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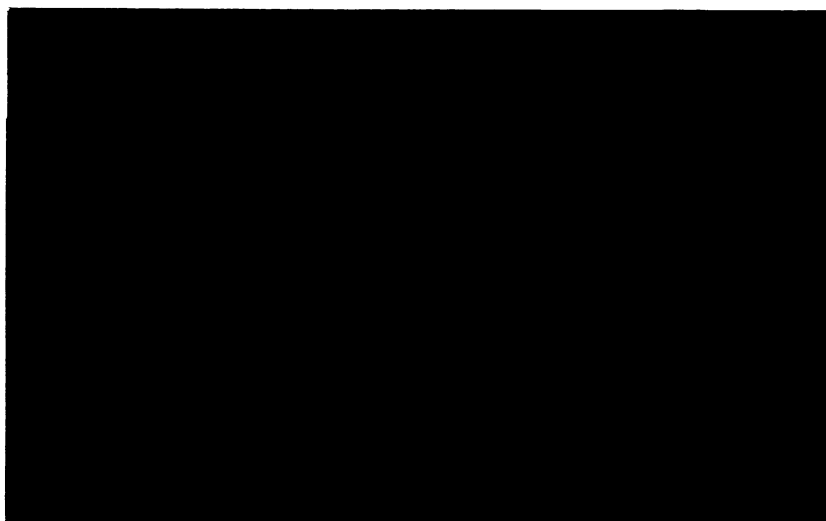
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**THE DEMAND FOR QUALITY
DIFFERENTIATED GOODS: A SYNTHESIS**

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The Demand for Quality Differentiated Goods: A Synthesis

Benefit-cost analyses of public actions are often best construed as studies of the qualities of private goods induced by changes in public goods. Frequently, benefit-cost analysis of a natural resource or environmental policy leads to an analysis of some quality dimension of a resource, such as visibility, water pollution, fish catch, etc., which is simultaneously a quality characteristic of a related service, produced by the household and consumed privately.

The degree to which a market separates public actions from private consumption decisions is critical to the analysis of the welfare effects of quality changes. The familiar hedonic model of Rosen is appropriate when public actions change one or more characteristics of a good traded on a market. This model works well for housing and labor services, where the market clearing price reflects the characteristics of goods. Some goods whose characteristics are affected by public actions are consumed privately but not traded on the market. These goods, such as those associated with environmental quality characteristics, will not have prices which reflect higher quality units of the good. Price will not serve as the rationing agent and does not provide signals to producers to change the quality of goods.

The quality differentiated goods model has served as the common basis for valuing changes in public actions associated with non-market goods. For example, studies such as Smith and Desvousges, Binkley and Hanemann, and Feenberg and Mills implicitly rely on this model in valuing changes in environmental quality. These applications are based on elements of theoretical models by Bradford and Hildebrandt, by Willig, and by Mäler.

Yet, a consistent theory of welfare measurement for changes in the quality of goods has not been articulated. Economists think of welfare measures as synonymous with areas under demand curves, a correspondence well established for the welfare effects of price

changes. However, the absence of a similar correspondence for quality changes has led to confusion in the literature on the measurement of these welfare effects.

In this paper, we derive and illustrate the rather unexpected comparative statics result that, under plausible assumptions about preferences, one can sign the Marshallian but not the Hicksian quality effect on demand. We use these results to explore the conditions for welfare measurement of quality changes for goods which are traded without market clearing prices. We develop the conditions when exact measures can be derived from Marshallian demand functions, and we show that the presence of a condition established by Willig (1978) determines when the numerical approach of Vartia can be used.

The Quality Differentiated Goods Model

In the quality differentiated goods model the quantity of a public good that exogenously enters the individual's decision problem is treated as the quality of a privately consumed good. We begin with the individual's decision problem:

$$(1) \quad \max u(x,b) + \lambda(y-p'x)$$

and the solution of the first order conditions:

$$x^m = f(p,y,b)$$

which gives the Marshallian demands. In the general case, x is a vector of goods, p is a corresponding vector of prices, b is a matrix of quality characteristics associated with the goods, y is income, and λ is a Lagrangian multiplier. In most applications of resource economics, market prices for the goods of interest do not exist. Nonetheless we assume here that the marginal costs (p) of obtaining these goods are constant and can be treated as parametric prices by both consumers and researchers.

The cost minimization problem associated with (1) is

$$(2) \quad \min p'x + \mu(u-u(x,b)),$$

which includes in its solution the compensated demands,

$$x^h = g(p, b, u).$$

Substituting compensated demands into the minimization problem gives the expenditure function $m(p, b, u)$ where

$$(3) \quad m(p, b, v(p, b, y)) = y$$

and inversion gives the indirect utility function

$$(4) \quad v(p, b, m(p, b, u)) = u.$$

In the remainder of the paper, we deal with changes in a single dimension of quality, b , associated with a single x_i . This quality characteristic is assumed to enter the preference structure such that $\partial u / \partial b > 0$ which implies $\partial v / \partial b > 0$ and $\partial m / \partial b < 0$. The structure of the quality-differentiated goods model implies that the quality characteristic of interest is attached to goods. A classic example is the original application by Stevens: the good is salmon fishing trips on the Columbia River and the quality characteristic is the expected catch of salmon on those trips. Examples of other such relations include: trips to a beach and the secchi disk reading of water quality at the beach; water from a municipal water system and clarity of the water. The quality characteristics are exogenous to the individual and thus appear as parameters in the utility maximization problem. Such basic characteristics adhere to most services provided by the public sector.

The Comparative Statics of Quality Change

Customary thinking about the decomposition of the Marshallian effect of a price change into an income and substitution effect is so habitual that we often fail to recognize the narrow circumstances that yield this particular comparative static result. In this section we explore the comparative statics of changes in exogenous quality and find quite different results from the price change case.

To find the Marshallian effect of a change in b , differentiate the first order conditions arising from the maximization problem in (1):

$$(5) \quad \begin{bmatrix} \partial \mathbf{x}^m / \partial b \\ \partial \lambda / \partial b \end{bmatrix} = \begin{bmatrix} u_{ij} & -p \\ -p' & 0 \end{bmatrix}^{-1} \begin{bmatrix} u_{jb} \\ 0 \end{bmatrix}$$

where $u_{ij} = \partial^2 u / \partial x_i \partial x_j$ and $u_{jb} = \partial^2 u / \partial x_j \partial b$. The first matrix on the right hand side with the inclusion of the multiplicative factor, $-\lambda$, is the standard comparative statics matrix for a price change. Denote c_{ij} as the Hicksian price effect of a change in p_j on x_i . The quality effect can be written as

$$\frac{\partial x_i^m}{\partial b} = \sum_j -\frac{c_{ij} u_{jb}}{\lambda},$$

The term c_{ii} must be negative, because it is the Hicksian own price effect. The term c_{ij} will be positive/negative respectively as x_j is a substitute/complement to good x_i .

The signs of the cross partials of utility with respect to goods and quality, i.e. the u_{jb} , cannot be determined by axioms on the structure of preferences. However reasonable assumptions provide the basis of a few important results. Let x_1 be the good with which b is associated. Then it seems reasonable to assume that $u_{1b} > 0$ if b is a desirable characteristic. An additional unit of x_1 is valued more highly if it is of a higher quality. This rules out, for example, the case where quality and quantity are perfect substitutes.

With regard to the cross partials of utility with respect to quality and other goods, x_j , $j \neq 1$, let us first assume that $u_{jb} = 0$. The effect of a quality change on good 1 is clear:

$$\frac{\partial x_1^m}{\partial b} = -\frac{c_{11} u_{1b}}{\lambda},$$

which is positive.¹ The effect of a change in b on the demand for other goods will equal

$$\frac{\partial x_j^m}{\partial b} = -\frac{c_{1j}u_{1b}}{\lambda},$$

the sign of which is determined by the sign of $-c_{1j}$. It will be negative (positive) as goods x_1 and x_j are substitutes (complements).

More generally, one could assume that if b has any effect at all on the marginal utility of good j , an increase in b will likely decrease (increase) the marginal utility of good x_j if x_j is a net substitute (complement) to x_1 . With this plausible and less restrictive assumption, we can, at least, unambiguously sign the Marshallian own quality effect. The term $\partial x_1^m / \partial b = \sum (-c_{1j} / \lambda) u_{jb} > 0$, because $\lambda > 0$ by construction and the product of c_{1j} and u_{jb} will always be negative.²

The graphical treatment of the two good case in Figure 1 is illustrative. The original utility level, U^0 , can be reached with bundles that contain less x_1 , after x_1 's quality has been increased from b^0 to b^1 . At any point in $[x_1, x_2]$ space, the slope of the indifference curve will now be steeper. As a result, the consumer will increase purchases of x_1 to return to equilibrium, i.e. a tangency point with the initial income constraint.

The Hicksian quality effect can be compared to the Marshallian effect using duality results. For $y = m(p, b, u)$,

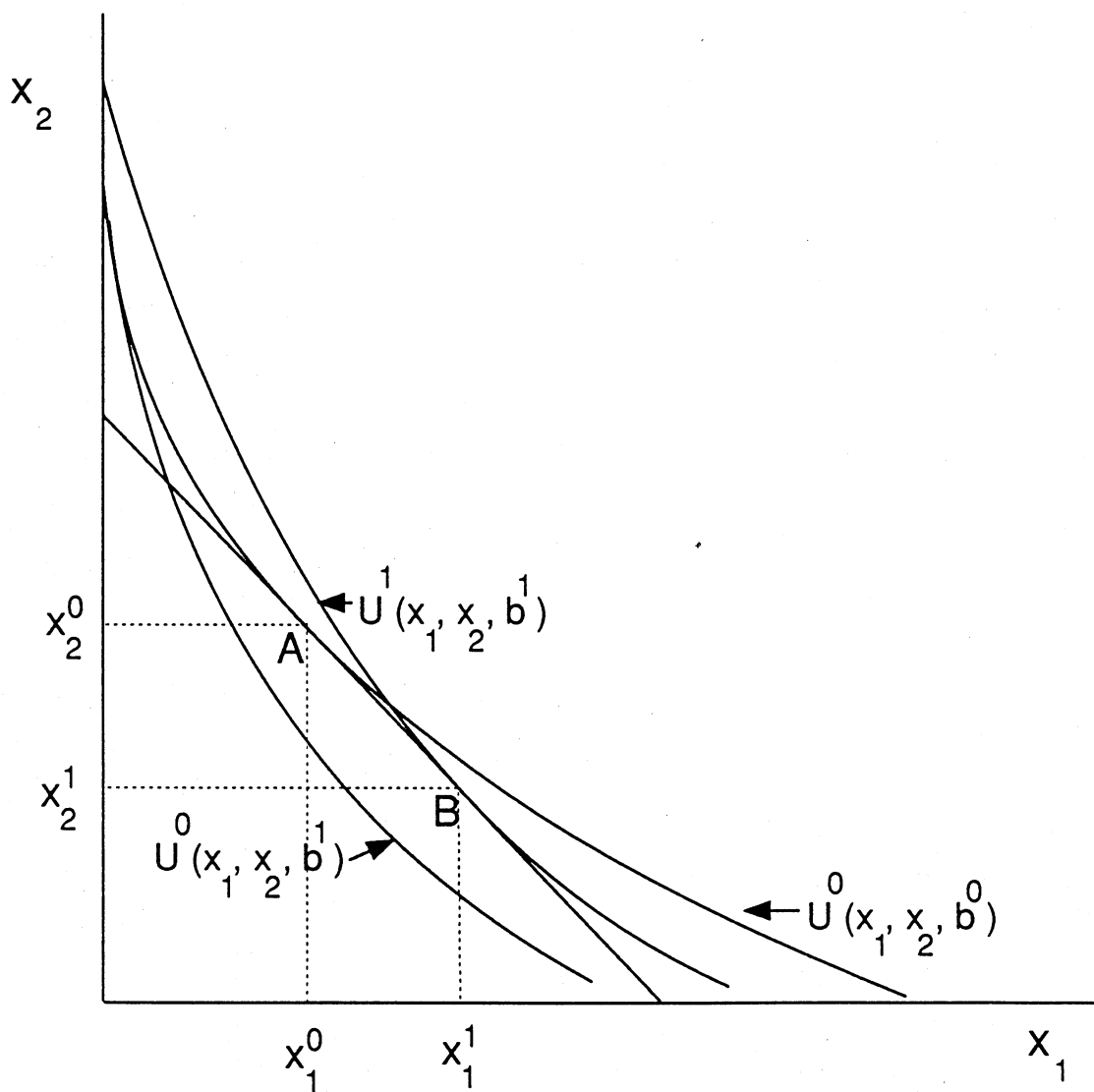
$$x_1^m(p, b, m(p, b, u)) = x_1^h(p, b, u).$$

Differentiating with respect to b gives

$$\partial x_1^h(p, b, u) / \partial b = \partial x_1^m / \partial b + (\partial x_1^m / \partial y) \partial m(p, b, u) / \partial b.$$

The first term on the right is the positive Marshallian effect. But the Hicksian response has an additional term that incorporates the income effect. Since $\partial x_1^m / \partial b$ is positive and $\partial m / \partial b$ is negative, the sign of $\partial x_1^h / \partial b$ can be guaranteed to be positive only if the income effect is negative. If the good is normal, the Hicksian response to a quality change will be smaller than the Marshallian response - and could potentially be negative.

Figure 1



This result, too, can be depicted graphically for the two good case. After the quality change in x_1 , the consumer purchasing the original bundle of goods (A in Figure 2) is out of "Hicksian equilibrium" in two ways: the level of utility is higher than the base utility level, U^0 , and the ratio of marginal utilities does not equal the price ratio. To return to the base utility level, the consumption of both goods will tend to be reduced. To correct the price ratio disequilibrium, x_1 must be increased relative to x_2 . At the new equilibrium (C in Figure 2), the change in x_1 is positive when the substitution effect is stronger than the income effect, or when the income effect for x_1 is negative. The graph is drawn for a positive income effect, sufficiently large to produce a decrease in the equilibrium consumption of x_1 .

The parallel between the effects of price changes and quality changes is striking; the own price slope is unambiguously signed for Hicksian demand functions while the Marshallian effect (under reasonable assumptions about preferences) is similarly unambiguous for quality changes. Likewise, the sign and size of the income effect influences the sign of the Hicksian quality effect but in a Marshallian world it influences the price effect. The reason for this reversal is, of course, that price changes alter the budget constraint but quality changes shift indifference curves, leaving the budget constraint unaltered.

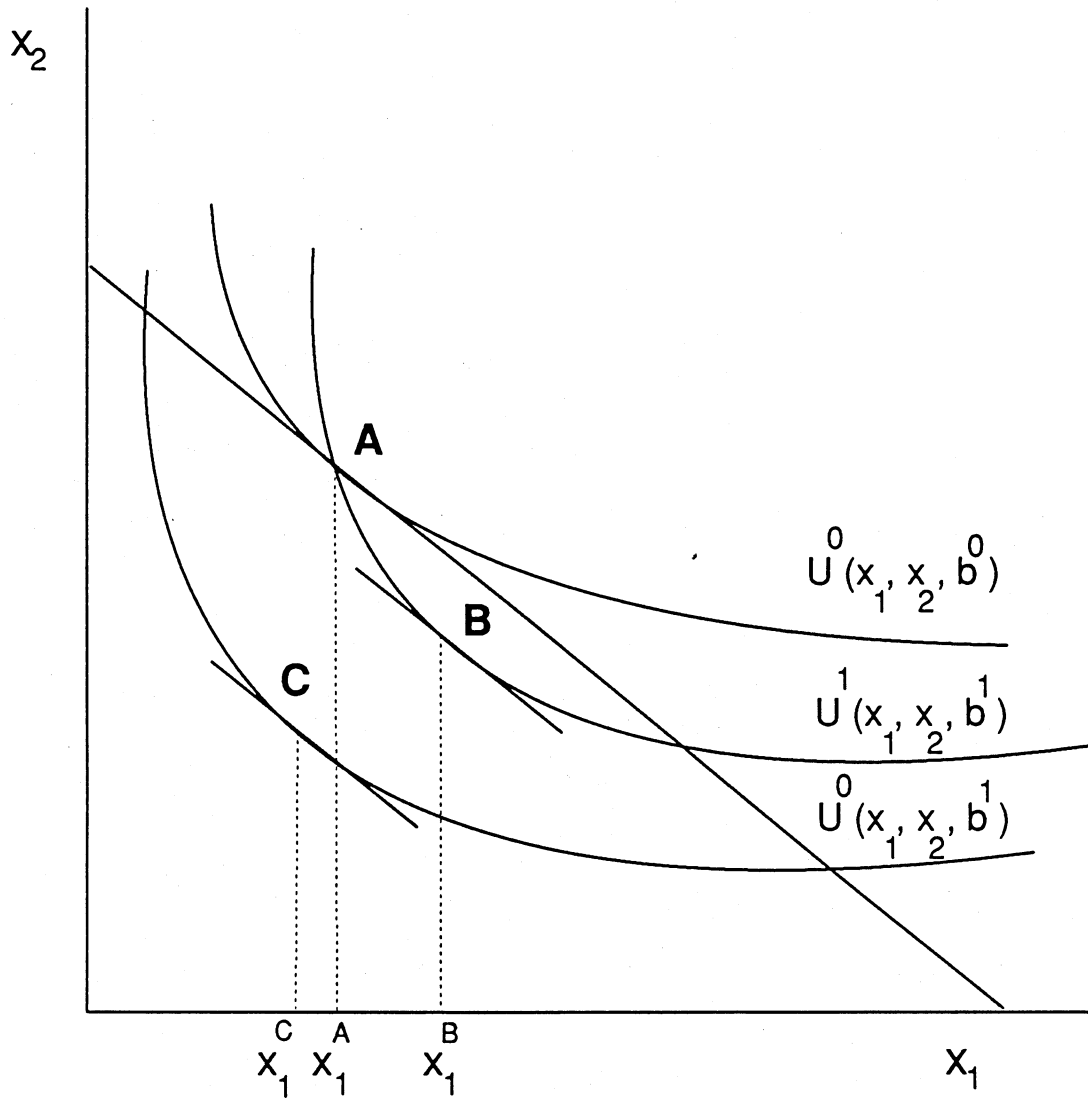
Welfare Measures of Quality Change in a Hicksian World

These comparative statics results are interesting in their own right but lead to equally interesting welfare theory results. For price changes, the Slutsky equation is the culmination of comparative statics analysis and the beginning of welfare analysis, because it describes how the Hicksian and Marshallian demand functions diverge from their common point as price changes. The corresponding results for quality changes are more difficult to establish.

As long as quality is a parameter to the individual, the compensating variation measure of the welfare effect of a quality change for good x_1 is defined as

$$(6) \quad CV(\Delta b) = m(p_1^0, b^0, \hat{p}, u^0) - m(p_1^0, b^1, \hat{p}, u^0)$$

Figure 2



where b is the quality characteristic of good x_1 and \hat{p} is a vector of the prices of all other goods.³

If we were evaluating a price change for x_1 , we could draw on the fortuitous envelope result that m_p equals compensated demand. In order to construct a somewhat analogous story for quality, an artificial problem must be formulated. Define a marginal value function for quality by the following scheme (suggested by Hanemann). First

$$(7) \quad \min \pi^h b + p'x + \mu(u^0 - u(x, b))$$

with respect to x and b , as if there were an exogenous price, π^h , for the quality characteristic and b were endogenously determined as is x . But then solve the first order conditions for

$$x = g(p, b, u^0) \text{ and } \pi^h = \pi^h(p, b, u^0).$$

The π^h function is simply another name for $-m_b$, since from (2) m_b equals $-\mu u_b$, and from (7), first order conditions require that $\pi^h - \mu u_b = 0$.

If such a function were observable, the welfare measure in (6) could be obtained by integrating π^h over the change in b . Except for the fact that the integration would be over a quantity rather than a price change, this procedure resembles traditional welfare analysis.

In a world consistent with the quality differentiated goods story, π^h functions do not represent observable behavior. This dilemma has long been noted and a solution offered by Mäler, who suggested the following, which has become the conventional wisdom. Take the difference between the Hicksian demand function for good x_1 evaluated at b^0 and integrated over price, and the same integral where the demand function is evaluated at b^1 :

$$(8) \quad \int_{\bar{p}_1(b^1)}^{\bar{p}_1(b^0)} g(p, b^1, u^0) dp_1 - \int_{\bar{p}_1(b^0)}^{\bar{p}_1(b^1)} g(p, b^0, u^0) dp_1,$$

where \bar{p}_1 is a price high enough to cause demand to fall to zero and $u^0 = v(p^0, b^0, y)$. The intuitive rationale for this procedure is the following: the first term is the value of access to

good x_1 conditioned on a level of quality b^1 and the second, the value of access conditioned on quality b^0 , so that (8) is the change in the value of access due to a change in b , conditioned on the original utility level.

Performing the integration in (8) yields the equivalent expression

$$(9) \quad m(\tilde{p}_1(b^1), \hat{p}, b^1, u^0) - m(p_1^0, \hat{p}, b^1, u^0) - m(\tilde{p}_1(b^0), \hat{p}, b^0, u^0) + m(p_1^0, \hat{p}, b^0, u^0).$$

The common procedure of measuring the change in the area behind the Hicksian demand function with a change in quality yields the correct measure if the first and third terms in (9) together equal zero. It is well known that this depends on two properties: non-essentiality and weak complementarity.

A good is nonessential if combinations of other goods can be found that will compensate the individual for its complete absence (see Willig, 1978). If x_1 is nonessential, for example, then there exists a vector of x_j^1 's, $j > 1$, such that

$$u(x_1^0, x_2^0, \dots, x_n^0, b) = u(0, x_2^1, \dots, x_n^1, b)$$

where the x_j^0 's are the utility maximizing demands given a set of prices and income.⁴

It is clear that a good is nonessential if and only if the area inside its compensated demand curve is finite. The notion of nonessentiality turns on the ability to compensate an individual completely for the loss of access to x , which means that the definition is couched in terms of compensation, not behavior. If the good is nonessential and non-inferior, its Marshallian consumer surplus must be finite, since the Marshallian demand for non-inferior goods lies everywhere inside the Hicksian demand as price is increased.⁵

An assumption of nonessentiality is reasonable for quality differentiated goods, each one of which may have many close substitutes, and for environmental goods, many of which are recreational goods. Nonetheless it has implications both for the theoretical characterization of the individual's decision problem and for its empirical estimation. It is not enough simply to choose a functional form which has a choke price. Once we correctly

acknowledge the nonessentiality of some environmentally related good, we must allow for demands for this activity to equal zero. That is, interior solutions will not always prevail, and classical optimization no longer provides an easy means of characterizing decisions. Expenditure functions, indirect utility functions, and demand functions will be continuous only over those ranges of prices and income that cause the set of goods to be consumed at non-zero levels.

The second concept of interest, weak complementarity, was originally defined by Mäler to show the circumstances in which changes in the area under compensated demand curves capture the value of a change in a public good. The nature of weak complementarity has been completely characterized by Willig.⁶ If b is weakly complementary to x_1 then

$$\partial u(0, \hat{x}, b) / \partial b = 0$$

where \hat{x} is the set of other goods, x_2, \dots, x_n . A change in b does not affect an individual's utility if the individual consumes no x_1 . Under weak complementarity,

$$m(\bar{p}_1(b^1), \hat{p}, b^1, u^0) = m(\bar{p}_1(b^0), \hat{p}, b^0, u^0)$$

and the change in (8) equals the compensating variation in (6).⁷

Weak complementarity might appear to suggest that if an individual is not consuming x , then b does not matter, and no change in b can cause him to change his behavior. But a discrete improvement in b can cause the individual, when maximizing utility in the new context, to choose a positive value for x when previously he consumed none. The Hicksian demand curve $g(p, b, u)$ shifts out as quality increases and the finite limit price $\bar{p}_1(b)$ increases with the quality of the good.⁸ When preferences are consistent with weak complementarity, there are empirical implications for behavior because weak complementarity partially embodies a theory of participation.

If non-essentiality and weak complementarity are reasonable assumptions then we can use the area between the Hicksian demand curves as a measure of compensating variation. This result together with the comparative statics results of the previous section suggest an interesting result. Earlier we found that for normal goods with sufficiently positive income effects, $\partial x^h / \partial b$ can be negative, and neither weak complementary nor non-essentiality preclude this possibility. Thus, if $\int x^h(p, b^1, u^0) dp - \int x^h(p, b^0, u^0) dp$ is to measure the positive increase in welfare even when $x^h(p, b^1, u) < x^h(p, b^0, u)$, it must be true that the two Hicksian demand functions cross at a price below the original Hicksian choke price. These results have implications for the functional forms of demand specifications that can usefully be estimated in empirical work. For example, the price and quality effects cannot be additively separable in the Hicksian demand functions.

In a Hicksian world, the complications introduced by the concept of "quality" into welfare analysis come about because we cannot measure welfare in the "market" for the quality characteristic itself and must turn to a secondary market that is related in a specific way to the quality characteristic. The conditions for welfare measurement depend, as a consequence, on the existence of an appropriately related secondary market. We will see that further restrictions on behavior in this secondary market are necessary to obtain welfare measures from Marshallian functions.

Using Marshallian Demands to Measure the Value of Quality Changes

Now suppose only Marshallian functions are available, as is usually the case. Reasoning from an understanding of welfare measures of price changes, one would doubt whether "exact" measures of welfare could be obtained from ordinary demand curves unless income effects were zero. But one might expect to be able to draw on the well-known results of Willig (1976) to show that areas under ordinary curves will be close approximations to the compensated measures even for quality changes.

This latter contention is widely held but false. The usual argument is the following: when weak complementarity and non-essentiality hold, the welfare measure of a quality change can be expressed as in equation (8). This is the difference between two terms, each of which is the integral of the compensated demand function over a price range. Since Willig has a) shown the conditions under which such areas will be closely approximated by integrals of ordinary demand functions, and b) defined the error bounds on the estimates of compensating and equivalent variation derived from consumer surplus and income elasticity measures, why should these same results not hold when applied to each term in (8)?

The fault in the logic can be revealed through the mathematics. If one uses the Marshallian instead of the Hicksian demand function in (8), the error of approximation can be stated as:

$$\int_{p^0}^{\tilde{p}} (x^m(p, b^1, y) - x^m(p, b^0, y)) dp - \int_{p^0}^{\tilde{p}} (x^h(p, b^1, u^0) - x^h(p, b^0, u^0)) dp$$

where $u^0 = v(p, b^0, y)$; \tilde{p} denotes a price high enough such that all relevant demands equal zero. The integral of x over ranges of prices higher than the respective choke price is defined as zero over that range.

In the above, the first integral equals the consumer surplus measure and the second the Hicksian measure. Rearranging terms gives:

$$\int_{p^0}^{\tilde{p}} (x^m(p, b^1, y) - x^h(p, b^1, u^0)) dp - \int_{p^0}^{\tilde{p}} (x^m(p, b^0, y) - x^h(p, b^0, u^0)) dp.$$

Each integral appears to involve a straightforward application of the Willig bounds principle, but in fact only the second integral can be assessed in this way. For the second integral, $x^m(p^0, b^0, y) = x^h(p^0, b^0, u^0)$ at the lower bound of integration. However, this is not true for

the first integral, since $x^m(p^0, b^1, y)$ will not generally equal $x^h(p^0, b^1, u^0)$ for $u^0 = v(p^0, b^0, y)$. From the comparative statics results, $\partial x^m / \partial b > \partial x^h / \partial b$ so that $x^m(p, b^1, y) > x^h(p, b^1, u^0)$. The two new demand functions, $x^m(b^1)$ and $x^h(b^1)$, can cross at p^0 only if $\partial x^m / \partial y = 0$ (zero income effects) or $m_b = 0$ (a quality change does not matter). In the case of zero income effects, compensated and ordinary demands coincide so that $x^h(b^0)$ and $x^m(b^0)$ are identical and $x^h(b^1)$ and $x^m(b^1)$ are identical. When quality has no effect, $x^h(b^0)$ and $x^h(b^1)$ coincide as do $x^m(b^0)$ and $x^m(b^1)$. In both cases the compensating variation of a quality change equals the equivalent variation and the consumer surplus of the change. In the latter case this is true because all are zero.

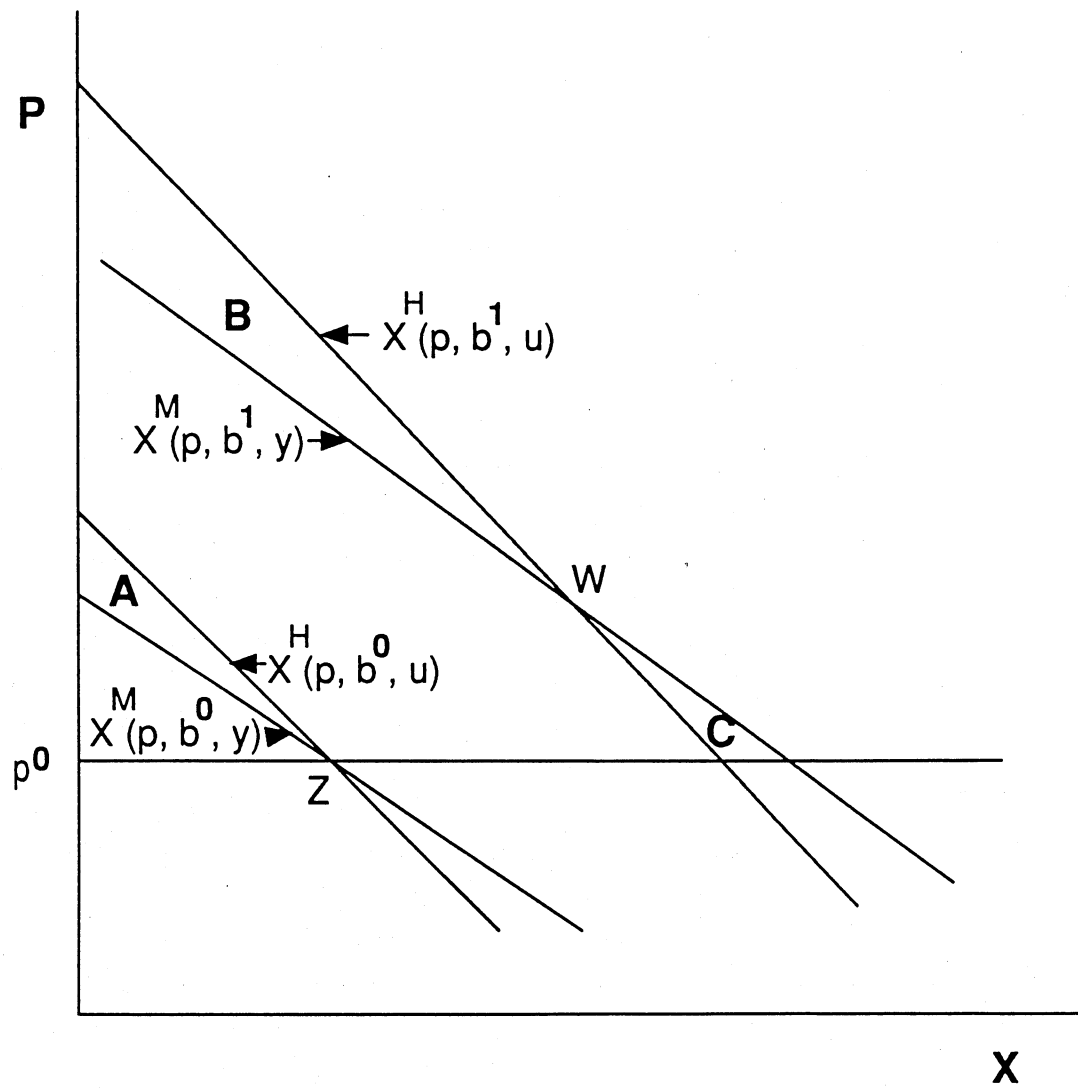
Figure 3 is drawn to represent one possible configuration of Hicksian and Marshallian demand functions. The difference between the consumer surplus and compensating variation measures of the quality change is given by areas A+C-B. In general, there would appear no way to determine the sign of this difference. In fact, it would seem possible for CS to equal CV even when income effects are not zero, but in such a case EV would not be equal to CS and CV.

To gain insight into the relationships in Figure 3, define a new function - the marginal value function for the quality characteristic, π^m . To do this, maximize the following with respect to x and b , giving back the consumer his expenditures on b , because b is actually unpriced in the market:

$$\max_{x, b} u(x, b) + \lambda (y^* - p'x - \pi^m b)$$

where $y^* = (y^0 + \pi^m b)$. This yields first order conditions $\partial u / \partial x - \lambda p = 0$, $\partial u / \partial b - \lambda \pi^m = 0$, and $y^0 - p'x = 0$, whose solution includes the same ordinary demands for the x 's, $x^m = h(p, b, y)$, and a marginal value function for quality, $\pi^m = \pi^m(p, b, y)$.

Figure 3



Note that

$$\pi^m = u_b/\lambda$$

which implies that

$$(10) \quad \pi^m = v_b/v_y,$$

since, by the envelope theorem, $v_b = u_b$ and $v_y = \lambda$.

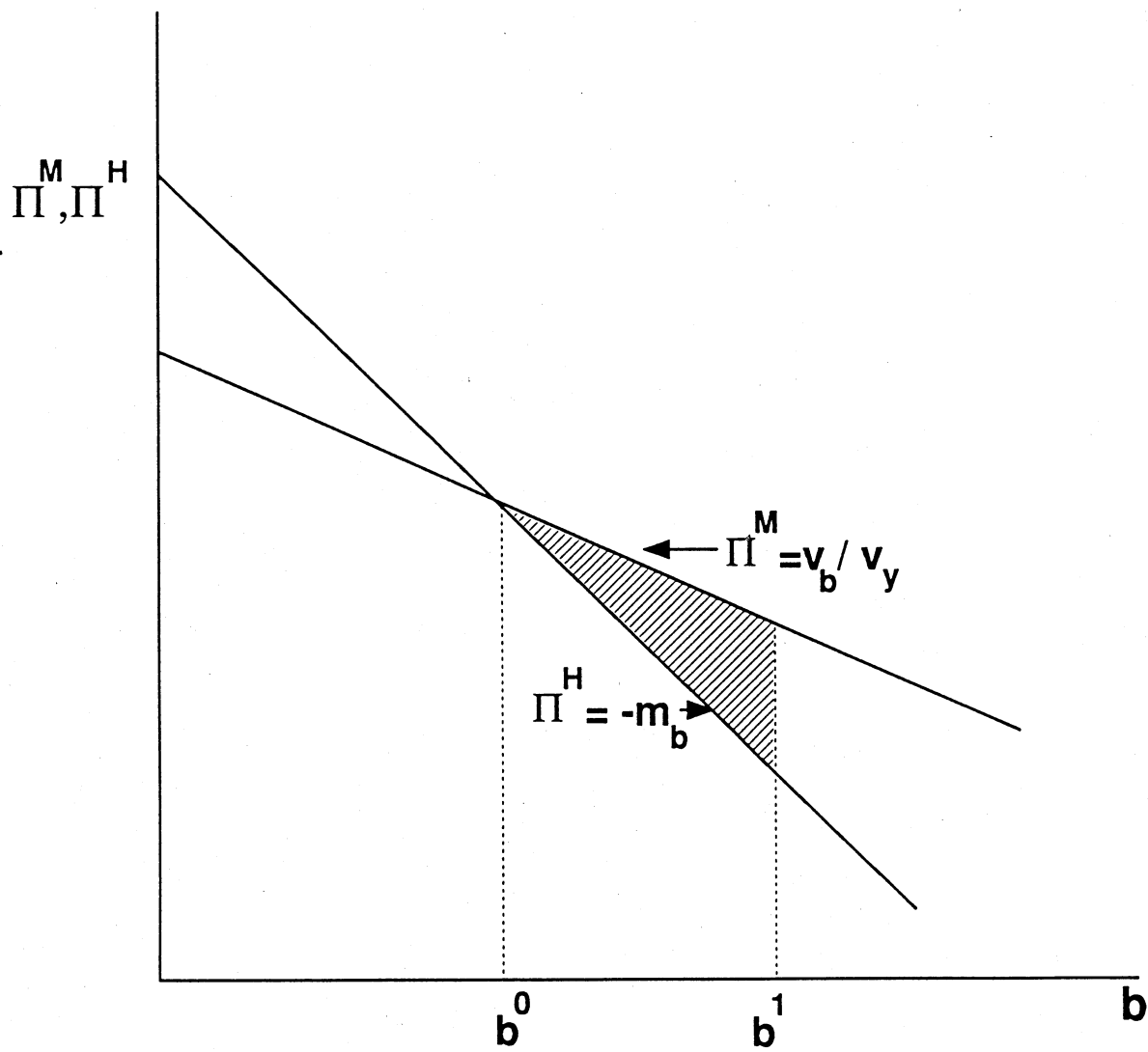
The relationship between π^h and π^m is completely analogous to the relationship between any pair of inverse Hicksian and Marshallian demand functions. The functions are drawn in Figure 4. As noted before, the only unusual feature is that welfare measures must be obtained by integrating over a quantity dimension (b) rather than a price dimension (π), since it is quantity that is exogenously determined.

The task of welfare measurement would be simple if the function π^m were observable. In an extension of the Willig argument, Randall and Stoll showed that the exact welfare effects of exogenous quantity changes can be approximated by areas under Marshallian schedules, and that errors of approximation are bounded by functions of relative price flexibilities (specifically, the income elasticity of relative marginal values.) This same argument could be used to show that the area under π^m is a good approximation of the exact welfare measure required here.

If these functions in quality $[b, \pi]$ space were observable, Willig's bounds - or more appropriately Randall and Stoll's generalization of Willig's bounds to the case of quantity rather than price changes - could be called upon to determine how close an approximation the Marshallian measure would be to the compensated one.

But quality space is not observable. We have already seen that to obtain a welfare measure equivalent to the Hicksian measure in quality space $[b, \pi^h]$ we must resort to a measure in a related goods space $[x, p]$, where x is nonessential and b is weakly complementary to x . What remains is to determine whether the areas under corresponding

Figure 4



Hicksian and Marshallian functions in goods space $[x, p]$ are approximately equal. That is, does

$$(11) \quad \int_{p^0}^{\tilde{p}} x^m(b^1) dp - \int_{p^0}^{\tilde{p}} x^m(b^0) dp$$

approximate

$$(12) \quad \int_{p^0}^{\tilde{p}} x^h(p, b^1, u^0) dp - \int_{p^0}^{\tilde{p}} x^h(p, b^0, u^0) dp ?$$

Clearly if expression (11) equals $\int \pi^m db$, then the Randall and Stoll results are equally applicable to the problem whether phrased in terms of $[b, \pi]$ space or $[x, p]$ space, since it has already been demonstrated that if weak complementarity holds

$$(13) \quad \int_{b^0}^{b^1} m_b(p, b, u^0) db = \int_{p^0}^{\tilde{p}} x^h(p, b^1, u^0) dp - \int_{p^0}^{\tilde{p}} x^h(p, b^0, u^0) dp.$$

From (10) π^m and v_b/v_y are synonymous, which implies $\int \pi^m db = \int v_b/v_y db$. This is useful because in a 1978 paper Willig asked the question: when is the change in the Marshallian money measure of quality equal to the change in the consumer's surplus for the quality-differentiated good? In our notation, this is equivalent to asking: when does the following equality hold?

$$(14) \quad v_b/v_y(p^0) = \int_{p^0}^{\bar{p}} \partial x^m / \partial b \, dp$$

If (14) holds, then the area between two Marshallian demands for x (eq. 11) equals $\int \pi^m db$, the exact equivalence sought. Willig showed that equation (14) will hold if and only if any of the following hold:

- a) v_b/v_p is independent of income;
- b) the average incremental consumer surplus, $\int x_b^m dp/x$, is independent of income;
- c) $v_b/v_p = \int x_b^m dp / x$.

All of these are simply mathematical equivalences to (14), and none provide much intuitive appeal. But looking back at (14), substituting π^m for v_b/v_y and differentiating both sides with respect to p , yields:

$$(15) \quad \partial \pi^m / \partial p = \partial x^m / \partial b.$$

When (15) holds, the usually unobservable change in consumer's surplus associated with a quality characteristic equals the change in the consumer's surplus brought about by that quality change for a related good, i.e. the Marshallian measure of welfare in quality space $[b, \pi]$ and goods space $[x, p]$ are identical. And, at the point, this is also the quality-induced change in the expenditure function.

The symmetry in (15) is intuitively appealing. Expression (15) states that at the equilibrium, the effect of a price change on the marginal price of quality equals the effect of quality on the quantity of the good. There is an analogy with homothetic preferences and path independence. When preferences are homothetic, one may take any path of price changes and calculate the same welfare measure. In the case of quality, the Willig condition makes it possible to use the change in consumer's surplus as a measure of welfare change, and this change will be unique.

The Role of the Willig Condition

The Willig condition shows when the incremental consumer surplus equals the marginal value of quality measured from the expenditure function. When the Willig condition holds, then the value of a change in quality measured by a change in consumer surplus will be bounded by the equivalent and compensating variation from a quality change. That is, the Willig condition gives the same kind of bounding results that exist for price changes. Since CV and EV differ only to the extent that changes in b change utility, this condition is quite valuable.

All of this is important only if the Hicksian function is not available to us. One might reasonably argue for integrating back to the Hicksian from the Marshallian function and thus avoiding all the trouble suggested by the previous discussion. Since analytical integration is not always possible, numerical integration as suggested by Vartia is commonly employed. But here again the Willig condition comes into play.

Consider the numerical approach. The algorithm of Vartia requires one to start at the point where the original Marshallian intersects the Hicksian, point z in Figure 3. The algorithm accumulates compensation as price increases, numerically finding the compensating variation of each small price step. When the Marshallian demand shifts, the new Marshallian and the appropriate Hicksian cross at some unknown point depicted as w in Figure 3. To apply the numerical integration techniques of Vartia, one needs some means of identifying the new intersection point. The Willig condition provides such a means.

Specifically, we need to know how the Hicksian shifts with quality:

$$(16) \quad \frac{\partial x^h}{\partial b} = \frac{\partial x^m}{\partial b} + \frac{\partial x^m}{\partial y} m_b.$$

Usually $\partial x^m/\partial b$ and $\partial x^m/\partial y$ can be estimated but m_b is unobservable. The Willig condition implies that

$$(17) \quad m_b = - \int_p^{\bar{p}} x_b^m(p, b, u) dp,$$

for which the right hand side, incremental consumer surplus, is observable. Consequently the Willig condition permits the use of Vartia's numerically exact results in integrating back, because it defines the difference between $\partial x^m/\partial b$ and $\partial x^h/\partial b$, and hence the intersection point of the new Marshallian and Hicksian curves. To use the condition in (16) to calculate the new level of x^h , one needs simply to take the definite integral of both sides from b^0 to b^1 . This yields

$$(18) \quad x_1^h = x_1^m - \int_{b^0}^{b^1} x_y^m(b) \int_p^{\bar{p}} x_b^m(p, b) dp db.$$

In practice, this can be accomplished with finite changes in b .⁹ Expression (18) describes how far apart the functions are at the current price. It can be recalculated at higher prices until $x^h(p^1, b^1) = x^m(p^1, b^1)$, the starting point for the Vartia numerical analysis.

This calculation points to the intuition behind Marshallian and Hicksian deviations. As quality is changed, the expenditure function measures the increase in well being and is given in (17). To keep utility constant, money income must be reduced at the rate given by m_b , which is approximated by $\int x_b(p, b, y) dp$ when the Willig condition holds. To stay on the original Hicksian demand curve, the Marshallian is adjusted at the appropriate rate in response to an income change, x_y .

When the Willig condition does not hold, there are two consequences. First, the consumer surplus change from a quality change is not bounded by the equivalent and compensating variation of a quality change. Second, the equality (17) cannot be used to help determine the new Hicksian demand curve. That is, the new level of x^h cannot be found using the relationship (18) because the facilitating relationship $m_b = - \int x_b^m dp$ no longer

holds. We then have to rely on the more basic intuition that small income effects from small quality changes will produce changes in consumer surplus which are 'close' to compensating and equivalent variation. We have no guarantee of this, however.

Nature has been benevolent with regard to violations of the Willig condition. Most popular functional forms which meet the non essentiality and weak complementarity conditions also satisfy the Willig condition. Specifying a demand function which satisfies the other conditions but fails the Willig condition requires a function so rich in parameters that it would be hopeless to estimate.

Conclusion

For purposes of benefit-cost analysis of public projects, economists have found it useful to model changes in public goods as changes in the quality of privately consumed goods which are weakly complementary to the public goods. The private goods are quality-differentiated goods, where different levels of the public good yield different qualities of the private good. With this model, economists have measured the benefits of changes in public goods as the change in the area under the demand for the complementary private good.

While economists have pursued this strategy for some time, the theory behind it has not been fully articulated. Intuitive ideas about the welfare effects of price changes have driven research in the valuation of nonmarket goods. But these intuitive ideas can be misleading when quality changes. Comparative statics show that Hicksian and Marshallian demand functions shift in opposite ways for quality changes when compared with price changes. When the welfare effects are defined in a logically consistent way via the expenditure function, we can establish a correspondence between the Hicksian compensating variation of a change in the public good (or equivalently, quality of the private good) and the change in the private good's compensating variation brought about by the quality change. But it has usually been assumed that a similar correspondence exists between (Marshallian)

consumer surpluses. To justify this, economists have relied on Willig's bounds for the price change case and proceeded accordingly. But, this is wrong on two counts. First, although trivial, Willig's bounds refer to situations when quantity is chosen and finite price changes are being valued. But here, the relevant integration is over the entire range of prices so as implicitly to drive the individual out of the market. The relevant bounds are given by Randall and Stoll in an extension of Willig's work - price change bounds to quantity space.

More important, neither Willig's bounds nor Randall and Stoll's extension applies in the circumstances depicted in Figure 3, because the Hicksian and Marshallian demands after the quality change are not equal at p^0 , the lower bound of integration. A further condition must be called upon to establish for the quality change case the bounding results that are well-known for prices changes. This condition is set out in Willig's 1978 paper and characterizes the case when the marginal effect of quality on consumer's surplus equals the marginal effect of quality on the expenditure function.

It is equally fallacious is to assume that numerical techniques such as those set by Vartia can be used to integrate back from the Marshallian to the Hicksian demand functions and thus obtain compensating variation measures directly. Without information on where the shifted Marshallian and Hicksian curves intersect, this approach is impossible. The relevant information is provided by the Willig condition. This paper integrates our results on comparative statics with the results of Willig, Vartia, and Bradford and Hildebrandt, among others, to formulate a consistent theory of the welfare effects of quality changes.

Endnotes

1. Superficially it appears that the result: $\text{sign } \partial x_1 / \partial b = \text{sign } (-c_{11} u_{1b} / \lambda) = u_{1b}$ would change with a monotonic transformation that made $u_{jb} \neq 0$ for $j \neq 1$. If so, then such a relationship is of little value. It turns out that a monotonic transformation of u does not affect the sign of $\partial x_1 / \partial b$. Let $F(\cdot)$ be the monotonic transformation, with the only restriction $F'(\cdot) > 0$.

The FOC conditions with the transformed utility function are

$$(i) \quad F'(u) u_i - \lambda^* p_i = 0 \quad i=1, \dots, n$$

$$(ii) \quad x \cdot p - y = 0$$

Differentiating with respect to b yields (for each i)

$$(iii) \quad F''(u) u_i \sum_j (u_j \partial x_j / \partial b) + F'(u) \sum_j (u_{ij} \partial x_j / \partial b) - p_i \partial \lambda^* / \partial b = -F'(u) u_{ib} - F''(u) u_i u_b$$

From the budget constraint, it follows that

$$(iv) \quad \sum_j p_j \partial x_j / \partial b = 0.$$

This can be simplified as follows. From (i), $u_j = p_j \lambda^* / F'(u)$, so

$$(v) \quad F''(u) u_i \sum_j u_j \partial x_j / \partial b = F''(u) u_i \frac{\lambda^*}{F'} \sum_j p_j \partial x_j / \partial b = 0 \quad \text{for all } i.$$

After differentiating both sides of (v) by $F'(u)$, expression (iii) can be written

$$(vi) \quad \sum_j u_{ij} \partial x_j / \partial b - p_i \frac{\partial \lambda}{\partial b} = -u_{ib} - \frac{F''(u)}{F'(u)} u_i u_b \quad i = 1, n$$

where λ is the multiplier from the untransformed utility function, i.e. $\lambda = \lambda^* F'(u)$.

Together (iv) and (vi) yield the standard form solution

$$\frac{\partial x_1}{\partial b} = -\sum c_{1j} u_{jb} / \lambda - \frac{F''(u)}{F'(u)\lambda} \sum c_{1j} u_j u_b$$

The second term on the right hand side can be written $\frac{-u_b F''(u)}{F'(u)} \sum c_{ij} p_j$ because $u_j = \lambda p_j$, but by (iv), this term equals zero, so the effect is

$$\frac{\partial x_1}{\partial b} = -\sum c_{1j} u_{jb} / \lambda.$$

If the assumption made in the text is imposed, i.e. $u_{jb} = 0$ for $j \neq 1$, then

$$\partial x_1 / \partial b = -c_{11} u_{1b} / \lambda.$$

Hence the comparative static effect is not changed by monotonic transformations of a utility function that is strongly separable in b and x_j , $j \neq 1$, and $\text{sign } \partial x_1 / \partial b = \text{sign } u_{1b}$.

2. As one would expect the cross quality effects, $\partial x_j^m / \partial b$ where $j \neq 1$, cannot be unambiguously signed because they depend on patterns of substitution and complementarity among all goods.
3. Throughout we adopt the convention of defining variation measures so that they have the same sign as the utility change. Thus if $b^1 > b^0$, the $CV(\Delta b)$ in (6) will be positive.
4. By the equivalent, dual characterization, x_1 is nonessential if and only if there exists some \hat{p}^1 and y^1 such that

$$(b) \quad v(p_1^0, \hat{p}^0, b^0, y) = \lim_{t \rightarrow \infty} v(t, \hat{p}^1, b^0, y^1)$$

where $t \rightarrow \infty$ is shorthand for $t \rightarrow \bar{p}_1$, the price that sets the Marshallian demand to zero.

Expression (b) implies an equivalent condition for the expenditure function, that the limit of the expenditure function as $p_1 \rightarrow \infty$ is finite.

5. However, it is not necessarily true that a finite consumer surplus implies nonessentiality. Denoting x^m as the Marshallian demand, it is possible for the limit of x^m to equal 0 as $p \rightarrow \infty$ even when the limit of x^h does not equal 0 as $p \rightarrow \infty$. Given prices and income, the individual may maximize utility at $x^m = 0$ even if x is essential, but there will be no prices of other goods low enough nor income high enough to compensate for his being forced out of the market for x .
6. Willig (1978) proves that weak complementarity holds if and only if:

$$(a) \quad \lim_{p_1 \rightarrow \infty} \frac{\partial v}{\partial b}(p_1, \hat{p}, b, y) = 0$$

or alternatively

$$(b) \quad \lim_{p_1 \rightarrow \infty} \frac{\partial m}{\partial b}(p_1, \hat{p}, b, y) = 0$$

7. There is another way of looking at the concept. The indefinite integral of the compensated demand function yields

$$\int x^h(p, b, u) dp = e(p, b, u) + c(\cdot)$$

where $e(p, b, u) + c(\cdot) = m(p, b, u)$ and $c(\cdot)$ is the constant of integration which cannot be recovered. This constant poses no problem in assessing price changes. It cannot be a function of price since the integration is over price. Consequently the welfare change associated with a price change can as easily be defined with the $e()$ function as with the complete expenditure function. However, there is no guarantee that $c(\cdot)$ will not be a function of quality, and if it is, then

$$\begin{aligned} CV(\Delta b) &= m(p^0, b^0, u^0) - m(p^0, b^1, u^0) \\ &= e(p^0, b^0, u^0) - e(p^0, b^1, u^0) + c(b^0) - c(b^1) \\ &\neq e(p^0, b^0, u^0) - e(p^0, b^1, u^0), \end{aligned}$$

the former being the true measure while the latter is the recoverable measure.

Consequently, in the general case, $CV(\Delta b)$ cannot necessarily be calculated even with knowledge of the compensated demand function.

8. The effect of quality changes on the participation decision can be shown by differentiating the expenditure function. An individual whose preferences exhibit weak complementarity in x and b and who faces choke price \bar{p} will have an expenditure function with the property that

$$\partial m(\bar{p}, \hat{p}, b, u) / \partial b = \partial m(\bar{p}) / \partial p * \partial \bar{p} / \partial b + \partial m(\bar{p}) / \partial b$$

Whether quality exhibits weak complementarity or not, the first term on the right-hand side equals zero for $p = \bar{p}$, by definition; at such points $x = 0$ and p drops out of the expenditure function. The weak complementarity condition requires that the second term be zero, i.e. $\partial m(\bar{p}) / \partial b = 0$. But nothing about the equality requires that $\frac{\partial \bar{p}}{\partial b} = 0$, so that even for someone not consuming x , changes in b influence the choke price. If the change in b is large enough to move $\bar{p}(b)$ above current price, then the individual will enter the market.

9. For finite changes in b , the change in the Hicksian quantity is

$$\Delta x^h = \sum_{i=1}^n \left[\Delta x^m \left(\frac{b_i + b_i - 1}{2} \right) + x_y^m \left(\frac{b_i + b_i - 1}{2} \right) \int_p^{p^*} x_b \left(\frac{b_1 + b_i - 1}{2} \right) dp \right] \Delta b_i$$

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