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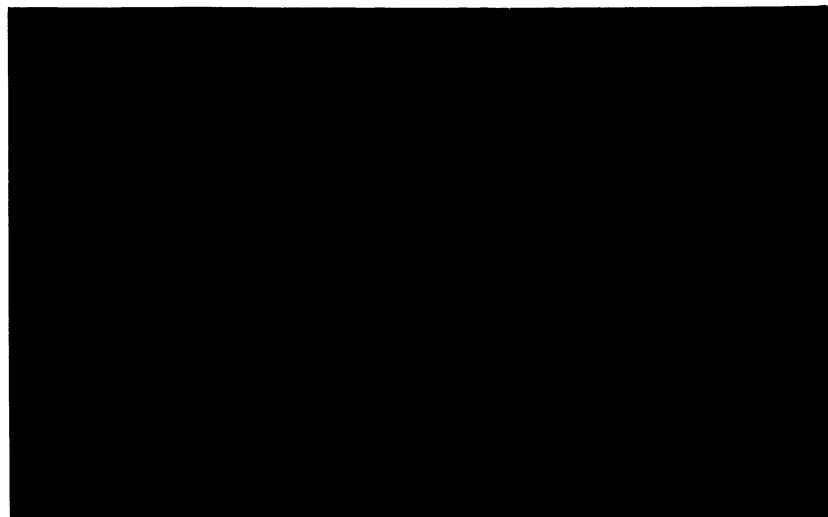
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THE GENERAL EQUILIBRIUM CONSEQUENCES
OF PAYMENT-KIND-PROGRAMS

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Working Paper No. 90-22

August 1990

Revised May 1991

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Introduction

In 1983, the United States Government resurrected the payment-in-kind concept from the early 1960s. Under this program farmers who removed land from production received a compensatory payment from CCC commodity stocks. The objectives of this payment-in-kind (PIK) program were: (1) to reduce production and stocks; (2) to guarantee adequate supplies; (3) to reduce U.S. Government budget exposure; (4) improve conservation of land; (5) to increase farm income; and (6) to ease storage problems (ERS, p. 6). The impacts of this program on agriculture and the economy were, and remain, controversial. One contentious issue centered about an inherent conflict in the program. On one hand farmers were required to remove land from production to qualify for the payment. However, the payment was made in commodities from CCC stocks which were isolated from the market according to the specified release rules. It was unclear whether market supplies would be smaller or larger, yet answering that question was central to the impacts. The initial ERS analysis argued for a net supply reduction and higher grain prices. The analysis also emphasized the benefits to farmers through input cost savings.

[This article examines a payment-in-kind program using a two-good general equilibrium model. The results show that the conflict between the acreage reduction effect and the stocks release are critical to assessing the impacts of a PIK program.] When the PIK is first introduced the production effect dominates and the price of agricultural goods rises. Thus, the initial ERS analysis was correct in arguing for a price increase. But if a PIK payment is increased for an existing program, the agricultural price can move either direction. The model also demonstrates that public stocks cannot rise under the PIK. If market prices are "riding" the loan rate when the program is

introduced stocks released are repurchased leaving the total unaffected. Otherwise government stocks are reduced.

The paper is organized as follows. First, the 1983 U.S. payment-in-kind program rules are covered to define the central features to consider in designing the model. Then the model used in the analysis is developed. Next the comparative statics results for an increase in the payment rate are presented. Finally, the interaction of the PIK with a nonrecourse loan program across time is considered.

Operation of the 1983 PIK

The 1983 payment-in-kind was added to preexisting U.S. Government commodity programs under the 1981 legislation. That legislation authorized target prices, loan rates, and public stocks release prices for program crops. Acreage reduction programs (ARP) were discretionary. In July 1982 a 20 percent ARP was announced. This was revised in September to include a 5 percent paid diversion with a payment rate of \$2.70 per bushel. The feed grains program was announced in September. It contained a 20 percent ARP including a 10 percent paid diversion at \$1.50 per bushel.

The outlook for wheat feed grains during late 1982 was pessimistic. Income was expected to be down, U.S. Government costs high, and U.S. Government surplus stocks extremely large. In January 1983, the U.S. Government announced the payment-in-kind (PIK) program to deal with this bleak outlook. For wheat, farmers who participated in the 20 percent ARP could remove an additional 10 to 20 percent of their base acres and receive a compensatory payment in wheat from public stocks. This payment was 95 percent of the normal yield on the PIK acreage idled. Also they could bid to idle their entire wheat base in return for a PIK payment. For corn and sorghum a

similar PIK program was announced with a payment rate of 85 percent of the normal yield.

Because U.S. commodity programs are voluntary, the extent of participation is often critical to their success. In the 1983 wheat program 78 percent of farmers enrolled in the ARP, while 51 percent also joined the PIK program. Participation in the feed grains ARP was lower than for wheat at 71 percent, but participation in the PIK was greater at 60 percent.

The model used to analyze the PIK must reflect these basic features of how the program operated. It needs to allow for the participation decision. To enroll in the program, farmers must comply with the ARP percentage. That is, the established ARP percentage of base must be idled for the protection offered by the target price and the loan rate. For participants, the PIK acreage idled is a choice variable. A farmer can decide not to join the PIK by not enrolling any land beyond the ARP. Or a farmer can decide to idle any acreage up to the established PIK limit. The next section presents a two-good general equilibrium model which includes these features.

The Model

There are two countries and two internationally traded commodities. The two commodities are an agricultural good and a nonagricultural good. In the home country production relations are governed by the input-nonjoint production possibilities set

$$Y = \left\{ (y_a, y_n, L, K_a, K_n) : (y_i, L_i, K_i) \in Y_i (i = a, n), L_a + L_n = L; \right. \\ \left. y_i \in \mathbb{R}_+ (i = a, n), L_i \in \mathbb{R}_+ (i = a, n), K_i \in \mathbb{R}_+ (i = a, n) \right\}$$

Here $Y_i (i = a, n)$ is a production possibilities set which is closed, convex, and bounded from above for finite $K_i (i = a, n)$. Each Y_i satisfies constant returns to scale so that if $(y_i, K_i, L_i) \in Y_i$ then for $\mu > 0$ $(\mu y_i, \mu K_i, \mu L_i) \in Y_i$.

Each y_i is to be interpreted as production of the i th commodity where the subscript "a" denotes agricultural and the subset "n" denotes nonagricultural. K_i is a factor of production specific to the production of commodity i . For agriculture we assume that this factor of production is land. L is a mobile factor of production which can be used in the production of either y_a or y_n and L_i denotes the allocation of the mobile factor of production to the production of y_a or y_n . At the economy level L , K_a , and K_n are in fixed supply and fully employed, and without confusion L , K_a , and K_n denote these endowments. All factors are immobile internationally. Producers are risk neutral and price takers.

Nonagricultural Producers

Because the focus is on the general equilibrium effects of a PIK program, assume that all nonagricultural producers are identical. A dual representation of the aggregate nonagricultural technology is the restricted profit function

$$\hat{R}_n(p_n, w; K_n) = \text{Max}\{p_n y_n - wL_n : (y_n, L_n, K_n) \in Y_n\}$$

where $p_n \in \mathbb{R}_{++}$ and $w \in \mathbb{R}_{++}$ are the price of the nonagricultural commodity and the mobile factor of production, respectively. $\hat{R}_n(p_n, w; K_n)$ is the Marshallian quasi-rent accruing to the owners of K_n . We shall typically refer to $\hat{R}_n(p_n, w; K_n)$ as nonagricultural income. \hat{R}_n is convex and positively linearly homogeneous in p_n and w . Constant returns to scale implies that \hat{R}_n is positively linearly homogeneous in K_n , i.e., $\hat{R}_n(p_n, w; \mu K_n) = \mu \hat{R}_n(p_n, w; K_n)$ for $\mu > 0$. Letting $\mu = 1/K_n$ gives

$$\begin{aligned} \hat{R}_n(p_n, w; K_n) &= K_n \hat{R}_n(p_n, w; 1) \\ &\equiv K_n R_n(p_n, w). \end{aligned}$$

We shall always presume that \hat{R}_n is at least twice differentiable which implies via the Shepherd-Hotelling lemma that

$$\begin{aligned} y_n(p_n, w; K_n) &= \partial_{p_n} \hat{R}_n(p_n, w; K_n) \\ &= K_n \partial_{p_n} R_n(p_n, w) \end{aligned}$$

and

$$\begin{aligned} L_n(p_n, w; K_n) &= - \partial_w \hat{R}_n(p_n, w; K_n) \\ &= - K_n \partial_w R_n(p_n, w) \end{aligned}$$

where $y_n(p_n, w; K_n)$ and $L_n(p_n, w; K_n)$ are, respectively, the profit maximizing supply of the nontraded good and the derived demand for the mobile factor by industry n . The notation $\partial_k F(\cdot)$ denotes the partial derivative of the function $F(\cdot)$ with respect to argument K .

Agricultural Producers

Agricultural producers are not identical. Assume that there exists a fixed number of agricultural producers $m(m \in \mathbb{N}_+)$. Producer k 's technology is given by Y_a^k which satisfies the same general properties as Y_1 . Corresponding to the distribution of producer types there is a fixed distribution of K_a across farmers. The k farmer thus has a fixed land endowment equalling K_a^k . For simplicity we prohibit trades of K_a amongst farmers during the length of run of the analysis. Y_a is thus characterized by

$$\begin{aligned} Y_a &= \left\{ (y_a, L_a, K_a) : (y_a^i, L_a^i, K_a^i) \in Y_a^i (i = 1, \dots, m), \right. \\ &\quad \left. y_a = \sum_{i=1}^m y_a^i; L_a = \sum_{i=1}^m L_a^i; K_a = \sum_{i=1}^m K_a^i \right\}. \end{aligned}$$

Agricultural production is carried out in an environment in which farmers have available to them a "government program" with the following characteristics: a target price $p_a^t = p_a + s$ where s is the per unit deficiency payment; a paid (in cash) land diversion (PLD) with a payment rate of r ; a

nonpaid acreage retirement (ARP); and a PIK land diversion program. The program functions in the following way: farmers are free to participate in the program or not. However, if they choose to participate they are eligible to receive the target price of p_a^t for all their production but in return they must agree to idle T^k acres (T^k is fixed but specific to each farmer) in a paid land diversion in return for a cash payment of $\$r$ per acre idled; they must idle H^k (H^k is also fixed but farmer specific) acres in an unpaid acreage retirement program; and they have the further option of idling a further $b^k \left(b^k \in [0, K_a^k - T^k - H^k] \right)^1$ acres in a PIK program where the payment rate is θ units of y_a per acre of land idled.

If farmer k does not participate in the program his rent from farming his K_a^k acres of land is given by

$$\hat{G}^k(p_a, w; K_a^k) = \text{Max} \left\{ p_a y_a^k - w L_a^k : (y_a^k, L_a^k, K_a^k) \in Y_a^k \right\}.$$

Here $p_a \in \mathbb{R}_{++}$ is the market price of the agricultural commodity.

$\hat{G}^k(p_a, w; K_a^k)$ is positively linearly homogeneous and convex in p_a and w by standard properties of profit functions. By constant returns to scale it is also positively linearly homogeneous in K_a^k whence

$$\begin{aligned} \hat{G}^k(p_a, w; K_a^k) &= K_a^k \hat{G}^k(p_a, w; 1) \\ &\equiv K_a^k G^k(p_a, w). \end{aligned}$$

We always assume that \hat{G}^k is at least twice continuously differentiable so that

$$y_a^k(p_a, w; K_a^k) = K_a^k \partial_p G^k(p_a; w)$$

and

$$L_a^k(p_a, w; K_a^k) = -K_a^k \partial_w G^k(p_a; w)$$

where $y_a^k(p_a, w; K_a^k)$ and $L_a^k(p_a, w; K_a^k)$ are the profit maximizing supply and derived demand for the mobile factor of production, respectively, for a non-participating farmer.

Producers who participate in the program choose y_a^k , L_a^k , and b^k according to

$$\begin{aligned}
& \hat{g}^k(p_a, w, s, r; \theta, T^k, H^k, K_a^k) \\
&= \max_{y_a^k, L_a^k, b^k} \left\{ (p_a + s)y_a^k - wL_a^k + rT^k + p_a \theta b^k : (y_a^k, L_a^k, K_a^k - T^k - H^k - b^k) \in Y_a^k \right\} \\
&= rT^k + \max_{y_a^k, L_a^k, b^k} \left\{ (p_a + s)y_a^k - wL_a^k + p_a \theta b^k : (y_a^k, L_a^k, K_a^k - T^k - H^k - b^k) \in Y_a^k \right\} \\
&= rT^k + \max \left\{ (p_a + s)y_a^k - wL_a^k + p_a \theta b^k : K_a^k \left(\frac{y_a^k}{K_a^k}, \frac{L_a^k}{K_a^k}, 1 - \frac{T^k}{K_a^k} - \frac{H^k}{K_a^k} - \frac{b^k}{K_a^k} \right) \in Y_a^k \right\} \\
&= rT^k + K_a^k \max \left\{ (p_a + s) \frac{y_a^k}{K_a^k} - w \frac{L_a^k}{K_a^k} + p_a \frac{\theta b^k}{K_a^k} : \left(\frac{y_a^k}{K_a^k}, \frac{L_a^k}{K_a^k}, 1 - \frac{T^k}{K_a^k} - \frac{H^k}{K_a^k} - \frac{b^k}{K_a^k} \right) \in Y_a^k \right\} \\
&= rT^k + K_a^k g^k(p_a, s, w; \theta, t^k, h^k)
\end{aligned}$$

where the third equality follows by constant returns to scale and

$t^k \equiv \frac{T^k}{K_a^k}$ and $h^k \equiv \frac{H^k}{K_a^k}$. (Because the technology is only defined over the

positive orthant the requirement that $b^k \leq K_a^k - T^k - H^k$ is automatically

satisfied.) g^k , which is the profit (rent) function for a single acre of land, is positively linearly homogeneous and convex in p_a , w , and s .

Moreover, because the objective function is affine in θ g^k is also convex in

θ . Assume that g^k is at least twice differentiable so that:

$$\partial_p g^k(p_a, s, w; \theta, t^k, h^k) = \frac{1}{K_a^k} \left(y_a^k(p_a, s, w; \theta, t^k, h^k) + \theta b^k(p_a, s, w; \theta, t^k, h^k) \right),$$

$$\partial_\theta g^k(p_a, s, w; \theta, t^k, h^k) = \frac{p_a b^k(p_a, s, w; \theta, t^k, h^k)}{K_a^k}$$

$$\partial_w g^k(p_a, s, w; \theta, t^k, h^k) = - \frac{1}{K_a^k} L_a^k(p_a, s, w; \theta, t^k, h^k).$$

Here $y_a^k(p_a, s, w; \theta, t^k, h^k)$, $b^k(p_a, s, w; \theta, t^k, h^k)$, and $L_a^k(p_a, s, w; \theta, t^k, h^k)$ are, respectively, the profit maximizing supply, PIK acreage retirement, and

derived demand for the mobile factor of a participating farmer.

To insure that no PIK acreage is retired when there is no PIK payment assume that $\left. \partial_{\theta} g^k(p_a, s, w; \theta, t^k, h^k) \right|_{\theta=0} = 0$.

Farmer k participates in the program if

$$rt^k + g^k \geq G^k.$$

Each farmer, therefore, solves

$$R_a^k(p_a, s, w, r; \theta) = \text{Max}\{G^k, rt^k + g^k\}$$

where for notational convenience we have suppressed in R_a^k the arguments t^k and h^k . Define

$$R_a(p_a, s, w, r; \theta, K_a) \equiv \sum_{k=1}^m K_a^k R_a^k(p_a, s, w, r; \theta)$$

where $K_a = (K_a^1, \dots, K_a^m)$ and R_a is the aggregate agricultural rent function which we shall refer to as agricultural income. R^a is positively linearly homogeneous and convex in p_a , s , w , and r and convex in θ . We shall assume that R_a is at least twice differentiable² so that

$$\partial_p R_a = \sum_{j \in N} y_a^j(p_a, w; K_a^j) + \sum_{i \in P} \left[y_a^i(p_a, w, s; \theta, t^i, h^i) + \theta b^i(p_a, w, s; \theta, t^i, h^i) \right]$$

$$\partial_w R_a = - \left[\sum_{j \in N} L_a^j(p_a, w; K_a^j) + \sum_{i \in P} L_a^i(p_a, w, s; \theta, t^i, h^i) \right],$$

$$\partial_{\theta} R_a = \sum_{i \in P} p_a b^i(p_a, w, s; \theta, t^i, h^i), \text{ and}$$

$$\partial_r R_a = \sum_{i \in P} T^i.$$

Here N denotes the set of nonparticipants and P the set of participants. (An Appendix discusses the properties of R_a .)

Domestic Consumer and Foreign Demand

Domestic demand for the agricultural and nonagricultural products are assumed to be governed by the integrable demand functions $D_i(p_a, p_n; E)$

$i = (a, n)$ where E represents consumer expenditure on commodities. Each D_i is assumed homogeneous of degree zero in its arguments. Because econometric evidence and Engel's law suggest that the income elasticity for agricultural commodities is low we assume in what follows that $\partial_E D_a(p_a, p_n, E) = 0$. This assumption makes some results which would be ambiguous in its absence unambiguous. Thus, it may lead some to question the generality of the results. However, its main analytic contribution is to force direct price effects on agricultural demands to dominate general equilibrium indirect price effects entering through the income term. As long as this latter condition is satisfied (as econometric evidence suggests that it is) then the qualitative nature of the results presented below should not change.

Foreign demand for the agricultural and nonagricultural products is governed by the excess demands $q_i(p_a^*/p_n^*)$ ($i = a, n$) where p_i^* is the international price of commodity i . We assume that these excess demands are negatively sloped in that $q'_a(\cdot) < 0$ and $q'_n(\cdot) > 0$.

Government Behavior

The domestic government raises its revenue (T) through a nondistortionary income tax. To focus the analysis on commodity programs we assume that all of this revenue is used on the agricultural program specified above so that it faces the budget constraint

$$T = (\partial_s R_a) s + (\partial_r R_a) \cdot r.$$

Several things should be noted about this budget constraint. By the envelope theorem

$$\partial_s R_a^k(p_a, s, w, r; \theta, K_a^k) = y_a^k(p_a, s, w; \theta, t^k, h^k)$$

for participants and zero otherwise. Also

$$\partial_r R_a^k(p_a, s, w, r; \theta, K_a^k) = T^k$$

for participants and zero otherwise. And finally the budget constraint contains no expression for the expenditures on the PIK portion of the land retirement. This follows by the assumption that the commodity stocks used to finance the PIK program do not comprise a portion of flow income or flow production. This closely approximates the situation that existed in the 1983-84 PIK program. When a nonrecourse loan program is introduced below this assumption will be modified.

Equilibrium

The equilibrium of the foregoing model is represented by the following equations

Mobile Factor

$$\partial_w R_a = -L$$

Agricultural Commodity

$$\partial_p R_a - D_a(p_a, p_n, E) = q_a(p_a^*/p_n^*)$$

Nonagricultural Commodity

$$\partial_p R_n - D_n(p_a, p_n, E) = q_n(p_a^*/p_n^*)$$

Consumer Budget Constraint

$$R_a + R_n - T = E$$

Government Budget Constraint

$$T = (\partial_s R_a)s + (\partial_r R_a)r$$

Assuming that there are no price barriers to trade, i.e., $p_i^* = p_i$ ($i = a, n$), this system of equations can be reduced to three equations by substituting the government budget constraint into the consumer-budget constraint to get

$$R_a - (\partial_s R_a)s - (\partial_r R_a)r + R_n = E$$

and then substituting this equation (which imposes a balanced trade condition on the model) using M as shorthand notation for the left-hand side of the above into the consumer demands to get

$$\frac{\partial R_a}{\partial w} + \frac{\partial R_n}{\partial w} = -L$$

$$\frac{\partial R_a}{\partial p_a} - D_a(p_a, p_n, M) = q_a(p_a/p_n)$$

$$\frac{\partial R_n}{\partial p_n} - D_n(p_a, p_n, M) = q_n(p_a/p_n).$$

This is a system of three equations in three variables (p_a , p_n , and w).

However, the homogeneity properties of the system permit further reduction by normalizing all prices. We normalize by the price of the variable factor. Therefore, set $w = 1$ in what follows and let p_a and p_n be counted in units normalized by w .

Comparative Static Results

This section analyzes a PIK program in the absence of a nonrecourse loan program. What emerges is a story about the interaction of two opposing effects. The first effect is that the PIK program by retiring acreage cuts production of the agricultural good and puts upward pressure on the price. Second, the PIK program replaces (at least partially) this production with CCC stocks, creating downward pressure on the price. Whether the agricultural good's price rises or falls depends on the outcome of these conflicting forces. Thus, the initial dispute by analysts over the direction of change in the market price for the 1983 program is not surprising.

This section proceeds as follows. Initially, the system described above is used to find the expressions for the price changes with respect to changes in the PIK payment rate, θ . It is shown that the sign of these expressions depend on certain critical parameters. Subsequently, special cases are

introduced to sign the critical parameters. The first special case corresponds to the introduction of the program, while the second is a change in a previously existing PIK program.

General Results

The impact of altering the PIK payment rate is found by differentiating the equilibrium conditions. This yields

$$\partial_{\theta} p_a = \left[- \left[\partial_{w\theta} R_a \left(-D_{12} + q'_a \frac{p_a}{p_n^2} \right) \right] + \partial_{wp_n} R_n \left(\partial_{p\theta} R_a \right) \right] \frac{1}{\Delta}$$

$$\partial_{\theta} p_n = \left[- \left(\partial_{p\theta} R_a \right) \left(\partial_{wp} R_a \right) + \partial_{w\theta} R_a \left(\partial_{pp} R_a - D_{11} - q'_a p_n^{-1} \right) \right] \frac{1}{\Delta}$$

where

$$\Delta = \partial_{wp} R_a \left(-D_{12} + q'_a \frac{p_a}{p_n^2} \right) - \partial_{wp_n} R_n \left(\partial_{pp} R_a - D_{11} - q'_a p_n^{-1} \right)$$

Consider the denominator, Δ . Because R_n is convex and positively linearly homogeneous in p and w , $\partial_{wp} R_n \leq 0$. The mobile factor is nonregressive in the production of the nonagricultural good. Because R_a is positively linearly homogeneous in p_a , s , w , and r , however, homogeneity does not insure that $\partial_{wp} R_a < 0$. However, by the derivative properties of R_a

$$\partial_{wp} R_a = \frac{\partial}{\partial p_a} \left[\sum_{i \in N} L_a^i(p_a, w; K_a^k) + \sum_{i \in P} L_a^i(p_a, w, s; \theta, t^k, K^k) \right].$$

Therefore, so long as an increase in the price of the agricultural commodity draws more of the mobile factor into the agricultural sector, $\partial_{wp} R_a < 0$.

Because one usually expects a price rise to divert resources toward production of the commodity whose price rises we make the assumption that $\partial_{wp} R_a < 0$.

The reason for the ambiguity is clear however. When p_a rises increased production of the agricultural good as well as an increase in the PIK payment

are more price attractive to farmers. But the latter can only come about as a result of retiring acreage which tends to diminish production and, therefore, the need for L by participants. Which effect dominates is an empirical question but we feel safe in making this assumption because $\frac{\partial}{\partial p} L_a^1(p, w; K_a^k) > 0$ in all cases which only the effect on $L_a^1(p_a, w, s, \theta, t^k, h^k)$ is ambiguous. The terms D_{ij} measure the own and cross price effects in domestic demand. Accordingly, the own-price effect, D_{11} , is nonpositive. With two consumption goods the cross-price effect and a negligible income effect, D_{12} , is nonnegative. The term $\partial_{pp} R_a$ is the own-price effect in production of the agricultural good and is nonnegative by R_a 's convexity. The remaining term in Δ is the change in excess demand for the agricultural good facing the country with respect to p_a/p_n . This effect, q'_a , is nonpositive. Δ is, therefore, nonnegative.

The major ambiguity in the above expressions occurs in the numerator. The effect of an increase in the PIK payment on the negative of the mobile factor demand in agriculture, $-L_a(\cdot)$, is given by $\partial_{w\theta} R_a$. As θ rises less and less agricultural acreage is used to produce y_a . To see why recall that

$$\partial_{\theta} R_a = p_a \sum_{k \in P} b^k(p_a, s, w; \theta, t^k, h^k)$$

and further that R_a is convex in θ . As a result, less and less of the mobile factor is needed to cooperate with land and one expects mobile factor use to decline. (However, it is also possible and likely to some extent that the remaining acreage is farmed more intensively as b^k rises so that more of the mobile factor is probably devoted to the remaining acres. But it seems implausible that this "intensification" effect dominates the cutback due to the diminished acreage associated with PIK cutback, so that we assume that $\partial_{w\theta} R_a > 0$.) The term $\partial_{p\theta} R_a$ measures the effect of the change in θ on total

supply. Because total supply consists of program and nonprogram production --

$$\sum_{i \in P} y_a^i(p_a, s, w; \theta, t^k, h^k) + \sum_{j \in N} y_a^j(p_a, w; K_a^j) \text{ -- as well as the PIK payments --}$$

$$\theta \sum_{i \in P} b^i(p_a, s, w; \theta, t^k, h^k) \text{ -- the term } \partial_{p\theta} R_a \text{ consists of three separate}$$

effects: The first is the impact of the increase in θ on agricultural production. This is negative as acreage is removed from production and $\partial_{w\theta} R_a > 0$. The second effect is the addition to total supply to pay for the PIK acreage already removed -- or $b^k(\cdot)$ in this formulation -- which is positive. The third influence is that an increase in θ encourages more land to be enrolled in the PIK program, as the PIK acreage rises so does the PIK payment further adding to supply. Thus, an increase in θ has one negative influence and two positive influences on total supply of the agricultural good. To go further with the analysis requires establishing scenarios which allow this conflict to be resolved.

Introduction of a PIK Program

The first case represents the introduction of a PIK program corresponding roughly to the situation in 1983. Analytically, introducing a PIK implies that initially the PIK payment rate equals zero. Hence, all comparative static expressions are evaluated at $\theta = 0$. When $\theta = 0$, both of the positive effects of a change in θ on total supply are eliminated. (Recall $b^k(\cdot) \Big|_{\theta=0} = 0$.)

All that remains is the PIK induced acreage decrease which decreases production. This implies $\partial_{p\theta} R_a \Big|_{\theta=0} \leq 0$:

$$\partial_{\theta} p_a \Big|_{\theta=0} \geq 0.$$

Introducing a PIK program raises, or leaves unchanged, p_a the agricultural price. What happens is that introducing a PIK program cuts total resources

allocated to agricultural production, thus diminishing production and putting upward pressure on p_a .

In general, the effect introducing a PIK program has on p_n is ambiguous. However, if the introduction of the PIK program leaves the sets P and N unchanged,³ then one can show that (see Appendix)

$$\left. \frac{\partial p_n}{\partial \theta} \right|_{\theta=0} = \frac{1}{\Delta} (\partial_{p\theta} R_a) (D_{11} + \frac{q'_a}{p_n}) \bigg|_{\theta=0}.$$

This expressions is positive. The reason is that the supply cutback in the agricultural market caused by introducing the PIK program causes p_a to rise. Consumers are diverted from the agricultural market toward the nonagricultural market. To soak up the initial excess demand thus created in the nonagricultural market its real price must rise also. Generally, however, one expects that resources diverted away from agricultural production as a result of the PIK-associated acreage retirement would flow to the nonagricultural market thus enhancing supply there and tending to mitigate the demand induced rise in p_n . Which effect dominates generally is ambiguous.

Agricultural producers separate naturally into participants and non-participants. The effect of introducing a PIK on nonparticipants is straightforward. Because their real land rents depend only on K_a^k and p_a (real rents equal $K_a^k G^k(p_a, 1)$) a rise in p_a implies that these rents rise. Hence, relative to the owners of the mobile factor nonparticipants are better off. Notice, moreover, that the percentage change in nonparticipant land rent is

$$\frac{\partial \ln \hat{G}^k}{\partial \theta} = \frac{\partial \ln G^k}{\partial \ln p_a} \frac{\partial \ln p_a}{\partial \theta}.$$

By Hotelling's lemma $\frac{\partial \ln G^k}{\partial \ln p_a}$ represents land revenue divided by rent which must exceed one. Hence, land rents rise at a more rapid rate for nonparticipants than p_a .

Participants' real rent also rises as a result of the introduction of PIK payments. Participants' per acre real rents are given by $g^k(p_a, 1, s; 0, t^k, h^k)$. Hence, introducing PIK payments has two effects, the direct change in rent caused by raising θ -- $\partial_\theta g^k(p_a, 1, s; 0, t^k, h^k)$ -- and an indirect effect induced through the rise in the real agricultural price -- $\partial_p g^k(p_a, 1, s; 0, t^k, h^k) \cdot \partial_\theta p_a$. The first of these effects is measured by PIK acreage while the second is measured by total supply for the participant. The former is zero when θ equals zero but the latter will generally be positive. Thus, the overall effect is positive.

Analytically it is interesting to note that the major effect on participant income of an introduction of a PIK program is not an input cost saving. In fact, the input cost saving must be exactly balanced by the change in revenues associated with introducing the PIK. To see this notice that the homogeneity and derivative properties of \hat{g}^k insure that

$$rt^k + (p_a + s)y_a^k + p_a \theta b^k - wL_a^k = \hat{g}^k.$$

Differentiating this expression with respect to θ and evaluating the result at $\theta = 0$ gives (holding p_a constant)

$$\left[(p_a + s)\partial_\theta y_a^k - w\partial_\theta L_a^k \right] \bigg|_{\theta=0} = 0.$$

Hence, any cost saving must be exactly balanced by a revenue loss. Participants instead gain as a result of the PIK-induced price rise. Thus, both participants and nonparticipants gain at the expense of owners of the mobile factor. Moreover, proceeding exactly as we did with nonparticipants it is easy to show that \hat{g}^k rises more rapidly than p_a .

The ambiguous impact of the PIK on p_n hampers interpretation of the consequences for nonagricultural producers. However, by the above one can say

that if the introduction of the PIK program does not affect the sets P and N, introducing a PIK increases real nonagricultural income. Moreover, proceeding as we did with nonparticipants one can easily show that real nonagricultural income grows at a faster rate than p_n . Turning to the owners of the mobile factor of production, the above indicates that introducing a PIK program diminishes their purchasing power in terms of the agricultural commodity. Thus, relative to participants and nonparticipants their relative income falls. However, because of the ambiguities involved it is not clear whether mobile factor owners gain relative to participants and owners of K_n . And although we have not explicitly modelled the foreign component of the world economy, our results indicate that international producers of the agricultural commodity gain from introduction of the PIK program. Essentially what happens is that they "free-ride" on the price rise caused by the domestic country's PIK induced supply curtailment. Of course, this result is not surprising given the wide recognition that the biggest gainers from U.S. withdrawal from international grain markets are foreign producers.

Introducing the PIK program causes the real agricultural price to rise because the output contraction effect dominates the supply expansion effects. Farmers always benefit from the price increase.

Augmenting an Existing PIK

If a PIK program already exists and if it is increased then raising θ corresponds to making the PIK program more lucrative. The supply enhancing terms of the expression $\partial_{p\theta} R_a$ then become important to the analysis. As long as the output effect of increasing the PIK payment rate dominates, i.e.,

$\left| \frac{\partial Y_a^k}{\partial \theta} \right| \geq b^k(\cdot) + \theta \frac{\partial b^k}{\partial \theta}$, the previous results on p_a still apply. (In other words as long as more production is curtailed then new PIK commodities come

onto the market.) If the effects in $\partial_{p\theta} R_a$ cancel each other -- this last expression holds as an equality -- then p_a will rise. The real return to agricultural land also rises and that rise will exceed the increase in p_a .

If the supply enhancing effects of the PIK program dominate the output reduction caused by acreage retirement ($\partial_{p\theta} R_a > 0$) -- the case where the initial PIK acreage retirement or θ is large relative to total production -- then the real agricultural price can fall. For this to occur, the following condition must be satisfied:

$$\frac{-\frac{\partial}{\partial \theta} R_a}{\partial_{p\theta} R_a} \geq \frac{-\frac{\partial}{\partial p} R_n}{\left(-C_{12} + q'_a \frac{p_a}{p^2}\right)}$$

where it is explicitly assumed that $\partial_{p\theta} R_a > 0$. Homogeneity and the basic assumptions of the model insure that the right hand side of this last expression is negative. (The numerator representing $\partial_{p_n} L(p_n, w, K_n)$ is always positive while the denominator is negative.) Hence, for p_a to fall the numerator of the left hand side can be either positive or negative. If the numerator of the left hand side is positive, raising θ leads to such intensive farming on the remaining acreage that more (not less) of the mobile factor is used. The inequality is thus always satisfied. The reason is that both the effect on resource utilization and total supply tend to enhance supply implying a price depressing effect.

If agricultural demand for the mobile factor is discouraged by an increase in θ but is very inelastic with respect to θ the left hand side approaches zero and one again expects the inequality to be satisfied. To see why this makes sense notice that given that $\partial_{p\theta} R_a > 0$, the only price depressing effect in the agricultural market is a possible withdrawal of the mobile factor from production. But to now actually increase p_a this effect

must be large relative to $\partial_{p\theta} R_a$, if it is not then the price will fall.

Also notice that if $\partial_{p\theta} R_a > 0$ then solving for the real price change for the nonagricultural good shows p_n rising in this case. In the earlier case of introducing the PIK, the term $\partial_{p\theta} R_a$ is negative implying generally that $\partial_{\theta} p_n$ is ambiguous. But when $\partial_{p\theta} R_a > 0$, an unambiguous result is obtained. Hence in this case the quasi-rent to K_n rises and rises faster than does p_n . Owners of that factor of production gain.

The effects on nonparticipating farmers depends upon what happens to p_a . As before, the rent to participating farmers is even more complex. Differentiating the per acre rent with respect to θ gives the same two terms as before -- $\partial_{\theta} g^k$ and $(\partial_p g^k \cdot \partial_{\theta} p_a)$. The first term which by the derivative properties is equivalent to $p_a \cdot b^k(\cdot)$ is always nonnegative. However, as discussed above the second term is ambiguous. If $\partial_{p\theta} R_a > 0$, the agricultural price falls which induces a corresponding drop in agricultural rents. Two things count here: how large the original PIK acreage is relative to total supply and how responsive the agricultural price is to the change in θ . Using the derivative properties of g^k we see that the effect on agricultural per acre rent is measured by $p_a b^k + (y_a^k + \theta b^k) \partial_{\theta} p_a$. Hence, a necessary and sufficient condition for the k th farmer's rent to rise is that

$$\frac{\theta b^k}{y_a^k + \theta b^k} > - (\partial_{\theta} p_a) \frac{\theta}{p_a}.$$

The term on the left-hand side of this expression is naturally positive and less than one. Hence, if the agricultural price rises it is obvious (see above) that rent goes up, but if p_a falls the term on the right is positive. Thus when p_a falls as a result of augmenting the PIK the percentage fall in p_a must be less than the PIK share in total supply if agricultural rent is to rise to farmer k .

Temporal Payment-in-kind

The model so far assumes that the PIK stocks used for the subsidy are from previously accumulated public stocks; hence, they do not affect the government's flow income. While that assumption is probably appropriate for the introduction of a new PIK program (given the preexistence of a support program) or for increasing the payment rate during the early part of the program, a continual PIK operation requires a change in the model because payments will be made from public stocks continuously obtained through a price support program. Because the PIK program affects the real agricultural price, it also affects any price support operations and thus stock acquisitions from which the payment is made. This interaction needs to be reflected in the analysis for a PIK involving multiple time periods.

Determining the longer run consequences of a PIK requires introducing government price-support operations. A price-support program is, therefore, introduced into the model by assuming the government will accept delivery of all commodities offered by farmers at a guaranteed price, the support rate -- denoted p_ℓ .⁴ In the United States, since the CCC often obtains program crops at prices above the loan or support rate, these purchases are modeled by a continuous function -- $i(p_a/p_\ell)$. This function has two assumptions to reflect the actual operation of the U.S. nonrecourse loan program. First, $i'(\alpha) < 0$, or that accumulation of public stocks through forfeited loans declines as the market price of agricultural commodities rises relative to the support rate. Second, it is assumed that the CCC will accept all volume offered by farmers at the support rate. That is $i(\cdot)$ becomes perfectly elastic at p_ℓ . This is expressed as:

$$\lim_{\alpha \rightarrow 1} i(\alpha) = -\infty$$

The existence of this program changes the government's financial constraint. That constraint shown earlier now includes the cost of price support operations, or $p_\ell i(p_a/p_\ell)$. Unlike the previous scenarios, the PIK will now alter the government's current flow of income.

Another change in the model is necessary to capture the effects of price support operations. The market clearing for the agricultural commodity must now include government stock acquisitions. Thus, that condition is rewritten as:

$$\partial_p R_a - D_a(p_a, p_n, M) - i(p_a/p_\ell) = q_a(p_a/p_n)$$

where

$$M = R_a + R_n - (\partial_s R_a)s - (\partial_r R_a)r - p_\ell i(p_a/p_\ell).$$

The impact of the payment-in-kind program is again found by differentiating the equilibrium conditions with respect to θ . This gives:

$$\begin{aligned} \partial_\theta p_a &= \frac{1}{\phi} \left[-\partial_{w\theta} R_a \left(-C_{12} + q'_a \frac{p_a}{p_n^2} \right) + \partial_{wp} R_n \left(\partial_{p\theta} R_a \right) \right] \\ \partial_\theta p_n &= \frac{1}{\phi} \left[\partial_{wp} R_a \left(\partial_{p\theta} R_a \right) + \partial_{wp} R_a \left(\partial_{pp} R_a - D_{11} - q'_a p_n^{-1} \right) \right. \\ &\quad \left. - \partial_{w\theta} R_a i'(\cdot) p_\ell^{-1} \right] \end{aligned}$$

where,

$$\phi = \Delta + \frac{i'(\cdot)}{p_\ell} (\partial_{wp} R_n).$$

From the previous results, $\Delta \geq 0$. Since $i'(\cdot) \leq 0$ and $\partial_{wp} R_n \leq 0$, $\phi > \Delta$. This means that, if evaluated at the same prices, the denominators of the above expressions are larger than the ones calculated earlier. Comparing the numerators for the change in p_a shows that they are the same with and without the support program. Thus, the support program dampens change in the real

agricultural price (relative to no support program) through the larger denominator. Because the government stocks function satisfies:

$$\lim_{\alpha \rightarrow 1} i'(\alpha) = -\infty$$

then $\lim_{\alpha \rightarrow 1} \phi(\alpha) \rightarrow \infty$. Thus in the limit, when the market price is "riding" the loan rate ($\alpha \rightarrow 1$), the PIK payment has no effect on p_a or p_n . Rents for nonparticipants are unchanged. Participant's rent now rises unambiguously with an increase in θ . Because there is no effect on p_a the PIK payment approximates a lump-sum transfer from which participants unambiguously gain.

An important aspect of any PIK program is its impact on public stocks. One of the objectives of the 1983 U.S. PIK program was to curtail CCC stocks. Net stock acquisitions by the government consist of two parts -- acquisitions, through price support operations, $i(\cdot)$ -- and the stocks released to payments -- $\theta \sum_{k=1}^m b^k(\cdot)$. Differentiating with respect to θ gives:

$$\frac{\partial TS}{\partial \theta} = \frac{i'(\cdot)}{p_\ell} \partial_\theta p_a - \sum_{k \in P} b^k(\cdot) - \theta \sum_{k \in P} \frac{\partial b^k}{\partial \theta}.$$

Increasing θ has three separate effects on government stocks: a price-induced accumulation (disaccumulation) of government stocks through the price-support operation, this is the first term on the right; a disaccumulation of stocks as the PIK rate is raised on existing PIK acreage; and an expansionary effect on PIK acreage which also decreases government stocks. The second two effects are always negative so that a sufficient condition for government stocks not to increase is that $\partial_\theta p_a > 0$. However, if p_a falls as θ rises, some inventories will be reaccumulated through price-support operations and the overall effect is ambiguous.

When a PIK is just introduced, it was shown that $\partial_\theta p_a > 0$ so no additional stocks are acquired through price-support operations. Moreover,

the second two effects are zero so that introducing a PIK program must diminish government stocks.

When the price is "riding the loan rate", $\partial_{\theta} p_a$ approaches zero asymptotically and there is no impact on public stocks from the first effect. And again one sees that government stocks diminish.

Consequently, when the PIK program is new or the market price is "riding the loan rate" government-owned stocks are reduced. If the PIK program is new and p_a is "riding" p_ℓ there is no change in government stocks because any stocks released are immediately reacquired by the government. The expression for the change in total stocks suggests that for a preexisting PIK stocks might rise if the real agricultural price falls sharply enough in response to an increase in θ and $i'(\cdot)$ is strongly negative. So long as altering θ leaves the sets P and N unchanged, however, this is not the case. $dTS/d\theta$ is unambiguously negative if:

$$\frac{\partial_{wp} R_n i'(\cdot)}{p_\ell \phi} - 1 < 0.$$

Rearranging this expression gives

$$\frac{\partial_{wp} R_n i'(\cdot)}{p_\ell \phi} - 1 = -\frac{\Delta}{\phi} \leq 0.$$

Thus, government stocks cannot rise because the direct effect of giving away stocks in the PIK program overwhelms any possible indirect effect through reaccumulation in response to a price decrease.

Turning to p_n , incorporating the support program changes both the denominator of the comparative static expression and the numerator. Evaluating all prices at the same level and subtracting the expression for the price change without the loan program, $\left(\frac{dp_n}{d\theta}\right)_0$, from that with the loan program, $\left(\frac{dp_n}{d\theta}\right)_1$, gives:

$$\left(\frac{dp_n}{d\theta}\right)_1 - \left(\frac{dp_n}{d\theta}\right)_0 = \frac{i'(\cdot)\partial_{wp} R_a}{\Delta p_\ell \phi} \left[\partial_{wp} R_n \left(\partial_{p_a} \theta R_a \right) - \left(\partial_{w\theta} R_a \right) \left(-D_{12} + q'_a \frac{p_a}{p_n^2} \right) \right]$$

The sign of this expression depends on the sign of $\partial_{p\theta} R_a$ which is determined by the effect of changes in θ on total agricultural supply. These are the same factors determining $\partial_{\theta} p_a$. If $\left| \frac{\partial y_a^k}{\partial \theta} \right| > b^k(\cdot) + \theta \frac{\partial b^k}{\partial \theta}$ then the increase in θ cuts total supply of the agricultural good and $\partial_{p\theta} R_a < 0$. In this case, the influence of a change in the payment rate on the nonagricultural price will be greater with the support program compared to the price change without the support program. If that term is positive, the differences in the impact of the PIK payment rate increase are ambiguous and depend on the price responsiveness of domestic and export demand as well as the relative influence of the nonagricultural price and θ on the use of the mobile factor. In the situation where the PIK is introduced, the result is clear as the loan program magnifies the adjustment in the nonagricultural good's price.

Conclusion

This paper considers the effect of a payment-in-kind program where the total payment is linked to farmers removing acreage from production. The payment then is a bribe to the farmer for this land retirement. Such a program creates an inherent conflict which must be resolved if the impacts on the economy are to be determined. One effect is that land is removed from agricultural production tends to lower output and put upward pressure on farm prices. Meanwhile this acreage was bribed out of production through the release of public stocks. These stocks add to supply and create downward pressure on the price.

Using a two-good general equilibrium model, this paper shows that the balance between these effects is the critical factor in determining the outcome. When the PIK program is first introduced, the output contraction effect dominates and the real agricultural price rises. As a result, the rents to nonparticipants and participants in the program rises.

If the payment rate is increased in the context of an existing program, the impact on the agricultural price is ambiguous. As long as the output effect dominates the supply enhancing stock release, real agricultural prices will rise. In this case, the real price of the nonagricultural good rises as well. Even if the real agricultural price falls, the rent to the fixed agricultural factor may rise because of the revenues generated by the PIK payment.

When a nonrecourse loan program is introduced into the model, the impact of the PIK on the agricultural price is dampened. Governments stocks cannot increase in such a program. If the price is riding the loan rate when the PIK program is introduced, stocks released are reaccumulated. In other circumstances, there is a net reduction in public stocks.

Endnotes

¹To conserve on notation and simplify the presentation we assume with little loss of generality that K_a^k represents the farmer's base acreage. The analysis that follows changes only slightly when we suppose that the acreage base does not equal K_a^k .

²See the Appendix for a discussion of this Assumption.

³Basically this implies that at the time of program introduction no producer is just indifferent between participating and nonparticipating.

⁴For most U.S. agricultural commodities price supports come in the form of nonrecourse loans which are only available to participating producers. Although the current analysis is suggestive of what would happen with a nonrecourse loan program, it does not apply exactly. However, approximating the nonrecourse loan program exactly would greatly increase the mathematical complexity of the model.

Appendix

The function $R_a(p_a, w, s, r; \theta, K_a)$

By definition

$$\begin{aligned} R_a(\cdot) &= \sum_k \text{Max}\{K_a^k G^k, r t^k + K_a^k g^k\} \\ &= \sum_k K_a^k \text{Max}\{G^k, r t^k + g^k\}. \end{aligned}$$

To establish positive linear homogeneity in p_a, w, s, r note that

$$\begin{aligned} &K_a^k \text{Max}\{G^k(\mu p_a, \mu w), \mu r t^k + g^k(\mu p_a, \mu s, \mu w; \theta, t^k, h^k)\} \\ &= K_a^k \text{Max}\{\mu G^k, \mu r t^k + \mu g^k\} \\ &= \mu K_a^k \text{Max}\{G^k, r t^k + g^k\} \end{aligned}$$

by the homogeneity properties of G^k and g^k . To complete the proof simply recall that summation preserves linear homogeneity. To establish convexity note that

$$\text{Max}\{G^k, r t^k + g^k\} = \text{Max}\{\delta G^k + (1 - \delta)(r t^k + g^k)\}, \quad \delta \in [0, 1]$$

The maximand of the right-hand side of the above satisfies convexity because it is the positively weighted average of two convex functions. Thus the optimal value of the maximand must inherit convexity by standard results in optimization theory (Chambers, p. 316). Convexity of $R_a(\cdot)$ is then obvious. So long as we restrict ourselves to the positive orthant convexity insures continuity of $R_a(\cdot)$ in p_a, w, s, r , and θ . In general, however, each $\text{Max}\{G^k, r t^k + g^k\}$ function will be nondifferentiable at those parameter values for which

$$G^k = r t^k + g^k$$

because no unique solution exists to the optimization problem. (Put another way δ can assume any value in the unit interval without changing the value of

the objective function. In all other cases $\delta = 0$ or $\delta = 1$.) But if a unique solution to the optimization problem does exist and G^k and g^k are themselves differentiable then $\delta = 0$ or $\delta = 1$ and

$$\partial_z \text{Max}\{G^k, rt^k + g^k\} = \delta \partial_z G^k + (1 - \delta) \partial_z g^k$$

for $z = (p_a, w, s, \theta)$ by an application of the envelope theorem.

Because R_a is defined as a sum across a large number of farmers each of which possesses a different technology, at any particular (p_a, w, s, θ) constellation only a small number of the $\text{Max}\{G^k, rt^k + g^k\}$ functions should be nondifferentiable. Therefore, little generality seems lost by assuming that R_a which is continuous is also differentiable.

$$\text{Derivation of } \partial_{\theta} p_n \Big|_{\theta=0} = \frac{1}{\Delta} (\partial_{p\theta} R_a) \left(D_{11} + \frac{q'_a p_a}{p_n} \right) \Big|_{\theta=0}.$$

If changing θ leaves P and N unchanged then

$$\partial_{w\theta} R_a = \sum_{k \in P} \partial_{w\theta} \hat{g}^k$$

because $\partial_{w\theta} G^k = 0$. Recall that $\hat{g}^k = K_a^k(rt^k + g^k)$ where g^k is positively

linearly homogeneous in p_a , s , and w . This latter fact gives

$$p_a (\partial_p g^k) + s (\partial_s g^k) + w (\partial_w g^k) = g^k$$

which by the derivative properties of g_k implies

$$p_a \theta b^k + (p_a + s) y_a^k(p_a, w, s; \theta, t^k, h^k) - w L_a^k(p_a, s, w; \theta, t^k, h^k) = g^k.$$

Differentiate this last expression with respect to θ and evaluate the result at $\theta = 0$ to get

$$\left[(p_a + s) \frac{\partial y_a^k}{\partial \theta} - w \frac{\partial L_a^k}{\partial \theta} \right] \Big|_{\theta=0} = 0$$

whence

$$(p_a + s) \left. \frac{\partial R_a}{\partial \theta} \right|_{\theta=0} + w \left. \frac{\partial R_a}{\partial \theta} \right|_{\theta=0} = 0.$$

Now differentiate the earlier expression with respect to p_a and evaluate the result at $\theta = 0$ to get

$$(p_a + s) \left(\left. \frac{\partial R_a}{\partial p} \right|_{\theta=0} \right) + \left(\left. \frac{\partial R_a}{\partial p} \right|_{\theta=0} \right) \cdot w = 0.$$

Recalling that $w = 1$ by the normalization then gives the expression in the text when these equations are substituted into the general comparative static expression. Also notice that if the sets P and N are unchanged by changes in p_a then this result also implies $\left. \frac{\partial R_a}{\partial p} \right|_{\theta=0} \leq 0$.

