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# OPTIMAL FARM CREDIT POLICY UNDER 

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Nonlinear pricing models have been applied to a variety of markets (Spence; Goldman, Leland, and Sibley; Guesnerie and Seade; Maskin and Riley). An important presumption of these models is that informational asymmetries exist among the parties to the transaction. In credit markets, information asymmetries between lenders and borrowers are an important source of lender risk. Even without moral hazard, adverse-selection problems arising from asymmetric information can be important. For example, Stiglitz and Weiss have shown that informational asymmetries and default risk can lead to credit rationing.

Nonlinear pricing models, on the other hand, virtually never use rationing devices to cope with asymmetric information. Rather, price schedules are designed in a fashion which elicits truthful revelation of asymmetric information through the purchaser's choice of a particular price-quantity bundle. Eliciting this information, however, comes at a cost, and asymmetric information inevitably leads to divergences from efficient, marginal-cost pricing. Only in certain bounding cases do these inefficiencies will disappear and buyers are priced at marginal cost (Seade; Roberts).

One reason that Stiglitz and Weiss isolate rationing is that their model uses only interest rates to screen borrowers of differing quality: All borrowers are presumed to finance investment projects of the same size. But this precludes consideration of a potentially important and realistic screening mechanism, namely, loan size. It is easy to think of instances where the size of investment project that a borrower wishes to finance can convey important imformation about the borrower to the lender under asymmetric information. This paper explores such possibilities using a nonlinear pricing
model of farm lending.
Two market structures are considered: cooperative and monopolistic. Cooperative markets are examined because of the important role cooperative lenders play in agriculture throughout the world ( for example, Credit Agricole and the U.S. Farm Credit System). In what follows, assumptions and the borrower's maximization process are presented. For the two market structures considered, optimal pricing rules are then derived and examined. Finally, distributional implications of these rules are examined and the paper closes with conclusions.

## Assumptions

Farmers possess different farming abilities summarized by a scalar z. z represents those borrower characteristics, like entrepreneurial ability, which lenders cannot measure completely. Lenders are aware that these ability differences exist and that they affect the profitability of a farmer's operation. But lenders cannot ascertain a priori the exact farming ability (z) of any farmer at loan initiation. Lenders do know, however, the population distribution of ability $z$.

To focus the analysis on asymmetric information, the paper abstracts from other important determinants of farm lending. Most importantly, production is presumed certain, and lenders can ascertain whether funds lent are applied to production. Although production uncertainty can be incorporated in the model, doing so makes it difficult to isolate the results that arise from information asymmetries. Throughout the analysis, marginal lending cost of credit is assumed constant and equal to $r$.

## Farmer Behavior

Production is entirely credit-financed, and financing credit incurs costs. In making production decisions, therefore, a farmer must consider the
cost of credit. A convenient analytic device in this context is the expenditure-constrained variable profit function (Lee and Chambers). This function relates variable profit to expenditure on variable inputs. Denote this function as $\pi(1)$, where 1 is expenditure. ${ }^{1 /}$ Although our conceptual framework uses $\pi(1)$, the developments apply equally well to other conceptualizations of firm decisionmaking under credit constraints. For example, $\pi(1)$ could, with minor modifications, be interpreted as a version of the wealth maximization problem considered by Boehlje and Eidman.

Lee and Chambers derive $\pi(1)$ for a representative farmer. However, since this study focuses on how lending patterns vary among farmers, a continuous index $z$ ranging from $z_{0}$ to $z^{*}$ is introduced into $\pi(1) . \pi(z, l)$ is assumed strictly increasing and strictly concave in 1 and $z$. Further, the marginal quasirent of credit, $\pi_{1}(z, l)-R_{1}(1)$, is assumed to be increasing with $z$, i.e., $\pi_{z 1}>0$ and $\pi(z, 1)$ is twice continuously differentiable. Because production is entirely credit financed, $\pi(z, 1)$, therefore, represents profit before interest is paid. ${ }^{2 /}$ All farmers face the same differentiable credit price schedule $R(1)$. Each farmer maximizes variable profit net of borrowing cost, $\pi(z, 1)$ $R(1)$, by choosing 1. Define

$$
\begin{equation*}
y(z)=\underset{1}{\operatorname{Max}}\{\pi(z, 1)-R(1)\} \tag{1}
\end{equation*}
$$

where $y(z)$ is the $z$ farmer's profit. The envelope theorem applied to (1) gives:

$$
\begin{equation*}
y_{z}(z)=\pi_{z}(1(z), z) \tag{2}
\end{equation*}
$$

Because $\pi_{z}>0$, equation (2) establishes that $y_{z}>0$, i.e., higher ability results in higher profits.

## Monopolistic Lending

Commercial agricultural banks are treated here as monopolistic lenders. Although not completely realistic, this assumption best highlights how information asymmetries affects lending practices across different market. structures and provides a point of comparison for the cooperative structure to be discussed later. But the assumption is not completely unrealistic. A unique aspect of most agricultural banks is their localized lending. This characteristic along with others -- barriers to entry and scale advantages in operations -- can afford a certain degree of market power to farm lenders.

A monopolist allocates credit only to that subset of the credit-seeking population, say, $\left[z^{\mathrm{P}}, z^{*}\right]$ that maximizes the monopolist's return. Remember that since the lender has no way of exactly identifying borrowers, the lender must offer all the same price schedule. Thus, a profit-maximizing, credit price schedule makes it unprofitable for farmers whom the monopolist wants to exclude from the market to take contracts. The monopolist's profit is:

$$
\begin{equation*}
\int_{z^{p}}^{z^{*}}[R(l(z))-r l(z)] d G(z)-b \tag{3}
\end{equation*}
$$

where $b$ is the monopolist's fixed cost of lending, and $G(z)$ is the commulative distribution of $z$ (number of farmers with ability lower than $z$ ), and the indices $z^{p}$ and $z^{*}$ index the lowest and highest ability among credit-obtaining farmers, respectively.

In designing $R(1)$ the lender must account for the borrower's profit-maximizing behavior. In other words, lender maximization is subject to the farmer's maximization condition (2). ${ }^{3 /}$

Using the identity $R(l(z))=\pi(l(z), z)-y(z)$, the monopolist's problem can be written as:

$$
\underset{y(z) \geq 0,}{\operatorname{Max}} l(z) \geq 0 \quad \int_{z^{p}}^{z^{*}}[\pi(l(z), z)-y(z)-r l] d G(z)-b
$$

s.t.

$$
y_{z}(z)=\pi_{z}(z, l(z))
$$

Appending a vector of differentiable costate variables, $\mu(z)$, to the constraint yields the Lagrangean:
(4)

$$
\begin{aligned}
L & =\int_{z^{p}}^{z^{*}}[\pi(1(z), z)-y(z)-r l(z)] d G(z)-b \\
& +\int_{z^{p}}^{z^{*}} \mu(z)\left[y_{z}(z)-\pi z(z, l(z))\right] d z .
\end{aligned}
$$

Integration by parts gives:

$$
\begin{aligned}
\int_{z^{\circ}}^{z^{*}}\{ & {\left.[\pi(1(z), z)-y(z)-r l(z)] g(z)-\mu_{z}(z) y(z)-\mu(z) \pi_{z}(l(z), z)\right\} d z-b } \\
& +\mu\left(z^{*}\right) y\left(z^{*}\right)-\mu\left(z^{\circ}\right) y\left(z^{\circ}\right) .
\end{aligned}
$$

First-order conditions are:
(5) $\partial L / \partial y(z)=-g(z)-\mu_{z}(z) \leq 0$ and $-\left(g(z)+\mu_{z}\right) y(z)=0$;
(6) $\partial L / \partial 1(z)=\left[\pi_{1}(l(z), z)-r\right] g(z)-\mu(z) \pi_{z l}(1(z), z) \leq 0$ and

$$
\left\{\left[\pi_{1}(l(z), z)-r\right] g(z)-\mu(z) \pi_{z 1}(l(z), z)\right\} l(z)=0 ;
$$

the boundary conditions,

$$
\begin{align*}
& \partial \mathrm{L} / \partial \mathrm{y}\left(z^{\mathrm{p}}\right)=-\mu\left(z^{\mathrm{p}}\right) \leq 0 \text { and }-\mu\left(z^{\mathrm{p}}\right) \mathrm{y}\left(z^{\mathrm{p}}\right)=0 ;  \tag{7}\\
& \partial \mathrm{L} / \partial \mathrm{y}\left(z^{*}\right)=\mu\left(z^{*}\right) \leq 0 \quad \text { and } \mu\left(z^{*}\right) y\left(z^{*}\right)=0 ; \tag{8}
\end{align*}
$$

and the constraint, $y_{z}(z)-\pi_{z}(z, l(z))=0$.
Because no farmer accepts a negative profit, $y\left(z^{p}\right) \geq 0$. The strict monotonicity of $y(z)$ then implies $y(z)>0$ for all $z>z^{p}$ and $\mu\left(z^{*}\right)=0$ from (8). 4/ For $y(z)>0$, (5) then implies that $\mu_{z}(z)=-g(z)$. Integrating gives

$$
\begin{align*}
\mu(z) & =\mu\left(z^{p}\right)-\int_{z^{p}}^{z} g(z) d z  \tag{9}\\
& =\int_{z}^{z^{*}} g(z) d z=G\left(z^{*}\right)-G(z)
\end{align*}
$$

The second equality uses the fact that $\mu\left(z^{*}\right)=0$. The co-state $\mu(z)$ is thus the number of farmers with index values exceeding $z$. The exact shape of $\mu(z)$ depends on the distribution of $\mathbf{z}$.

The monopolist would extract all quasirents if possible. But the monopolist is prevented from doing so by asymmetric information and the monotonicity of $y(z), \mu(z)$ can be interpreted as the shadow price of insuring that $y(z)$ is strictly increasing. Put another way, $\mu(z)$ is the lender's opportunity cost of being unable to identify individual farmer's productive ability. By (9), this cost depends upon the distribution of ability. To see why this cost must occur, suppose that the monopolist offered borrowers a price schedule that entailed borrowing an 1 that maximized quasirents and paying all quasirents to the lender. This represents perfect price discrimination and implies $y(z)$ is zero for all $z$. But equation (2) implies $y_{z}>0$ which means that if $y\left(z^{*}\right)=0$ then $y(z)<0$ for all $z<z^{*}$. Hence, equation (2) rules out perfect discrimination. Thus, the monopolist can only extract all quasirents from the last farmer $\left(z^{p}\right)$ in the market, and $y\left(z^{p}\right)=0$.

Assuming an interior solution for $l(z)$, condition (6) can be solved for $\pi_{1}(l(z), z)$ and then combining (9) and the first order conditions for (1) $\pi_{1}=$ $R_{1}$ yield the following optimal marginal lending schedule:

$$
\begin{equation*}
\pi_{1}(1(z), z)=R_{1}(1(z))=r+\frac{\pi_{z 1}(1(z), z)\left(G\left(z^{*}\right)-G(z)\right)}{g(z)} \tag{10}
\end{equation*}
$$

Expression (10), lets us to compare the monopolist's marginal pricing rule to the Pareto-optimal marginal cost pricing. Because $\pi_{21}>0$ and
$\left(G\left(z^{*}\right)-G(z)\right) / g(z)>0$ for all $z<z^{*}$, the monopolist's marginal price always exceeds marginal cost except at $z^{*}$. Only $z^{*}$ farmers are marginal cost-priced. As a result, credit is underutilized compared to the situation that would prevail under marginal cost pricing.

The result that a monopolist only charges the most able marginal cost is familiar from the literature on nonlinear pricing (Maskin and Riley). But some interpretation of the results is still merited. Generally, a monopolist expects to contract sales to increase profit. With nonlinear pricing, a monopolistic lender does just that: everybody obtains less credit than what could be obtained under marginal-cost pricing. But there is an important difference here because each farmer "sees" a different marginal price (note (10) depends implicitly upon $z$ ) designed to best enhance lender profit. The amount by which this marginal price diverges from marginal cost is governed by $\mu(z)=G\left(z^{*}\right)-G(z)$. So, if $\mu(z)$ is large (i.e., there are many farmers with more ability), relatively little credit is extended to the $z$ farmer. When there are many farmers with higher ability many opportunities exist to extract quasi rents from those higher ability farmers.

Lending to low ability farmers has an opportunity cost. The low-ability farmers' incomes must be kept low enough to permit the lender to extract as much quasi rent from high ability farmers as possible without violating the income monotonicity constraint. As $\mu(z)$ goes to zero (as $z$ approaches $z^{*}$ ), opportunities to extract quasi rent from higher ability farmers vanish, and the lender has the incentive to lend the most efficient farmer type the amount that maximizes his or her producer surplus and then to extract as much of this producer surplus as possible. Hence, $\mathbf{z}^{*}$ is marginally cost priced.

## Cooperative Lending

A cooperative lender is defined as a decisionmaker who maximizes the
welfare of farm borrowers through its lending practices. Farmers' welfare is taken as the weighted sum of individual farmer incomes. The model, therefore, is general enough to incorporate redistributional objectives by the lender. This notion encompasses a variety of lending objectives: a group of farmers or farm groups banding together to ensure that they are not monopolistically exploited, or a government agency (or the cooperative management) seeking to enhance the welfare of a given group of farmers (perhaps even at the expense of other farmers).

Weights depend on $z$ and reflect the lender's goal for farm income distribution. Three weight structures are considered: 1) uniform, 2) monotonically decreasing, and 3) monotonically increasing in $z$. The uniform weight scheme involves no income redistribution while the others redistribute income. ${ }^{5 /}$ We first discuss some intuition behind nonuniform weight structures.

The decreasing weight scheme is designed to transfer income from high $z$ to low $z$ farmers. Thus, such a scheme can be viewed as a subsidy program for low-income farmers. Because the least efficient ( $z^{\circ}$ farmers) must make at least zero profits to stay in business, the introduction of a decreasing weight scheme which favors low-income farmers should ensure $y\left(z^{0}\right)>0$.

Increasing weights favor high $z$ farmers over low $z$ farmers. Because the $z^{*}$ farmers receive the largest weight, transferring income from the rest of the population to the $z^{*}$ farmers is desirable. But such a transfer cannot be freely accomplished because, loosely speaking, the second highest $z$ (say, $z^{*}-\varepsilon$ ) farmers may prefer the $z^{*}$ farmer contract to the one for $z^{*}-\varepsilon$ farmers. If the $z^{*}-\varepsilon$ farmers adopt $z^{*}$, s contract, quasirents available to be redistributed to the $z^{*}$ farmers go down and the program collapses. This same type of incentive constraint applies to all $z$ 's. So, the $z-\varepsilon$ farmers must
have no incentive to take the $\mathbf{z}$ farmer's contract for all $\mathbf{z}$ if the price schedule is optimal. Pricing for the lowest $z^{\circ}$ farmers, however, is not restricted by this incentive constraint because no farmer exists below $z^{\circ}$. Thus, the lender can take all quasirents from the $z \circ$ farmers, leaving them with zero profits. So long as these farmers generate positive marginal quasirents, credit will be extended to them and all their quasirents will be transferred to higher $z$ farmers.

The above discussion addresses intuitively the core of the cooperative lender's problem when lending to a group of self-interested borrowers. Increasing weights transfer income to high $z$ farmers by charging them favorable prices -- yet these prices cannot be so favorable as to attract lower $z$ farmers. In the decreasing weight case, the reverse is true; pricing to the low $z$ farmers cannot be so favorable as to attract high $z$ farmers to the contracts meant for the low $z$ farmers.

The cooperative lender maximizes,

$$
\begin{equation*}
\int_{z^{0}}^{z^{*}} y(z) w(z) d G(z) \tag{12}
\end{equation*}
$$

subject to the following constraints,

$$
\begin{align*}
& y_{z}(z)=\pi_{z}(1(z), z), \text { for all } z \text { and }  \tag{13}\\
& \int_{z_{0}}^{z^{*}} R(1) d G(z) \geq r L+b \tag{14}
\end{align*}
$$

The weights $w(z)$ are positive over the entire farm population $\left[z^{\bullet}, z^{*}\right]$. Expression (14) is the cost constraint and guarantees that the lender's revenue at least covers the cost of providing loans. The cost constraint, therefore, implies that if the cooperative lender enhances the relative position of, say, the $z^{\circ}$ farmers with subsidized credit, it does so by effectively taxing some other farmers.

Forming a Lagrangean by appending a Lagrangean multiplier $\gamma$ and a vector of costate variables ( $\lambda(z)$ ) to equations (14) and (13), respectively, and by substituting $R(1(z))=\pi(1(z), z)-y(z)$ into (14) gives:

$$
\begin{aligned}
W= & \int_{z^{0}}^{z^{*}}\left[y(z) w(z) g(z)+\lambda(z)\left(y_{z}(z)-\pi_{z}(1(z), z)\right)\right] d z \\
& +\gamma\left\{\int_{z^{0}}^{z^{*}}[\pi(1(z), z)-y(z)-r l(z)] d G(z)-b\right\}
\end{aligned}
$$

First-order conditions are: ${ }^{6 /}$

$$
\begin{align*}
& \partial W / \partial y(z)=-\lambda z(z)-\gamma g(z)+w(z) g(z) \leq 0,  \tag{15}\\
& {\left[-\lambda_{z}(z)-\gamma g(z)+w(z) g(z)\right] y(z)=0 ;} \\
& \partial W / \partial l(z)=-\lambda(z) \pi_{z l}(l(z), z)+\gamma \pi_{1}(l(z), z) g(z)-r \gamma g(z) \leq 0,  \tag{16}\\
& {\left[-\lambda(z) \pi_{z 1}(l(z), z)+\gamma \pi_{1}(l(z), z) g(z)-r \gamma g(z)\right] l(z)=0 ;} \\
& \partial \mathrm{W} / \partial \mathrm{y}\left(\mathrm{z}^{*}\right)=\lambda\left(\mathrm{z}^{*}\right) \leq 0, \quad \lambda\left(z^{*}\right) y\left(z^{*}\right)=0 \text {; }  \tag{17}\\
& \partial W / \partial y\left(z^{\circ}\right)=-\lambda\left(z^{\circ}\right) \leq 0, \lambda\left(z^{\circ}\right) y\left(z^{\circ}\right)=0 ;  \tag{18}\\
& \partial W / \partial \gamma=\int_{z^{0}}^{z^{*}} \pi(l(z), z)-y(z)-r l(z) d G(z)-b \geq 0, \\
& \left\{\int_{z_{0}}^{z^{*}} \pi(l(z), z)-y(z)-r l(z) d G(z)-b\right\} \gamma=0,
\end{align*}
$$

$y(z) \geq 0, \quad l(z) \geq 0$, and $\gamma \geq 0$.
Applying an argument analogous to the monopolist's case yields $\lambda\left(z^{*}\right)=0$.
For all $y(z)>0$, (15) can be rewritten in equality form:

$$
\begin{equation*}
\lambda_{z}(z)=g(z)(w(z)-\gamma) . \tag{20}
\end{equation*}
$$

Integrating (20) back for $z$ and using the fact $\lambda\left(z^{*}\right)=0$ gives:
(21)

$$
\lambda(z)=-\int_{z}^{z^{*}}(w(m)-\gamma) g(m) d m .
$$

Expression (21) measures the cost and benefits associated with a $\$ 1$ lump-sum income redistribution from the lender to farmers above $z$. Such a transfer increases weighted income by:

$$
\int_{z}^{z^{*}} w(m) g(m) d m,
$$

while its shadow value in terms of the budget constraint is:

$$
\gamma \int_{z}^{z^{*}} g(m) d m,
$$

Thus, if $\lambda(z)>0$, the cost-benefit ratio is greater than one, i.e., the lender effectively loses by redistributing income to high $z$ farmers.

Suppose the lender makes a strictly positive profit, implying that $\gamma=0$ in (19). When $\gamma=0$, equation (21) gives:

$$
\lambda\left(z^{\circ}\right)=-\int_{z^{0}}^{z^{*}} w(z) d G(z)<0,
$$

since $w(z)$ and $g(z)$ are strictly positive. This contradicts equation (18). Hence, the optimal cooperative pricing rule never yields the lender a profit and a cooperative lender always disburses any profit it makes to its members. ${ }^{7 /}$

Marginal Pricing Rules
For $l(z)>0$, the optimal marginal pricing rule is:

$$
\begin{equation*}
R_{1}(l(z))=\pi_{1}(l(z), z)=r+\lambda(z) \frac{\pi_{z l}(l(z), z)}{\gamma g(z)} \tag{22}
\end{equation*}
$$

The term ( $\left.\lambda \pi_{z l} /(\gamma g)\right)$ represents the optimal divergence of marginal price from marginal cost. The sign of this term is determined by the sign of $\lambda(z)$, because $\pi_{z 1} \geq 0, \gamma>0$, and $g(z)>0$. Hence, the structure of $\lambda(z)$ needs to be examined.

From (20), it is easy to check that for $z$ where $w(z)>\gamma, \lambda(z)$ is increasing (i.e., $\lambda_{z}(z)>0$ ) and when $w(z)<\gamma \quad \lambda(z)$ is decreasing (i.e., $\left.\lambda_{z}(z)<0\right)$ with a turning point where $w(z)=\gamma$. So, when $w(z)$ decreases monotonically $\lambda(z)$ is a nonnegative function with a single peak. Likewise, when $w(z)$ increases monotonically $\lambda(z)$ is a negative function for $z<z^{*}$ (remember that $\lambda\left(z^{*}\right)=0$ ) with a trough. The shape of $\lambda(z)$ for a uniform $G(z)$ is illustrated in Figure 1.

Figure 1 suggests that cooperative marginal prices exceed marginal cost for the decreasing weight scheme and are below marginal cost for the increasing weight scheme. Thus, compared to marginal-cost pricing, credit underutilization prevails with a cooperative lender under decreasing weights and credit overutilization occurs with increasing weights. In all cases, $\lambda\left(z^{*}\right)=0$; the cooperative lender always marginally prices the highest $z^{*}$ farmers at marginal cost.

Under uniform weights, cooperative lending maximizes the simple sum of all farmer incomes. But, marginal-cost pricing also maximizes quasirents net of the cost of funds (the simple sum of farm incomes). Under uniform weights, therefore, marginal-cost pricing is optimal. (Formally, note that $\lambda(z)=0$ when $w(z)=\hat{w}$ for all $z$ (see Appendix A).) Furthermore, when $b>0$, for the cost constraint (14) to be satisfied, $b$ has to be distributed among all loans. Marginal-cost pricing can be only maintained by charging a fixed fee decoupled from the loan amount. Defining bo as $b / G\left(z^{*}\right)$, the optimal lending policy, therefore, requires a two-part tariff, $R(1)=b \circ+r l$ : The first part is an entry fee bo that spreads the lender's fixed cost evenly among farmers regardless of the size of the loan, the second bases actual loan demand on the marginal cost of funds. A formal proof is presented in Appendix B. Notice that charging all a common fixed fee implies that the pricing rule is actually
regressive since the average cost of credit declines as 1 rises. (This presumes that all farmers can make a strictly positive profit when only charged r. For small b this is likely valid but as $b$ increases the ability to spread fixed costs among all farmers decreases forcing an adjustment to this rule.)

## Distributional Implications

This section examines the distributional implications of nonlinear pricing for the cooperative and monopolistic cases. Marginal-cost pricing is used as a point of comparison. Throughout, the lender's cost structure is assumed identical across all lending practices.

Define $\tau(1)$ as $R(1)-r 1$. When $\tau_{1}(1)>0\left(\tau_{1}<0\right)$, every farmer pays (receives) a marginal markup (markdown) over marginal-cost pricing. Because $\tau(1)$ is nonlinear, each farmer faces different marginal markups or markdowns.

When the weights decline with $z$, the optimal marginal price for each borrower is always greater than or equal to marginal cost. Hence, marginal losses to the borrowers emerge. Since these marginal losses prevail over all $1<1\left(z^{*}\right)$, the loss to farmer $z(\tau(1(z)))$ which is the integral of $\tau_{1}(1)$ over $1(z)$ would be the largest to the one who borrows most. Thus, the biggest losers over marginal cost pricing, or the largest taxpayers, are those demanding the most credit--the $z^{*}$ farmers. A formal derivation is shown as follows. The lender's total markup (markdown) revenue is:

$$
\int_{z^{0}}^{z^{*}} \tau(l(z)) d G(z)=\int_{z^{0}}^{z^{*}}\left\{\int_{0}^{l(z)} \tau(1) d l\right\} d G(z) .
$$

To break even, this total markup (markdown) has to equal the fixed cost (b). Suppose $b=0$. Rewrite the term inside the braces noting that $[0,1(z)]$ can be decomposed into two intervals, $\left[0,1\left(z^{\circ}\right)\right]$ and $\left[1\left(z^{\circ}\right), l(z)\right]$, to get:

$$
\int_{z^{0}}^{z^{*}}\left\{\int_{0}^{1\left(z^{0}\right)} \tau_{1}(1) \mathrm{d} l+\int_{1\left(z^{0}\right)}^{1\left(z_{1}\right)}(1) \mathrm{d} l\right\} d G(z)=0
$$

Total loss (gain) is now divided into two parts: that over $\left[0,1\left(z_{0}\right)\right]$ and that over $\left[l\left(z^{\circ}\right), l(z)\right]$, which are measured by the first and second terms inside the braces, respectively. The losses (gains) over [ $0,1\left(z^{\circ}\right)$ ] are experienced by everybody because all loans are at least as large as $l\left(z^{\circ}\right)$. On the other hand, the second term depends on the individual's loan amount.

Simplifying gives,

Consider the case where $\mathrm{w}_{\mathbf{z}}(\mathrm{z})<0$. For loan levels above $l\left(\mathrm{z}^{\circ}\right)$, borrowers pay marginal markups, meaning that the first term in the above equality is positive. For the equality to hold, loans below $l\left(z_{0}\right)$ must be marked down, i.e., $\tau$ - must be negative enough to offset the total markups.

The $z$ farmer's net gain (or loss) from the decreasing weight scheme can be expressed as the difference between the per loan markdown ( $\tau \circ$ ) and the markup imposed on the loan above $1\left(z^{\circ}\right)$ :

$$
\tau_{0}-\int_{1\left(z_{0}\right)}^{1(z)} \tau_{1}(1) d l
$$

When weights decrease, the second term increases with $l(z)$. Thus, as 1 rises above $l\left(z^{\circ}\right)$, the uniform markdown is eroded by the second term; high $z$ farmers actually incur net losses. So, the greatest gainers are the $z$ ofarmers and the biggest losers are the $\mathrm{z}^{*}$ farmers.

A similar logic applies to the increasing weight case. In transferring income to high $z$ farmers, the lender charges a higher price than is necessary
to cover costs for all loans up to $1\left(\mathrm{z}^{\circ}\right)$ and charges lower marginal prices to the borrowers with larger loan sizes, i.e., $\tau_{1}(1)<0$ for $1>1\left(z_{0}\right)$. Therefore, the biggest gain goes to the $z^{*}$ farmers who demand the largest $1 .{ }^{8 /}$ However, charging too high a price for $l\left(z_{0}\right)$ erodes potential loan demand. The resulting price for $1\left(\mathrm{z}^{\circ}\right)$, therefore, will be just enough to keep the $z^{\circ}$ farmers in the market, i.e., to give them zero profit.

The monopolist's pricing can be analyzed analogously. The monopolist's margin over marginal-cost pricing from the loan size $l(z)$, denoted by $\eta(l(z))$ (=R(l(z))-rl(z)) can be also broken down into two components: that over $\left[0,1\left(z^{\mathrm{p}}\right)\right]$ and that over $\left[1\left(z^{\mathrm{p}}\right), l(z)\right]$ :

$$
\begin{aligned}
\eta(l(z)) & =\int_{0}^{l\left(z^{p}\right)}\left(R_{1}(1)-r\right) d l+\int_{l\left(z^{p}\right)}^{l\left(z_{1}\right)}\left(R_{1}(1)-r\right) d l \\
& =\left[R\left(l\left(z^{p}\right)\right)-r l\left(z^{p}\right)\right]+\int_{l\left(z^{\mathrm{p}}\right)}^{l\left(\mathrm{z}_{1}\right)}\left(\mathrm{R}_{1}\right) \mathrm{d} l
\end{aligned}
$$

The two components above can be also viewed as two-part monopolistic tariff, $\eta \circ$ and $\phi(1(z))$, respectively. $\eta \circ$ is uniformly collected from each loan. The monopolist refuses to grant any loans smaller than $l\left(z^{\mathrm{p}}\right)$. The term $\phi(l(z))$ is the additional, nonlinear margin for loan levels above $l\left(z^{\mathrm{p}}\right)$. No borrower gains over marginal-cost pricing under monopoly lending, as anticipated. The most quasirents are extracted by the monopolist from the $z^{*}$ farmers even though they receive the lowest marginal price.

## Conclusions

This paper has investigated agricultural credit pricing under asymmetric information for two market structures. Optimal loan-pricing rules have been derived for monopolistic and cooperative lenders. These rules are expressed in terms of the properties of the distribution of farms and the expenditure-
constrained variable profit function. Because the expenditure-constrained profit function can be estimated and the distribution of farms is at least potentially observable from cross-section data, the rules outlined in equations (10) and (22) can be determined empirically.

Among the important results established in this paper are: monopolistic lending is always characterized by markups over marginal cost, i.e., farm credit is underutilized; when income is redistributed by cooperative lenders, the contracts redistributing income toward low-z farmers are characterized by credit-underutilization and those redistibuting toward high-z farmers are characterized by credit-overutilization; finally, regardless of the market structure or lending objectives, the most efficient farmers are always marginally cost priced.

To close it is interesting to note some broader implications for agricultural economics from this paper's methodology. Although this paper was cast in an optimal lending framework, the methodology applies to a much broader array of agricultural problems than to farm lending. In particular, the methodology can apply to the pricing of any perfectly divisible input to an agricultural production process. As just one example, with minor modifications, the analysis applies to the optimal rationing and pricing of a scarce input such as groundwater.

Boelhlje, Michael, and Vernon Eidman. "Financial Stress in Agriculture: Implications for Producers," American Journal of Agricultural Economics, Vol. 65, December 1983, pp. 937-44.

Goldman, M., H. Leland, and D. Sibley. "Optimal Nonuniform Prices," Review of Economic Studies, 1984, pp. 305-19.

Guesnerie, R. and J. Seade. "Nonlinear Pricing in a Finite Economy," Journal of Public Economics, 17. 1982, pp. 157-79.

Lee, Hyunok and Chambers, Robert G. "Expenditure Constraints and Profit Maximization in US Agriculture, American Journal of Agricultural Economics, Vol. 68, November 1986, pp. 857-65.

Maskin, E. and J. Riley. "Monopoly with Incomplete Information," Rand Journal of Economics, 15. 1984, pp. 171-96.

Mirrlees, J. "An Exploration in the Theory of Optimal Taxation," Review of Economic Studies, 1971, pp. 175-208.

Roberts, K. "Welfare Considerations of Nonlinear Pricing," Economic Journal, 89. 1979, pp. 66-83.

Seade, J. "On the Shape of Optimal Tax Schedules," Journal of Public Economics, 7. 1977, pp. 203-35.

Spence, M. "Nonlinear Prices and Welfare," Journal of Public Economics, 8. 1977, pp. 1-18.

Stiglitz, J. and A. Weiss. "Credit Rationing in Markets with Imperfect Information," American Economic Review, June, 1982, pp. 393-409.

## Footnotes

1/ Mathmatically, the expenditure-constrained variable profit function can be expressed as:

$$
\pi(p, w, 1)=\operatorname{Max}\{p y-w x: w x \leq 1\}
$$

where $y, x$ are output and input vectors and $p, w$ are the respective price vectors. Therefore, $\pi(p, w, l)$ maximizes short run profit, given prices and expenditure available.

2/ While 100 -credit financing may not be totally realistic, the analysis can be generalized by incorporating own financing. It is likely that lenders have no way to find out how much of own financing will be put into farm production of credit seeking farmers. Thus, the lender has to 'guess' the level of own financing from the information collected by the lender. For instance, the lender believes that the level of this year's own financing (e)is correlated with the credit seeking farmer's previous year income statement (i), information which the lender can easily obtain. Then, equation (1) can be rewritten as:

$$
y(z)=\operatorname{Max}_{1}\{\pi(z, 1)-R(1-e(i))\}
$$

where $e(i)$ relates own financing to $i$ and the farmer pays the interest only for credit $1-e(i)$. When $1-e(i)$ is denoted by $\hat{l}$, the first order condition for the above problem becomes $\pi_{1}=R_{\hat{1}}$. Therefore, qualitative results obtained from the analysis assuming $\pi_{1}=R_{1}$ (i.e., $e(i)=0$ ) apply to the problem incorporating equity financing. Or, put differently, once the function $e($.$) is determined, R((1-e(i))$ can be rewritten as $R(1: i)$.

3/ Mirrlees was the first to formulate the nonlinear pricing problem in this fashion. To see why it makes sense, consider a discrete analog of this problem where the lender chooses a separate vector ( $1, R$ ) for each farmer type. Suppose that the lender wants to lend the $z^{*}$ farmer $1^{*}$ and charge
$R^{*}$, but the monopolist's price schedule is such that there exist an $l^{\prime}$ and R':

$$
\pi\left(1^{\prime}, z^{*}\right)-R^{\prime}>\pi\left(1^{*}, z^{*}\right)-R^{*}
$$

The $z^{*}$ farmer will always adopt the contract ( $1^{\prime}, R^{\prime}$ ) over ( $1^{*}, \mathrm{R}^{*}$ ) and the monopolist's plan will be thwarted. Therefore, for $\left(1^{*}, R^{*}\right)$ to be freely chosen by the $z^{*}$ farmer, the lender has to ensure that $\left(1^{*}, R^{*}\right)$ yields the most profit to $z^{*}$ farmers among all available contracts. That is, if such a contract is denoted by $\left(1\left(z^{*}\right), R\left(z^{*}\right)\right)$, it must be true that

$$
\pi\left(l\left(z^{*}\right), z^{*}\right)-R\left(z^{*}\right) \geq \pi\left(l(z), z^{*}\right)-R(z) \quad \text { for all } z \neq z^{*} \text {, }
$$

which is precisely the meaning of (2). And, since this applies to all $z$, (2) must hold for all $z$. The nonlinear (continuous) equivalence of a set of optimally chosen contracts $(l(z), R(z))$ would be $R(l(z))$.

4/ This parallels the transversality condition of the calculus of variations with a free-end value and means that a tiny change in the optimal path at the end point cannot enhance the objective value. Put another way, there is no need to modify the optimal pricing schedule when a small exogenous change in $y\left(z^{*}\right)$ occurs because no farmers above $z^{*}$ exist to be affected.

5/ The uniform weight scheme is introduced to make a comparison to the income position that wouldemerge if the cooperative lender were displaced by a competitive market.

6/ Integration by parts yields:

$$
\begin{aligned}
W= & \int_{z^{\bullet}}^{z^{*}}\left[y(z) w(z) g(z)-\lambda z^{y}(z)-\lambda(z) \pi_{z}(1, z)\right] d z \\
& +\left[\int_{z^{0}}^{z^{*}}(\pi(1, z)-y(z)-r l(z)) d G(z)-b\right] \gamma \\
& +\lambda\left(z^{*}\right) y\left(z^{*}\right)-\lambda\left(z^{\circ}\right) y\left(z^{\circ}\right)
\end{aligned}
$$

7/ All the qualitative results which follow remain intact if one assumes that the lending authority can run a deficit so long as the lower bound of the deficit is not minus infinity.

8/ Nonetheless, in the increasing weight case, it is not clear as to who the greatest losers are. Unlike the decreasing weight case, $z$ - farmers may not be the biggest losers. In this case since the corner solutions $\left(y\left(z^{\circ}\right)=0\right)$ are not ruled out. We cannot conclude $\lambda\left(z^{\circ}\right)=0$. Consequently, $\tau_{1}\left(1\left(z^{\circ}\right)\right)$ may be greater than zero.

## Appendix A

The optimality of marginal cost pricing under uniform weights can be also demonstrated by showing $\lambda(z)=0$ from (21). Under the unform weight, $w(z)$ is a constant, say, $\hat{W}$. Therefore (21) can be rewritten as:

$$
\lambda\left(z^{\circ}\right)+(\hat{w}-\gamma)\left[G(z)-G\left(z^{\circ}\right)\right]=-(\hat{w}-\gamma)\left[G\left(z^{*}\right)-G(z)\right]
$$

Suppose $\hat{w} \neq \gamma$. Then it is true for all $z$ that:

$$
\lambda\left(z^{\circ}\right) /(\hat{w}-\gamma)=\left[G\left(z^{*}\right)-G(z)\right] /\left[G(z)-G\left(z^{\bullet}\right)\right],
$$

which is not possible. Thus, by contradiction, $\hat{w}=\gamma$.

## Appendix B

With the condition $R_{1}(1)=r$ for all levels of 1 , one can infer the structure of $R(1)$ under uniform weights. Integrating over 1 yields $R(1)=b \circ+r l$, where $b o$ is a constant of integration. The zero profit constraint to the lender implies:

$$
\int_{z_{0}}^{z^{*}} R(1) d G(z)=\int_{z_{0}}^{z^{*}}(b \bullet+r l) d G(z)=b \bullet G\left(z^{*}\right)+r \int_{z_{0}}^{z^{*}} 1 d G(z)=b+r \int_{z^{0}}^{z^{*}} 1 d G(z)
$$

Therefore, $R(1)$ becomes $b \bullet+r l$ with bo equal to $b / G\left(z^{*}\right)$. Therefore, the fixed cost $b$ is distributed equally among all loans.

Figure 1. The Shape of $\lambda(z)$

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\begin{aligned}
& \because \\
& \therefore
\end{aligned}
$$

