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THE SIMPLE ANALYTICS OF NONJOINT PRODUCTION RELATIONS

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The Simple Analytics of Nonjoint Production Relations

A series of empirical tests for agricultural input nonjointness (Weaver, 1977; Ray; Lopez; Shumway; Weaver, 1983; Moschini; and Ball) have been conducted: the bulk of the empirical evidence weighs against input nonjointness. Shumway, Pope, and Nash (SPN) show, however, that some dual tests for input nonjointness are invalid in the presence of allocatable-fixed inputs. SPN also speculate that dual methods cannot recapture variable-input allocations when allocatable-fixed inputs are present. This speculation has proven particularly controversial spawning a series of studies demonstrating approaches to resurrecting variable-input allocations.

Just, Zilberman, and Hochman (JZH) accept SPN's position and develop an approach to recapture variable input allocations using the first-order conditions for a set of input-nonjoint Cobb-Douglas production functions. Paris rejects the SPN claim and demonstrates a "purified profit function" approach to recapture the variable input allocations. (As Paris points out he, SPN, and JZH have different interpretations of what constitutes a "dual" and what constitutes a "primal" approach.) Chambers and Just (CJ) develop a crop-specific profit function approach to recapturing the variable input allocations.

This paper also studies nonjoint production relationships. Its first contribution is a new nonjointness definition (q-nonjointness) which has input-nonjoint and output-nonjoint as special cases. The new definition is motivated by the recognition that traditional nonjointness definitions (both input and output) are logically irrelevant to many agricultural industries. Thus, the uncritical application of traditional nonjointness tests to agriculture (characteristic of most empirical agricultural jointness studies) has limited scientific interest because these tests ignore the basic realities of agricultural production.

This paper's second contribution is a generalization and simplification of the CJ multistage approach to input-nonjoint production relations. A resulting pedagogical benefit is a simple graphical exposition of the economics of nonjoint production. Visualization of netput allocation decisions clarifies the essential issues involved in the SPN controversy. An immediate by-product is an easy comparison and evaluation of the three solutions offered to the SPN problem (JZH; Paris; and CJ). The CJ approach alone is found to be capable of achieving full econometric efficiency and functional flexibility. A new LeChatelier result is also deduced.

The Model

Agricultural netputs are divided into three categories: nonallocatable ($q \in \mathbb{R}^k$), allocatable-fixed ($z \in \mathbb{R}^m$), and allocatable-variable ($x \in \mathbb{R}^n$). Agricultural technology is represented by a nonempty, compact[1], convex set $T \subseteq \mathbb{R}^k \times \mathbb{R}^m \times \mathbb{R}^n$ exhibiting free disposal of netputs called the production possibilities set and defined by

$$T = \{\ (q,z,x):\ q, z, \text{ and } x \text{ are producible }\}.$$

By convention, a netput is an input if it is negative and an output if it is positive. Several points should be stressed here. First, this representation of the technology is more general than previous representations of nonjoint, agricultural technologies. It avoids the strong differentiability assumptions on the primal technology used by SPN, JZH, and Paris. This is important because as Paris and Knapp have recently pointed out many well-recognized agricultural technologies are not smoothly differentiable (e.g, the von Liebig technology). And second it avoids the arbitrary classification of netputs into inputs and outputs characteristic of previous discussions of nonjoint agricultural technologies (SPN, JZH, Paris, CJ, Lynne). This permits a new notion on nonjointness (next section) which recognizes the realities of agricultural production relations.

Dual to T is the long-run profit function

$$R(p,w,r) = \text{Max} \{ pq + rz + wx : (q,z,x) \in T\}.$$

Here $p \in \mathbb{R}^k_{++}$, $w \in \mathbb{R}^n_{++}$, and $r \in \mathbb{R}^m_{++}$ denote vectors of netput prices. The notation pq means the inner product of the vector p and the vector q. R is positively linearly homogeneous, continuous, and convex in its arguments. If a unique solution exists to the profit maximization problem, R(p,w,r) is differentiable and

(1) $$\nabla_p R(p,w,r) = q(p,w,r),$$
$$\nabla_r R(p,w,r) = z(p,w,r), \text{ and}$$
$$\nabla_w R(p,w,r) = x(p,w,r).$$

Here the notation $\nabla_j R$ denotes the gradient of R(p,w,r) with respect to the vector j and q(p,w,r), z(p,w,r), and x(p,w,r) are the long-run profit maximizing levels of the nonallocatable, allocatable-fixed, and allocatable-variable netputs, respectively. Differentiability of R(p,w,r) does not imply differentiability for T. (As a simple example, Leontief technologies have differentiable cost functions even though the primal functions defining T are not differentiable.)

The short-run, variable profit function is defined

$$R(p,w,z) = \text{Max} \{ pq + wx : (q,z,x) \in T\}.$$

R(p,w,z) is positively linearly homogeneous, convex, and continuous in p and w. Moreover, the convexity of T implies that R(p,w,z) is concave and continuous in z (Diewert). If a unique solution to the short-run, variable profit maximization problem exists, $R(p,w,z)$ is differentiable in p and w (we shall typically presume it is also differentiable in z as well) with

(2)
$$\nabla_p R(p,w,z) = q(p,w,z), \text{ and}$$
$$\nabla_w R(p,w,z) = x(p,w,z).$$

Here $q(p,w,z)$ and $x(p,w,z)$ represent the short-run, variable profit maximizing nonallocatable and allocatable-variable netput vectors.

The dual relationship between $R(p,w,r)$ and $R(p,w,z)$ is reflected by the conjugate relations

(3)
$$R(p,w,r) = \operatorname*{Max}_z R(p,w,z) + rz$$
$$R(p,w,z) = \operatorname*{Min}_r R(p,w,r) - rz.$$

Presuming that $z \in \mathbb{R}_-$ (i.e., z is a scalar input) the choice of profit-maximizing z is represented pictorially in Figure 1. The left panel of that figure portrays both parts of the maximand as well as the maximand itself. The right panel presents the first-order condition for that maximization problem of equating the shadow-price of the allocatable-fixed netput to its market price.

The conjugate, minimization problem is represented pictorially in Figure 2. The left panel there presents a graphical depiction of both parts of the minimand and the minimand itself. The right panel illustrates the first-order condition for the minimization problem requiring that the derivative of $R(p,w,r)$ with respect to r be equated to the profit maximizing z level. (This captures pictorially part of Shephard's lemma (equations (2)).)

The conjugate optimization problems in (3) represent different ways of looking at the same reality: Figure 1 takes r as given and seeks which z maximizes profits; Figure 2 takes z as given and seeks the r for which z would be profit maximizing. Both potentially yield the same information. (For technologies meeting the differentiability requirements of Paris and JZH, Figure 2 is obtained by applying the inverse-function theorem to Figure 1.)

q-Nonjointness Defined

The technology is *q-nonjoint* if

$$T = \{(q,x,z) : (q_i,\ z^i,\ x^i) \in T_i \ (i = 1,\dots,k),\ x = \sum x^i,\ z = \sum z^i\}$$

where the $T_i \subseteq \mathbb{R} \times \mathbb{R}^m \times \mathbb{R}^n$ are nonempty, compact, and convex production possibilities sets defined by

$$T_i = \{\ (q_i,x^i,z^i)\ :\ (q_i,x^i,z^i) \text{ are producible }\}.$$

The technology is q-nonjoint if there exist separate production possibilities sets for each nonallocatable netput. To see that input nonjointness is a special case of q-nonjointness let $q \in \mathbb{R}^k_+$, $x \in \mathbb{R}^n_-$, $z \in \mathbb{R}^m_-$. That is q is a vector of outputs, and both z and x are input vectors. As will be shown, q-nonjointness then implies the existence of separate production functions for each of the outputs (the usual notion of input nonjointness). Output nonjointness is the special case where the technology is q-nonjoint and q is a vector of inputs while both z and x are output vectors.[2] q-nonjointness then implies the existence of separate input-requirement functions for each input.

Input and output nonjointness, although apparently distinct notions, share one common aspect: both clearly dichotomize between inputs and outputs. The realities of agricultural production do not obey such neat dichotomies. As an example, coarse grains are typically outputs for grains farmers but inputs for livestock producers. The concept of netputs has no trouble accommodating this reality but the usual input-output dichotomy cannot. Suppose that a researcher has data on farmers producing livestock and wheat as outputs while some of these farmers are also coarse-grain producers and others use coarse grains solely as feed.[3] Is there a way to use observations on these farmers' operations to test for nonjointness in the production of wheat and livestock? Using the traditional notion of input nonjointness, which requires that grains be categorized as either an input or an output, the answer is no. However, q-nonjointness is both testable and meaningful.

Other examples from the farming sector and outside farming abound, but perhaps the most telling evidence on the shortcomings of the traditional notions of jointness for agriculture comes from common empirical practices in the agricultural economics literature which ignore the realities of agricultural production. A series of studies (Ray; Lopez; Ball; Shumway, Saez, and Gothret; Moschini) test for input nonjointness between livestock and grains sectors even though common observation reveals that outputs from the grain sector are used as inputs by the livestock sector. Some tests have even found support for the hypothesis that livestock and grains are input nonjoint (Lopez). So although these tests perhaps are legitimate for a very highly aggregated technology they give only scanty information about production relations within agriculture. Indeed, given the obvious realities of agricultural production one should be surprised, a priori, if such tests offer support for nonjointness.

Dual methods for testing for q-nonjointness are obvious: T is q-nonjoint if and only if the long-run profit function takes the form

$$R(p,w,r) = \sum R^i(p_i,w,r)$$

where $R^i(p_i,w,r)$ is the long-run profit function dual to T_i. (A proof is in the Appendix). If the technology is q-nonjoint its long-run profit function's Hessian matrix in p is diagonal.

For the remainder of the paper, assume that the technology is q-nonjoint.

Allocating Fixed and Variable Netputs: A Graphical Presentation

We now derive a representation of a q-nonjoint technology amenable to graphical illustration. Define the q_i-aggregator function $h^i(x,z)$ by

$$h^i(x,z) = \text{Max}\ \{q_i\colon\ (q_i, x, z) \in T_i\}.$$

If q_i is an output and both x and z are input vectors in T_i, $h^i(x,z)$ is a production function. If q_i is an input and both z and x are output vectors in T_i, $h^i(x,z)$ is an input-requirement function. Other interpretations of the q_i-aggregator function are also possible: suppose $q_i \in \mathbb{R}_+$, $x \in \mathbb{R}^n_-$, and $z \in \mathbb{R}^m_+$ in T_i, then $h^i(x,z)$ is an asymmetric transformation function in the sense of Diewert.

Each $h^i(x,z)$ is concave and continuous (see Appendix). For graphical convenience I depict the $h^i(x,z)$ as though they are differentiable. Dual to each $h^i(x,z)$ (and to T_i) is the q_i-specific profit function

$$R^i(p_i,\ w,\ z^i) = \text{Max}\ \{\ p_i h^i(x,z^i) + wx\ \}. \tag{4}$$

Each $R^i(p_i,w,z^i)$ is convex, continuous, and positively linearly homogeneous in p_i and w. $R^i(p_i,w,z^i)$ is also concave in the fixed netput allocation z^i (Diewert). If a unique solution to (4) exists, $R^i(p_i,w,z^i)$ is differentiable in p_i and w with

$$\frac{\partial R^i(p_i,w,z^i)}{\partial p_i} = q_i(p_i,w,z^i),\ \text{and} \tag{4$'$}$$

$$\nabla_w R^i(p_i w, z^i) = x^i(p_i w, z^i).$$

Here $q_i(p_i,w,z^i)$ and $x^i(p_i,w,z^i)$ are the q_i-specific profit maximizing q_i and variable netput allocation.

If $q_i \in \mathbb{R}_+$ and $x \in \mathbb{R}_-$, Figure 3 pictorially represents (4). (Notice the similarity with Figure 1.) The left panel presents graphical illustrations of both components of the maximand and the maximand itself; the right panel presents the first-order condition of equating marginal-value product of x to its price.

The conjugate to (4) is

$$h^i(x,z^i) = \underset{w/p_i}{\text{Min}} \{ R^i(1,w/p_i,z^i) - (w/p_i)x\}. \tag{5}$$

Figure 4 pictorially represents (5) for $x \in \mathbb{R}_-$. The left panel portrays both parts of the minimand and the minimand itself. The right panel depicts the first-order condition for the minimization problem in (5): the derivative of $R^i(1,w/p_i,z^i)$ with respect to w/p_i equals the profit maximizing allocation of x to the q_i production process. (Again this panel represents part of Shephard's lemma (4′) and generalizes to the n-dimensional case).

A special case of (5) is the input-nonjoint production technology. The allocation of variable inputs to the ith output has been the focus of the recent controversy on input-nonjoint production relations (SPN; JZH; Paris; and CJ). Figures 4 and 5 imply that these allocations can be recaptured either by inverting the first-order conditions for (4) or more directly from the first-order conditions for (5) (equations (4′)).

When T is q-nonjoint, R(p,w,z) can be derived in either of two ways: the first is to allocate the fixed and variable netputs to solve

$$R(p,w,z) = \text{Max} \{ pq + wx : q_i = h^i(x^i,z^i) \ (i = 1,\dots,k),\ x = \textstyle\sum x^i,\ z = \textstyle\sum z^i\} \tag{6}$$

The second is to solve

$$R(p,w,z) = \text{Max} \{ \textstyle\sum R^i(p_i,w,z^i) : \textstyle\sum z^i = z\}. \tag{6′}$$

Presuming that z is a scalar input and that $q \in \mathbb{R}^2_+$, Figure 5 graphically represents the first-order conditions for (6′) in terms of the "beaker" diagram familiar from trade theory (Dixit and Norman). The horizontal dimension of the beaker is given by the total amount of the allocatable-fixed netput (input) to be allocated across production activities, the right vertical axis measures the shadow price of the allocatable-fixed netput in the 1st production process, and the left vertical axis measures the shadow price of the allocatable-fixed netput in the 2d production process. The allocation of the allocatable fixed netput is determined by the equalization of these two shadow prices. So, for example, $0\hat{z}$ is allocated to activity 1 while $z'\hat{z}$ is allocated to activity 2.

A graphical depiction of the allocation decision across q_i is now available. From Figure 1 the total, allocatable-fixed netput utilization can be obtained as z′ when its price is r′. The amount z′ determines the horizontal dimension of the beaker in Figure 5. From Figure 5, one determines the allocations of the allocatable-fixed netput to the various production activities. Once these allocations are determined, either Figure 3 or Figure 4 can be used to determine the allocation of the allocatable-variable netput to the ith production process.

A simple example illustrates the use of Figures 1, 3, 4, and 5. Suppose r rises. From Figure 1 (this is true for an arbitrary number of allocatable-fixed netputs) total utilization of z falls. The horizontal dimension of Figure 5 shrinks. As a result, less (or more precisely no more than before) of the allocatable-fixed input is allocated to both activities. Two polar cases delimit the possibilities consistent with concavity of $R^i(p_i, w, z^i)$: one where the shadow-price is constant (constant returns to scale) and the other where the shadow-price function is perfectly vertical. In the former, all the decrease in the allocatable-fixed netput is absorbed by the constant-returns industry. In the latter, all the netput decrease is absorbed by the industry whose shadow price curve is not perfectly vertical. In between these polar cases, netput utilization falls in both industries. The decrease in z^i (remember the graphical illustration is for an input) decreases (increases) the value-marginal product of the allocatable-variable input in producing q^i if the allocatable-fixed and allocatable-variable netputs are cooperative (competitive). The allocation of the allocatable-variable netput to the ith production process falls (rises), accordingly.

A LeChatelier Result

Expression (6′) suggests a LeChatelier relationship exists between the short-run allocations and the q_i-specific allocations. For any feasible but otherwise arbitrary allocation of the allocatable-fixed netputs

$$R(p,w,z) \geq \sum R^i(p_i,w,z^i).$$

This inequality becomes an equality at the optimal allocation. The two functions are, therefore, tangent in (p,w) space implying that $q_i(p,w,z)$ is $q_i(p_i,w,z^i)$ evaluated at the optimal z^i. Because both $R(p,w,z)$ and $\sum R^i(p_i,w,z^i)$ are convex and tangent at the optimal allocation the inequality suggests that $R(p,w,z)$ must be more convex than $\sum R^i(p_i,w,z^i)$ (Figure 6 illustrates the case where $q_i \in \mathbb{R}_+$.) implying the LeChatelier result

$$\frac{\partial q_i(p,w,z)}{\partial p_i} \geq \frac{\partial q_i(p_i,w,z^i(p,w,z))}{\partial p_i}.$$

(A proof is in the Appendix.) Thus, long-run netput allocations are more own-price elastic than short-run allocations [the usual LeChatelier relationship (Sakai)], and short-run netput allocations in turn are more own-price elastic than q_i-specific allocations.

The SPN Problem

The pictorial representation of the first-order conditions for (6′) (Figure 5) contrasts markedly with the pictorial representations of the other first-order conditions. Figure 5 depicts the intersection of two upward sloping q_i-specific, shadow prices to determine the allocatable-fixed netput allocation. Figures 1 and 3 portray the intersection of an upward sloping shadow price or value marginal productivity with a constant price. For Figures 1 and 3 one can change the axes of the maximization problem to obtain a conjugate minimization problem in the prices that is the mirror image of the original problem. Thus, knowledge of $R(p,w,z)$ and r lets one infer $R(p,w,r)$ by optimizing over z. Conversely, the conjugate relation lets one obtain $R(p,w,z)$ from $R(p,w,r)$ and z by optimizing over r.

For Figure 5, two shadow prices are equated to get $R(p,w,z)$. Solving for $R(p,w,z)$, therefore, requires knowledge of <u>both</u> $R^1(p_1,w,z^1)$ and $R^2(p_2,w,z^2)$ (in general, of all the $R^i(p_i,w,z^i)$). Knowing $R(p,w,z)$ alone, therefore, is not sufficient to recapture <u>both</u> $R^1(p_1,w,z^1)$ and $R^2(p_2,w,z^2)$. Knowledge of $R(p,w,z)$ along with knowledge of either $R^1(p_1,w,z^1)$ or $R^2(p_2,w,z^2)$ is sufficient to recapture the other, but both cannot be inferred from $R(p,w,z)$ alone. Put another way, no natural conjugate to (6′) exists which can be used to infer the $R^i(p_i,w,z^i)$ from $R(p,w,z)$.

Expression (6′) also clarifies why information on either the long or the short-run profit functions is not sufficient to allow the researcher to capture exact information on q_i-specific allocatable-variable netput allocations. If Shephard's lemma applies, a unique solution to (6′) exists and the gradient of the short-run profit function in w equals the profit maximizing allocatable-variable netput level. But presuming each $R^i(p_i, w, z^i)$ is also differentiable and that a strictly interior solution to (6′) exists, applying the envelope theorem gives

$$\nabla_w R(p,w,z) = \sum \nabla_w R^i(p_i, w, z^i) = \sum x^i(p_i, w, z^i). \tag{7}$$

Shephard's lemma only gives the total allocation of the allocatable-variable netputs and not the specific allocations of the allocatable variable netputs.

Reconciling Approaches to Recapturing Allocatable-Variable Netput Allocations--The Flexibility-Efficiency Tradeoff

Three separate methods have been developed to recapture the $x^i(p_i, w, z^i)$ given information on q, z^i, z, x, p, and w. The first, suggested by JZH, is to specify the q_i-aggregator functions parametrically and use the first-order conditions for problem (6) as estimating equations. Once the $h^i(x,z)$ functions are estimated, the first-order conditions associated with (4) can be inverted to obtain estimates of the $x^i(p_i, w, z^i)$. (This requires that the inverse function theorem applies which, in turn, requires strong differentiability assumptions on the $h^i(x,z)$.) If the technology is captured by the chosen $h^i(x,z)$, this approach can achieve full econometric efficiency (presuming an efficient estimation procedure is utilized).

The main difficulty with the JZH approach is that obtaining closed-form solutions for the first-order conditions in (4), in terms of the x^i, for anything but the simplest specifications is always difficult and often impossible. For example, JZH use a single-output, Cobb-Douglas production function. The approach, thus, has extremely limited flexibility in representing agricultural technologies (because of the required differentiability assumptions it cannot handle well documented agricultural technologies like the von Liebig) and even in classes where it applies it cannot be implemented using most flexible functional forms.

Although Paris' contribution predates Chambers and Just, our purposes are best served by discussing the CJ approach next. CJ take a similar path to JZH but they use representation (6') and not (6). (CJ also develop and estimate a model capable of testing for input-nonjointness in the presence of allocatable fixed netputs.) Their approach is to specify flexible parametric representations of the q_i-specific profit functions and use the first-order conditions for (6') along with equations (4') to estimate the technology. Because all information available from optimization is used, the CJ approach also can achieve full econometric efficiency. However, allocatable-variable input allocations are now obtained via Shephard's lemma. Unlike the JZH approach, allocatable-variable netput allocations can be obtained without inverting first-order conditions. Instead the first-order conditions for the dual problem are estimated. Weaker differentiability and invertibility assumptions are involved so the approach also covers a broader class of technologies. Moreover, flexible functional forms can be used to represent the technology and the CJ approach dominates the JZH approach in terms of flexibility. CJ demonstrate this dominance by statistically rejecting the JZH specification in favor of a more general specification.

In principle, however, the CJ and JZH approaches are equivalent if the same differentiability assumptions are employed. One just uses the mirror image representations of the optimal allocatable-variable netput allocations of the other. In terms of this paper, one (JZH) uses Figure 3 the other (CJ) uses Figure 4.

The Paris approach,[4] however, is decidedly different from either the JZH approach or the CJ approach. Rather than relying on the stage-wise optimization approach outlined above which emphasises the distinct economic allocation decisions being made, the Paris approach is more algorithmic. It involves solving not the original optimization problem in (6) or (6′) but on solving another optimization problem which has the same solution as (6) and (6′). Paris recognizes that an optimization problem exists which has a natural conjugate relationship and which has the same solution as SPN, JZH, and CJ.

The approach is first to estimate a flexible functional specification of the short-run profit function $R(p,w,z)$. If this function is differentiable in z, $\nabla_z R(p,w,z)$ is the vector of shadow prices for allocatable-fixed inputs. Applying the envelope theorem (assuming an interior solution) to (6′) reveals that $-\nabla_z R(p,w,z)$ equals the optimal Lagrangian multipliers to which shadow prices are equated to solve that problem (see Appendix). (In Figure 5, $\nabla_z R(p,w,z)$ is the common value of $R_z^1(p_1,w,z^1)$ and $R_z^2(p_2,w,z^2)$ at their intersection.) Hence, the following optimization problem involves the same interior solutions as (6′).

$$\text{(8)} \qquad \text{Max } \{ \sum R^i(p_i,w,z^i) - \nabla_z R(p,w,z) \sum z^i \} = \sum_{i=1}^{k} R^i(p_i,w,-\nabla_z R(p,w,z))$$

where $R^i(p,w,-\nabla_z R(p,w,z))$ is the long-run profit function dual to T_i. The equality follows because the left-hand side of (8) is a separable programming problem. The last step in Paris' procedure is to recognize that Shephard's lemma, if applicable, implies,

$$q_i(p,w,-\nabla_z R(p,w,z)) = \frac{\partial R^i(p_i,w,-\nabla_z R(p,w,z))}{\partial p_i} \qquad i = 1,\ldots,k$$

$$x(p,w,-\nabla_z R(p,w,z)) = \sum_{i=1}^{k} \nabla_w R^i(p_i,w,-\nabla_z R(p,w,z))$$

$$z^i(p,w,-\nabla_z R(p,w,z)) = \nabla_r R^i(p_i,w,-\nabla_z R(p,w,z)).$$

After the $R^i(p,w,r)$ dual to T_i have been specified parametrically these last three sets of equations can then be used to estimate the $R^i(p,w,r)$ functions. Allocatable-variable input allocations can then be recaptured from the calculated expressions for $\nabla_w R^i(p_i,w,-\nabla_z R(p,w,z))$. While Paris frequently alludes to a "purified profit function," the purified profit function itself plays no direct role in estimation or in the resurrection of the allocatable-variable netput allocations. All this is accomplished via estimating the long-run profit functions for each T_i.

The intuition behind Paris' approach can best be seen by returning to (8). The recognition that $-\nabla_z R(p,w,z)$ equals the optimal Lagrangian multipliers for (6′) lets Paris "crack" that optimization problem into k-distinct suboptimization problems each of which only involves the $R^i(p_i,w,z^i)$ functions. Pictorially Figure 5 is split into two separate subproblems which mirror Figure 1. If $z \in \mathbb{R}-$, $\nabla_z R(p,w,z)$ on each side of Figure 5 plays the same role as -r′ in Figure 1. Just as Figure 1 has a conjugate representation in terms of the long-run profit function in Figure 2, so do each of these "cracked" suboptimization problems in terms of the q_i-specific long-run profit functions, the $R^i(p,w,-\nabla_z R(p,w,z))$. Once these $R^i(p,w,-\nabla_z R(p,w,z))$ are estimated they can be used to recapture the allocatable-variable input allocations via Shephard's lemma.

Because a duality exists both between T_i and $R^i(p,w,z^i)$ and T_i and $R^i(p,w,r)$, the Paris, JZH, and CJ approaches are, in principle, equivalent (apart from the different differentiability assumptions and the computational differences that arise from the different estimation procedures implied). The Paris approach, however, does not appear to achieve both full estimation efficiency and flexibility simultaneously. Recall that

$$R(p,w,-\nabla_z R(p,w,z)) = \sum_{i=1}^{k} R^i(p_i,w,-\nabla_z R(p,w,z)).$$

Hence, expressions (3) imply

$$(9) \qquad R(p,w,z) = \underset{\nabla_z R(p,w,z)}{\text{Min}} \quad R(p,w,-\nabla_z R(p,w,z)) + \nabla_z R(p,w,z) \cdot z.$$

$$= \underset{\nabla_z R(p,w,z)}{\text{Min}} \quad \sum_{i=1}^{k} R^i(p_i,w,-\nabla_z R(p,w,z)) + \nabla_z R(p,w,z) \cdot \sum z^i.$$

Expressions (9) imply that choosing a functional form for $R^i(p_i,w,r)$ has structural implications for $R(p,w,z)$. (The reader can see this explicitly by solving (9) for a Cobb-Douglas $R^i(p,w,r)$. $R(p,w,z)$ inherits the Cobb-Douglas parameters.) To achieve full econometric efficiency these restrictions (amounting to cross-equation restrictions) must be recognized and incorporated prior to estimating $R(p,w,z)$. The first stage of the Paris approach must capture these restrictions for full estimation efficiency to be achieved: arbitrary flexible specifications for $R(p,w,z)$ and the $R^i(p_i,w,r)$ will not satisfy (9). Efficiency requires the prior solution of (13) having specified the $R^i(p_i,w,r)$ functions. (One cannot start by specifying $R(p,w,z)$ and then using (9)'s conjugate relation to recapture the $R^i(p_i,w,-\nabla_z R(p,w,z))$. The SPN problem reappears in a different form.) As with the JZH approach, achieving closed form solutions for (9) is very difficult or impossible for all but the simplest specifications of the $R^i(p,w,r)$.[5] Full efficiency is available with Paris' approach but as with JZH it is purchased at the price of flexibility.

Conclusion

This paper generalizes the usual notions of nonjointness in a fashion consistent with the realities of agricultural production and amenable to graphical portrayal. This representation is then used to evaluate the three existing approaches (JZH, Paris, and CJ) to recapture allocatable-variable input allocations for nonjoint technologies. The CJ approach alone is shown to be capable of achieving full econometric efficiency and flexibility. Moreover, the CJ approach involves weaker differentiability and invertibility assumptions than either JZH or Paris so that it is applicable to a wider variety of technologies than either JZH or Paris.

Footnotes

1. Compactness is overly strong and could be replaced by closedness and semi-boundedness. Its purposes is to guarantee the existence of maxima for the maximization problems that follow. Note for example that compactness rules out T exhibiting constant returns to scale. In the case of constant returns compactness could be replaced by an appropriate modification such as closedness and boundedness for every z.
2. SPN have claimed that output-nonjoint technologies do not characterize agricultural technologies. This view seems to have been accepted by the profession (Lynne). Putting aside the fact that this is an issue which can only be resolved empirically, one should note that traditional examples of output-nonjoint industries are often drawn from the agricultural sector (e.g. cattle into meat, offal, and hide) (Lau).
3. A related example for time-series data would be: researchers have data on a sector (like agriculture) which produces more coarse grains than it uses in some periods (at the aggregate level coarse grains are an output) but which uses more coarse grains in some other periods than it produces (at the aggregate coarse grains are an input).
4. Paris sketches a procedure in his paper but does not present an empirical example. Thus, what follows involves some deduction on my part in arriving at an estimable procedure.
5. For both the JZH approach and the Paris approach if no closed-form solutions exist to the first-order condition's numerical procedures can be used to invert these equations locally. The extra computational burden than required for the JZH and Paris approaches to approximate functional flexibility emphasize the advantages of using representation (6′).

Appendix

Result: T is q-nonjoint if and only if

$$R(p, w, r) = \sum_{i=1}^{k} R^i(p_i, w, r).$$

If T is q-nonjoint then

$$R(p,w,r) = \text{Max}\ \{\ pq + wx + rz : (q_i, x^i, z^i) \in T_i\,(i=1,\dots,k),\ x = \sum x^i,\ z = \sum z^i\}$$

$$= \text{Max}\ \{\sum_{i=1}^{k} p_i q_i + wx^i + rz^i\ :\ (q_i, x^i, z^i) \in T_i\,(i=1,\dots,k)\}$$

$$= \sum_{i=1}^{k} \text{Max}\ \{\ p_i q_i + wx^i + rz^i\ :\ (q_i, x^i, z^i) \in T_i\}$$

$$= \sum_{i=1}^{k} R^i(p_i, w, r).$$

The converse follows by duality.

Result: $h^i(x,z)$ is concave.

Let $\hat{q}_i = h^i(\hat{x}, \hat{z})$ and $\bar{q}_i = h^i(\bar{x}, \bar{z})$. Concavity requires that for $\lambda \in (0,1)$

$$h^i(\lambda\hat{x} + (1 - \lambda)\bar{x},\ \lambda\hat{z} + (1 - \lambda)\bar{z}) \geq \lambda h^i(\hat{x}, \hat{z}) + (1 - \lambda)h^i(\bar{x}, \bar{z}).$$

By convexity of T_i $(\lambda\hat{q}_i + (1 - \lambda)\ \bar{q}_i,\ \lambda\hat{x} + (1 - \lambda)\bar{x},\ \lambda\hat{z} + (1 - \lambda)\bar{z}) \in T_i$ so the right-hand side is feasible. But the left-hand side is by definition the largest feasible element which completes the demonstration.

Result: The LeChatelier Result

The Lagrangian for (6′) is

$$L = \sum R^i(p_i, w, z^i) + \lambda \sum z^i - \lambda z$$

where $\lambda \in \mathbb{R}^m$ is a vector of Lagrangian multipliers. For notational convenience the result is demonstrated for the first nonallocatable netput.

By the envelope theorem (presuming Shephard's lemma and an interior solution)

$$\frac{\partial R(p,w,z)}{\partial p_1} = \frac{\partial R^1(p_1,w,\hat{z}^1)}{\partial p_1}$$

where (^) denotes optimal. Hence,

$$\frac{\partial^2 R(p,w,z)}{\partial p_1^2} = \frac{\partial^2 R^1(p_1,w,\hat{z}^1)}{\partial p_1^2} + \sum_{j=1}^{m} \frac{\partial^2 R^1(p_1,w,\hat{z}^1)}{\partial p_1 \; \partial z_j^1} \frac{\partial \hat{z}_j^1}{\partial p_1}$$

Differentiating the first-order conditions for the Lagrangian expression (not presented) gives the following expression

$$\frac{\partial \hat{z}_j^1}{\partial p_1} = - \sum_{t=1}^{m} h_{jt} \frac{\partial^2 R^1(p_1,w,\hat{z}^1)}{\partial z_t^1 \; \partial p_1}$$

where h_{jt} is the (j,t) element of the inverted Hessian matrix of the Lagrangian. Thus

$$\frac{\partial^2 R(p,w,z)}{\partial p_1^2} = \frac{\partial^2 R^1(p_1,w,\hat{z}_1)}{\partial p_1^2} - \sum_{j=1}^{m} \sum_{t=1}^{m} \frac{\partial^2 R^1}{\partial p_1 \partial z_j^1} h_{jt} \frac{\partial^2 R^1}{\partial p_1 \partial z_t^1}$$

The Lagrangian is concave in the z^i and, therefore, the inverted Hessian must be negative definite. The second term on the right of this last expression is thus minus a quadratic from in a principal submatrix of a negative definite matrix. It must be positive which establishes the derived result.

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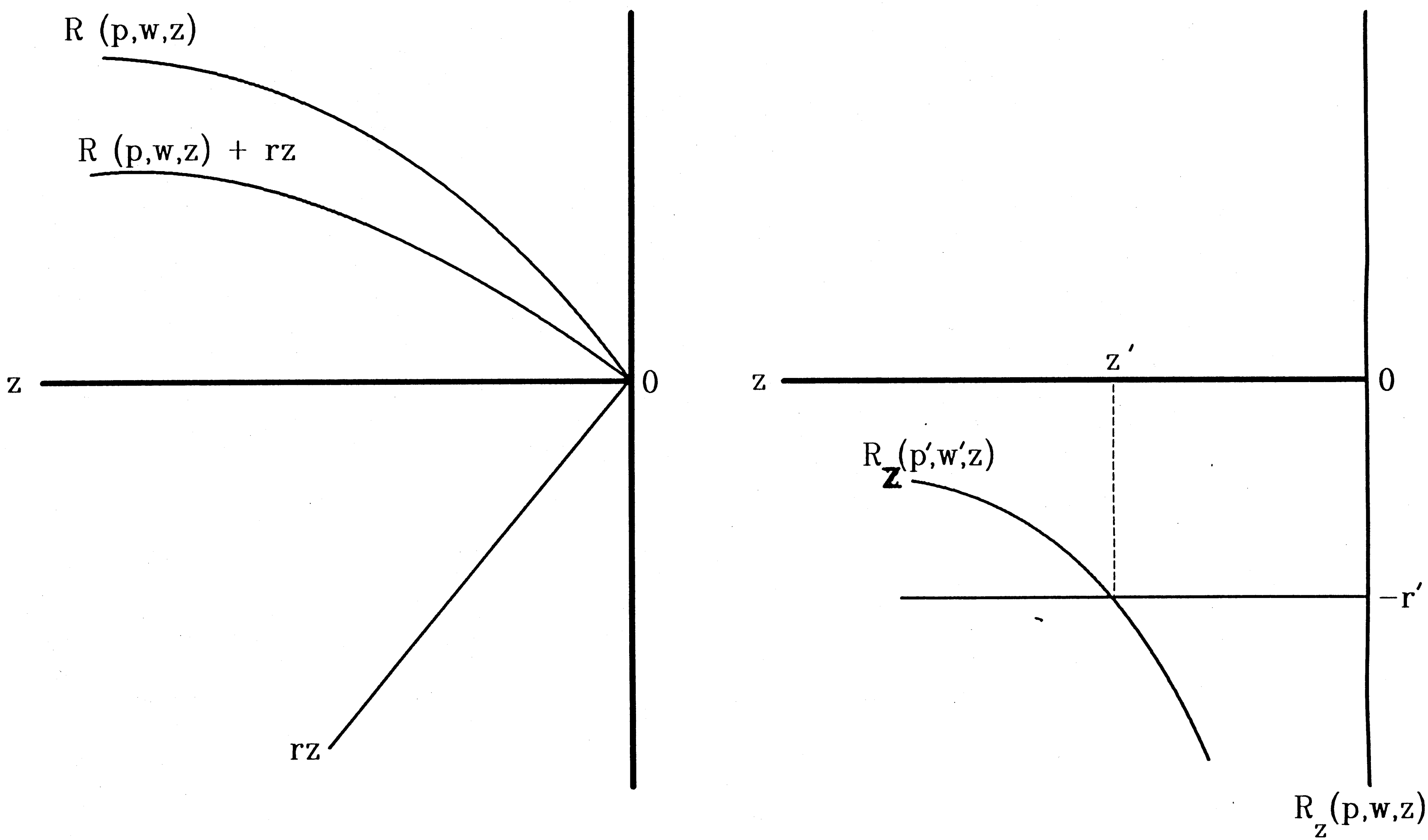

Figure 1: Long–Run, Allocatable–Fixed, Netput Choice

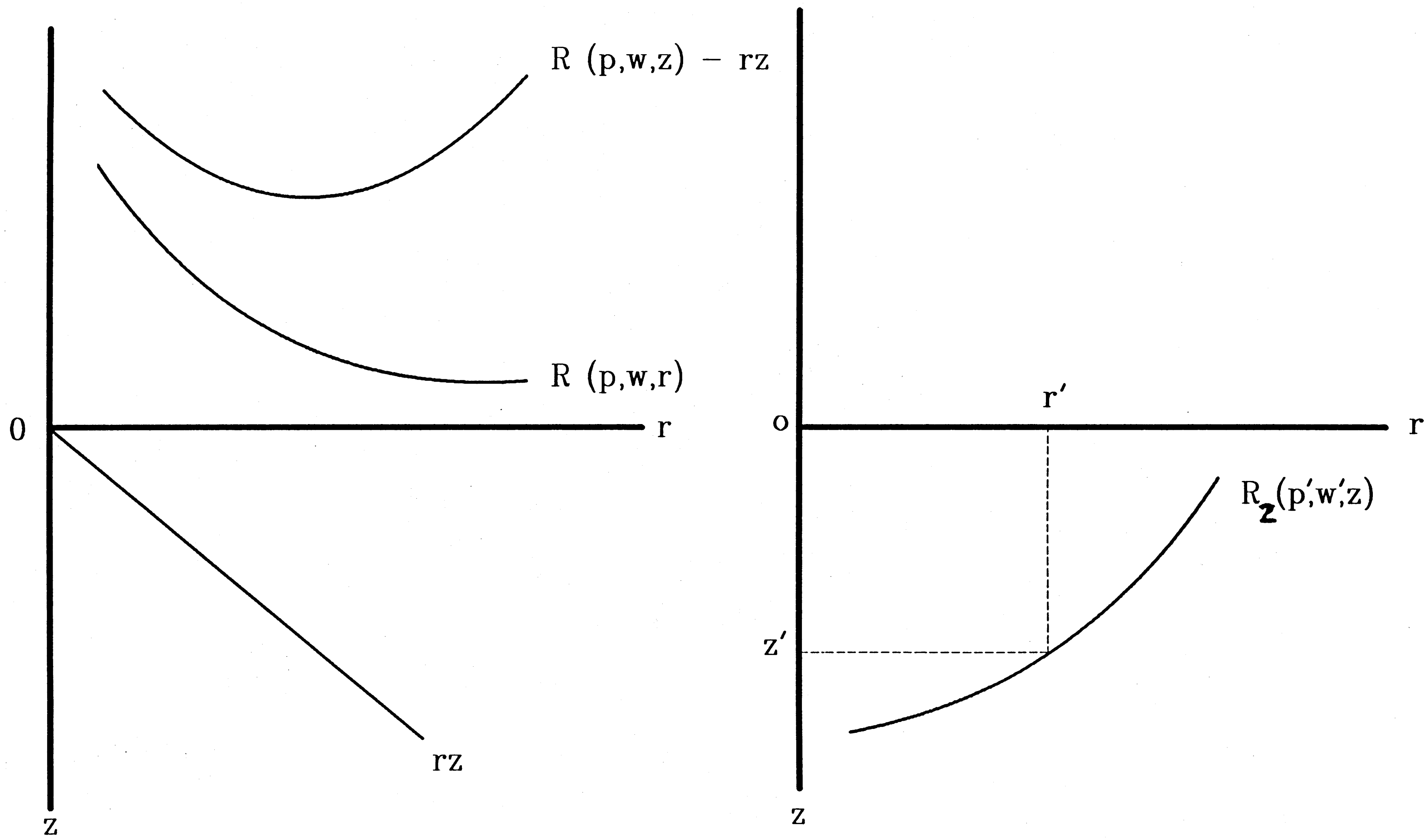

Figure 2: Long–Run, Short–Run Conjugate Relation

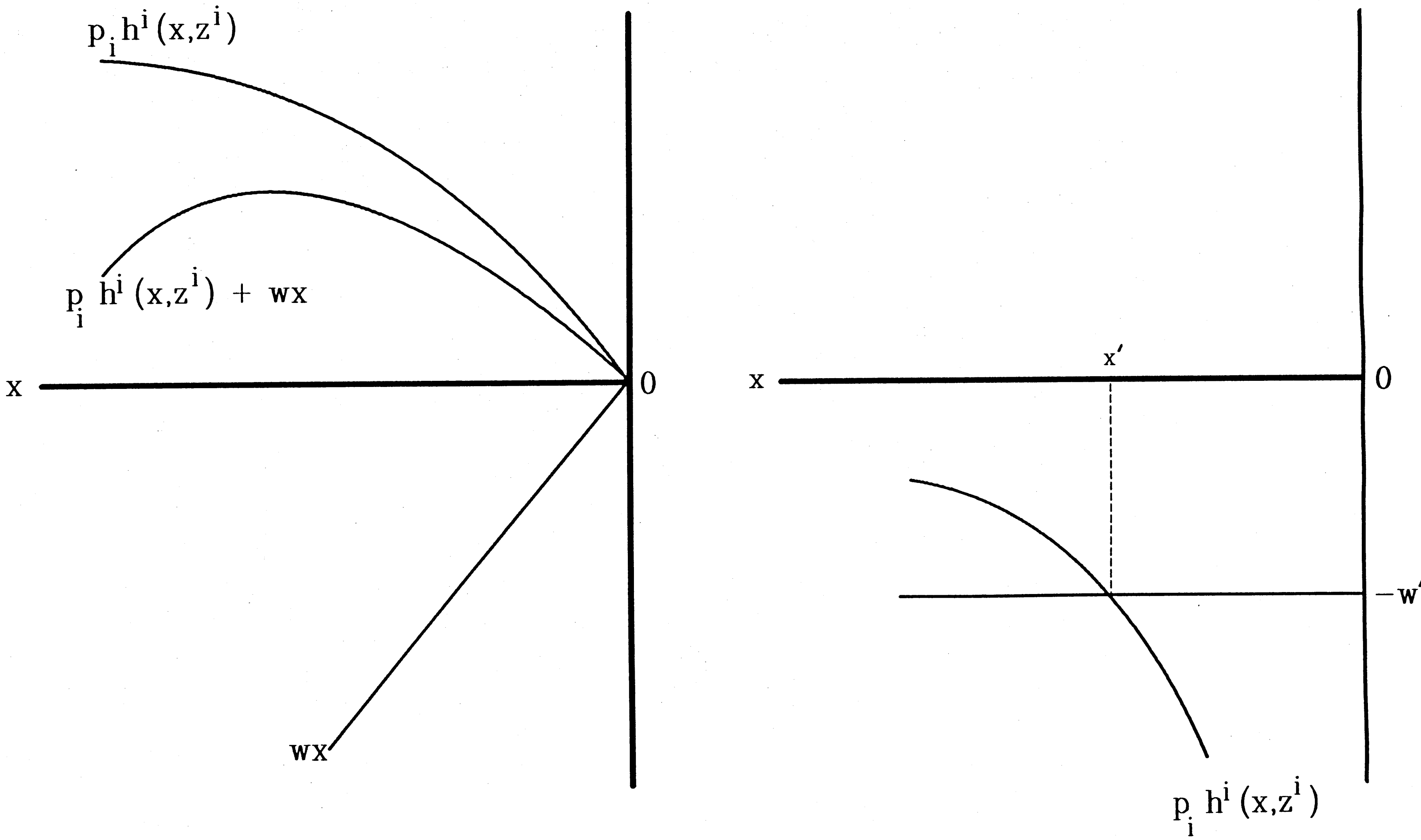

Figure 3: Allocating the Variable–Allocatable Netput

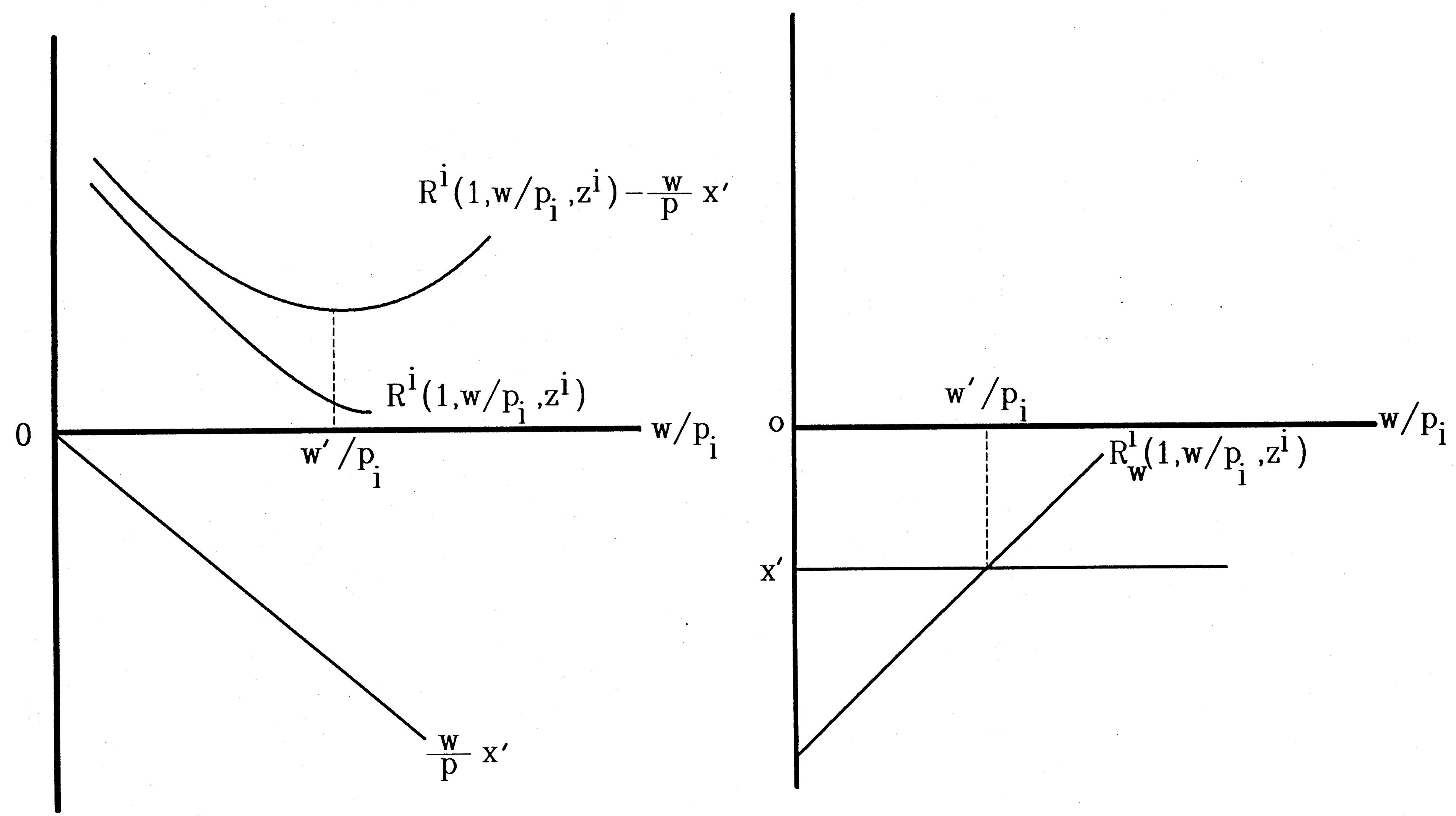

Figure 4: q^i–Specific Conjugacy

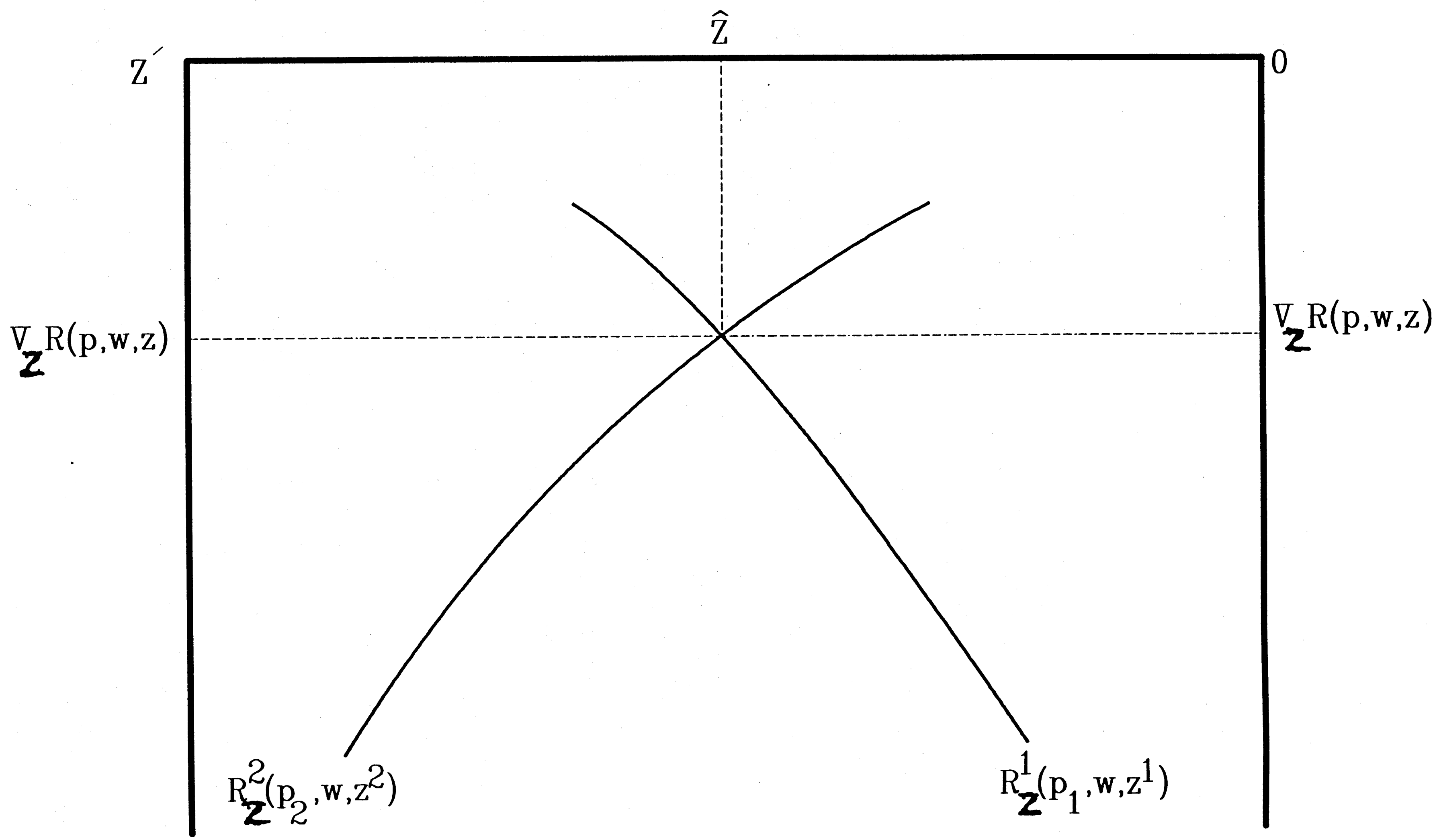

Figure 5: Allocating the Fixed–AllocatableNetput

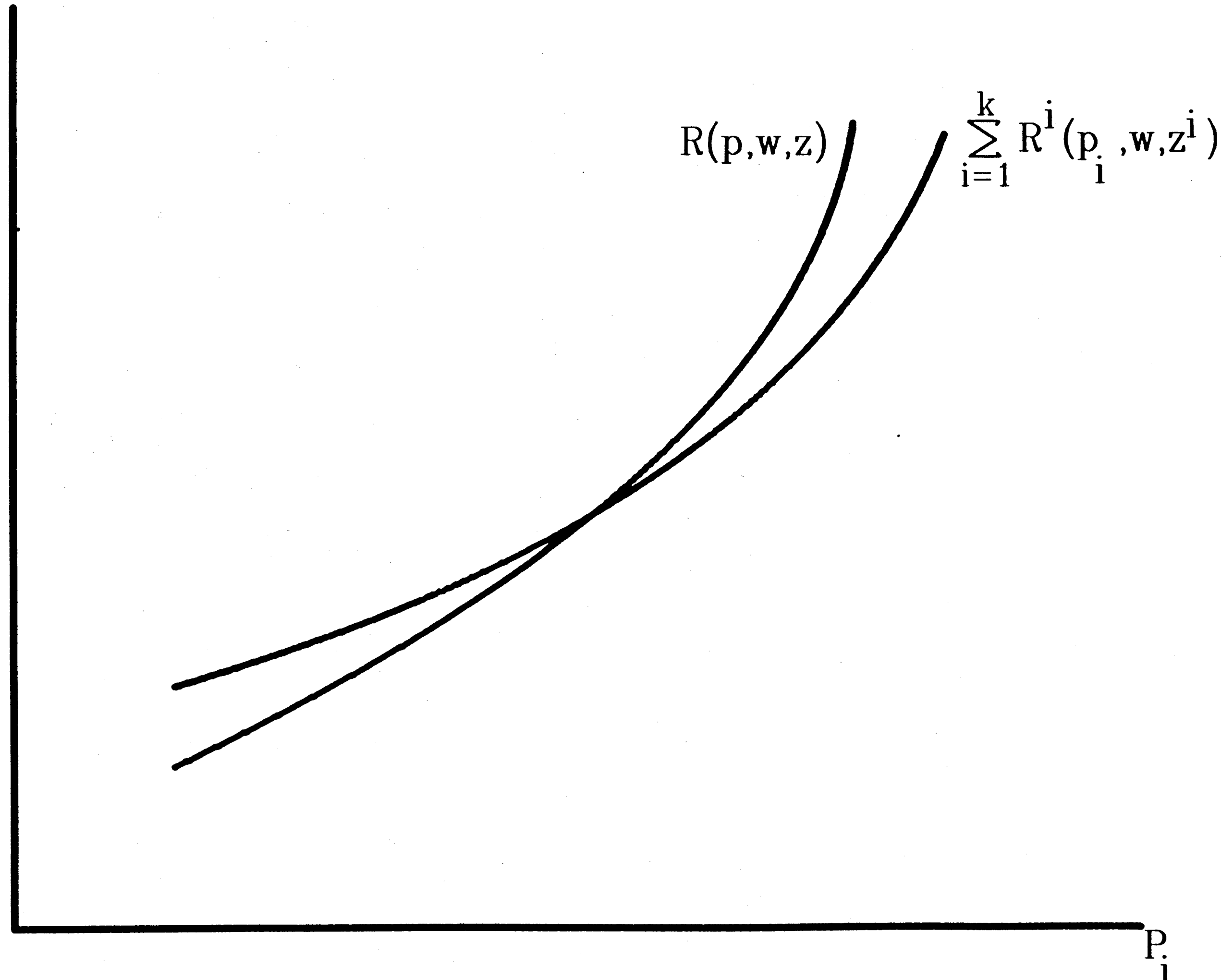

Figure 6: The LeChatelier Relationship