Dynamic Model of Capital Structure for the Noncorporate Firm

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Abstract: A dynamic model of capital structure for the noncorporate farm is developed and analyzed. In this model, the probability of bankruptcy increases as the farmer's debt/asset ratio increases. Funds invested outside agriculture earn a riskless rate of return. The farmer/proprietor is also able to obtain a riskless wage from off-farm employment; this wage may differ from the implicit wage received as a farmer. The expected return to equity on the farm depends on the leverage. The model examines the effects on optimal capital structure of (1) bankruptcy risk, (2) the difference between the riskless rate and the expected return in agriculture, and (3) the difference between the off-farm wage and the implicit on-farm wage. The third element introduces an incentive for the proprietor to change leverage over the proprietor's lifetime; this occurs if the difference between the wages is nonzero but constant. If the difference between the wages is zero, a constant leverage is optimal under reasonable circumstances. The model predicts that older farmers require more leverage to induce them to remain in farming and that they tend to reduce their leverage as retirement approaches. The model is tested with cross-sectional data.

Introduction

Current financial difficulties in commercial agriculture highlight the importance of the capital structure of agricultural firms. This paper develops a theory to explain the noncorporate firm's choice of capital structure at a point in time and the evolution of capital structure over the lifetime of the proprietor.

The Modigliani-Miller theorem (Modigliani and Miller, 1958) provides conditions under which the value of a publicly traded firm is independent of its capital structure. Hellwig (1981) reexamines the theorem and concludes, "From a practical point of view, it seems reasonable to suppose that the Modigliani-Miller principle fails when there is a chance of bankruptcy" (p. 167). Myers (1984) reviews the various explanations that have been advanced to explain the capital structure of publicly traded firms. In answer to the question, "How do firms choose their capital structures?," he replies, "We don't know" (p. 575).

The theory of the capital structure of public forms is, at best, unresolved; theories about the capital structure of noncorporate firms are virtually nonexistent. The incompleteness of markets for noncorporate equity means that one cannot appeal to market arbitrage forces, which form the basis for models in the Modigliani-Miller tradition. The market value of proprietary equity is the liquidating value of the firm (the market value of assets minus liabilities). The value of proprietary equity is determined in the asset markets and does not necessarily reflect the management of the individual firm. Rather than attempting to maximize the market value of equity, the proprietor is likely to concentrate on the stream of income that can be withdrawn from the firm or on the expected liquidation value of the firm at retirement. The choice of capital structure affects these goals.

The farmer's only source of external financing is assumed to be debt; given a level of equity, the farmer decides whether to remain in farming and, if so, how much debt to acquire. Since debt is, by an overwhelming margin, the most significant source of external finance for US farms, the assumption is reasonable. Innes (1987) models the debt/external equity option for firms as an agency problem; his model provides conditions under which an all-debt contract results in equilibrium.

By assumption, bankruptcy occurs when debt equals assets. Bulow and Shoven (1978) demonstrate that in a variety of circumstances this assumption does not provide the optimal foreclosure rule. Their model describes the publicly held firm, but similar arguments hold for the proprietary firm. For the purposes of the present model, foreclosures must occur under prescribed conditions; for simplicity, this is taken to be where assets net of debt are less than or equal to zero.

The chief concern here is with the dynamics of capital structure. In order to keep the model tractable, a simple description of the stochastics is used and the farmer is assumed to be risk neutral. This permits identification on the effects on capital structure of the:

underlying riskiness of the enterprise, constraints on the reinvestment of income, level of equity, time until retirement, and opportunity cost of managing the enterprise.

The next section assumes that the opportunity cost of managing the enterprise is zero. This results in a very simple problem for which a closed-form solution can be obtained. The optimal leverage is independent of equity; under plausible circumstances, the optimal leverage is a constant that balances risks and returns.

In the subsequent section, the opportunity cost of managing the firm is assumed to be positive; an approximate solution for this problem is obtained. Young farmers who go bankrupt have greater possibilities of starting a second career than do old farmers, so the opportunity cost of managing the firm decreases over time. The optimal leverage depends on the level of equity and the time until retirement. This model provides two explanations for why young farmers would be expected to be more highly leveraged than old farmers: they tend to have less equity, and their opportunities outside farming are greater. In neither model is it optimal to plan to retire free of debt.

The model abstracts from small variations in income in order to concentrate on a single catastrophic event, bankruptcy. The probability that bankruptcy occurs over a unit of time depends on the leverage. For example, a firm with a debt/asset ratio of 0.8 will not survive a 25-percent loss in assets; the same loss with a debt/asset ratio of 0.7 is tolerable. If the firm does not go bankrupt, it earns a nonstochastic rate of return on assets. The evolution of the equity in continuous time is modelled using a jump process. This model is consistent with Bulow and Shoven's description of a financial crisis. They compare such a crisis with an earthquake. "The important assumption is that the expected future productivity of the firm's plant decreased discontinuously" (note 12, p. 442). This corresponds in the present model to a discontinuous decrease in the value of assets due, for example, to a fall in land prices.

The following two sections elaborate the versions of the present model with and without opportunity cost of managing the firm. Proofs are contained in a longer version of this paper available upon request. The subsequent section provides an empirical test of the hypotheses implied by the theory. A conclusion follows.

Simple Model with Bankruptcy Risk

The farmer's equity at a point in time is $E(t)$ and his debt/asset ratio is δ. The probability of bankruptcy occurring over an interval of time dt is proportional to the increasing convex function $\gamma(\delta)$.

The rate of return on equity net of borrowing costs is the concave function $I(\delta)$ that first increases and then decreases. If the rate of return on assets exceeds the cost of borrowing, an increase in financial leverage increases the rate of return on equity since more assets work for each dollar of equity; this is the leverage multiplier effect. The expected net rate of return on assets declines, however, because of the higher borrowing costs associated with the firm's increased probability of bankruptcy. As long as the multiplier effect exceeds the increased borrowing costs, $I(\ )$ increases; at some value of δ less than 1, $I(\ )$ reaches a maximum and thereafter decreases.

The proportion of income withdrawn from the firm is w. With these definitions, the stochastic differential for equity is:

$$(1)\ dE = (1 - w)\, I[\delta(t)]\, E(t)\, dt + E d\pi,$$

where:

$$Pr(d\pi=-1) = \gamma[\delta(t)]\, dt + o(dt), \text{ and } Pr(d\pi=0) = 1 - \gamma[\delta(t)]\, dt + o(dt),$$

where $o(dt)$ denotes terms of order dt. The deterministic portion of dE is $(1-w)IEdt$, which is the retained portion of earnings over an interval dt; when $d\pi = 0$, equity increases at the expected rate; $(d\pi = -1)$ means that the firm is bankrupt. Equation (1) is a jump process.

The manager is assumed to maximize the expectation of the present value of withdrawals plus the liquidation value at retirement. At time t, given equity $E(t)$, his problem is:

$$(2)\ J(E,t) = \max_{\substack{\delta\epsilon[0,1] \\ w\epsilon[\underline{w},\infty]}} \xi_t\{\int_t^T e^{-\rho s} wIEds + e^{-\rho T}E(T)\},$$

subject to (1). The value function $J(\ )$ gives the farmer's expectation of the present value of the firm. Provided that this is greater than the liquidation value, $e^{-\rho t}E$, the farmer wishes to remain in business.

The optimization problem in (2) can be reinterpreted as one of maximizing the expected value of a retirement portfolio consisting of equity in the firm and riskless bonds. With this interpretation, a withdrawal of $wIEdt$ implies an investment of the same amount at the riskless rate ρ.

The withdrawal rate w is unbounded above, so that at any point the farmer can liquidate the firm and receive equity $E(t)$. The lower bound on w is $\underline{w}$. In many cases, one can reasonably assume that $\underline{w} = 0$; i.e., the farmer is capable of retaining all earnings of equity but has no source of outside funds other than debt. A positive value of $\underline{w}$ is appropriate if the proprietor is required to consume more as the proprietor's equity increases, or if the proprietor is committed to a particular balance in the retirement fund consisting of riskless bonds and equity in the firm. A negative value of $\underline{w}$ implies that the proprietor is able to obtain funds at the riskless rate (e.g., by drawing on the proprietor's retirement fund). The additional equity the proprietor can obtain in this manner is proportional to the proprietor's earnings, $I(\ )E$. $\underline{w}$ is assumed to be given, although in a more general model it might be regarded as a control variable.

The important assumptions implicit in (1) and (2) are:

(i) The farmer is risk neutral.

(ii) The world has two states: either bankruptcy occurs or the farmer earns a nonstochastic return.

(iii) Bankruptcy occurs whenever $E(t) \leq 0$.

(iv) Functions $I(\ )$ and $\gamma(\ )$ do not depend on time.

For the remainder of this section, the following is also assumed:

(v) The value of the programme without equity is zero: $J(0, t) = 0$.

Assumption (v) is not innocuous. It implies that the opportunity cost of the farmer's labour equals the (implicit) wage received as a proprietor. More typically, the wage under alternative employment may be greater than the implicit wage. The effect of relaxing assumption (v) is considered in the next section. The assumption is useful because it clarifies the effect of the functions $I(\ )$ and $\gamma(\ )$ and the parameter $\underline{w}$ on the choice of δ.

The quantity $\hat{\delta}$ is defined as the leverage that maximizes $I(\delta) - \gamma(\delta)$. At $\hat{\delta}$, the marginal risk and the marginal expected return of increasing the leverage are equal. The quantity $I(\hat{\delta}) - \gamma(\hat{\delta})$ gives the maximized risk-adjusted expected rate of return to equity. The optimal leverage equals $\hat{\delta}$ at retirement, provided that the firm is still in operation. The optimal leverage and withdrawal policy is described in the following proposition.

PROPOSITION 1.

(i) If $\rho = I(\hat{\delta}) - \gamma(\hat{\delta})$, setting $\delta = \hat{\delta}$ is optimal; withdrawal policy is irrelevant.

(ii) If $\rho > I(\hat{\delta}) - \gamma(\hat{\delta})$, liquidating immediately is optimal.

(iii) If $\rho < I(\hat{\delta}) - \gamma(\hat{\delta})$, setting $w = \underline{w}$ is optimal. (a) For $\underline{w} > 0$, the optimal δ increases over time; (b) for $\underline{w} = 0$, maintaining $\delta = \hat{\delta}$ is optimal; and (c) for $\underline{w} < 0$, the optimal δ decreases over time.

This proposition has an intuitive interpretation. If the discount rate ρ equals the risk adjusted rate $I - \gamma$, where the latter is maximized, then the farmer is indifferent between liquidation and staying in business. If asset markets were perfect and management ability homogenous, competitive pressure would cause the liquidation value of the firm to equal its value as a going concern. If the discount rate is greater than the maximized value of the risk-adjusted expected return, the farmer does better to liquidate immediately.

In the case where the discount rate is less than the maximized risk-adjusted expected rate, the value of the firm as a going concern exceeds the liquidation value. The optimal leverage depends on the lower bound of the proportion of withdrawals. If $\underline{w} = 0$, the farmer chooses the leverage that maximizes the risk-adjusted expected return on equity and holds it constant.

Bankruptcy Risk with Positive Opportunity Cost

This section replaces assumption *(v)* with

(v′) $J(0, t) = e^{-\rho t}(c/\rho)(1-e^{-\rho\tau}),$

which uses the definition $\tau = T - t$, the time to retirement. This assumption can be interpreted as the statement that either the wage in the alternative employment exceeds the implicit wage by the amount c or that, in the alternative employment, the farmer accumulates a pension fund at the rate c. Replacing the constant c by a function $c(\tau)$ would complicate the solution without adding insight. The following assumptions are also made:

(vi) $I(\hat{\delta}) > \rho + \gamma(\hat{\delta}) \equiv \alpha.$

(vii) $\underline{w} = 0.$

An appropriate solution to this problem is obtained, which is valid for small c. This solution implies the following two results:

PROPOSITION 2. Older farmers require higher levels of equity to induce them to remain in farming.

PROPOSITION 3.
(i) Farmers with high equity are less highly leveraged than farmers with low equity.
(ii) Given the same level of equity, old farmers are less highly leveraged than middle-aged farmers; the latter may be more highly leveraged than very young farmers.
(iii) An individual farmer tends to decrease leverage over time, conditional on not going bankrupt.

Empirical Test of Model

The principle implications of the model are stated in Propositions 2 and 3. Survey data from Arkansas farmers in 1986 (Collins, 1987) was used to test the model. The survey consisted of a stratified random sample of 2,500 farms selected from the nine crop and livestock reporting districts in Arkansas; the survey resulted in 989 usable survey forms. The average annual earnings of farm labourers in each county of Arkansas (Bureau of the Census, 1982) was used as a proxy for the individual farmer's opportunity cost (the constant c in the previous section).

Since the data did not include time series, one could not determine the extent to which the probability of future financial difficulty depends on current capital structure. That is, the function $\gamma(\delta)$ could not be estimated.

The model assumes constant returns to scale. Previous empirical tests of this hypothesis have been ambiguous. Using OLS, we regressed the rate of return on equity, defined as net cash flows divided by equity, against δ (debt/assets), δ^2, assets, and age. The results strongly support the hypotheses that the return to equity is increasing in leverage and is independent of scale. Weak evidence exists that $I(\delta)$ is concave. The F statistic for the null hypothesis that all coefficients are insignificant exceeds 30, so that hypothesis is strongly rejected.

The data suggest that younger farmers have a higher expected rate of return on equity than older farmers; this may be due to different levels of education. This does not contradict the model, which allows the function $I(\delta)$ to vary across individuals; it does, however, suggest an additional reason why older farmers may be less highly leveraged.

Proposition 2 states that older farmers require more equity to keep them from retiring than do younger farmers. The minimum equity level $\bar{E}$ is directly observable for those who choose to leave farming voluntarily. A sample of farmers who leave farming voluntarily was created by taking farmers who leave because of financial problems, better alternative occupation, or other nonhealth related reasons and who had positive equity. Farmers who indicated they were leaving because of health problems or retirement or those who had nonpositive equity were eliminated from the sample. The results are shown in Table 1. The coefficient on age has the expected sign and is significant at the 3.5-percent level for a one-tailed test. This provides a moderate level of support for Proposition 2.

Proposition 3 states that older farmers and farmers with higher levels of equity tend to be less highly leveraged. In addition, a higher opportunity cost implies higher optimal leverage. The observed leverage was regressed against age, equity, and opportunity cost. All coefficients were highly significant and had the expected sign. Since equity is an explanatory variable and also appears in the denominator of the independent variable, the potential exists for spurious correlation. Proposition 3 was therefore interpreted in terms of debt rather than debt/assets. The proposition implies that the elasticity of debt with respect to equity is less than 1, that the derivative of debt with respect to age is negative, and that the derivative with respect to opportunity cost is positive. Debt was regressed against age, equity, and opportunity cost; we used Tobit analysis since debt is constrained to be nonnegative. The results, shown in Table 2, are consistent with the theory. The derivatives of debt with respect to age and opportunity cost have the expected sign and are significant. The elasticity of debt with respect to equity was calculated using Thraen, Hammond, and Buxton (1978). The elasticity at the sample mean was 0.0526 and was less than 0.2 at all data points.

Table 1—Dependent Variable: Equity

Independent Variable	$\hat{\beta}$	Standard Error	t
Intercept	−176,403	--	--
Age	7,307.6	3,910.69	1.87

[R^2 = 0.0907; R^2 = 0.0647; Equity = $\beta_0 + \beta_1 AGE + \varepsilon$; N = 36.]

Table 2—Dependent Variable: Debt

Independent Variable	Estimated Coefficient	T-Ratio
Constant	12.5256	2.570
Opportunity cost ($1,000)	3.12345	5.263
Age (years)	−0.384612	−4.427
Equity ($10,000)	0.048492	10.974

Conclusion

This paper has provided a model to explain the evolution of the capital structure of a noncorporate firm over the lifetime of the proprietor. Even if the expected rate of return and probability of failure are stationary, optimal capital structure is likely to change as retirement approaches. The cost of failure is likely to be greater for older farmers, because their opportunities for alternative employment are less attractive; to the extent that they have more equity, they also have more to loose.

The theory is consistent with cross-sectional data of Arkansas farmers. A more comprehensive test will require time series data.

Note

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DISCUSSION OPENING—*Shankar Narayanan* (Agriculture Canada)

Karp and Collins' model of optimal capital structure for the noncorporate farm firm is conceptually sound in terms of investment and finance theory and encompasses key capital and investment variables; i.e., net cash flow, debt/equity, rate of return to equity, opportunity costs of the absence of market arbitrage for noncorporate assets, and risk of bankruptcy. The assumptions and limiting conditions also appear quite relevant. The prepositions that follow validate traditional theories and observations relating to business and capital investment. Specifically, propositions 2 and 3 follow from the analysis of optimal leverage under bankruptcy risk with positive opportunity cost to the owner, and the dynamic relationship between current financial leverage and future financial condition present very useful and interesting policy and programme implications.

Including a well-defined net revenue function that incorporates unrealized capital appreciation in the rate of return to equity component of the value function *J( )* is suggested for consideration. Such explicit representation of the production environment will not only help to enhance the model's analytical ability but also facilitate obtaining determinate measurements of expected future values of prices (inputs and outputs) and capital gains and losses from available medium and long-term forecasts of the market situation and macroeconomic environment.

By the same token, a discount rate function can also be endogenized to reduce the dependence on exogenous expected discount rate values used for present value purposes. This function will generally depend on financial leverage and therefore could be linked to the bankruptcy risk function under a simultaneous system.

Consideration of liquidation value of equity at the time of retirement in the model implies allowing for liquidation losses due to sale under duress (losses varying according to type of assets—more for some and less for others). Expected future market value seems more logical in this regard.

Some of my professional colleagues are of the view that experimental simulations using this dynamic model could be tried for representative farm situations. Hypothetical or probable values for changing equity levels, withdrawal rates, rates of returns to equity, or, in the case of endogenized revenue function, the corresponding production coefficients, costs, and prices, discount rates or discount rate function parameters, expected time path to retirement, etc., will have to be assumed in this regard. In this way, the impacts on maximizing the value function for alternative leverage situations for representative farm firms could be evaluated. The results from these experiments can be used to determine the probability of future financial difficulty based on the current capital structure for typical farm situations. The estimation of the probability at the aggregate level is reportedly not feasible due to lack of time series data with adequate degrees of freedom. In this regard, in Canada, data from three farm credit surveys (6,000 sample farms) conducted in 1981, 1984, and 1986 supplemented with the census and national farm survey data for 1980, 1982, 1983, 1985, and 1987 provide a good eight-year series of matched time series/cross section farm finance data for such estimation.

Defining the rate of return to equity as net cash flow divided by equity (as shown in the empirical analysis) precludes noncash expenses (depreciation) in a conventional accounting sense. Strictly speaking, total return to equity is the residual from farm cash income plus real capital gains on assets and debts after depreciation and interest charges.

The effect of inflation on real interest rates in determining the optimal capital structure in this model is not clear and needs clarification. This is important because, as observed in the 1980s, real interest rates were lowest (negative) when the land value and nominal capital gains were at the peak, followed by a rapid slide in land values in the early to mid-1980s when real interest rates rose sharply.

Government policy and programmes (credit and income policy) impact directly and indirectly on the firm's behaviour with respect to acquisitions, income generation, and liquidation of assets over time. This has taken a special significance after the 1980s financial crisis of the industry in the USA and Canada, resulting in new credit and debt review programmes to facilitate restructuring. How to allow for the impact of government policy and programmes in the model warrants consideration.

This model is clearly relevant to large noncorporate commercial farms in the developed economies (North America, EC, and some Latin American countries). Its applicability to commercial farm situations in developing country agriculture is perhaps questionable.

GENERAL DISCUSSION—*Manuel Cabanes, Rapporteur* (Escuela Superior de Técnica Empresarial Agrícola, Universidad de Córdoba)

Karp was asked about the time horizon considered in his dynamic model of capital structure for the noncorporate farm. In his answer, he stated that the time horizon used was the farmer's life expectancy. Karp was asked whether the constant consumption hypothesis contemplated in the model implied the reinvestment of the surplus obtained. In his reply, Karp stated that reinvestment was not a significant aspect in his model.

Participants in the general discussion included C. Alves, G. Corazza, G.T. Jones, E. Soliman, and H. von Witzke.