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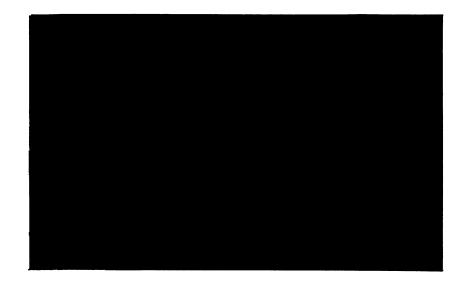## UNIVERSITY OF MARYLAND
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PIGLOG AGGREGATION

AND

ENGEL'S LAW

by

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PIGLOG Aggregation and Engel's Law

For at least four decades, economists have been conc
useful, if not ideal, index numbers (e.g., Fisher, Gorman
This research was born of a desire to develop not only conc ___, but also
empirically superior ways to measure aggregate response and welfare. Because
the most prevalent aggregate measure is the simple sum or mean of micro data,
aggregation theory often led to the nihilistic conclusion that existing
measures or indices were inadequate. A major proponent of a more prospective
approach is Gorman: He has systematically argued that economists should
consider the calculation of "economic indices" rather than "mechanical
indices" (Gorman, 1988). The difference between the two is that the former
are developed to be consistent with the behavior of microeconomic agents
whereas the latter are usually convenient and even reflective statistics
constructed for a purpose other than measuring aggregate response or welfare.
The mean is a good example of such a mechanical index. It is a useful summary
measure of the location of the distribution of income. But for most
aggregation problems, it is only a meaningful economic index if individual
responses are linear with a common, constant marginal or slope effect with
respect to the variable being aggregated (Gorman, 1953).

Deaton and Muellbauer (1980a), in pursuing economic indices, estimate
systems of aggregable share equations which possess nonlinear Engel curves
using the Almost Ideal Demand System (AIDS). The AIDS is a special case of a
family of share equations which Muellbauer calls the Price Independent
Generalized Log (PIGLOG) system. The associated aggregate income index is
independent of consumer prices, relatively easy to compute, and has some other
very attractive properties (Muellbauer, 1975). The aggregation criterion

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employed is that a commodity's aggregate budget share (total expenditure on the good divided by the sum of incomes) be representable as an aggregate function of an aggregate income index and consumer prices. The aggregate function is interpreted as an aggregate or representative budget share. Simple algebraic manipulation, however, allows reinterpreting Muellbauer's aggregate quantity as a weighted average of individual budget shares where the weights are each individual's share in total consumer income.

Because aggregation rules are inherently arbitrary, alternative aggregation rules should be considered: For example, one can plausibly specify a weighted average of individual budget shares as an aggregation rule, following Muellbauer, or just as plausibly one could also consider aggregate expenditure as a weighted average of individual expenditures. Weighted aggregation of expenditures or budget shares place different behavioral restrictions on individual behavior and thus are observationally different rules. Hence, one can discriminate empirically whether a weighted average of expenditures or budget shares is better supported by data.

This paper uses this observation to propose a simple nested procedure to test the appropriateness of Muellbauer's aggregation rule using microeconomic data. Thus, unlike virtually every other study on aggregation this paper does not impose an arbitrary aggregation rule and then force the data to accommodate it. Rather, Muellbauer's aggregation rule is generalized by adding a single parameter to individual demands and then tested statistically. For parsimony, however, we preserve the Muellbauer aggregate income index unchanged.

Our empirical application is the U.S. Engel curve for food. Because food expenditure is a relatively large proportion of income, it has had

extensive study (e.g., Smallwood). However, relatively little attention has been paid to aggregation issues [see Deaton and Muellbauer, 1980b]. This is unfortunate because Engel's Law implies aggregate food demand must depend upon more than mean income. Aggregate food demand also should depend on the dispersion of income if Engel's law is valid. We concentrate on a single equation approach to the aggregation of Engel curves. The single-equation approach is especially useful because of the extensive amount of empirical work on food Engel curves done in a single-equation context. Moreover, the maintained assumptions of a single-equation approach are very weak because the maximization hypotheses implicit in a systems approach place cross-equation restrictions (often empirically rejected, e.g., Deaton and Muellbauer, 1980b) on each aggregable equation.

In what follows, we first introduce our notation, generalize Muellbauer's aggregation rule, and derive a micro-form consistent with the generalized aggregation rule (a generalized PIGLOG). We then specify a parametric representation of the form and suggest a nested testing procedure. The approach is then applied to survey data on food expenditures. The results support the generalization of the generalized PIGLOG equation introduced here. That is, the Muellbauer rule for aggregation of shares or expenditures is rejected by the data in favor of a more general specification. We close with an illustration of the effects of a mean preserving spread in the distribution of income on aggregate quantities to illustrate the importance of income distribution changes on food expenditures.

II. Notation and Proposed Test

The single product analogue of Muellbauer's (1975) aggregation rule is

$$(1) \qquad \bar{w}_j(y_{oj}, p) = \frac{\sum_{h=1}^{H} p_j X_{hj}}{\sum_{h=1}^{H} y_h} = \frac{\sum_{h=1}^{H} y_h w_{hj}}{\sum_{h=1}^{H} y_h}$$

for $j=1,\ldots,m$, where y_{oj} is the aggregate income index, p_j is the price of good

j an element of the m vector p, X_{hj} is the demand by consumer h for good j, y_h

is income or total expenditure of consumer h, and w_{hj} is the budget share of

consumer h for good j, m is the number of goods and H is the number of

consumers and $\bar{w}_j$ is a monotonic function (Equation (1) differs from

Muellbauer's systems approach only in that (1) must hold for only commodity j

rather than for all in commodities. Second, y_{oj} in Muellbauer is independent

of j because a single index is used for all commodities). Muellbauer's

aggregation rule creates an aggregate expenditure share, where the last

expression in (1) indicates an interpretation of (1) as a weighted average of

micro budget shares but not of expenditures. It is this latter interpretation

of exact aggregation that leads to the interpretation of the function on the

left side of (1) as an average or representative budget share. Indeed

treating $y_h / \sum_h y_h$ as the relevant probability of y_h leads to the

interpretation of $\bar{w}_j$ as an expected budget share. Aggregation requires the

search for a reasonable y_{oj} and $\bar{w}_j$ which satisfies (1). As is apparent, the

search inextricably depends on the micro responses X_{hj} and w_{hj}.

A simple y_{oj} is average income, $\bar{y}$. Gorman (1953) showed that when the

right side of (1) is simply the numerator and $y_{oj} = \bar{y}$, then micro demands must

be of the Gorman Polar Form. Engel curves are linear though they do not

necessarily emanate from the origin (affine in income). In fact, even if $\bar{w}_j$

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is retained when y_{oj} is still chosen as average income, micro demands must still assume the Gorman Polar Form. To see this, multiply (1) by $\sum_{h=1}^{H} y_h$ and set $y_{oj} = (1/H) \sum_{h=1}^{H} y_h = \bar{y}$

$$(1') \quad \sum_{h=1}^{H} y_h \bar{w}_j (\bar{y}; p) = p_j \sum_{h=1}^{H} X_{h_j} (y_h, p).$$

This is a Pexider equation, where $\bar{w}_j$ and $X_{h_j}: R_+^{m+1} \to R$, and whose solution is (Aczel, Theorem 3.1):

$$p_j X_{h_j} (y_h; p) = \gamma_{h_j}(p) + \beta_j(p)y_h \quad h = 1,\ldots, H$$

which is the Gorman Polar Form.[1] In order to highlight this implication, it is stated as the following result.

> *Result 1: Given that the measure of aggregate income is the mean, then demands aggregate as in (1) if and only if they assume the Gorman Polar Form.*

Thus, any attempt to do aggregate work using mean income in (1) implies that a mean preserving spread in the distribution of income will not alter aggregate demand or the aggregate budget shares.

In contrast, the more general approach in (1) with $y_{oj} \neq \bar{y}$ allows for nonlinear Engel curves, an empirical "law" for many commodities such as food [e.g., Houthakker (1957), Blanciforti and Green (1983), Deaton and Muellbauer (1980b)]. Substantial effort has been devoted to the search for alternative income indexes. The PIGLOG index used in the AIDS is among the most popular alternatives where

$$\ln y_{oj} = \frac{\sum_{h} y_h \ln(y_h / c_{hj})}{\sum_{h} y_h}.$$

We adopt this index throughout this paper. However, at issue here is the appropriate aggregation rule.

A reasonable but alternative aggregation rule to (1) is

$$(2) \qquad \bar{E}_j(y_{oj}, p) = \frac{\sum_{h=1}^{H} y_h p_j X_{hj}}{\sum_{h=1}^{H} y_h}$$

for some j. In this case, the interpretation of (1) as a weighted average of budget shares is extended to expenditures. Equation (1) does not imply (2) and it is an empirical matter as to whether (1) or (2) is the more appropriate description of the data. It may well be that (1) is descriptive for some goods while (2) is appropriate for other goods.[2]

Here the focus is on a parsimonious extension of (1) which allows (2) as a special case. A simple generalization replaces y_h in the numerator of (1) or (2) with an arbitrary transformation of y_h, $g_h(y_h)$. For example, when $g(y_h)$ in (2) is 1, then Muellbauer's rule (1) is obtained for that good. When $g_h(y_h) = y_h$, then the expenditure weighted average in (2) is obtained. In other cases near these extremes, one obtains "nearly" weighted averages of expenditures or budget shares. Therefore, the focus here is on the existence of a $y_{oj}(y_1, \ldots, y_H)$ and $\bar{z}_j$ such that

$$(3) \qquad \bar{z}_j(y_{oj}(y_1, \ldots, y_H), p) = \frac{\sum_{h=1}^{H} g_h(y_h) z_{hj}(y_h, p)}{\sum_{h=1}^{H} y_h},$$

where z_{hj} is any arbitrary micro response function of interest. Since the approach varies substantively from Muellbauer, the Appendix derives the micro functions consistent with (3) and the PIGLOG index for income where $\ln y_h$ denotes the natural logarithm of y_h.

Result 2: Given that the measure of aggregate income is the PIGLOG

index,

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$$y_{oj} = \frac{\sum_{h=1}^{H} y_h \ln(y_h/c_{h_j})}{\sum_{h=1}^{H} y_h},$$

then the aggregate responses defined in (3) exist if and only if micro responses assume the form

(4) $\quad g_h(y_h) z_{hj}(y_h, p) = \alpha_j(p) y_h + \beta_j(p) \sigma_h(y_h) + \gamma_{hj}(p) \quad h=1,\ldots,H,$

where $\sum_{h=1}^{H} \gamma_{hj}(p) = 0.$

It is important to note that this is derived for a single-equation problem and that no cross equation restrictions such as Slutsky symmetry or Engel aggregation conditions are required as in Muellbauer. The form in (4) virtually dominates empirical analyses of demand as the AIDS system is ubiquitous (e.g., Fulponi, Green and Alston). Interpreting z_{hj} as expenditures and parameterizing g_h as y_h^θ yields:

(5) $\quad y_h^\theta p_j X_{hj} = \alpha_j(p) y_h + \beta_j(p) y_h \ln(y_h/c_{hj}) + \gamma_{hj}(p) \quad h=1,\ldots,H$

where $\sum_{h=1}^{H} \gamma_{hj}(p) = 0.$ We will refer to the left side of (5) as the *augmented expenditure*. Two polar cases of interest in (5) are $\theta = 0$ where (1) is obtained and $\theta = 1$ where (2) holds.[3]

When $\theta = 0$, (5) becomes the PIGLOG expenditure equation consistent with (1). As in Muellbauer, the aggregate budget share becomes

$$\overline{w}_j = \alpha_j(p) + \beta_j(p) \left[\frac{\sum_h y_h \ln y_h / c_{h_j}}{\sum_h y_h} \right]$$

where the bracketed term may be interpreted as the log of aggregate income. When $\theta = 1$, a PIGLOG expenditure equation is defined by (2). In this case

$$\overline{E}_j = \alpha_j(p) + \beta_j(p)\left[\frac{\sum\limits_{h} y_h \ln y_h/c_{h_j}}{\sum\limits_{h} y_h}\right].$$

Though homogeneity will not be of concern in our empirical study (our data are cross-sectional), the ability to impose homogeneity on (5) should be demonstrated. Because demands are homogeneous of degree zero in p and y_h, $p_j X_{h_j}$ is homogeneous of degree one. Hence, the left side of (5) must be homogeneous of degree $\theta + 1$ in y_h and p. A simple way to impose like homogeneity on the right side of (5) is to set $\alpha_j(p) = -\beta_j(p) \ln\eta_j(p)$, where $\eta_j(p)$ is a positive function. This yields

$$-\beta_j(p) \, y_h \, \ln \eta_j(p) + \beta_j(p) \, y_h \, \ln(y_h/c_{hj})$$

$$+ \gamma_{hj}(p) = \beta_j(p) \, y_h \, [\ln(y_h/c_{hj}) - \ln \eta_j(p)]$$

$$+ \gamma_{hj}(p) = \beta_j(p) \, y_h \, \ln(y_h/c_{hj} \, \eta_j(p))$$

$$+ \gamma_{hj}(p), \ h = 1, \ldots, H.$$

Thus, for arbitrary t > 0, homogeneity requires

$$\beta_j(tP) t y_h \, \ln[t y_h/(c_{hj} \, \eta_j(tP)) + \gamma_{hj}(tP) =$$

$$t^{\theta+1} \{\beta_j(p) \, y_h \, \ln[y_h/(c_{hj} \, \eta_j(p))] + \gamma_{hj}(p)\}.$$

Hence, $\beta_j(tP) = t^\theta \beta_j(P)$, $\eta_j(tP) = t \, \eta_j(P)$ and $\gamma_{hj}(tP) = t^{\theta+1} \gamma_{hj}(p)$ are sufficient to ensure that demand functions corresponding to (4) are positively homogeneous of degree zero in p and y_h.

The macro form corresponding to (5) is (summing and dividing by $\sum\limits_{h=1}^{H} y_h$)

$$\overline{z}_j(y_{oj}, p) = \alpha_j(p) + \beta_j(p) \ln y_{oj}$$

where $\ln y_{oj}$ is the aggregate income index equal to $\sum\limits_{h=1}^{H} y_h \ln(y_h/c_{hj}) / \sum\limits_{h=1}^{H} y_h$. Therefore (5) not only generalizes Muellbauer but retains use of the PIGLOG income index. This index is among the easiest to compute other than merely average income.

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III. Empirical Test

The data are from a random sample of 763 households from the 1985 Consumer Expenditure Survey, U.S. Bureau of Labor Statistics. A rich set of consumer characteristics are available. However, for broad expenditure categories like food, commonly used socio-economic characteristics seem to be empirically unimportant in determining expenditure patterns.[4] However, family size is a salient characteristic and it enters (5) through the individual specific terms γ_{hj} and c_{hj} (see e.g., Lewbel, 1988). Food expenditure and income are mean scaled.

Because this is a cross-sectional analysis, prices are taken as constant across individuals. A preferred approach would have considered cross-sectional price differences (Cox and Wohlgenant) but the data were unavailable. Hence α_j, β_j and γ_{hj} do not vary with respect to prices. Thus, one is left to model the effects of family size fs_h on food expenditure. The specification adopted is $\gamma_{hj}(p) = \gamma_{oj}(p) + \gamma_{1j}(p)fs_h$ and $c_{hj} = fs_h^{c_j}$, $h = 1,\ldots,$ H. Hence, in our cross-section, the specification employed here is:

$$(6) \quad y_h^{\theta_j} \, pX_{hj} = \gamma_{oj} + \gamma_{1j}fs_h + \alpha_j y_h + \beta_j y_h \ln(y_h/fs_h^{c_j}) + \varepsilon_{hj} \quad h=1,\ldots,H,$$

where $E(\varepsilon_{jh}\,\varepsilon_{jh}') = \delta_{hh'}$, where $\delta_{hh'} = \sigma^2$ when $h = h'$ and zero otherwise and $E(\varepsilon_{jh}\,y_h) = 0.$[5] The form in (6) allows family size to affect expenditures in two distinct ways. First, $\gamma_{oj} + \gamma_{1j}fs_h$ allows variations in fs_h to affect average expenditures (an intercept effect) regardless of the income level. Second $c_{hj} = fs_h^{c_j}$ allows the marginal propensity to consume to vary with family size. Because $\sum_h \gamma_{hj} = 0$, $\gamma_{oj} = -\gamma_{1j}\overline{fs}$ where $\overline{fs}$ is average family size (2.6 persons). Thus (6) is estimated in the following form:

$$(6') \quad y_h^{\theta_j} \, p_j X_{h_j} = \gamma_{1j}(fs_h - \overline{fs}) + \alpha_j y_h + B_j y_h \ln(y_h/fs_h^{c_j}) + \varepsilon_{hj}.$$

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Table 1 presents the empirical results of nonlinear least squares estimation using a gradient procedure. Log likelihood values are also reported assuming ε_{hj} is normally distributed. Estimates for the unrestricted model are plausible with food being a normal good. Moreover, in accordance with Engel's Law, the unrestricted budget share declines with income. The unrestricted income elasticity evaluated at mean values of expenditure, income, and family size is .49.

As is apparent from the asymptotic t values in the unrestricted model, the null hypotheses of $\theta = 0$ or $\theta = 1$ are both rejected at traditional confidence levels. Though there is strong evidence for rejecting both the share and expenditure aggregations in (1) and (2), the log likelihood ratio is larger for the expenditure weighted average aggregation rule than for the share weighted average representation. Though the estimated parameters for Muellbauer's model are generally reasonable, the same cannot be said of the weighted average expenditure rule in (2). Note particularly the negative sign for γ: family sizes below the mean have higher average expenditures, _ceteris paribus_. This and the large log-likelihood ratio (or t in the unrestricted model) casts severe doubt on the appropriateness of the weighted expenditure aggregation rule for food expenditure.

Given these results, the aggregation rule empirically supported is the "nearly" weighted average

$$\bar{z}_j(y_{oj};p) = \frac{\sum_h y_h^{.153} p X_{hj}}{\sum_h y_h}$$

where $\ln y_{oj} = \sum_h y_h \ln(y_h/fs^{.711})/\sum_h y_h$ and $\bar{z}_j(y_o,p) = .786 - .439 \ln y_{oj}$.[6]

IV. The Impact of Changing Distribution

The estimates in Table 1 parameterize a coherent aggregation scheme for augmented expenditures. It is interesting to see how changes in the income distribution affect this new definition of aggregate food expenditure. A simple and useful way to do this is to examine the impact of a mean preserving spread in the income distribution using the estimated model.

The estimated augmented expenditure represented by Table 1 is concave in income because $\hat{B}_j = -.439 < 0$. Further, within the relevant data range, the parameter estimates in Table 1 imply that each of the estimated demands is concave in income. This is required by Engel's law. Jensen's inequality thus implies that a mean preserving spread of the income distribution should reduce aggregate demand.

To examine a similar effect on aggregate income, it is useful to note the connection of the PIGLOG income index to Shannon's entropy. Entropy as defined here is:

$$(7) \quad Ent = -\sum_h \lambda_h \log \lambda_h,$$

where $\lambda_h = y_h / \sum_h y_h$. Because $0 \leq \lambda_h \leq 1$ and $\sum_h \lambda_h = 1$, λ_h can be interpreted as the probability that income (normalized) will assume the value $y_h / \Sigma_h y_h$. Entropy is a standard measure of inequality or the variance of income (Shorrocks).

Using entropy (Theil, 1967), the PIGLOG index can be written

$$(8) \quad \ln y_{oj} = -Ent + \ln(\sum_h y_h) - \frac{\sum_h y_h \ln fs_h^c}{\Sigma_h y_h}.$$

When the estimate of the weighting parameter $\hat{\theta}$ is small, the interpretation of $\bar{z}_j$ is closer to an aggregate share than to an aggregate

expenditure. Thus, the interpretation of $\bar{z}_j$ here as an aggregate budget share, though rejected by the data, is more congruent with the value of $\hat{\theta}$ than interpreting $\bar{z}_j$ as an expenditure. Hence, with entropy and family size fixed, increasing mean income increases $\ln(\sum_h y_h)$ and thus reduces $\bar{z}_j$ because $\hat{B}_j(p) < 0$. A mean preserving spread, however, does not alter $\sum_h y_h$. Therefore the second term on the right in (8) is unchanged. The third term is the sum of income when family size is fixed under a mean preserving spread. It is well known that such a spread increases entropy (Theil, 1967). Therefore, $\ln y_{oj}$ falls as dispersion increases, *ceteris paribus*. Because $\bar{z}_j$ is negatively sloped in y_{oj}, a decrease in y_{oj} increases augmented expenditure. Were $\hat{\theta}$ zero, concavity of demand in income implies that Muellbauer's aggregate budget share and weighted expenditures would fall. Given that $\hat{\theta}$ is low we expected similar results. However, the quantitative effects of such an increase in dispersion with a nondegenerate distribution of family size is unclear.

Table 2 presents calculations representing the effects of a mean preserving spread in the income distribution on the predicted aggregate income index, augmented expenditure (3), Muellbauer's budget share (1), and weighted expenditure (2). Estimates from Table 1 are used to predict each quantity. Incomes are scaled with a mean preserving spread factor as in Sandmo.

As expected aggregate income rises with a mean preserving spread. Conversely, the predicted aggregate in (3) and the aggregate budget share in (1) fall as the spread of the income distribution increases, *ceteris paribus*. Finally a mean preserving spread in the distribution of income raises weighted average expenditures. That is, greater inequality in incomes lead to a greater "average" budget share.

A ten percent change in the standard deviation (or mean preserving spread) leads to a five percent change in the log of aggregate income. The mean preserving spread has the largest percentage effect on the aggregate budget share and the smallest effect on augmented expenditures. This is presumably due to the greater nonlinearity in income for both the micro and aggregate quantities. Since the changes for the second through the fifth columns are substantial, ignoring distributional considerations and using only mean income in aggregate work potentially induces substantial errors into the analyses.

V. Conclusion

This paper develops and applies an empirical procedure for discriminating among alternative aggregation rules for food consumption. An important special case, the Muellbauer aggregation rule, is soundly rejected for aggregate U.S. food demand in favor of a more general share weighting approach.

Table 1

Engel Curve Estimates[a]
Food

Parameters	Unrestricted	Restricted	
		$\theta = 0$	$\theta = 1$
	(Equation 6')	(Equation 1)	(Equation 2)
γ	1.040	.056	−.136
	(.023)	(.021)	(.041)
α	.786	.870	.190
	(.057)	(.058)	(.079)
β	−.439	−.522	.240
	(.031)	(.028)	(.045)
θ	.153	—	—
	(.014)		
c	.711	.437	−4.360
	(.128)	(.089)	(.855)
lnL	−793.65	−824.20	−1146.76
Income Elasticity	.49	.56	.43
H	763		

[a]All parameters are as defined in (6'). Reported standard errors use White's heteroskedastically robust covariance estimates. These standard errors do not differ substantively from the conventionally estimated standard errors. Income elasticity is calculated at the sample mean.

Table 2

Aggregate Effects of a Mean Preserving
Spread in the Distribution of Income

MPS Factor[a]	$\ln y_{oj}$[b]	$\bar{z}_j$[c]	$\bar{w}_j$[d]	$\bar{E}_j$[e]
1.0	.72	.94	.83	1.53
1.05	.74	.93	.74	1.56
1.10	.76	.91	.70	1.58
1.15	.79	.90	.63	1.66
1.20	.83	.88	.56	1.71
1.25	.86	.86	.49	1.75
1.30	.89	.85	.45	1.80

[a]Factor by which the data is scaled while preserving the mean. When the factor goes from 1 (the original data) to 1.5, the sample variance more than doubles. Micro incomes are generated by $\bar{y}_h = \bar{y} + \gamma(y_h - \bar{y})$ where γ is the mean preserving spread or MPS factor (e.g., Sandmo, 1970).

[b]Aggregate income (PIGLOG), $\ln y_{oj} = \sum_h y_h \ln(y_h/f_{sh}^{\hat{c}}) / \sum_h y_h$.

[c]Predicted aggregate augmented expenditure from Table 1 using (3) and (5).

[d]Predicted aggregate budget share from Table 1 using (1) and (5).

[e]Predicted aggregate expenditure from Table 1 using (2) and (5).

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Appendix

Let $y_{oj}: R_+^a \to R$, $\bar{z}_j: R \times R_+^m \to R$, $g_h: R_+ \to R$ and $z_{hj}: R_+^{m+1} \to R$ be twice continuously differentiable and strongly monotonic. Let $\bar{z}_j(y_{oj}, p) = \sum_{h=1}^H g_h(y_h)z_{hj} / \sum_{h=1}^H y_h$ be the aggregation rule. Let $t_h = g_h(y_h)z_{hj}$.

Differentiation of $\bar{z}_j$ with respect to y_h yields (dropping the j subscript):

$$(A.1) \qquad \frac{\partial \bar{z}}{\partial y_o}\frac{\partial y_o}{\partial y_h} = \frac{\partial t_h/\partial y_h - \bar{z}}{\sum_h y_h}.$$

Under price independence of y_o,

$$(A.2) \qquad \frac{\partial^2 \bar{z}}{\partial y_o \partial p}\frac{\partial y_o}{\partial y_h} = \frac{1}{\sum_h y_h}\left[\frac{\partial^2 t_h}{\partial y_h \partial p} - \frac{\partial \bar{z}}{\partial p}\right]$$

Hence,

$$(A.3) \qquad \frac{\partial^2 \bar{z}/\partial y_o \partial p}{\partial \bar{z}/\partial y_o} = \frac{\partial^2 t_h/\partial y_h \partial p - \partial \bar{z}/\partial p}{\partial t_h/\partial y_h - \bar{z}}$$

and from (A.1) and (A.3)

$$(A.4) \qquad \frac{\partial^2 \bar{z}/\partial y_o \partial p}{\partial \bar{z}/\partial y_o} = \frac{\partial^2 t_h/\partial y_h \partial p - \partial^2 t_k/\partial y_k \partial p}{\partial t_h/\partial y_h - \partial t_k/\partial y_k}$$

where terms similar to (A.1) and (A.3) are calculated for y_k and subtracted to form the ratio in (A.4). Because the right hand side of (A.4) holds for arbitrary h and k, it must be $\phi(p)$ [or, differentiate (A.4) by $y_j(j \neq k, h)$ to establish that $\partial^2(\ln \partial\bar{z}/\partial y_o)/\partial p \partial y_o = 0$]. Integrating $\partial \ln(\partial\bar{z}/\partial y_o)/\partial p = \phi(p)$ yields

$$(A.5) \qquad \bar{z} = \alpha(p) + \beta(p)\sigma(y_o)$$

as the macro form. This is the PIGL form in Muellbauer where $\sigma(y_o) = \ln y_o$ in the PIGLOG special case of (A.5). Using (A.5) in (A.3) yields (after some algebra)

$$\partial[\ln(\partial t_h/\partial y_h - \alpha(p))]/\partial p = \partial \ln \beta(p)/\partial p$$

and integrating

(A.6) $t_h = \alpha(p)y_h + \beta(p)\sigma_h(y_h) + \gamma_h(p),$ $h=1,\ldots,H.$

From the aggregation rule

$$\sigma(y_o) = \left[\sum_h \sigma_h(y_h)/\sum_h y_h\right] + \left[\sum_h \gamma_h(p)/\beta(p)\right]/\sum_h y_h.$$

Price independence requires that $\sum_h \gamma_h(p) = 0$ and the PIGLOG form is $\sigma(y_o) = \ln y_o$ while $\sigma_h(y_h) = y_h \ln(y_h/c_h).$

1. When derivatives exist, this result is easily seen by differentiating both sides of (1') with respect to y_k. The derivative of the left side depends upon $\bar{y}$ and p. The corresponding derivative of the right side depends on y_k and p. Thus, $p_j \, \partial x_{k_j}/\partial y_k = \beta_j(p)$ and integrating yields the Gorman Polar Form.

2. This aggregation rule is not as convenient as (1) because the data to form (2) (see the middle expression), are not readily available. However, if micro decisions aren't consistent with (1), the convenience associated with (1) is hardly a virtue.

3. There are two other generalizations that are not pursued here. One could reparameterize the micro function such that $z_{hj}(y_h,p) = f_{hj}(y_h^\theta,p)$ and proceed as in the Appendix where the aggregation rule is

$$\bar{z}_j = \sum_{h=1}^{H} y_h^\theta z_{hj} / \sum_{h=1}^{H} y_h^\theta$$

and differentiate and integrate with respect to y_h^j instead of y_h. This will generate a true weighted average for all values of θ but one could not obtain the simple PIGLOG income index. That is, the PIGLOG income index could not necessarily hold. Secondly, one could add a second parameter to the denominator to obtain $\sum_{h=1}^{H} y_n^\tau$. When $\tau = 1$ and $\theta = 0$, one obtains the Gorman rule. When $\tau = 0$ and $\theta = 1$, Muellbauer's rule is obtained and so on. However, this considerably complicates matters. Note that $g_h(y_h)$ could be parameterized as a Box-Cox transformation, $(y_h^\theta - 1)/\theta$. However, this seems unduly nonlinear for our purposes.

4. The random sample was chosen so as to conveniently fit in a modestly configured PC using GAUSS. Repeated random samples indicated that the coefficients were remarkably stable. A preliminary analysis of regions, sex, race, age and other socioeconomic variables led to the conclusion that they were not a critical part of the analysis here. The results with age and region added as in Nichols vary little from those reported here. (Blaylock and Smallwood find only income to be "significant" in explaining food expenditures.) Lewbel included only family size as well.

5. A number of authors have expressed concern about whether (6) can be normally distributed with scale invariant t ratios (e.g., Spitzer, 1984). The conventional maximum likelihood results are reported here. Two stage nonlinear least squares (Amemiya, 1974) estimates are not notably different than those reported here.

6. Note that β would be interpreted as negative when $\bar{z}_j$ is a share and the good is a non-luxury good, and y_{oj} is income.