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A Virtually Ideal Production System: Specifying and Estimating the VIPS Model

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A Virtually Ideal Production System: Specifying and Estimating the VIPS Model

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The class of profit functions, termed VIPS for "virtually ideal production system," consistent with all derived demands being linear in numeric functions of output prices is characterized. A flexible but parsimonious version of the VIPS profit function is specified and the implied supply-response system is estimated using aggregate U.S. agricultural data.

Key words: profit functions, production, inputs, outputs, derived demand, supply

A Virtually Ideal Production System: Specifying and Estimating the VIPS Model[1]

There are two distinct approaches to specifying estimable systems of equations for consumer-demand and derived-demand models. The more popular is to specify an appropriate dual indirect objective function with attractive properties and then use versions of Roy's identity, Hotelling's lemma, or Shephard's lemma, as appropriate, to derive functional specifications for the demand or supply relationships. Well-known examples of this approach include the transcendental logarithmic (translog) family of cost, profit, and indirect utility functions, the Generalized Leontief family, and McFadden's general linear model.

The second approach is to specify demand relationships with desirable properties and then impose upon these relationships the requisite properties for integrability. The Stone-Geary, Rotterdam, and Muellbauer's PIGL systems were originally derived in this fashion. Where the first approach involves specifying an indirect objective function which guarantees integrability but may not ensure desirable demand relationships (for example, in terms of empirical tractability), the latter starts with the desired demand or supply form and resurrects the implied indirect objective function and, along with it, any associated restrictions on the derived demands. The nexus between the two approaches is the envelope relationship in its various guises (Roy's identity, Hotelling's lemma, and Shephard's lemma).

This paper pursues the second approach in a production context. (To our knowledge, the only previous effort in this direction was the Laitenen and Theil extension of the Rotterdam model to production-response systems.) Our purpose is to start with a very general derived-demand relationship that satisfies a criterion which is particularly convenient for empirical production analysis. This criterion is that derived demands be linear in functions of output prices. These demands have other convenient properties, such as aggregate derived-demand models that are internally consistent with microeconomic models. That is, they can consistently "price aggregate" in the sense of Chambers and Pope (1991) and thus circumvent the Pope-Chambers impossibility result. For example, variations in product quality leading to different output prices can be easily, explicitly, and exactly accommodated using these demands and the corresponding supplies. Moreover, like the AIDS model from consumer demand theory, they are also consistent with second-order flexibility and can be specified to be linear in parameters. Important special cases of the class of these demands are the linear (in output price) derived demand and the linear-in-moments model introduced by Chambers and Pope (1992). Because they are simple, are second-order flexible, and can price aggregate we refer to them, in the spirit of Deaton and Muellbauer, as "virtually ideal".

Our first section defines and motivates the virtually ideal input demand functions. The next section characterizes the class of profit functions implied by the virtually ideal input demand system and derives from it the system of supplies consistent with the virtually ideal system. The third section specifies an estimable version of this general class of profit functions. The fourth section illustrates the empirical use of this system by applying it to a set of aggregate production data for the United States that has served as the basis of a number of empirical studies of U.S. agricultural supply response. The final section concludes.

A Virtually Ideal Input Demand System — Definition and Motivation

The virtually ideal input-demand system is the integrable subset of the class of input demands assuming the form:

$$-x^i(p,w) = a^i(w) + \sum_{r=1}^{R} b^{ir}(w) f^r(p) \qquad (i=1,2,\ldots,n) \tag{1}$$

where $x^i(\mathbf{p},\mathbf{w})$ represents the profit maximizing derived demand for the ith input, $\mathbf{p} \in \Re^m_{++}$ is a vector of output prices, and $\mathbf{w} \in \Re^n_{++}$ is a vector of input prices. (Throughout, superscripts are commodity or input indices unless otherwise noted.) Each $f^r(\mathbf{p})$ (r = 1, ..., R) represents a distinct numeric function of the output prices. Generally, choice of $f^r(\mathbf{p})$ and the magnitude of R will be dictated by the degree to which the researcher wants to approximate (in **p**) the derived demands. For example, if a researcher desired a first-order approximation, R could equal one and a natural candidate for $f^r(\mathbf{p})$ would be $f^r(p) = p$ (in the case of scalar p). Higher order approximations would require increasing R.

The reader will note that (1) contains no direct representation of the system of associated supplies. Because our focus is on derived demands, that exclusion is intentional. Generally, derived demands and supplies from a common profit function will not be in the same polynomial class (in **p**). Thus, for example, specification of the supplies in a form similar to (1) would generally limit the class of integrable derived demands so as to make (1) either degenerate or trivial. Consider an example. Suppose that p is a scalar, R = 1 and $f^r(p) = p$, and that the desired supply form is the same as (1). By Hotelling's lemma, the associated profit function must be quadratic.[1] As our results below indicate, this is overly restrictive. Thus, to preserve as much generality as possible while still retaining the tractable form in (1) for the derived demands, no additional restrictions are placed on supplies. Once the profit function consistent with (1) is deduced, supply functions can be derived via Hotelling's lemma: these supply functions will reflect the restrictions in (1).

What makes (1) attractive when other demand systems might be specified? First, (1) can be explicitly and nontrivially made to be integrable (derived from profit maximization). Further, it can be made second-order flexible. This is shown in the next section. Finally, empirical production analysis often involves some form of aggregation either over different microeconomic units or over different quantities. This happens for both inputs and outputs. System (1) has been specified to permit easy aggregation over different output prices.

We have chosen to highlight the aggregation of farm level output prices. This is not to say that input variations are not important, but we feel that output quality variation is more typically of concern. Clearly fruits and vegetables exhibit great intraseasonal and spatial variation in price. Moreover, even a commodity like wheat exhibits substantial quality and price variation (see Nuckton and Gardner for a recent discussion). Recent empirical work (Chambers and Pope, 1991) also indicates that output-price aggregation is a problem in analyzing wheat supply response. While we concentrate on output-price aggregation, future work must address the possibility of both input and output price aggregation. In any case, the approach discussed here should aid in other production aggregation problems regardless of the source of the heterogeneity.

To illustrate briefly how (1) might be used in aggregation, consider the single output case. Let there be $k = 1, \ldots, K$ agents facing K output prices $p_1, \ldots, p_K$. Suppose also that the "intercept" in (1) varies as $a_k^i(w)$ $k = 1, \ldots, K$. Hence,

$$(1') \qquad -x_k^i = \alpha_k^i(w) + \sum_{r=1}^{R} b^{ir}(w)\, f^r(p_k), \qquad k = 1,\ldots,K.$$

Average aggregate or "representative" demand is

$$-\bar{x}^i \equiv -\frac{1}{K}\sum_{k=1}^{K} x_k^i = \bar{\alpha}^i(w) + \sum_{r=1}^{R} b^{ir}(w)\, \bar{f}^r$$

where $\bar{\alpha}^i(w) = \frac{1}{K}\sum_{k=1}^{K} \alpha_k^i$ and $\bar{f}^r = \frac{1}{K}\sum_{k=1}^{K} f^r(p_k)$. If for example, $f^1(p_k) = p_k$ and $f^2(p_k) = p_k^2$ then $\bar{f}^1(p) = \frac{1}{K}\sum_{k=1}^{K} p_k$ and $\bar{f}^2 = \frac{1}{K}\sum_{k=1}^{K} p_k^2$, i.e., the first two moments about zero enter $\bar{x}^i$. Thus, the aggregate input demand equation would be linear in the moments (about zero).

Because the second moment about zero is the second central moment (variance) plus the mean squared, the above could easily be rewritten in terms of the variance.

Another possibility for aggregate price indices is the mean and $\frac{1}{K}\sum_{k=1}^{K} p_k \ln p_k$, which is similar to the PIGLOG income index used by Muellbauer but applied here to output prices. These indices are consistent for R = 2 with $f^1(p_k) = p_k$ and $f^2(p_k) = p_k \ln p_k$. Thus, the form in (1) can accommodate many price indices at the aggregate level. For that reason, we refer to the integrable version of (1) as a "virtually ideal input demand system" by analogy to the "almost ideal demand system" of Deaton and Muellbauer.[2]

Finally, a complete production system generally includes both demand and supply functions. Supply functions consistent with an integrable version of (1) are presented in the next section. The supply functions are themselves not aggregable using the same aggregators used in the aggregate demands. But they will generally be aggregable using a different set of price indices.

A Virtually Ideal Production System

Our main restriction for the derived demands beyond (1) is that they emerge from the following maximization problem:

(2) $$\pi(\mathbf{p}, \mathbf{w}) = \text{Max}\ \{\mathbf{py} - \mathbf{wx} : (\mathbf{x},\mathbf{y}) \in T\}$$

where $T \subseteq \Re^m_+ \times \Re^n_+$ is a compact and strictly convex production possibilities set. It is well-known that $\pi(\mathbf{p},\mathbf{w})$ is positively linearly homogeneous and convex in $\mathbf{p}$ and $\mathbf{w}$ (Chambers). Moreover, because our assumptions on T guarantee the existence of a unique solution to (2), Hotelling's Lemma implies that

(3) $$-x^i(\mathbf{p}, \mathbf{w}) = \pi_i(\mathbf{p}, \mathbf{w}) \qquad (i = 1, 2, ..., n)$$

$$y^s(\mathbf{p}, \mathbf{w}) = \pi_s(\mathbf{p}, \mathbf{w}) \qquad (s = 1, ..., m)$$

where $\pi_i \equiv \partial\pi/\partial w^i$ ($i = 1, ..., n$) and $\pi_s \equiv \partial\pi/\partial p^s$ ($s = 1, ..., m$). (Subscripts on functions denote partial derivatives.) Our main theoretical result is (the proof is in an appendix):

Result: The derived-demand structure derived from (2) satisfies (1) if and only if

$$\pi(\mathbf{p},\mathbf{w}) = A(\mathbf{w}) + C(\mathbf{p}) + \sum_{r=1}^{R} B^r(\mathbf{w})\, f^r(\mathbf{p}).$$

with the derived demands and supplies given by

$$-x^i = A_i(w) + \sum_{r=1}^{R} B_i^r(w)\, f^r(p) \qquad (i = 1, ..., n),$$

$$y^s = C_s(p) + \sum_{r=1}^{R} B^r(w)\, f_s^r(p) \qquad (s = 1, ..., m).$$

The profit function in the Result is referred to as the virtually ideal production system (VIPS) profit function. The result establishes that the system of derived demands in (1) is consistent with profit maximizing behavior if and only if the profit function (and hence the entire system) can be characterized in terms of $2(R + 1)$ independent functions: $A(\mathbf{w})$, $C(\mathbf{p})$, and R $B^r(\mathbf{w})$ and $f^r(\mathbf{p})$ functions. Notice, however, that the system as represented in (1) has $n + R(n + 1)$ functions. In most practical instances, therefore, the rank of the VIPS derived-demand system will be considerably smaller than (1) suggests. (So long as $n > 1$ and each demand actually depends on p, the rank of the VIPS derived-demand system will be smaller than that in (1).) Thus, the requirements for integrability of the derived demands in (1) substantially reduce the number of independent functions required to represent derived demands and profit maximizing supply.

The choice of R (i.e., the number of $f^r(\mathbf{p})$ functions) may usefully be discussed in the context of the Result. Generally R will be chosen with an eye toward making either $\pi(\mathbf{p}, \mathbf{w})$ or $x^i(\mathbf{p}, \mathbf{w})$ at least second-order flexible in both $\mathbf{w}$ and $\mathbf{p}$. Thus, the choice is somewhat arbitrary and will be dictated by the needs of the researcher and involves as much craft as theory. The usual choice is to make $\pi(\mathbf{p}, \mathbf{w})$ second-order flexible in $\mathbf{p}$. But a strength of the VIPS model is that appropriate choices of R and $f^r(\mathbf{p})$ permit making derived demand second-order flexible. For example, suppose that we choose R = 2 and the PIGLOG specification above, then both $\pi(\mathbf{p}, \mathbf{w})$ and $x^i(\mathbf{p},\mathbf{w})$ are second-order flexible in $\mathbf{p}$.

Another criterion which may guide the choice of R and $f^r(\mathbf{p})$ $(r = 1,\ldots,R)$ is the desired aggregation properties of the resulting system. Pope and Chambers have shown that no single price index can aggregate derived demands and supplies jointly so long as quantity aggregates are sums of individual quantities. The VIPS model has the ability to aggregate derived demands and supplies jointly. The Pope-Chambers impossibility result is circumvented by allowing supplies to be aggregated using a different set of multiple price aggregates than is used for the derived demands.

To illustrate this property of the VIPS model let us return to the aggregation formulation discussed earlier where aggregate derived demands were representable in terms of $\bar{f}^1(p) = \frac{1}{K}\sum_{k=1}^{K} p_k$ and $\bar{f}^2(p) = \frac{1}{K}\sum_{k=1}^{K} p_k^2$. Referring to the Result, it is now apparent that the corresponding supplies would be aggregable using the index $\bar{f}_s^2(p) = \frac{2}{K}\sum_{k=1}^{K} p_k$ and $\bar{C}_1(p) = \frac{1}{K}\sum_{k=1}^{K} C_1^k(p_k)$. Thus, by relaxing the requirement of a single aggregate price index we can specify meaningful aggregate supply-response systems. The desired properties of the price aggregators would then be a useful guide to the choice of $f^r(\mathbf{p})$ $(r = 1,\ldots,R)$.

For the VIPS profit function to be consistent with known properties of profit functions, it must be both convex and positively linearly homogeneous in prices. In making the Result operational, that is, in specifying a potentially estimable system based upon the VIPS profit function, the main difficulty is in specifying versions of the $B^r(\mathbf{w})$ and $f^r(\mathbf{p})$ functions consistent with the homogeneity and convexity properties of profit functions. Because we seek a system that can be estimated solely by linear regression methods, our focus in what follows will be on satisfying the simpler homogeneity properties. (However, by appropriate choice of functional specification our discussion can be extended to encompass convexity using the methods developed by Diewert and Wales.)

In generating candidate functions to satisfy the Result, the proof of the Result offers a clear strategy: Find 2(R +1) functions satisfying the properties of profit functions and then proceed to generate the functions in the Result by the use of reference vectors. As a practical matter, researchers often proceed conditionally: $f^r(\mathbf{p})$ and R are specified using some attractive criteria. Given this choice, $A(\mathbf{w})$ and $B^r(\mathbf{w})$ ($r = 1,\ldots,R$) are specified based upon differentiability, homogeneity, familiarity, and simplicity of estimation. In the following section, we pursue this strategy, which is similar in spirit to the work of Howe, Pollak, and Wales in developing the quadratic expenditure system of consumer demands. Because there are an infinite number of possible functions satisfying the Result, our search is guided by the principles of parsimony and flexibility as espoused by Fuss, McFadden, and Mundlak.

An Estimable VIPSLIM Profit Function

Attention here is restricted to the case of a scalar output. The extension to multiple outputs is straightforward. For a scalar output, the requirement of positive linear homogeneity virtually eliminates the term $C(\mathbf{p})$ in the VIPS model because it must assume the form cp, $c \in \Re$. $A(\mathbf{w})$, does not suffer from this same difficulty. An obvious, practical, and familiar choice is the Generalized Leontief function

$$A(\mathbf{w}) = (1/2)\sum_{i=1}^{n}\sum_{j=1}^{n} a_{ij}(w_i w_j)^{1/2}$$

with $a_{ij} = a_{ji} \in \Re$ (i = 1,2,..,n). This choice of A(**w**) is particularly convenient because its properties are well understood. Moreover, sufficient parametric restrictions for convexity are well-known: A(**w**) is convex if $a_{ij} < 0$ for all $i \neq j$.

In choosing a specification for $\sum_{r=1}^{R} B^r(\mathbf{w})\, f^r(p)$, we want to choose something that is simple but still informative. To highlight the aggregation properties of (1) we specify what we refer to as the VIPSLIM form (for virtually ideal production system — linear in moments). Our choice is predicated upon the ability of the VIPS model to price aggregate in the sense of Chambers and Pope (1991), and a desire to have price aggregators which are capable of fully characterizing the distribution of prices at the micro level. Under very weak regularity properties, the moments (about zero) of the price distribution provide this quality (Bickel and Doksum). Therefore, we choose

$$f^r(p) = p^r \qquad (r = 1,2,\ldots,R).$$

To satisfy homogeneity globally, each $B^r(\mathbf{w})$ must be homogeneous of degree 1 - r. For simplicity, we only address the case of R = 2, i.e., a quadratic profit function. (The reader can easily extend this to arbitrary R). As noted earlier, this allows aggregate demand to depend on the mean and variance (and hence on the mean squared) of the price distribution. Specify $B^1(\mathbf{w})$ as

$$B^1(\mathbf{w}) = \sum_{i \neq k} b_i (w_i/w_k)$$

and specify $B^2(\mathbf{w})$ as

$$B^2(\mathbf{w}) = g/\sum_{i=i} w_i$$

where b_i and g are parameters to be estimated. The single product VIPSLIM supply function is $y = c + B^1(\mathbf{w}) + 2B^2(\mathbf{w})p$. Thus, the supply function is linear in output price but as is obvious from (1), input demands are not. This highlights the fundamental aggregation property discussed above: to be aggregable, supply and input demands must be affected in fundamentally different fashions by output price heterogeneity and thus must rely on different price aggregators. In the VIPSLIM model specified here the aggregate derived demands and profit depend upon the first R moments while supply depends upon the first R-1 moments.

The VIPSLIM profit function has (n+2)(n+1)/2 parameters and is linearly homogeneous in input and output prices. Therefore, so long as the relevant Wronksian matrix (see Chambers, Chapter 5) is invertible, the VIPSLIM form provides a second-order approximation to an arbitrary differentiable profit function parsimoniously. Moreover, it belongs to the class of generalized linear profit functions introduced by McFadden. Consequently, its parameters can be estimated using linear multivariate regression techniques. Notice, however, that the VIPSLIM model treats (as does the original McFadden model) one input price asymmetrically in the specification of the $B^1(\mathbf{w})$ function. Hence, one of its factor demands (here factor demand k) will have a slightly different form than the other factors. All, however, remain linear in the moments of p, preserving the fundamental VIPSLIM property.

Asymmetries are encountered for other normalized forms such as the "normalized quadratic". This asymmetry is attractive because the form is simple yet allows for a variety of nonnested models with only changes in normalization. However, some may prefer symmetric model structures. Giving up linearity in parameters, it is easy to specify symmetric versions of the VIPSLIM model that closely approximate the Diewert and Wales formulation.[3] There are many reasonable ways that the functions of w can be parameterized. Our purpose here is not to delineate the best possible specification (this depends upon the needs of the study) but to illustrate one that is easily estimable.

An Illustrative VIPS Model

To illustrate the VIPSLIM model and its applicability, we make the strong assumption that the United States agricultural production sector can be accurately modeled using a single-output aggregate profit function. While a number of other authors have made similar assumptions in estimating aggregate profit and cost functions (Ball and Chambers; Antle; Shumway; Capalbo and Denny; Ball), doing so here should not be interpreted as reflecting a belief on our part that such an aggregate profit function is plausible. Specifically, we do not assume that the estimated model represents a model derived using the LIM aggregation procedure discussed above. Rather our purpose is to illustrate the estimation of a model in the VIPSLIM form using a data set that has been the basis for a number of other applied production studies. For this illustration, assume that aggregate U.S. agricultural output is produced using a constant returns to scale technology (this assumption is imposed because of the manner in which the data were constructed) using six aggregate inputs: land (A), labor (L), pesticides(P), fertilizer (F), materials (M), and capital (K). Land was treated as a fixed input, so that the assumption of constant returns to scale implies that the variable profit function (land-rent) function can be represented as:

$$\pi^*(p, w_L, w_P, w_F, w_M, w_K; A, t) = A\,\pi(p, w_L, w_P, w_F, w_M, w_K; t).$$

Here t stands for time. Its inclusion reflects the fact that our data are time series in nature and captures the possibility of technical change over the sample period. We accommodate the presence of technical change by modifying A(**w**) and c(p) as follows:

$$A(\mathbf{w}; t) = (1/2) \sum_i \sum_j a_{ij} (w_i w_j)^{1/2} + t \sum \tau_i w_i \text{ , and}$$

$$c(p, t) = cp + t\tau_s p.$$

where the τ_k are parameters to be estimated.

In what follows, $\pi(p, w_L, w_P, w_F, w_M, w_K; t)$ is the focus of our attention. Applying the Shephard-Hotelling lemma yields the following per-acre supply and input demand functions, respectively:

$$
\begin{aligned}
y/A &= c + t\tau_s + \sum_{i\neq k}(w_i/w_k) + 2gp/\sum_{i=1}^{n} w_i, \\
-x_i/A &= t\tau_i + (1/2)\sum_{j=1}^{n} a_{ij}(w_j/w_i)^{(1/2)} + b_i(p_i/w_k) - p^2 g/\left(\sum_{j=1}^{n} w_j\right)^2 \qquad (i\neq k), \text{ and} \\
-x_k/A &= t\tau_k + (1/2)\sum_{j=1}^{n} a_{ij}(w_j/w_k)^{(1/2)} - (p/w_k)\sum_{i\neq k} b_i(w_i/w_k) - p^2 g/\left(\sum_{i=1}^{n} w_i\right)^2 .
\end{aligned}
\tag{4}
$$

Estimating equations were obtained by appending an additive error term to each equation in (4). These error terms were assumed to be contemporaneously correlated but intertemporally independent and identically distributed around a mean of zero to permit the application of simple constrained multivariate regression techniques.

The data used to estimate the model in (4) were annual data for U.S. agriculture and were provided by V. Eldon Ball of the Economic Research Service, United States Department of Agriculture. The data cover the period 1948-1989. Descriptions of the original data sources and the philosophy and methods used for the construction of variables, including the price of capital, are contained in Ball (1985) and Ball (1988).

Estimation Results

Because the VIPSLIM model treats input k asymmetrically, the choice of which input price to use as the numeraire in $B^1(\mathbf{w})$ is somewhat arbitrary. Therefore, we estimated five versions of the VIPSLIM model with each version corresponding to a different numeraire in $B^1(\mathbf{w})$. The estimated parameters and their asymptotic standard errors for the version using pesticide price as the numeraire are presented in Table 1. The parameters were estimated using the iterative, seemingly-unrelated regression routine available in SAS/PC. Estimated price elasticities derived from these parameter estimates are reported in Table 2. All are plausible and well within the range of elasticities reported in other studies.

We should note, however, that different versions of the VIPSLIM model performed differently. In particular, depending upon which input price was chosen as the numeraire, we encountered a fairly wide range of estimates for the supply elasticity (.09 to .24 at sample means) and a wider variation in the own-price elasticity of the pesticide demand equation (upward sloping derived demands were found for some sample periods). The difficulty with the pesticide equation is not surprising given the difficulty of correctly specifying an annual, aggregate model of pesticide use (Chambers and Lichtenberg). In principle, therefore, the asymmetry offers a path by which nonnested hypothesis testing or other model selection techniques could be used to isolate the "best" model within the VIPSLIM family.

For each version of the model difficulties were encountered in satisfying convexity properties of profit functions. One of the calculated eigenvalues for the Hessian matrix of the pesticide-normalized version of the VIPSLIM profit function evaluated at sample means was negative. Hence, the estimated VIPSLIM profit function is not convex at that point. Difficulties with convexity were encountered at other points in the sample. However, as most students of the existing empirical literature on agricultural supply-response systems know, failure to satisfy convexity in estimated profit functions is not unique to this study. Moreover, similar problems have been encountered previously for versions of this data set (Ball, 1988). Therefore, no attempt was made to correct for the failure to satisfy convexity of the VIPSLIM model because the purpose of the present study is not to explain aggregate response but to illustrate how the VIPSLIM model might be applied.

Conclusion

This paper's contributions are two: to characterize exhaustively the family of profit functions consistent with derived-demand functions being affine in functions of the output prices; and to propose and estimate one member of this class of profit functions using aggregate U.S. data. The profit function chosen, which we have dubbed VIPSLIM, has the attractive property of being able to aggregate consistently in the sense of Chambers and Pope (1991) as well as being linear in the sample moments of the output price.

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Endnote

1. In a related paper we show that the VIPS profit function derived below is consistent with derived demands assuming the form in (1) and supplies of the form:

$$y^i(p, w) = \alpha^i(p) + \sum_{\ell=1}^{L} \beta^\ell(p)\alpha^\ell(w)$$

2. Both of these specifications employ two price aggregators thus circumventing the impossibility result of Pope and Chambers which is for single price aggregations.

3. A symmetric version of the VIPSLIM model was specified and estimated with results similar to those reported below. Giving up linearity in parameters, $B^2(w)$ could be made to be the simple Cobb-Douglas homogeneous of degree minus one. However, this would also add additional parameters to be estimated.

Table 1: Coefficients and Standard Errors of the US VIPSLIM Model

Parameter	Estimate	Standard Error
a_{LL}	-8.242582	0.983050
a_{LP}	1.493558	0.395451
a_{LF}	-0.394250	0.274661
a_{LM}	-3.646507	0.775624
a_{LK}	0.219843	0.484850
a_{PP}	0.892916	0.358189
a_{PF}	-0.996692	0.349708
a_{PM}	1.304924	0.774904
a_{PK}	-2.297284	0.535933
a_{FF}	0.247581	0.380703
a_{FM}	0.043853	0.401890
a_{FK}	1.971695	0.201058
a_{MM}	-0.718115	1.856245
a_{MK}	-1.915917	0.616212
a_{KK}	0.918254	0.763407
γ_L	0.041067	0.004392

γ_P	-0.014625	0.001897
γ_F	-0.021272	0.002238
γ_M	-0.052898	0.005380
γ_K	-0.045110	0.003729
c	2.117302	0.342188
γ_S	0.276324	0.008206
b_L	0.589044	0.110945
b_F	-0.351864	0.075438
b_M	0.140474	0.191226
b_K	-0.463031	0.122426
g	0.173032	0.018957

Table 2: Elasticities Calculated at the Sample Mean

price→ ↓quantity	Output	Labor	Pesticide	Fertilizer	Materials	Capital
Output	0.19432	-0.007255	-0.011881	-0.069030	-0.027107	-0.079051
Labor	0.021525	-0.27236	0.025111	0.023383	0.21613	-0.013785
Pesticide	0.52508	0.37403	-0.40311	-0.26807	-0.85572	0.62779
Fertilizer	1.36546	0.15589	-0.11998	-0.46745	-0.019229	-0.91469
Materials	0.089626	0.24086	-0.064021	-0.003214	-0.41172	0.14847
Capital	0.35653	-0.020955	0.064068	-0.20855	0.20252	-0.39361

Appendix: Proof of the Result

Sufficiency is clear: apply Hotelling's lemma to the profit function in the Result to obtain (1). The following establishes necessity. Integrability requires Hotelling's lemma. Hence,

$$\pi_i(\mathbf{p},\mathbf{w}) = - x^i(\mathbf{p},\mathbf{w}) = a^i(\mathbf{w}) + \Sigma_r\, b^{ir}(\mathbf{w})\, f^r(\mathbf{p}).$$

There are two approaches to establishing the necessary conditions for integrability. One can assume differentiability of the demands, as is typically done in consumer demand theory (Howe, Pollak and Wales; Lewbel). Or, one can use the reference vector approach pioneered by Gorman and used extensively by Blackorby et al. (see, e.g., pp. 56-7).

Because the latter does not require differentiability of demands and thus is more general, we employ it here. While more general, the reference vector approach also turns out to be simpler mathematically because it does not require the deployment of the relatively exotic Frobenius theorems on integrability of system of partial differential equations (see e.g. Lewbel). Because its use may not be familiar to some readers, before proceeding with a complete proof we illustrate its use for the simple case where R = 1 and n = 1. Then, (1) degenerates to

$$x(\mathbf{p}, \mathbf{w}) = a(\mathbf{w}) + b(\mathbf{w})\, f(\mathbf{p})$$

(superfluous superscripts are dropped). Now find two separate output price vectors (the reference vectors), call them $\mathbf{p}^1$ and $\mathbf{p}^2$, such that $x(\mathbf{p}^1, \mathbf{w}) \neq x(\mathbf{p}^2, \mathbf{w})$. Then if (1) and Hotelling's lemma are to apply it must be true that

$$-\pi_w(\mathbf{p}^1, \mathbf{w}) = a(\mathbf{w}) + b(\mathbf{w})\, f(\mathbf{p}^1)$$

$$-\pi_w(\mathbf{p}^2, \mathbf{w}) = a(\mathbf{w}) + b(\mathbf{w})\, f(\mathbf{p}^2)$$

and $f(\mathbf{p}^1) \neq f(\mathbf{p}^2)$. If we now set $f^1 = f(\mathbf{p}^1)$, $f^2 = f(\mathbf{p}^2)$, $\alpha(\mathbf{w}) = -\pi_w(\mathbf{p}^1, \mathbf{w})$, and $\beta(\mathbf{w}) = -\pi_w(\mathbf{p}^2, \mathbf{w})$ the above can be reinterpreted as two functional equations in two unknow functions $(a(\mathbf{w}), b(\mathbf{w}))$. And it can be rewritten in matrix form as

$$m(\mathbf{w}) = [e, F]\begin{bmatrix} a(\mathbf{w}) \\ b(\mathbf{w}) \end{bmatrix}$$

where

$$m(\mathbf{w}) = \begin{bmatrix} \alpha(\mathbf{w}) \\ \beta(\mathbf{w}) \end{bmatrix} \quad e = \begin{bmatrix} 1 \\ 1 \end{bmatrix} \quad F = \begin{bmatrix} f^1 \\ f^2 \end{bmatrix}$$

which allows us to solve for $a(\mathbf{w})$ and $b(\mathbf{w})$ as

$$a(\mathbf{w}) = (f^2 - f^1)^{-1} (f^2 \alpha(\mathbf{w}) - f^1 \beta(\mathbf{w}))$$

$$b(\mathbf{w}) = (f^2 - f^1)^{-1} (\beta(\mathbf{w}) - \alpha(\mathbf{w}))$$

This establishes that $x(\mathbf{p}, \mathbf{w})$ can be expressed completely in terms of the parameters f^2, f^1, and the functions $\beta(\mathbf{w})$ and $\alpha(\mathbf{w})$ which themselves are derivatives of $\pi(\mathbf{p}, \mathbf{w})$ evaluated at the reference vectors $\mathbf{p}^2$ and $\mathbf{p}^1$ respectively, and hence are integrable.

Substitute these results into (1) and use Hotelling's lemma to get

$$[f^2 - f^1]\, \pi_w(\mathbf{p}, \mathbf{w}) = f^2 \pi_w(\mathbf{p}^1, \mathbf{w}) - f^1 \pi_w(\mathbf{p}^2, \mathbf{w}) + f(\mathbf{p})\, (\pi_w(\mathbf{p}^2, \mathbf{w}) - \pi_w(\mathbf{p}^1, \mathbf{w})).$$

Now integrate over w to establish that

$$(f^2 - f^1)\, \pi(\mathbf{p}, \mathbf{w}) = f^2 \pi(\mathbf{p}^1, \mathbf{w}) - f^1 \pi(\mathbf{p}^2, \mathbf{w}) + f(\mathbf{p})\, (\pi(\mathbf{p}^2, \mathbf{w}) - \pi(\mathbf{p}^1, \mathbf{w})) + h(\mathbf{p})$$

where $h(\mathbf{p})$ is a constant of integration. This is the general form found in the Result.

In the general case, set the $\mathbf{p}$ vector in (1) to the R+1 distinct reference vectors $\mathbf{p}^1, \ldots, \mathbf{p}^{R+1}$, where the reference vectors are now chosen so that the matrix $[\mathbf{e} : \mathbf{F}]$ is invertible. Here e is now a (1xR) column vector of ones and $\mathbf{F}$ is a (1+r)xR matrix with typical element, $f^r(\mathbf{p}^k)$. This operation gives the following invertible system of functional equations:

$$m^k_i(\mathbf{w}) = a^i(\mathbf{w}) + \textstyle\sum_r b^{ir}(\mathbf{w})\, f^{rk} \qquad (i = 1, \ldots, n), \ (k = 1, \ldots, R+1)$$

where $m^k(\mathbf{w}) = \pi(\mathbf{p}^k, \mathbf{w})$ (hence $m^k_i(\mathbf{w}) = \pi_i(\mathbf{p}^k,\mathbf{w})$) and $f^{rk} = f^r(\mathbf{p}^k)$. Invert this (R+1)x(R+1) system to get

$$a^i(\mathbf{w}) = \Sigma_k\, n_k m^k_i(\mathbf{w}),$$

$$b^{ir}(\mathbf{w}) = \Sigma_k\, v_{rk} m^k_i(\mathbf{w}),$$

where n_k and $v_{rk} \in \mathbb{R}$ ($k = 1, 2, \ldots, r+1$, $r = 1,2,\ldots,R$). Substitute these results back into the Hotelling-Shephard lemma to get:

$$\pi_i(\mathbf{p},\mathbf{w}) = \Sigma_k\, n_k m^k_i(\mathbf{w}) + \Sigma_r(\Sigma_k\, v_{rk} m^k_i(\mathbf{w}))\, f^r(\mathbf{p}),$$

$i = 1, 2, \ldots, n$. Now integrability of π requires that each m^k_i is integrable. Integrating this system of derived demands yields:

$$\pi(\mathbf{p},\mathbf{w}) = h(\mathbf{p}) + \Sigma_k\, n_k m^k(\mathbf{w}) + \Sigma_r(\Sigma_k\, v_{rk} m^k(\mathbf{w}))\, f^r(\mathbf{p}).$$

Now define $A(\mathbf{w}) = \Sigma_k\, n_k m^k(\mathbf{w})$ and $B^r(\mathbf{w}) = \Sigma_k\, v_{rk} m^k(\mathbf{w})$ $r = 1,\ldots,R$ to achieve the form in the Result. This establishes necessity.

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