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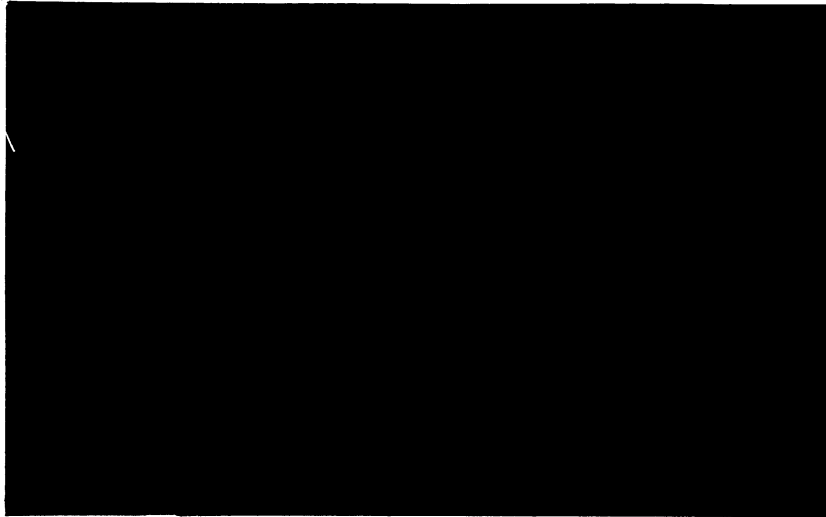
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**PRODUCER WELFARE MEASUREMENT UNDER  
ALTERNATIVE BEHAVIORAL HYPOTHESIS**

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## 1. Introduction.

The most widely used behavioral hypothesis in production economics is that of profit maximization under certainty and perfect competition. Under these assumptions, Just, Hueth, and Schmitz (1982) (JHS) have shown that the relevant welfare measure for a producer is the quasi-rent function (defined as profits plus total fixed costs) since this function captures the firm's compensating variation (Cv) or equivalent variation (Ev) associated with a multiple price change. As different behavioral hypothesis are used, however, quasi-rents are no longer be based on the firm's profit function; when the objective of the firm changes, so does the basis for calculating Cv and Ev. Relaxing the certainty assumption, for example, leads to a measure of Cv based on the agent's von-Neumann Morgenstern utility function (see *e.g.* JHS; Pope, Chavas, and Just (1983); and Larson (1987)), which can be estimated by measuring the consumer surplus associated with a necessary input.

Some of the other behavioral hypothesis have been employed in the literature are: output constrained cost minimization, input constrained revenue maximization, expenditure constrained profit maximization, and sales maximization. The first two have been used widely in empirical studies of production economics, especially with the widespread use of duality techniques (see Chambers (1988a)). Expenditure constrained profit maximizing behavior has been analyzed by Lee and Chambers (1986); the authors found empirical support for this behavioral hypothesis over unconstrained profit maximizing behavior. Additionally, Aldunate (1975) points out that an entrepreneur who has decision power but no share in profits will maximize the firms sales as long as a

minimum level of profits is reached.

This paper develops the producer welfare measures that are associated with the different behavioral hypothesis under the assumptions of certainty and perfectly competitive output and input markets. The results show that an exact measurement of welfare can be obtained under the different objective functions by using the concepts of producer and consumer surplus; the differences in Cv or Ev between behavioral hypothesis lies in the optimal output supply and derived input demand functions that are used in the calculation of these surplus measures.

The structure of the paper is as follows. In the second section the technological assumptions that will be used repeatedly are presented. The welfare measurement associated with the different objective functions are analyzed in sections three through six. Finally, the last section concludes the paper.

## 2. Firm's Cost Function.

When the producers are a price takers in the input market their technology may be represented by the multiproduct cost function which is defined as:

$$C(\mathbf{w}, \mathbf{q}, \mathbf{Z}) \equiv \underset{X}{\text{Min}} \left\{ \mathbf{w}' X \mid \mathbf{q} \leq F(X, \mathbf{Z}) \right\}, \quad (1)$$

where  $\mathbf{w} \in R_{++}^n$  is a vector of variable input prices,  $\mathbf{q} \in R_+^m$  is a vector of outputs,  $X \in R_+^{nm}$  and  $\mathbf{Z} \in R_+^{pm}$  are matrices of variable and fixed inputs, respectively,  $\mathbf{I}$  is the unit vector, and  $F(X, \mathbf{Z})$  represents the producer's neoclassical non-joint multiproduct production function.  $C(\mathbf{w}, \mathbf{q}, \mathbf{Z})$  is non-decreasing, positively linearly homogeneous,

concave, and continuous in input prices and non-decreasing in output. Assuming Shephard's lemma is satisfied (*i.e.* the firm's input requirement set is strictly convex) then

$$\nabla_{\mathbf{w}} C(\mathbf{w}, \mathbf{q}, Z) = [x^c_1(\mathbf{w}, \mathbf{q}, Z), \dots, x^c_n(\mathbf{w}, \mathbf{q}, Z)] , \quad (2)$$

where  $x^c_j(\mathbf{w}, \mathbf{q}, Z)$  are the total output compensated demands for input  $j$  which are given by

$$x^c_j(\mathbf{w}, \mathbf{q}, Z) = \sum_{i=1}^m x^c_{ij}(\mathbf{w}, \mathbf{q}_i, z_i) , \quad \forall j = 1, \dots, n , \quad (3)$$

and the  $x^c_{ij}(\mathbf{w}, \mathbf{q}_i, z_i)$  are the cost minimizing allocations of input  $j$  to output  $i$ .

### 3. Welfare Measurement for a Cost Minimizing Firm.

The welfare implications of a multiple price change for a cost minimizing firm can be expressed as

$$\Delta W(\mathbf{p}^0, \mathbf{p}^1, \mathbf{w}^0, \mathbf{w}^1, \mathbf{q}^0, Z) = C(\mathbf{w}^0, \mathbf{q}^0, Z) - C(\mathbf{w}^1, \mathbf{q}^0, Z) , \quad (4)$$

where  $C(\mathbf{w}^k, \mathbf{p}^0, Z)$  is the firm's cost function,  $(\mathbf{p}^0, \mathbf{w}^0)$  and  $(\mathbf{p}^1, \mathbf{w}^1)$  are vectors of initial and final input and output prices, respectively. It is important to note that  $\Delta W(\mathbf{p}^0, \mathbf{p}^1, \mathbf{w}^0, \mathbf{w}^1, \mathbf{q}^0, Z)$  represents the compensating (equivalent) variation associated with the multiple price change given that it is the maximum (minimum) amount the firm is willing to pay for (receive in lieu of) the price change. This welfare measure implicitly assumes that the multiple price change will not induce the firm to shutdown; theoretically, there is no shutdown input price for a cost minimizing firm.

From (4),  $\Delta W(\mathbf{p}^0, \mathbf{p}^1, \mathbf{w}^0, \mathbf{w}^1, \mathbf{q}^0, Z)$  can be rewritten as

$$\begin{aligned}
\Delta W(p^0, p^1, w^0, w^1, q^0, Z) &= - \int_L d C(w, q^0, Z) \\
&= - \int_L \sum_{j=1}^n \left\{ \frac{\partial C(w, q^0, Z)}{\partial w_j} dw_j \right\} \\
&= - \int_L \sum_{j=1}^n x^c_j(w, q^0, Z) dw_j,
\end{aligned} \tag{5}$$

where L is an arbitrary path of integration. Given (2), the integrand of (5) is an exact differential and, thus,  $\Delta W(p^0, p^1, w^0, w^1, q^0, Z)$  is a path independent measure of welfare change for the producer. Consider the following path of integration

$$\hat{w}_j(w_j) = [p^0, w^1_1, \dots, w^1_{j-1}, w_j, w^0_{j+1}, \dots, w^0_n], \tag{6}$$

then the welfare effect of a multiple price change becomes

$$\Delta W(p^0, p^1, w^0, w^1, q^0, Z) = - \sum_{j=1}^n \left\{ \int_{w^0_j}^{w^1_j} x^c_j(\hat{w}_j(w_j), q^0, Z) dw_j \right\}. \tag{7}$$

Equation (7) implies that the welfare implications of a multiple price change are given by the sum of consumer surplus (CS) changes in each input market, where the CS changes are given by the area behind the output compensated input demand curve above the input price. Therefore, in order to obtain an exact measure of welfare change (*i.e.* the compensating or equivalent variation) for a cost minimizing firm the welfare economist must use the cost minimizing input demands to calculate the CS changes in each input market. This area is clearly different than the CS associated with the profit maximizing input demands since the profit maximizing producer adjusts its output level as its input



prices change while the cost minimizing firm faces a fixed level of output.

This is not the only difference between the two welfare measures associated with the two behavioral hypothesis. The welfare implications of a multiple price change for an unconstrained profit maximizing firm are given by

$$\Delta W(p^0, p^1, w^0, w^1, Z) = \sum_{i=1}^m \int_{p_i^0}^{p_i^1} q^p(\hat{p}_i(p_i), Z) dp_i - \sum_{j=1}^n \int_{w_j^0}^{w_j^1} x^p(\hat{w}_j(w_j), Z) dw_j, \quad (8)$$

$$\hat{p}_i(p_i) = [p^1_1, \dots, p^1_{i-1}, p_i, p^0_{i+1}, \dots, p^0_n, w^1],$$

$$\hat{w}_j(w_j) = [p^0, w^1_1, \dots, w^1_{j-1}, w_j, w^0_{j+1}, \dots, w^0_n];$$

where  $q^p_i(p, w, Z)$  and  $x^p_i(p, w, Z)$  are the profit maximizing output supply and input demand functions, respectively (see Just, Hueth, and Schmitz (1982)). Expression (8) can be rewritten as

$$\Delta W(p^0, p^1, w^0, w^1, Z) = \sum_{i=1}^m \int_{p_i^0}^{p_i^1} q^p(\hat{p}_i(p_i), Z) dp_i - \sum_{j=1}^n \int_{w_j^0}^{w_j^1} x^c(\hat{w}_j(w_j), q, Z) dw_j$$

$$- \sum_{j=1}^n \int_{w_j^0}^{w_j^1} \sum_{i=1}^m \frac{\partial C(w, q, Z)}{\partial q_i} \frac{\partial q^p(\hat{w}_j(w_j), Z)}{\partial w_j} dw_j. \quad (9)$$

In the special case that  $q^p(p^0, w, Z) = q^0$ , the difference in welfare measures will be given by

$$\begin{aligned}
& \sum_{i=1}^m \int_{p_i^0}^{p_i^1} q^p_i(\hat{p}(p_i), Z) dp_i \\
& - \sum_{j=1}^n \int_{w_j^0}^{w_j^1} \sum_{i=1}^m \frac{\partial C(w, q, Z)}{\partial q_i} \frac{\partial q^p_i(\hat{w}_j(w_j), Z)}{\partial w_j} dw_j .
\end{aligned} \tag{10}$$

The first term of the error term represents the change in welfare for a profit maximizing firm of a change in its output prices which is given by the sum of producer surplus (PS) changes in each output market (the area behind the profit maximizing output supply function below output price). The second term represents the difference in CS changes of using a profit maximizing versus a cost minimizing input demand function.

#### 4. Welfare Measurement for an Input Constrained Revenue Maximizing Firm.

The input constrained revenue maximization problem can be formulated as

$$R(p, x^0, Z) \equiv \text{Max}_q \{ p'q : x^0 = h(q, Z) \} \tag{11}$$

where  $x^0$  is the original endowment of inputs. The revenue function defined in expression (11) is non-negative, non-decreasing, positively linearly homogeneous, convex, and continuous in output prices, and non-decreasing in  $x^0$ . When  $R(p, x^0, Z)$  is differentiable in output prices the following derivative property holds (Chambers 1988b):

$$\nabla_p R(p, x^0, Z) = [q_1^r(p, x^0, Z), \dots, q_m^r(p, x^0, Z)] , \tag{12}$$

where  $q_i^r(p, x^0, Z)$  represents the input constrained revenue maximizing output supply functions.

Given the above development, the compensating (or equivalent) variation associated with a multiple price change for an input constrained revenue maximizing firm is given by

$$\Delta W(p^0, p^1, w^0, w^1, x^0, Z) = R(p^1, x^0, Z) - R(p^0, x^0, Z) = \int_L dR(p, x^0, Z). \quad (13)$$

Employing the derivative property (equation 12)) and considering the following integration path

$$\bar{p}_i(p_i) = [p^1_1, \dots, p^1_{i-1}, p_i, p^0_{i+1}, \dots, p^0_m, w^0], \quad (14)$$

the welfare implication of a multiple price change can be rewritten as follows

$$\Delta W(p^0, p^1, w^0, w^1, x^0, Z) = \sum_{i=1}^m \left\{ \int_{p_i^0}^{p_i^1} q^r_i(\bar{p}_i(p_i), x^0, Z) dp_i \right\}. \quad (15)$$

Equation (15) implies that the exact welfare effect of a multiple price change for an input constrained revenue maximizing firm is given by the sum of changes in PS in each output market; note that the PS in output market  $i$  is the area behind the input constrained output supply curve ( $q^r_i(p, x^0, Z)$ ) below output price. As was the case with the cost minimizing firm, this area is different to the PS associated with the profit maximizing output supply function; the difference in PS changes presents itself since the profit maximizing producer adjusts the optimal input levels as output price changes while the input constrained revenue maximizer faces a fixed input endowment. The total difference in the measures of welfare change for a profit maximizing firm and an input

constrained revenue maximizing firm is given by

$$\sum_{i=1}^m \int_{p_i^0}^{p_i^1} \sum_{j=1}^n \frac{\partial R(p, x^p, Z)}{\partial x_{ij}} \frac{\partial x_{ij}^p(\bar{p}_i(p_i), Z)}{\partial p_i} dp_i - \sum_{j=1}^n \int_{w_j^0}^{w_j^1} x_{ij}^p(\bar{w}_j(w_j), Z) dw_j \quad (16)$$

$$\bar{w}_j(w_j) = [p^1, w^1_1, \dots, w^1_{j-1}, w_j, w^0_{j+1}, \dots, w^0_n]$$

for the special case where  $x^p(p^0, w^0, Z) = x^0$ . The first term of the above error term represents the discrepancy between the PS changes of the two behavioral hypothesis. The second term, on the other hand, represents the change in welfare for a profit maximizing firm of a change in its input prices; this welfare change is given by the sum of CS changes in each input market.

### 5. Welfare Measurement for an Expenditure Costrained Profit Maximizing Firm.

The producer that is faced with an expenditure constraint determines the optimal output level so as to

$$\Pi(p, w, Z, E) = \text{Max}_q \left\{ p'q - C(w, q, Z) - r'ZI : C(w, q, Z) + r'ZI \leq E \right\}, \quad (17)$$

where  $r \in R_{++}^p$  is a vector of fixed input prices and  $E$  is the maximum level of expenditure that the producer cannot exceed.  $\Pi(p, w, Z, E)$  is positively linearly homogeneous in  $(p, w, E)$ , non-decreasing and convex in  $p$ , non-increasing and quasi-convex in  $w$ , and non-decreasing in  $E$  (Chambers (1988b) and Lee and Chambers (1986)). The producer's exact measure of compensating (equivalent) variation is given by  $R(p, w, Z, E) = \Pi(p, w, Z, E) + r'ZI$  since profits underestimates the firm's welfare

measurement in the case that the firm is forced to shutdown (JHS); that is, the producer's welfare measure is given by profits plus total fixed costs.

All of the properties of the expenditure constrained profit function carry over to the quasi-rent function since they differ only by a constant term. Furthermore, applying the envelope theorem to (17) yields the following derivative properties:

$$\frac{\partial R(\mathbf{p}, \mathbf{w}, Z, E)}{\partial p_i} = q^e_i(\mathbf{p}, \mathbf{x}^*, Z, E), \quad (18)$$

where

$$x^*_j = \begin{cases} x^p_j(\mathbf{p}, \mathbf{w}, Z) & \text{if the constraint isn't binding} \\ x^e_j(\mathbf{p}, \mathbf{w}, Z, E) & \text{if the constraint is binding} \end{cases}, \quad (19)$$

and  $x^p_j(\mathbf{p}, \mathbf{w}, Z)$  and  $x^e_j(\mathbf{p}, \mathbf{w}, Z, E)$  are the unconstrained profit maximizing and expenditure constrained profit maximizing derived demands for input  $j$ . Additionally, these derived demands are given by

$$\left\{ \begin{array}{l} x^p_j(\mathbf{p}, \mathbf{w}, Z) = - \frac{\partial R(\mathbf{p}, \mathbf{w}, Z)}{\partial w_j} \\ x^e_j(\mathbf{p}, \mathbf{w}, Z, E) = \frac{\frac{\partial R(\mathbf{p}, \mathbf{w}, Z, E)}{\partial w_j}}{\sum_{k=1}^n \frac{\partial R(\mathbf{p}, \mathbf{w}, Z, E)}{\partial w_k} \frac{w_k}{E^*}} \end{array} \right\}, \quad (20)$$

where  $R(\mathbf{p}, \mathbf{w}, Z)$  and  $R(\mathbf{p}, \mathbf{w}, Z, E)$  represent the producer's unconstrained and expenditure constrained quasi-rent functions, respectively, and  $E^* = E - \text{total fixed costs}$ . The first derivative property is a result of Hotelling's lemma; the second derivative property, on the

other hand, is referred to as a modified Roy's identity (Lee and Chambers).

The welfare implications of a multiple price change will be given by the change in the producer's expenditure constrained quasi-rent function from initial prices  $(p^0, w^0)$  to final prices  $(p^1, w^1)$  which is

$$\Delta W(p^0, p^1, w^0, w^1, Z, E) = \int_L dR(p, w, Z, E) . \quad (21)$$

Using the derivative properties, (21) can be rewritten as

$$\begin{aligned} \Delta W(p^0, p^1, w^0, w^1, Z, E) &= \int_L \left\{ \sum_{i=1}^m \frac{\partial R(p, w, Z, E)}{\partial p_i} dp_i + \sum_{j=1}^n \frac{\partial R(p, w, Z, E)}{\partial w_j} dw_j \right\} \quad (22) \\ &= \int_L \left\{ \sum_{i=1}^m q^e_i(p, x^*, Z, E) dp_i + \sum_{j=1}^n \frac{\partial R(p, w, Z, E)}{\partial w_j} dw_j \right\} \end{aligned}$$

In the special case that at initial and final prices the constraint is not binding, (22) reduces to the change in quasi-rents associated with unconstrained profit maximization which is path independent and can be measured sequentially as the sum of producer surplus changes in each output market and the sum of consumer surplus changes in each input market (see equation (8)).

However, for the case in which the expenditure constraint is binding at both initial and final prices, equation (22) can be rewritten as

$$\Delta W(p^0, p^1, w^0, w^1, Z, E) = \int_L \left\{ \sum_{i=1}^m q_i^e(p, w, Z, E) dp_i - (1+\lambda) \sum_{j=1}^n x_j^e(p, w, Z, E) dw_j \right\}. \quad (23)$$

Thus, when the expenditure constraint is binding, the producer's compensating or equivalent variation is given by equation (23) which is path independent since the integrand is an exact differential. However, as it stands, equation (23) is not useful for empirical measurement of compensating (equivalent) variation given that  $\lambda$  is unobservable.

Alternatively, the welfare implications of a multiple price change for an expenditure constrained profit maximizing behavior can be obtained from the change in producer surplus associated with a necessary output; a necessary output is one for which a shutdown price exists. Suppose output  $i$  is a necessary output, then the change in the expenditure constrained quasi-rent function will be given by

$$\begin{aligned} \Delta W(p^0, p^1, w^0, w^1, Z, E) &= R(p^1, w^1, Z, E) - R(p^0, w^0, Z, E) \\ &= \int_{\hat{p}_i^1}^{p_i^1} \frac{\partial R(p_i^1, Z, E)}{\partial p_i} dp_i - \int_{\hat{p}_i^0}^{p_i^0} \frac{\partial R(p_i^0, Z, E)}{\partial p_i} dp_i \quad (24) \\ &= \int_{\hat{p}_i^1}^{p_i^1} q_i^e(p_i^1, Z, E) dp_i - \int_{\hat{p}_i^0}^{p_i^0} q_i^e(p_i^0, Z, E) dp_i, \end{aligned}$$

where  $p_i^k$  ( $k=0,1$ ) is given by  $[p_1^k, \dots, p_{i-1}^k, p_i, p_{i+1}^k, \dots, p_m^k, w^k]$ ,  $\hat{p}_i^k$  is the shutdown price for output  $i$ , and  $q_i^e(p, w, Z, E)$  is the expenditure constrained supply function for output  $i$ . Equation (24) implies that the producer's compensating (equivalent) variation for the case

when the expenditure constraint is binding can be measured in a necessary output market; this welfare measure is given by the difference in producer surplus in that output market at the different price levels.

## 6. Welfare Measurement for a Sales Maximizing Firm.

Assuming that the firm is a price taker in its output market, the decision problem of an entrepreneur that maximizes sales subject to the attainment of a minimum level of profits can be formulated as follows

$$S(p, w, Z, K) \equiv \text{Max}_q \left\{ p'q : p'q - C(w, q, Z) - r'Z \geq K \right\}, \quad (25)$$

where  $S(p, w, Z, K)$  is the indirect sales function and  $K$  is the minimum level of profits. The indirect sales function is non-decreasing, positively linearly homogeneous and convex in  $p$ .

Even though the entrepreneur's objective is to maximize sales subject to a minimum level of output, this is not necessarily the original objective of the 'owners' of the firm. If their objective is to maximize the returns over the fixed costs (which is equivalent to maximizing profits), then the relevant quasi-rent function for welfare measure is given by

$$R(p, w, Z, K) = \sum_{i=1}^m p_i q^s(p, w, Z, K) - C(w, q^s(p, w, Z, K), Z, K). \quad (26)$$

It is important to note that given the binding constraint the quasi-rent function defined in (26) is constant and equal to  $K^* = K - \text{total fixed costs}$ . Therefore, the welfare of the



owners of the firm is not affected by a multiple price change, as long as the minimum desired level of profits does not change. To see this, note that the welfare level at final prices is given by  $R(p^1, w^1, Z, K) = K^*$  and the welfare level at initial prices is  $R(p^0, w^0, Z, K) = k^*$ ; thus, the compensating (equivalent) variation, which is the difference in the quasi-rent functions, is given by  $K^* - k^* = 0$ .

## 7. Concluding Remarks.

When other behavioral hypothesis are taken into account, besides profit maximization, the producer's welfare measures change. However, the concepts of producer and consumer surplus changes (associated with output supplies and input demands that are specified in accordance with the firm's objective function) can still be used to estimate the Cv and Ev associated with a multiple price change.

In order to obtain an exact measure of welfare change, it is important to use output supply and input demand functions that are consistent with the firm's behavioral hypothesis. If this is not the case, then the integrability conditions are not satisfied and the concepts of surpluses are no longer related to the firm's quasi-rent function. This issue also implies that the input demand system and output supply system must be consistent with the underlying objective function; that is, they must satisfy certain properties that are imposed through the maximization (minimization) procedure.

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