } \{f_{k+1}(x_{k+1}), ..., f_n(x_n)\}.$$

Any output greater than y^* is not obtainable with $\mathbf{x}^{-1}$, and in what follows we shall, therefore, define input bundle $\mathbf{x}^{-1}$ as being *limitational* at y^*. Because no output higher than y^* is achievable with $\mathbf{x}^{-1}$, the variable cost-minimization problem for output levels higher than y^* is not well defined because the feasible set is empty. Therefore, define the variable cost of achieving these higher output levels as infinity. For output levels less than or equal to y^*, an easy extension of previous arguments shows that the short-run, variable cost function is given by $\sum_{i=1}^{k} w_i g_i(y)$. Hence, the vLP short-run, variable cost function is:

$$\text{(3)} \qquad \begin{aligned} c(\mathbf{w}^1, y, \mathbf{x}^1) &= \text{Min}\{\mathbf{w}^1\mathbf{x}^1 : \mathbf{x} \in V(y)\} \\ &= \sum_{i=1}^{k} w_i g_i(y) \qquad y \le y^*(x^{-1}) \\ &= \infty \qquad \text{otherwise.} \end{aligned}$$

where $\mathbf{w}^1 = (w_1, ..., w_k)$ is the subvector of $\mathbf{w}$ corresponding to the variable inputs. Thus, the short-run, variable cost function only depends upon the fixed inputs in the way that they limit the domain (in y) of the short-run, variable-cost function. Expression (3) offers an easy way to test for the validity of the von Liebig hypothesis given observations on variable inputs, variable input prices, and output or observations on variable input price, output, and variable-cost. Because all actual ouptut observations must be less than or equal to y^*, one can always fit flexible functional forms for the short-run, variable cost function and then impose the structural restrictions implied by (3) and test whether they are statistically valid for a given data set.

Characterizing the von Liebig and Mitscherlich Technologies with Data Envelopment Methods and Experimental Data

So far, we have suggested several econometric approaches based on dual representations of the vLP technology to test for the vLP specification against more general approaches. Unlike previous tests, these tests do not use nonnested hypothesis testing procedures because the dual implications of the vLP hypothesis are particularly stark and can easily be formulated in terms of parametric restrictions on the cost function: All derived demands are perfectly price inelastic. In this section we develop nonparametric methods for examining the validity of the vLP hypothesis. The methods developed in this section, however, should not be interpreted as "statistical tests"; probabilistic interpretations cannot be attached to them. They do, however, make it much simpler to isolate yield plateaus suggested by the vLP hypothesis for data from agronomic experiments, which lack the price information necessary for the econometric tests discussed above. Methods suited for use with experimental data are extremely important because of the central role such data play in deriving recommendations for agricultural practices (e.g., fertilizer use), in evaluating new agricultural technologies, and in similar uses where knowledge about non-substitution and yield plateaus is critical.

To proceed it is necessary to introduce some further notation. Let there be K observations on inputs and output with each observation being denoted $(\mathbf{x}_k, y_k)$, i.e., $\mathbf{x}_k$ denotes the kth observation of **x**. Therefore, the kth observation on input i is denoted x_{ki}.. Let $T(K) = \{(\mathbf{x}_k, y_n)$: $k = 1, ..., k\}$. It is well-known (e.g. Färe, Grosskopf, and Lovell) that for any such set of input-output observations, one can construct a piecewise linear approximation of the underlying technology satisfying free disposability of inputs, free disposability of outputs, convexity of the technology set (concavity of the production function) as:

$$T^* = \{(\mathbf{x}, y): \mathbf{x} \geq \sum_i \lambda_i \mathbf{x}_i,\ y \leq \sum_i \lambda_i y_i,\ \sum_i \lambda_i = 1,\ \lambda_i \in \mathbb{R}_+ \qquad (i = 1, 2, \ldots, K)\}.$$

T^* obviously has the characteristic that all input bundles are trivially limitational at

$$y' = \max\ \{y_1, \ldots, y_K\}$$

because no output higher than the highest observed output is consistent with this technology set.

T^* is often referred to as the free-disposal, convex hull of T(K). As such, it delineates the most conservative best-practice technology consistent with the data and consistent with the axioms of free disposability of inputs and outputs and concavity of the production function. Its graph or outer frontier represents the most optimistic approximation of the technology consistent with T(K).

A more conservative representation of the technology consistent with T(K) can be obtained by introducing the notion of vector dominance. Formally, $\mathbf{z} \in \mathbb{R}^k$ *dominates* $\mathbf{z}' \in \mathbb{R}^k$ if $\mathbf{z} \leq \mathbf{z}'$, i.e., each element of $\mathbf{z}$ is no larger than the corresponding element of $\mathbf{z}'$. Define $N(K) \subseteq T(K)$ as the set of nondominating observations, i.e.,

$$N(K) = \{(\mathbf{x}_k, y_k) \in T(K) : \nexists\ (\mathbf{x}_i, y_i) \in T(K) \text{ for which } (\mathbf{x}_k, -y_k) \text{ dominates } (\mathbf{x}_i, -y_i)\}$$

A more conservative approximation of the technology is offered by the free-disposal convex hull of N(K), i.e.,

$$T^N = \{(\mathbf{x}, y): \mathbf{x} \geq \sum_{i \in N(k)} \lambda_i \mathbf{x}_i,\ y \leq \sum_{i \in N(k)} \lambda_i y_i,\ \lambda_i \in \mathbb{R}_+,\ \sum_{i \in N(k)} \lambda_i = 1\}.$$

The production frontier for T^N lies everywhere on or below the production frontier for T^*, or perhaps more intuitively, the production function derived from T^N lies below the production function derived from T^*. This latter fact implies that there will be some observed input-output combinations which would not be technically feasible if T^N were the true technology. Therefore, instead of an approximation of the best-practice technology consistent with T(K), T^N might be interpreted as an approximation of the worst-practice technology consistent with T(K). For a representative T(K), the difference between T^* and T^N is portrayed graphically in Figure 1[1]. In that figure T(K) = {A, B, C, D}.

Given T^* and T^N, it is easy to determine whether any particular bundle of inputs is limitational (i.e., exhibits a yield plateau) at any $y^* < y'$ (as appropriately defined) for either of the technologies. Suppose that one wishes to check whether a fixed subvector of inputs x^{-1} is limitational in T^*. If the input bundle is limitational within the range of the observed data, then by previous developments there must be some y^* no greater than y' for which

$$\text{(4)} \qquad \text{argmin } \{\mathbf{w}^1\mathbf{x}^1 : (\mathbf{x}^1, \mathbf{x}^{-1}, y^*) \in T^*\}$$

is the empty set. Because T^* is defined by the intersection of halfspaces, this variable-cost minimization problem is a simple linear program. Therefore, for any given subvector of inputs one can determine $y^*(\mathbf{x}^{-1})$ by solving this linear program for successively increasing values of y. We shall say the input bundle $\mathbf{x}^{-1}$ is *not limitational within the data set* if one can find a solution to the linear program (the program for T^N is defined analogously):

$$\text{Min } \{\mathbf{w}^1\mathbf{x}^1 : (\mathbf{x}^1, \mathbf{x}^{-1}, y') \in T^*\}.$$

If, however, there exists a $y < y'$ for which,

$$\text{argmin } \{\mathbf{w}^1\mathbf{x}^1 : (\mathbf{x}^1, \mathbf{x}^{-1}, y) \in T^*\}$$

is the empty set, $\mathbf{x}^{-1}$ will be said to be *limitational within the data set.*

The second major component of the vLP hypothesis is that there is no substitutability between inputs. And from an economic perspective, this constitutes the main difference between the Mitscherlich-Baule technology and the vLP technology. Again there is an easy nonparametric check (we only demonstrate for T^*) for this property: compute the solution to:

$$\text{Min } \{\mathbf{w}\mathbf{x} : (\mathbf{x}, y) \in T^*\}$$

for differing values of **w**. If the solution changes for differing values of **w**, the technology exhibits substitutability and thus violates the vLP hypothesis.

Data

We demonstrate our approach using data from a three-year field study comparing yields of no-till corn following four cover crops (hairy vetch, crimson clover, Austrian peas and winter wheat) and winter fallow in the Maryland Coastal Plain (for a complete description see Decker et al.). The experiment examined only applied nitrogen use, so our study is for a single nutrient and cannot examine the non-substitution component of the vLP hypothesis.[2] Here the fixed inputs are yield of the cover crop (to measure the organic nitrogen content of the soil), precipitation, and temperature. Weather records were used to construct two variables measuring precipitation and temperature: total precipitation during the early growing season and the number of days during the late growing season between 70° and 86°F.

Four different nitrogen fertilizer rates were used on each winter cover crop. Fertilization rates of 0, 40, 80 and 120 pounds of nitrogen per acre were used on the vetch system, rates of 0, 60, 120 and 180 pounds of nitrogen per acre on the clover and peas systems and rates of 0, 120, 180 and 240 pounds per acre on the wheat and winter fallow systems. The experiment was conducted on different plots on the farm each year. Each cover crop followed no-till corn. Corn was planted in the spring between 5 and 15 days after the covers were killed with a knockdown herbicide, depending upon the condition of the killed cover crop growth, soil condition and rainfall. Samples of corn grain were dried to 15.5 percent moisture and used to estimate yield per acre. Samples of the cover crop were also harvested and dried and used to estimate yield per acre.

Results

The nonparametric methodology developed above was applied to each of the cover crops using both nonparametric representations of the technology. For each cover crop, we considered 5 different levels of the fixed input bundle (temperature, precipitation, and cover-crop yield) with each input fixed respectively at: (1) maximums observed over the three years; (2) points midway between the maximum and the mean observations; (3) sample means; (4) points midway between the sample means and the sample minimum; and (5) the sample minimum. Nitrogen cost functions and input requirement functions were then derived for each of the five fixed input bundles for each crop using both T^* and T^N.

Although the cost functions identified differed significantly across crops and across versions of the technology, all versions shared some common features: First, and perhaps most importantly, the data all appear to be consistent with the vLP technology in that in all instances when the fixed-input vector was held constant at below the sample maximum nitrogen-yield plateaus were identified by reaching an output level less than y' for which there was no feasible solution for the cost minimization (input-requirement) problem. Hence, at least for this experimental data set, the vLP representation of the technology seems reasonable. Second, the cost functions we calculated typically are convex and manifest increasing marginal cost suggesting that the more restrictive linear von Liebig representation, which implies constant marginal cost up to the yield plateau, is inappropriate for all cover crops for this data set.

Turning to the results from the vetch system, we have reported the numerical values for the five versions of the cost function corresponding to different fixed input levels for T^* in Appendix Table 1; the numerical values for the cost for T^N are reported in Appendix Table 2. These cost functions are portrayed graphically in Figures 2a and 2b. The most important thing to notice about these two figures, apart from the presence of the yield plateaus, is the significant difference between the versions of the cost function for T^* and T^N. For example, the cost function for T^n is ∞ for all positive output values when the fixed-input bundle is evaluated at the sample minimum and at a point midway between the sample minimum and the sample mean. (In the figures, the yield plateau is reached at the last plotted output-cost point. For example, for the mean vetch technology in Figure 2a, the yield plateau is reached at 170 bushels of corn.) This implies that no level of nitrogen application would achieve a positive output response for the more conservative approximation of the technology. Second, even when the cost function is well defined for T^N, it reaches a yield plateau (becomes infinity) at much lower output levels than the one for T^*. Third, the cost of achieving even modest output levels for T^N is much higher than for T^*. For example, for all fixed input bundles for T^*, the nitrogen cost of obtaining yield levels less than 135 bushels per acre is zero. This points out the wide disparity between the most optimistic nonparametric approximations to the technology and the most pessimistic, and the wide range for error involved in using either approximation uncritically. In particular, recommendations made to farmers or policy makers may be quite sensitive to the representation of the technology chosen. Consider fertilizer recommendation for corn following a vetch winter cover. Suppose that the farmer's yield goal is 130 bushels per acre, and that average levels of the weather and vetch yield are expected. Under T^*, this yield can be obtained without applying any nitrogen fertilizer; under T^N, this yield is unattainable at any cost.

To conserve space, we do not report graphical representations of the technology for all cover crops. Tabular presentations of our results for the remaining cover crops are also reported in the appendix. The basic pattern reported for the vetch system repeats itself: There is a fairly wide variation between the more conservative and more optimistic approximations to the technology. In what follows, we concentrate, therefore, on the intercrop differences.

Figures 3a and 3b, portray graphically the variable-cost functions associated with each cover crop (fixed inputs evaluated at their sample maximum) for T^* and T^N, respectively. Some interesting patterns emerge: For both T^* and T^N, the winter-wheat cover and fallow system are considerably less cost-efficient in achieving all yield levels in terms of applied-nitrogen cost. For T^*, winter wheat is the least cost-efficient, while for T^N the fallow system is least cost-effective in terms of applied nitrogen cost. Which cover crop is most cost-effective depends upon the yield level and the version of the technology chosen: For example, for T^* Austrian peas is always more cost-effective than hairy vetch, and is more cost-effective than crimson clover for all but the highest yield levels. On the other hand, crimson clover is at least as cost-effective as hairy vetch for all yield levels up to 165 bushels per acre, is then dominated in cost terms by hairy vetch for yield levels between 165 and 190 bushels per acre, but then has a higher yield plateau than either hairy vetch or Austrian peas. For T^*, the highest yield plateau is associated with crimson clover and is approximately 200 bushels per acre. Similar, although not duplicate, patterns emerge for T^N.

One of the most striking results for all versions of the technology is the high degree of yield response obtainable with zero applied nitrogen cost for all the legume cover crops except winter wheat, indicating that relying on legumes to provide nitrogen instead of chemical fertilizers is feasible.

These last two sets of results suggest that legume cover crops could play a valuable role in helping meet new targets for reducing non-point source nitrogen runoff from agriculture under the new Clean Water Act. Use of these cover crops may permit reductions in applications of chemical fertilizers. They may also reduce leaching and runoff by increasing the organic matter content of the soil, and thus the soil's ability to hold nutrients. Since they also appear to reduce production cost, further research into their potential seems warranted.

Concluding Remarks

We have developed the dual implications of the vLP technology and developed both parametric and nonparametric approaches for evaluating two of the key hypotheses of the vLP technology: yield plateaus and input nonsubstitutability. We discuss the nonparametric approach in the context of two different representations of the technology corresponding to an optimistic and pessimistic nonparametric approximation. The methodology for determining the presence of yield plateaus has been applied to an experimental data set for corn grown in the Maryland Coastal Plain. In all instances, we have confirmed the presence of yield plateaus for limiting bundles of fixed inputs. Our data set has not allowed us to conduct the nonparametric evaluation of input nonsubstitutability. Our results also suggest that legume cover crops may have significant potential both for increasing farm profitability and for reducing environmental spillovers from agriculture.

Footnotes

1. It might help the reader's intuition to note that T^* is the free-disposal convex hull of the set of *dominating* observations within T(K).

2. Because we only have a single variable input, our variable cost minimization problem is equivalent to isolating the applied-nitrogen input requirement function for different output levels and different bundles of fixed inputs.

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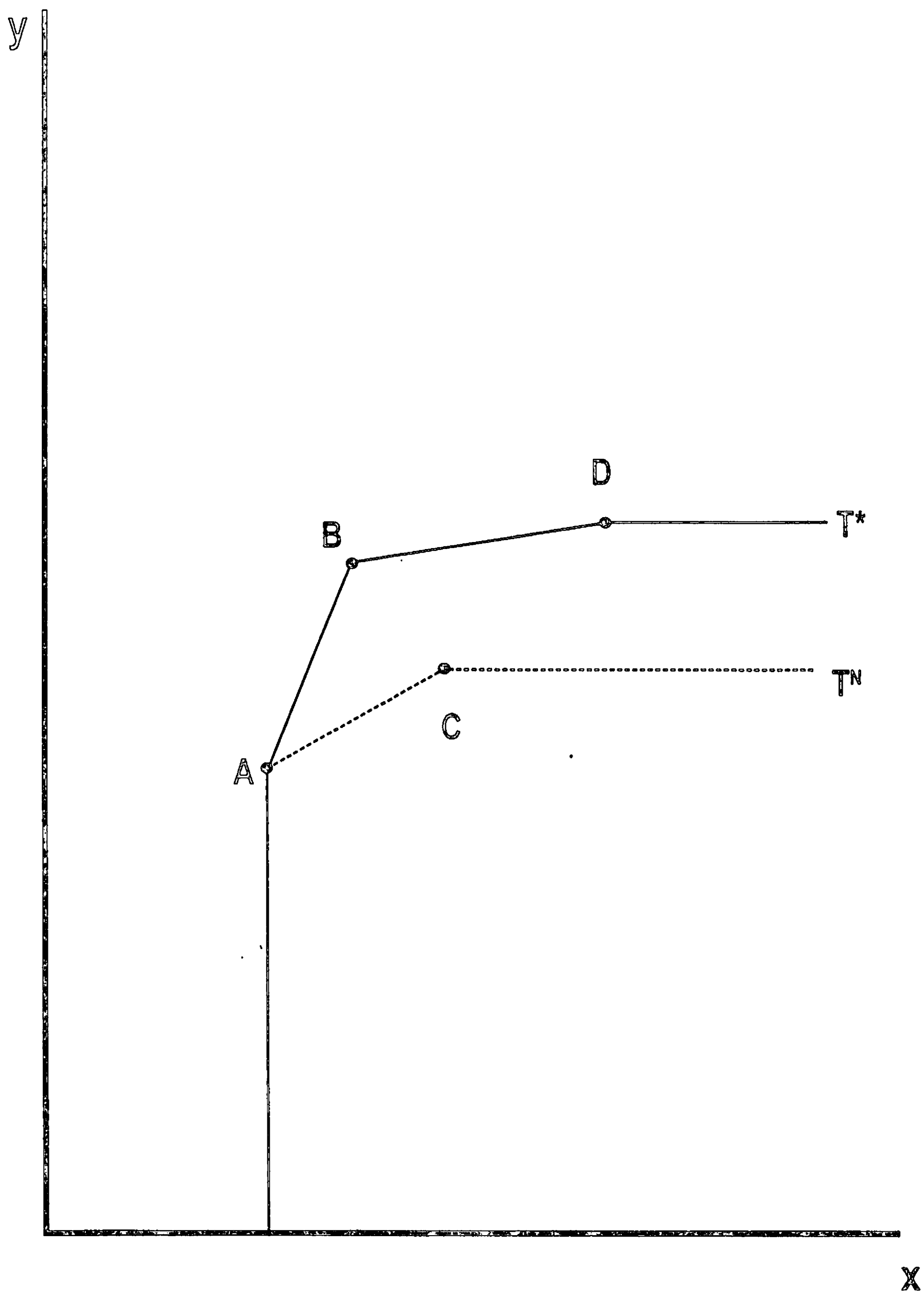


Figure 1: T* and T^N for Points A, B, C and D

Figure 2a: Comparative Costs for Fixed Inputs with Vetch Technology T*

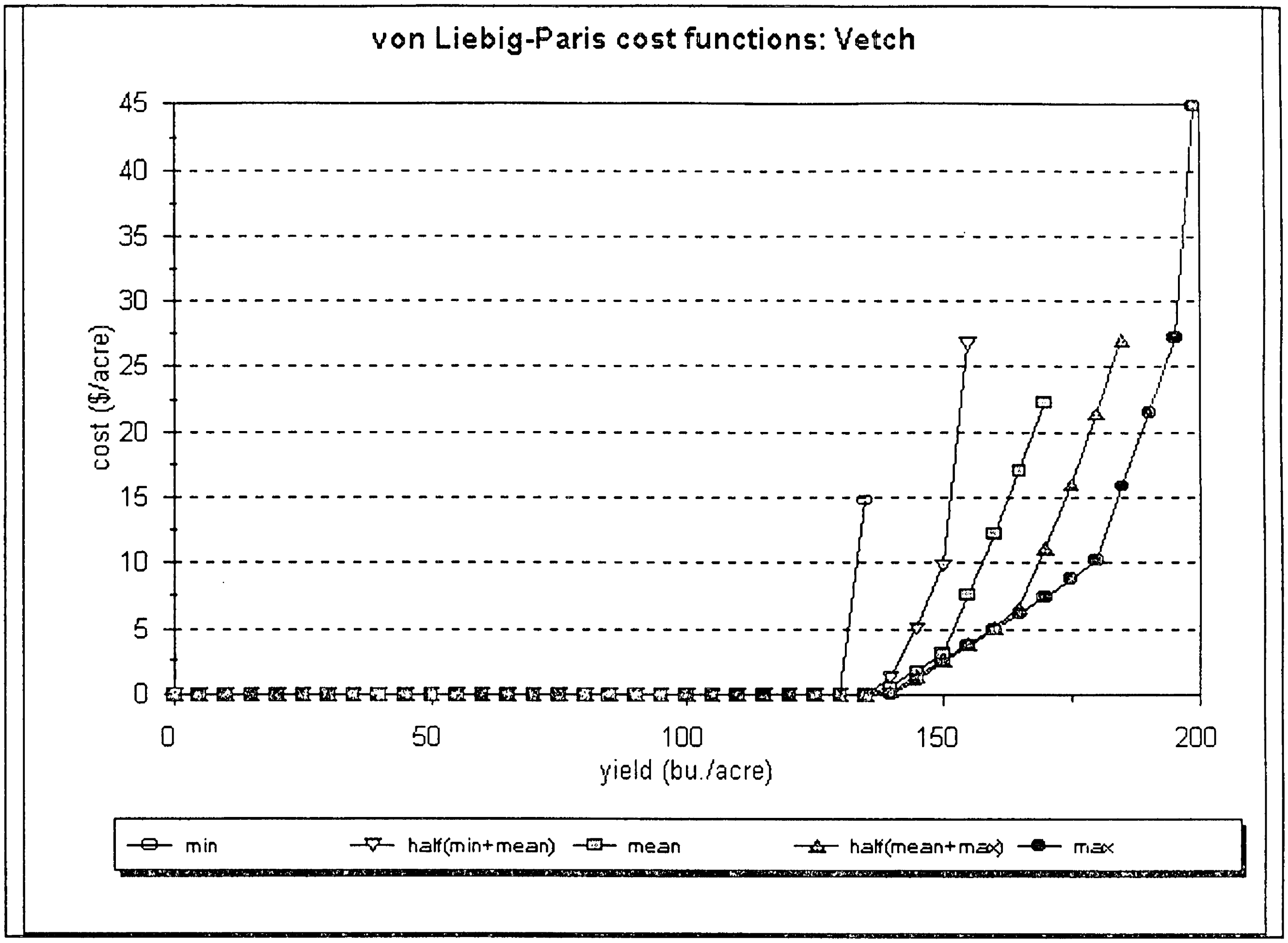


Figure 2b: Comparative Costs for Fixed Inputs with Vetch Technology T^V

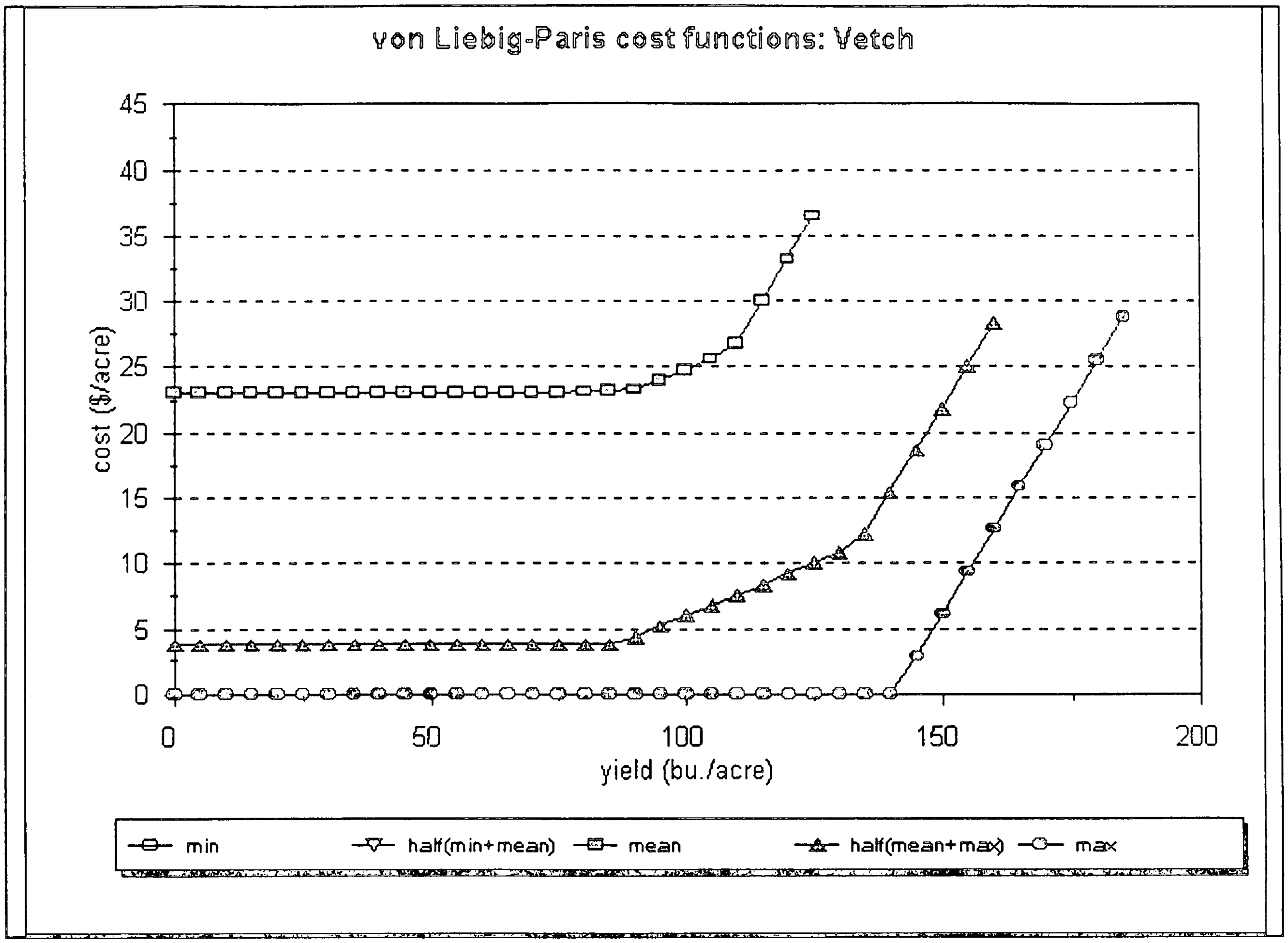


Figure 3a: Comparative Costs for Five Technologies under T*

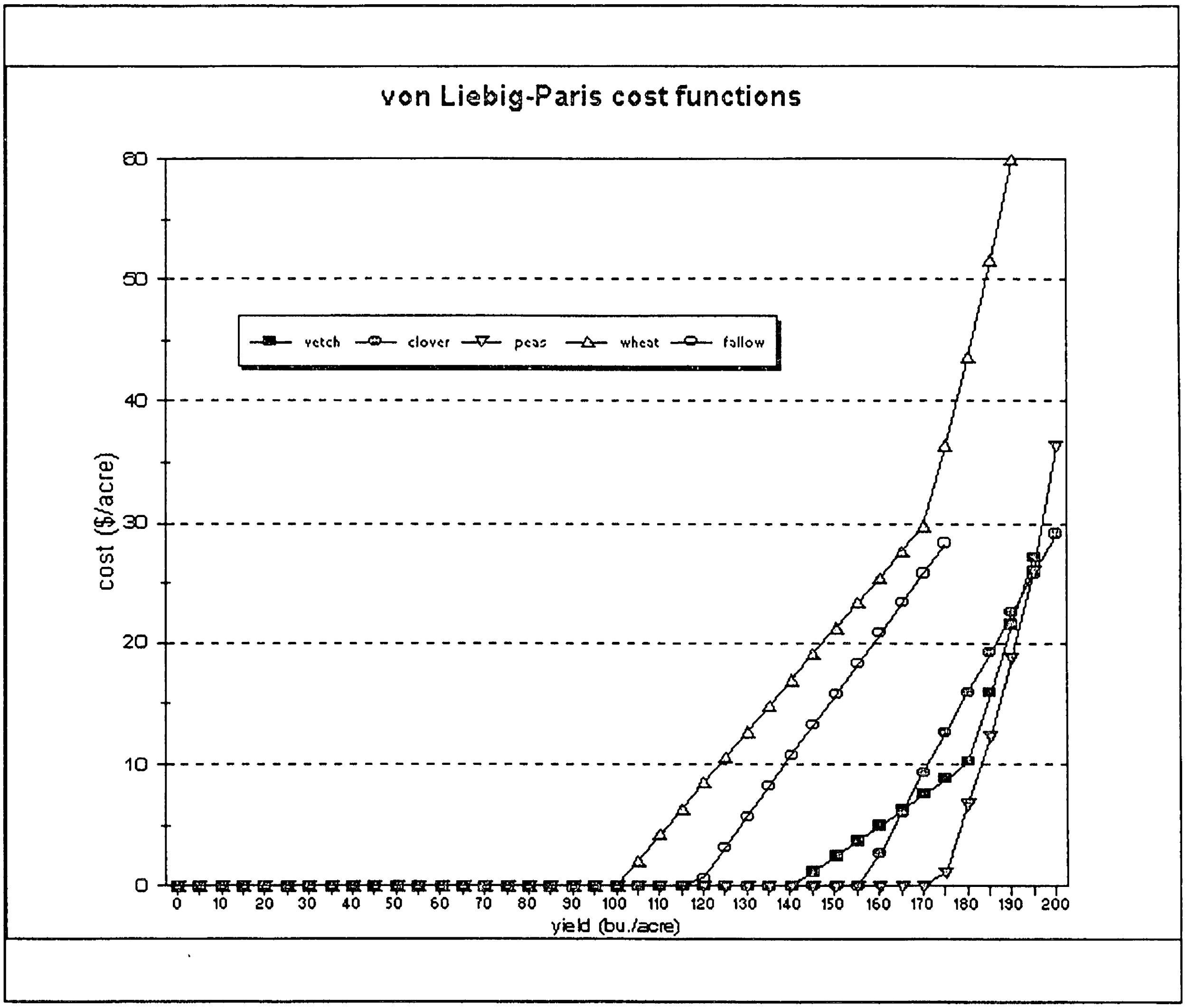
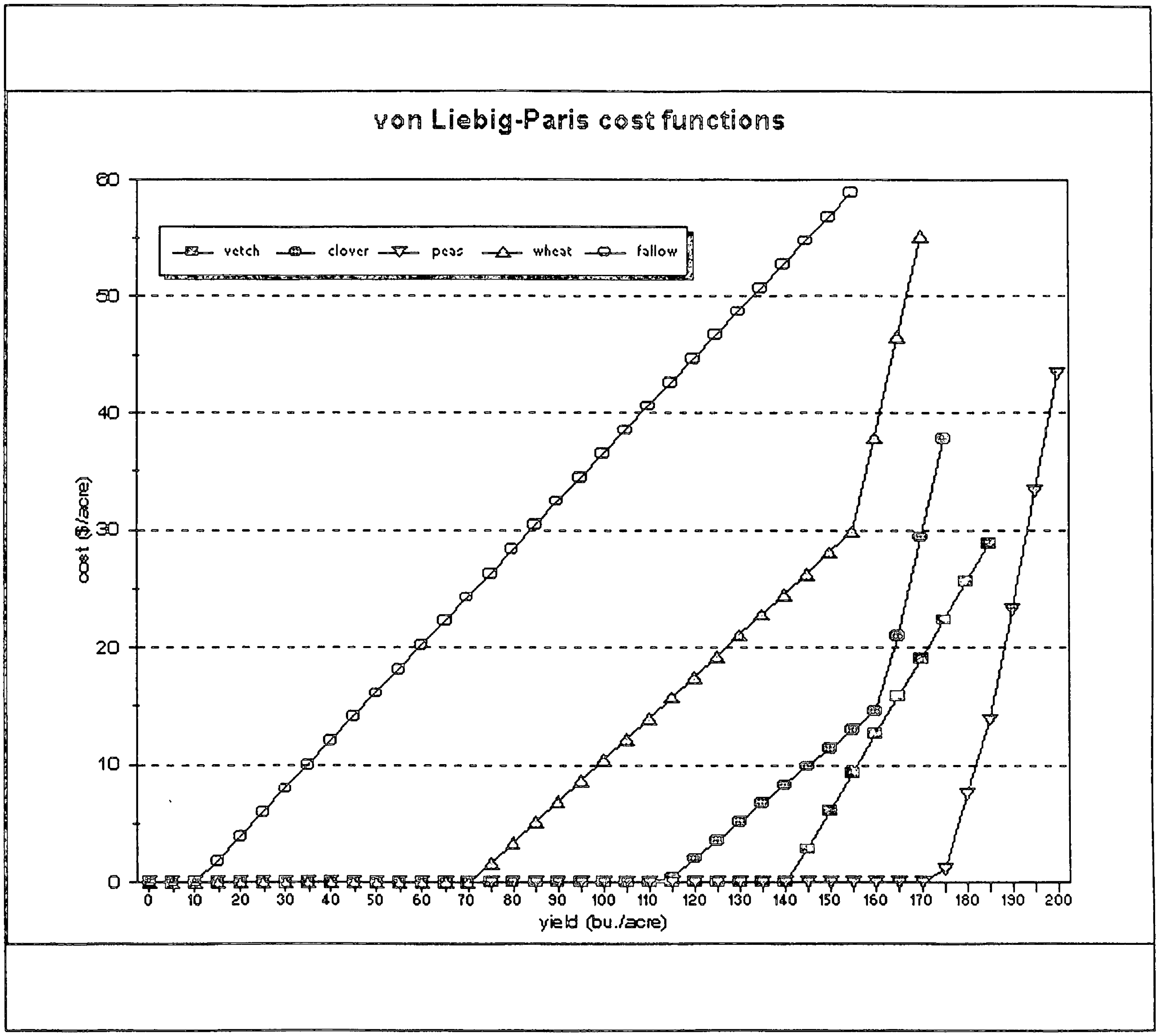


Figure 3b: Comparative Costs under T^N



Appendix

Table 1

Von Liebig: Cost and input functions for various levels of fixed inputs

VETCH (Temp = days between 70 - 86 degrees)

(All figures are per acre)

	Fixed inputs set at: Minimum		Half(min + mean)		Mean		Half(mean + max)		Maximum	
Yield (bu.)	Cost ($)	Nitrogen (lbs.)	Cost ($)	Nitrogen (lbs.)	Cost ($)	Nitrogen (lbs.)	Cost ($)	Nitrogen (lbs.)	Cost ($)	Nitrogen (lbs.)
0	0	0	0	0	0	0	0	0	0	0
5	0	0	0	0	0	0	0	0	0	0
10	0	0	0	0	0	0	0	0	0	0
15	0	0	0	0	0	0	0	0	0	0
20	0	0	0	0	0	0	0	0	0	0
25	0	0	0	0	0	0	0	0	0	0
30	0	0	0	0	0	0	0	0	0	0
35	0	0	0	0	0	0	0	0	0	0
40	0	0	0	0	0	0	0	0	0	0
45	0	0	0	0	0	0	0	0	0	0
50	0	0	0	0	0	0	0	0	0	0
55	0	0	0	0	0	0	0	0	0	0
60	0	0	0	0	0	0	0	0	0	0
65	0	0	0	0	0	0	0	0	0	0
70	0	0	0	0	0	0	0	0	0	0
75	0	0	0	0	0	0	0	0	0	0
80	0	0	0	0	0	0	0	0	0	0
85	0	0	0	0	0	0	0	0	0	0
90	0	0	0	0	0	0	0	0	0	0
95	0	0	0	0	0	0	0	0	0	0
100	0	0	0	0	0	0	0	0	0	0
105	0	0	0	0	0	0	0	0	0	0
110	0	0	0	0	0	0	0	0	0	0
115	0	0	0	0	0	0	0	0	0	0
120	0	0	0	0	0	0	0	0	0	0
125	0	0	0	0	0	0	0	0	0	0
130	0	0	0	0	0	0	0	0	0	0
135	14.8111	59.24	0	0	0	0	0	0	0	0
140	∞	∞	1.1954	4.78	0.5046	2.02	0.1781	0.71	0	0
145	∞	∞	5.0382	20.15	1.7348	6.94	1.4083	5.63	1.1209	4.48
150	∞	∞	9.8372	39.35	3.115	12.46	2.6385	10.55	2.3955	9.58
155	∞	∞	26.8558	107.42	7.6096	30.44	3.8687	15.47	3.6701	14.68
160	∞	∞	∞	∞	12.2359	48.94	5.1041	20.42	4.9447	19.78
165	∞	∞	∞	∞	17.0413	68.17	6.6053	26.42	6.2193	24.88
170	∞	∞	∞	∞	22.3187	89.27	11.1961	44.78	7.4939	29.98
175	∞	∞	∞	∞	∞	∞	16.0402	64.16	8.7685	35.07
180	∞	∞	∞	∞	∞	∞	21.4573	85.83	10.1918	40.77
185	∞	∞	∞	∞	∞	∞	27.1333	108.53	15.8679	63.47
190	∞	∞	∞	∞	∞	∞	∞	∞	21.5439	86.18
195	∞	∞	∞	∞	∞	∞	∞	∞	27.2199	108.88
198.72	∞	∞	∞	∞	∞	∞	∞	∞	45	180.00

Table 2

Von Liebig: Cost and input requirement functions for various levels of fixed inputs

Using the nondominating set as the reference technology

VETCH (Temp = days between 70 - 86 degrees)

(All figures are per acre)

	Fixed inputs set at: Minimum		Half(min + mean)		Mean		Half(mean + max)		Maximum	
Yield (bu.)	Cost ($)	Nitrogen (lbs.)	Cost ($)	Nitrogen (lbs.)	Cost ($)	Nitrogen (lbs.)	Cost ($)	Nitrogen (lbs.)	Cost ($)	Nitrogen (lbs.)
0	∞	∞	∞	∞	23.06	92.24	3.807	15.23	0	0
5	∞	∞	∞	∞	23.06	92.24	3.807	15.23	0	0
10	∞	∞	∞	∞	23.06	92.24	3.807	15.23	0	0
15	∞	∞	∞	∞	23.06	92.24	3.807	15.23	0	0
20	∞	∞	∞	∞	23.06	92.24	3.807	15.23	0	0
25	∞	∞	∞	∞	23.06	92.24	3.807	15.23	0	0
30	∞	∞	∞	∞	23.06	92.24	3.807	15.23	0	0
35	∞	∞	∞	∞	23.06	92.24	3.807	15.23	0	0
40	∞	∞	∞	∞	23.06	92.24	3.807	15.23	0	0
45	∞	∞	∞	∞	23.06	92.24	3.807	15.23	0	0
50	∞	∞	∞	∞	23.06	92.24	3.807	15.23	0	0
55	∞	∞	∞	∞	23.06	92.24	3.807	15.23	0	0
60	∞	∞	∞	∞	23.06	92.24	3.807	15.23	0	0
65	∞	∞	∞	∞	23.06	92.24	3.807	15.23	0	0
70	∞	∞	∞	∞	23.06	92.24	3.807	15.23	0	0
75	∞	∞	∞	∞	23.08	92.34	3.807	15.23	0	0
80	∞	∞	∞	∞	23.16	92.62	3.807	15.23	0	0
85	∞	∞	∞	∞	23.23	92.91	3.807	15.23	0	0
90	∞	∞	∞	∞	23.3	93.19	4.383	17.53	0	0
95	∞	∞	∞	∞	24.05	96.2	5.187	20.75	0	0
100	∞	∞	∞	∞	24.85	99.42	5.99	23.96	0	0
105	∞	∞	∞	∞	25.66	102.6	6.794	27.18	0	0
110	∞	∞	∞	∞	26.78	107.1	7.597	30.39	0	0
115	∞	∞	∞	∞	30.02	120.1	8.401	33.6	0	0
120	∞	∞	∞	∞	33.25	133	9.205	36.82	0	0
125	∞	∞	∞	∞	36.49	146	10.01	40.03	0	0
130	∞	∞	∞	∞	∞	∞	10.81	43.25	0	0
135	∞	∞	∞	∞	∞	∞	12.2	48.81	0	0
140	∞	∞	∞	∞	∞	∞	15.44	61.76	0	0
145	∞	∞	∞	∞	∞	∞	18.68	74.7	2.857	11.43
150	∞	∞	∞	∞	∞	∞	21.91	87.65	6.106	24.42
155	∞	∞	∞	∞	∞	∞	25.15	100.6	9.355	37.42
160	∞	∞	∞	∞	∞	∞	28.39	113.6	12.6	50.42
165	∞	∞	∞	∞	∞	∞	∞	∞	15.85	63.41
170	∞	∞	∞	∞	∞	∞	∞	∞	19.1	76.41
175	∞	∞	∞	∞	∞	∞	∞	∞	22.35	89.4
180	∞	∞	∞	∞	∞	∞	∞	∞	25.6	102.4
185	∞	∞	∞	∞	∞	∞	∞	∞	28.85	115.4
190	∞	∞	∞	∞	∞	∞	∞	∞	∞	∞
195	∞	∞	∞	∞	∞	∞	∞	∞	∞	∞
198.7	∞	∞	∞	∞	∞	∞	∞	∞	∞	∞

Table 3

Von Liebig: Cost and input requirement functions for various levels of fixed inputs

CRIMSON (Days within 70-86 degrees)

(All figures are per acre)

	Fixed inputs set at: Minimum		Half(min + mean)		Mean		Half(mean + max)		Maximum	
Yield (bu.)	Cost ($)	Nitrogen (lbs.)	Cost ($)	Nitrogen (lbs.)	Cost ($)	Nitrogen (lbs.)	Cost ($)	Nitrogen (lbs.)	Cost ($)	Nitrogen (lbs.)
0	0	0	0	0	0	0	0	0	0	0
5	0	0	0	0	0	0	0	0	0	0
10	0	0	0	0	0	0	0	0	0	0
15	0	0	0	0	0	0	0	0	0	0
20	0	0	0	0	0	0	0	0	0	0
25	0	0	0	0	0	0	0	0	0	0
30	0	0	0	0	0	0	0	0	0	0
35	0	0	0	0	0	0	0	0	0	0
40	0	0	0	0	0	0	0	0	0	0
45	0	0	0	0	0	0	0	0	0	0
50	4.0671	16.27	0	0	0	0	0	0	0	0
55	8.8205	35.28	0	0	0	0	0	0	0	0
60	13.574	54.30	0	0	0	0	0	0	0	0
65	18.4642	73.86	0	0	0	0	0	0	0	0
70	23.4131	93.65	0	0	0	0	0	0	0	0
75	28.3619	113.45	0	0	0	0	0	0	0	0
80	∞	∞	0	0	0	0	0	0	0	0
85	∞	∞	0	0	0	0	0	0	0	0
90	∞	∞	0	0	0	0	0	0	0	0
95	∞	∞	0.734	2.94	0	0	0	0	0	0
100	∞	∞	3.3738	13.50	0	0	0	0	0	0
105	∞	∞	6.0136	24.05	0	0	0	0	0	0
110	∞	∞	8.6535	34.61	0	0	0	0	0	0
115	∞	∞	11.2933	45.17	0	0	0	0	0	0
120	∞	∞	14.9693	59.88	0	0	0	0	0	0
125	∞	∞	∞	∞	0	0	0	0	0	0
130	∞	∞	∞	∞	0	0	0	0	0	0
135	∞	∞	∞	∞	3.2347	12.94	0	0	0	0
140	∞	∞	∞	∞	7.2508	29.00	0	0	0	0
145	∞	∞	∞	∞	11.266	45.07	0.287	1.15	0	0
150	∞	∞	∞	∞	15.283	61.13	3.6177	14.47	0	0
155	∞	∞	∞	∞	19.327	77.31	6.9483	27.79	0	0
160	∞	∞	∞	∞	28.640	114.56	10.2789	41.12	2.6766	10.71
165	∞	∞	∞	∞	∞	∞	13.6095	54.44	5.9885	23.95
170	∞	∞	∞	∞	∞	∞	17.4357	69.74	9.3003	37.20
175	∞	∞	∞	∞	∞	∞	21.4518	85.81	12.6122	50.45
180	∞	∞	∞	∞	∞	∞	25.7671	103.07	15.924	63.70
185	∞	∞	∞	∞	∞	∞	∞	∞	19.2358	76.94
190	∞	∞	∞	∞	∞	∞	∞	∞	22.5477	90.19
195	∞	∞	∞	∞	∞	∞	∞	∞	25.8595	103.44
200	∞	∞	∞	∞	∞	∞	∞	∞	29.1714	116.69

Table 4

Von Liebig: Cost and input requirement functions for various levels of fixed inputs

Using the nondominating set as the reference technology

CRIMSON (Temp = days between 70 - 86 degrees)

(All figures are per acre)

	Fixed inputs set at: Minimum		Half(min + mean)		Mean		Half(mean + max)		Maximum	
Yield (bu.)	Cost ($)	Nitrogen (lbs.)	Cost ($)	Nitrogen (lbs.)	Cost ($)	Nitrogen (lbs.)	Cost ($)	Nitrogen (lbs.)	Cost ($)	Nitrogen (lbs.)
0	∞	∞	29.0003	116.001	5.9698	23.879	0	0	0	0
5	∞	∞	29.0003	116.001	5.9698	23.879	0	0	0	0
10	∞	∞	29.0003	116.001	5.9698	23.879	0	0	0	0
15	∞	∞	29.0003	116.001	5.9698	23.879	0	0	0	0
20	∞	∞	29.0003	116.001	5.9698	23.879	0	0	0	0
25	∞	∞	29.0003	116.001	5.9698	23.879	0	0	0	0
30	∞	∞	29.0003	116.001	5.9698	23.879	0	0	0	0
35	∞	∞	29.0051	116.02	5.9698	23.879	0	0	0	0
40	∞	∞	29.1975	116.79	5.9698	23.879	0	0	0	0
45	∞	∞	29.3898	117.559	5.9698	23.879	0	0	0	0
50	∞	∞	29.5822	118.329	5.9698	23.879	0	0	0	0
55	∞	∞	29.7746	119.098	5.9698	23.879	0	0	0	0
60	∞	∞	31.7788	127.115	5.9698	23.879	0	0	0	0
65	∞	∞	36.0773	144.309	5.9698	23.879	0	0	0	0
70	∞	∞	40.3758	161.503	5.9698	23.879	0	0	0	0
75	∞	∞	44.6744	178.698	5.9817	23.927	0	0	0	0
80	∞	∞	∞	∞	6.1741	24.696	0	0	0	0
85	∞	∞	∞	∞	6.3664	25.466	0	0	0	0
90	∞	∞	∞	∞	6.5588	26.235	0	0	0	0
95	∞	∞	∞	∞	9.3856	37.542	0	0	0	0
100	∞	∞	∞	∞	13.6841	54.737	0	0	0	0
105	∞	∞	∞	∞	17.9827	71.931	0	0	0	0
110	∞	∞	∞	∞	22.2812	89.125	1.2997	5.199	0	0
115	∞	∞	∞	∞	26.5798	106.319	2.8706	11.482	0.4178	1.671
120	∞	∞	∞	∞	30.8783	123.513	4.4414	17.766	1.9887	7.955
125	∞	∞	∞	∞	35.1768	140.707	7.2059	28.823	3.5595	14.238
130	∞	∞	∞	∞	39.4754	157.902	11.798	47.192	5.1304	20.522
135	∞	∞	∞	∞	43.7739	175.096	16.3901	65.56	6.7012	26.805
140	∞	∞	∞	∞	∞	∞	20.9822	83.929	8.2721	33.088
145	∞	∞	∞	∞	∞	∞	25.5743	102.297	9.8429	39.372
150	∞	∞	∞	∞	∞	∞	30.1664	120.666	11.4138	45.655
155	∞	∞	∞	∞	∞	∞	37.0175	148.07	12.9846	51.938
160	∞	∞	∞	∞	∞	∞	∞	∞	14.5555	58.222
165	∞	∞	∞	∞	∞	∞	∞	∞	21.0124	84.05
170	∞	∞	∞	∞	∞	∞	∞	∞	29.3979	117.592
175	∞	∞	∞	∞	∞	∞	∞	∞	37.7834	151.134
180	∞	∞	∞	∞	∞	∞	∞	∞	∞	∞
185	∞	∞	∞	∞	∞	∞	∞	∞	∞	∞
190	∞	∞	∞	∞	∞	∞	∞	∞	∞	∞
195	∞	∞	∞	∞	∞	∞	∞	∞	∞	∞
200	∞	∞	∞	∞	∞	∞	∞	∞	∞	∞

Table 5

Von Liebig: Cost and input requirement functions for various levels of fixed inputs

Using the set of all observations as the reference technology

PEAS (Temp = days between 70 - 86 degrees)

(All figures are per acre)

Fixed inputs set at:

	Minimum		Half(min + mean)		Mean		Half(mean + max)		Maximum	
Yield (bu.)	Cost ($)	Nitrogen (lbs.)	Cost ($)	Nitrogen (lbs.)	Cost ($)	Nitrogen (lbs.)	Cost ($)	Nitrogen (lbs.)	Cost ($)	Nitrogen (lbs.)
0	0	0	0	0	0	0	0	0	0	0
5	0	0	0	0	0	0	0	0	0	0
10	0	0	0	0	0	0	0	0	0	0
15	0	0	0	0	0	0	0	0	0	0
20	0	0	0	0	0	0	0	0	0	0
25	0	0	0	0	0	0	0	0	0	0
30	0	0	0	0	0	0	0	0	0	0
35	0	0	0	0	0	0	0	0	0	0
40	0	0	0	0	0	0	0	0	0	0
45	0	0	0	0	0	0	0	0	0	0
50	0	0	0	0	0	0	0	0	0	0
55	0	0	0	0	0	0	0	0	0	0
60	0	0	0	0	0	0	0	0	0	0
65	0	0	0	0	0	0	0	0	0	0
70	0	0	0	0	0	0	0	0	0	0
75	0	0	0	0	0	0	0	0	0	0
80	0	0	0	0	0	0	0	0	0	0
85	0	0	0	0	0	0	0	0	0	0
90	0	0	0	0	0	0	0	0	0	0
95	0	0	0	0	0	0	0	0	0	0
100	0	0	0	0	0	0	0	0	0	0
105	0	0	0	0	0	0	0	0	0	0
110	0	0	0	0	0	0	0	0	0	0
115	0	0	0	0	0	0	0	0	0	0
120	0	0	0	0	0	0	0	0	0	0
125	0	0	0	0	0	0	0	0	0	0
130	0	0	0	0	0	0	0	0	0	0
135	∞	∞	0	0	0	0	0	0	0	0
140	∞	∞	0.3509	1.404	0	0	0	0	0	0
145	∞	∞	7.3991	29.596	0	0	0	0	0	0
150	∞	∞	26.8414	107.366	2.1329	8.532	0	0	0	0
155	∞	∞	∞	∞	8.3131	33.252	0	0	0	0
160	∞	∞	∞	∞	20.0029	80.012	0	0	0	0
165	∞	∞	∞	∞	40.0029	160.012	4.4018	17.607	0	0
170	∞	∞	∞	∞	∞	∞	10.0274	40.11	0	0
175	∞	∞	∞	∞	∞	∞	17.1616	68.647	1.0452	4.181
180	∞	∞	∞	∞	∞	∞	27.8167	111.267	6.6708	26.683
185	∞	∞	∞	∞	∞	∞	∞	∞	12.2964	49.185
190	∞	∞	∞	∞	∞	∞	∞	∞	18.7638	75.055
195	∞	∞	∞	∞	∞	∞	∞	∞	26.0101	104.041
200	∞	∞	∞	∞	∞	∞	∞	∞	36.3618	145.447

Table 6

Von Liebig: Cost and input requirement functions for various levels of fixed inputs

Using the nondominating set as the reference technology

PEAS (Temp = days between 70 - 86 degrees)

(All figures are per acre)

	Fixed inputs set at: Minimum		Half(min + mean)		Mean		Half(mean + max)		Maximum	
Yield (bu.)	Cost ($)	Nitrogen (lbs.)	Cost ($)	Nitrogen (lbs.)	Cost ($)	Nitrogen (lbs.)	Cost ($)	Nitrogen (lbs.)	Cost ($)	Nitrogen (lbs.)
0	∞	∞	3.7125	14.85	0	0	0	0	0	0
5	∞	∞	3.7125	14.85	0	0	0	0	0	0
10	∞	∞	3.7125	14.85	0	0	0	0	0	0
15	∞	∞	3.7125	14.85	0	0	0	0	0	0
20	∞	∞	3.7125	14.85	0	0	0	0	0	0
25	∞	∞	3.7125	14.85	0	0	0	0	0	0
30	∞	∞	3.7125	14.85	0	0	0	0	0	0
35	∞	∞	3.7125	14.85	0	0	0	0	0	0
40	∞	∞	3.7125	14.85	0	0	0	0	0	0
45	∞	∞	3.7125	14.85	0	0	0	0	0	0
50	∞	∞	3.7125	14.85	0	0	0	0	0	0
55	∞	∞	3.7125	14.85	0	0	0	0	0	0
60	∞	∞	3.7125	14.85	0	0	0	0	0	0
65	∞	∞	6.3339	25.336	0	0	0	0	0	0
70	∞	∞	9.9407	39.763	0	0	0	0	0	0
75	∞	∞	13.5474	54.19	0	0	0	0	0	0
80	∞	∞	17.1542	68.617	0	0	0	0	0	0
85	∞	∞	21.9983	87.993	0	0	0	0	0	0
90	∞	∞	26.883	107.532	0.0503	0.201	0	0	0	0
95	∞	∞	32.8104	131.242	3.657	14.628	0	0	0	0
100	∞	∞	39.0471	156.189	7.2638	29.055	0	0	0	0
105	∞	∞	∞	∞	10.8705	43.482	0	0	0	0
110	∞	∞	∞	∞	14.4773	57.909	0	0	0	0
115	∞	∞	∞	∞	18.4056	73.623	0	0	0	0
120	∞	∞	∞	∞	22.3603	89.441	0	0	0	0
125	∞	∞	∞	∞	26.4341	105.736	0	0	0	0
130	∞	∞	∞	∞	36.969	147.876	0	0	0	0
135	∞	∞	∞	∞	∞	∞	0	0	0	0
140	∞	∞	∞	∞	∞	∞	1.7006	6.802	0	0
145	∞	∞	∞	∞	∞	∞	5.7336	22.934	0	0
150	∞	∞	∞	∞	∞	∞	12.2713	49.085	0	0
155	∞	∞	∞	∞	∞	∞	19.9486	79.795	0	0
160	∞	∞	∞	∞	∞	∞	29.7391	118.956	0	0
165	∞	∞	∞	∞	∞	∞	40.092	160.368	0	0
170	∞	∞	∞	∞	∞	∞	∞	∞	0	0
175	∞	∞	∞	∞	∞	∞	∞	∞	1.1785	4.714
180	∞	∞	∞	∞	∞	∞	∞	∞	7.5216	30.086
185	∞	∞	∞	∞	∞	∞	∞	∞	13.8646	55.458
190	∞	∞	∞	∞	∞	∞	∞	∞	23.2618	93.047
195	∞	∞	∞	∞	∞	∞	∞	∞	33.3248	133.299
200	∞	∞	∞	∞	∞	∞	∞	∞	43.3879	173.552

Table 7

Von Liebig: Cost and input requirement functions for various levels of fixed inputs

Using the set of all observations as the reference technology

WHEAT (Temp = days between 70 - 86 degrees)

(All figures are per acre)

	Fixed inputs set at: Minimum		Half(min + mean)		Mean		Half(mean + max)		Maximum	
Yield (bu.)	Cost ($)	Nitrogen (lbs.)	Cost ($)	Nitrogen (lbs.)	Cost ($)	Nitrogen (lbs.)	Cost ($)	Nitrogen (lbs.)	Cost ($)	Nitrogen (lbs.)
0	∞	∞	0	0	0	0	0	0	0	0
5	∞	∞	0	0	0	0	0	0	0	0
10	∞	∞	0	0	0	0	0	0	0	0
15	∞	∞	0	0	0	0	0	0	0	0
20	∞	∞	0	0	0	0	0	0	0	0
25	∞	∞	0	0	0	0	0	0	0	0
30	∞	∞	0	0	0	0	0	0	0	0
35	∞	∞	0	0	0	0	0	0	0	0
40	∞	∞	0	0	0	0	0	0	0	0
45	∞	∞	0	0	0	0	0	0	0	0
50	∞	∞	0	0	0	0	0	0	0	0
55	∞	∞	0	0	0	0	0	0	0	0
60	∞	∞	0	0	0	0	0	0	0	0
65	∞	∞	0.4004	1.602	0	0	0	0	0	0
70	∞	∞	1.6542	6.617	0	0	0	0	0	0
75	∞	∞	2.9079	11.632	0	0	0	0	0	0
80	∞	∞	4.476	17.904	0	0	0	0	0	0
85	∞	∞	6.3077	25.231	0	0	0	0	0	0
90	∞	∞	8.1395	32.558	1.2393	4.957	0	0	0	0
95	∞	∞	9.9712	39.885	2.6653	10.661	0	0	0	0
100	∞	∞	11.803	47.212	4.0913	16.365	1.7247	6.899	0	0
105	∞	∞	13.6348	54.539	6.2188	24.875	3.858	15.432	2.0369	8.148
110	∞	∞	15.6412	62.565	8.3521	33.408	5.9913	23.965	4.1702	16.681
115	∞	∞	19.5136	78.054	10.4854	41.942	8.1247	32.499	6.3035	25.214
120	∞	∞	23.386	93.544	13.0372	52.149	10.258	41.032	8.4368	33.747
125	∞	∞	27.2584	109.034	16.1083	64.433	12.3913	49.565	10.5702	42.281
130	∞	∞	31.3041	125.217	19.1793	76.717	14.5246	58.098	12.7035	50.814
135	∞	∞	35.3627	141.451	22.5251	90.101	16.6579	66.632	14.8368	59.347
140	∞	∞	39.7777	159.111	26.3975	105.59	18.7913	75.165	16.9701	67.88
145	∞	∞	∞	∞	30.2829	121.132	20.9522	83.809	19.1034	76.414
150	∞	∞	∞	∞	34.4484	137.793	23.8601	95.441	21.2368	84.947
155	∞	∞	∞	∞	47.3267	189.307	27.0234	108.094	23.3701	93.48
160	∞	∞	∞	∞	∞	∞	30.3374	121.35	25.5034	102.014
165	∞	∞	∞	∞	∞	∞	36.552	146.208	27.6367	110.547
170	∞	∞	∞	∞	∞	∞	44.3172	177.269	29.77	119.08
175	∞	∞	∞	∞	∞	∞	∞	∞	36.4162	145.665
180	∞	∞	∞	∞	∞	∞	∞	∞	43.6077	174.431
185	∞	∞	∞	∞	∞	∞	∞	∞	51.6703	206.681
190	∞	∞	∞	∞	∞	∞	∞	∞	59.9421	239.768

Table 8

Von Liebig: Cost and input requirement functions for various levels of fixed inputs

Using the nondominating set as the reference technology

WHEAT (Temp = days between 70 - 86 degrees)

(All figures are per acre)

	Fixed inputs set at: Minimum		Half(min + mean)		Mean		Half(mean + max)		Maximum	
Yield (bu.)	Cost ($)	Nitrogen (lbs.)	Cost ($)	Nitrogen (lbs.)	Cost ($)	Nitrogen (lbs.)	Cost ($)	Nitrogen (lbs.)	Cost ($)	Nitrogen (lbs.)
0	∞	∞	∞	∞	0	0	0	0	0	0
5	∞	∞	∞	∞	0	0	0	0	0	0
10	∞	∞	∞	∞	0	0	0	0	0	0
15	∞	∞	∞	∞	0	0	0	0	0	0
20	∞	∞	∞	∞	0	0	0	0	0	0
25	∞	∞	∞	∞	0	0	0	0	0	0
30	∞	∞	∞	∞	0	0	0	0	0	0
35	∞	∞	∞	∞	0	0	0	0	0	0
40	∞	∞	∞	∞	0	0	0	0	0	0
45	∞	∞	∞	∞	0	0	0	0	0	0
50	∞	∞	∞	∞	0.2894	1.158	0	0	0	0
55	∞	∞	∞	∞	2.6549	10.62	0	0	0	0
60	∞	∞	∞	∞	5.0203	20.081	0	0	0	0
65	∞	∞	∞	∞	7.7056	30.822	0.8964	3.585	0	0
70	∞	∞	∞	∞	13.3793	53.517	2.6578	10.631	0	0
75	∞	∞	∞	∞	19.0529	76.212	4.4193	17.677	1.653	6.612
80	∞	∞	∞	∞	24.7266	98.906	6.1808	24.723	3.4144	13.658
85	∞	∞	∞	∞	32.3183	129.273	7.9423	31.769	5.1759	20.704
90	∞	∞	∞	∞	∞	∞	9.7037	38.815	6.9374	27.75
95	∞	∞	∞	∞	∞	∞	11.4652	45.861	8.6989	34.795
100	∞	∞	∞	∞	∞	∞	13.5604	54.241	10.4603	41.841
105	∞	∞	∞	∞	∞	∞	15.9258	63.703	12.2218	48.887
110	∞	∞	∞	∞	∞	∞	18.2913	73.165	13.9833	55.933
115	∞	∞	∞	∞	∞	∞	23.5632	94.253	15.7448	62.979
120	∞	∞	∞	∞	∞	∞	29.3112	117.245	17.5062	70.025
125	∞	∞	∞	∞	∞	∞	37.9747	151.899	19.2677	77.071
130	∞	∞	∞	∞	∞	∞	46.6382	186.553	21.0292	84.117
135	∞	∞	∞	∞	∞	∞	∞	∞	22.7906	91.163
140	∞	∞	∞	∞	∞	∞	∞	∞	24.5521	98.208
145	∞	∞	∞	∞	∞	∞	∞	∞	26.3136	105.254
150	∞	∞	∞	∞	∞	∞	∞	∞	28.0751	112.3
155	∞	∞	∞	∞	∞	∞	∞	∞	29.8365	119.346
160	∞	∞	∞	∞	∞	∞	∞	∞	37.8595	151.438
165	∞	∞	∞	∞	∞	∞	∞	∞	46.523	186.092
170	∞	∞	∞	∞	∞	∞	∞	∞	55.1866	220.746
175	∞	∞	∞	∞	∞	∞	∞	∞	∞	∞
180	∞	∞	∞	∞	∞	∞	∞	∞	∞	∞
185	∞	∞	∞	∞	∞	∞	∞	∞	∞	∞
190	∞	∞	∞	∞	∞	∞	∞	∞	∞	∞

Table 9

Von Liebig: Cost and input requirement functions for various levels of fixed inputs

Using the set of all observations as the reference technology

WINTER FALLOW (Temp = days between 70 - 86 degrees)

(All figures are per acre)

	Fixed inputs set at: Minimum		Half(min + mean)		Mean		Half(mean + max)		Maximum	
Yield (bu.)	Cost ($)	Nitrogen (lbs.)	Cost ($)	Nitrogen (lbs.)	Cost ($)	Nitrogen (lbs.)	Cost ($)	Nitrogen (lbs.)	Cost ($)	Nitrogen (lbs.)
0	0	0	0	0	0	0	0	0	0	0
5	0	0	0	0	0	0	0	0	0	0
10	0	0	0	0	0	0	0	0	0	0
15	0	0	0	0	0	0	0	0	0	0
20	0	0	0	0	0	0	0	0	0	0
25	0	0	0	0	0	0	0	0	0	0
30	0	0	0	0	0	0	0	0	0	0
35	0	0	0	0	0	0	0	0	0	0
40	0	0	0	0	0	0	0	0	0	0
45	0	0	0	0	0	0	0	0	0	0
50	0	0	0	0	0	0	0	0	0	0
55	0	0	0	0	0	0	0	0	0	0
60	0	0	0	0	0	0	0	0	0	0
65	0	0	0	0	0	0	0	0	0	0
70	1.1692	4.677	0	0	0	0	0	0	0	0
75	3.4835	13.934	0	0	0	0	0	0	0	0
80	5.7977	23.191	0.9452	3.781	0	0	0	0	0	0
85	8.112	32.448	2.7982	11.193	0	0	0	0	0	0
90	10.4263	41.705	4.6513	18.605	0.9541	3.817	0	0	0	0
95	12.7406	50.962	6.5556	26.223	2.8072	11.229	0	0	0	0
100	15.0548	60.219	8.4764	33.905	4.6603	18.641	0	0	0	0
105	17.3691	69.477	10.3971	41.588	6.5134	26.053	0.7035	2.814	0	0
110	19.6834	78.734	12.3178	49.271	8.3664	33.466	2.5566	10.226	0	0
115	21.9977	87.991	14.2385	56.954	10.2202	40.881	4.4097	17.639	0	0
120	24.312	97.248	16.1592	64.637	12.141	48.564	6.3052	25.221	0.6165	2.466
125	26.6262	106.505	18.0799	72.32	14.0617	56.247	8.2259	32.904	3.1389	12.556
130	28.9405	115.762	20.0006	80.002	15.9824	63.929	10.1466	40.586	5.6613	22.645
135	36.5127	146.051	21.9213	87.685	17.9031	71.612	12.2697	49.079	8.1837	32.735
140	∞	∞	23.842	95.368	19.8238	79.295	14.7921	59.168	10.7061	42.824
145	∞	∞	25.7627	103.051	21.7445	86.978	17.3145	69.258	13.2285	52.914
150	∞	∞	27.7159	110.864	23.9228	95.691	19.8369	79.348	15.7509	63.004
155	∞	∞	30.1568	120.627	26.4452	105.781	22.3593	89.437	18.2733	73.093
160	∞	∞	∞	∞	28.9676	115.871	24.8817	99.527	20.7957	83.183
165	∞	∞	∞	∞	∞	∞	27.4041	109.616	23.3181	93.273
170	∞	∞	∞	∞	∞	∞	29.9265	119.706	25.8406	103.362
175	∞	∞	∞	∞	∞	∞	∞	∞	28.363	113.452

Table 10

Von Liebig: Cost and input requirement functions for various levels of fixed inputs

Using the nondominating set as the reference technology

WINTER FALLOW (Temp = days between 70 - 86 degrees)

(All figures are per acre)

Fixed inputs set at:	Minimum		Half(min + mean)		Mean		Half(mean + max)		Maximum	
Yield (bu.)	Cost ($)	Nitrogen (lbs.)	Cost ($)	Nitrogen (lbs.)	Cost ($)	Nitrogen (lbs.)	Cost ($)	Nitrogen (lbs.)	Cost ($)	Nitrogen (lbs.)
0	∞	∞	∞	∞	0	0	0	0	0	0
5	∞	∞	∞	∞	0	0	0	0	0	0
10	∞	∞	∞	∞	0	0	0	0	0	0
15	∞	∞	∞	∞	1.8379	7.352	1.8379	7.352	1.8379	7.352
20	∞	∞	∞	∞	3.876	15.504	3.876	15.504	3.876	15.504
25	∞	∞	∞	∞	5.914	23.656	5.914	23.656	5.914	23.656
30	∞	∞	∞	∞	7.952	31.808	7.952	31.808	7.952	31.808
35	∞	∞	∞	∞	9.9901	39.96	9.9901	39.96	9.9901	39.96
40	∞	∞	∞	∞	12.0281	48.113	12.0281	48.113	12.0281	48.113
45	∞	∞	∞	∞	14.0662	56.265	14.0662	56.265	14.0662	56.265
50	∞	∞	∞	∞	16.9329	67.732	16.1042	64.417	16.1042	64.417
55	∞	∞	∞	∞	19.9145	79.658	18.1423	72.569	18.1423	72.569
60	∞	∞	∞	∞	22.8961	91.584	20.1803	80.721	20.1803	80.721
65	∞	∞	∞	∞	25.8777	103.511	22.2183	88.873	22.2183	88.873
70	∞	∞	∞	∞	28.8593	115.437	24.2564	97.026	24.2564	97.026
75	∞	∞	∞	∞	31.8409	127.364	26.2944	105.178	26.2944	105.178
80	∞	∞	∞	∞	34.8225	139.29	28.3325	113.33	28.3325	113.33
85	∞	∞	∞	∞	37.8041	151.216	30.3705	121.482	30.3705	121.482
90	∞	∞	∞	∞	40.7857	163.143	32.4086	129.634	32.4086	129.634
95	∞	∞	∞	∞	43.7673	175.069	34.4466	137.786	34.4466	137.786
100	∞	∞	∞	∞	46.7489	186.996	36.4846	145.939	36.4846	145.939
105	∞	∞	∞	∞	49.7305	198.922	39.1549	156.62	38.5227	154.091
110	∞	∞	∞	∞	52.7121	210.849	42.1365	168.546	40.5607	162.243
115	∞	∞	∞	∞	55.6937	222.775	45.1181	180.472	42.5988	170.395
120	∞	∞	∞	∞	58.6753	234.701	48.0997	192.399	44.6368	178.547
125	∞	∞	∞	∞	∞	∞	51.0813	204.325	46.6749	186.699
130	∞	∞	∞	∞	∞	∞	54.0629	216.252	48.7129	194.852
135	∞	∞	∞	∞	∞	∞	57.0445	228.178	50.751	203.004
140	∞	∞	∞	∞	∞	∞	∞	∞	52.789	211.156
145	∞	∞	∞	∞	∞	∞	∞	∞	54.827	219.308
150	∞	∞	∞	∞	∞	∞	∞	∞	56.8651	227.46
155	∞	∞	∞	∞	∞	∞	∞	∞	58.9031	235.612
160	∞	∞	∞	∞	∞	∞	∞	∞	∞	∞
165	∞	∞	∞	∞	∞	∞	∞	∞	∞	∞
170	∞	∞	∞	∞	∞	∞	∞	∞	∞	∞
175	∞	∞	∞	∞	∞	∞	∞	∞	∞	∞