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On-Farm Reservoir Adoption in the Presence of Spatially Explicit Groundwater Use and Recharge

Kent Kovacs, Michael Popp, Kristofor Brye, and Grant West

Groundwater management is conducted in spatial aquifers where well pumping results in localized cones of depression. This is in contrast to the single-cell aquifer used in most economic analyses that assumes groundwater depletion occurs uniformly over a study area. We address two aspects of the optimal management of groundwater: a spatially explicit representation of the aquifer and the potential of on-farm reservoirs to recharge the underlying aquifer. A spatial-dynamic model of the optimal control of groundwater use and on-farm reservoir adoption is developed. Results suggest that a single-cell aquifer overestimates groundwater use and farm net returns over thirty years.

Key words: aquifer depletion, groundwater, on-farm reservoirs, spatial-dynamic optimization

Introduction

Economic studies of groundwater use have long observed a pumping cost or stock externality (Burt, 1964; Brown and Deacon, 1972). The withdrawal by one user lowers the water table and increases pumping costs for all users. This externality is typically modeled to operate in a single-cell or "bathtub" aquifer (Gisser and Sánchez, 1980; Burness and Brill, 2001) in which the pumping lift is assumed to be identical for every well and the spatial location of the well is irrelevant. Brozović, Sunding, and Zilberman (2010) demonstrate that the single-cell aquifer assumption incorrectly estimates the magnitude of the pumping externality relative to spatially explicit models. Pfeiffer and Lin (2012) find empirical evidence of a behavioral response to the spatial movement of groundwater in the agricultural region of western Kansas. While the potential differences in the pumping externality may be significant, the empirical ramifications for long-term groundwater use and farm profitability are left unexplored. Also, there is no investigation of how differences in the two representations of the aquifer change water use and farm production results when water conservation is practiced on the landscape.

This study addresses two aspects of the optimal management of a groundwater resource. First, we examine the empirical significance of the pumping externality associated with a spatial aquifer for groundwater depletion and farm net returns. Second, we evaluate the value of creating on-farm reservoirs to collect surface water for reuse throughout the season in the presence of a spatial aquifer. Reservoirs are allowed to recharge the aquifer, which lowers reservoir capacity but raises water table levels sufficiently to reduce groundwater pumping costs. To address these issues, we develop a

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Kent Kovacs is an assistant professor, Michael Popp is a professor, and Grant West is a program associate in the Department of Agricultural Economics and Agribusiness at the University of Arkansas. Kristofor Brye is a professor in the Department of Crop, Soil, and Environmental Sciences at the University of Arkansas.
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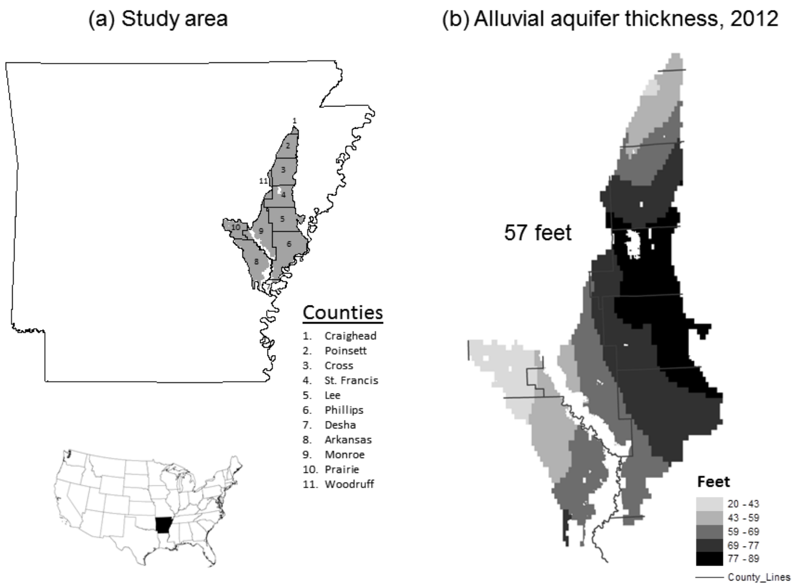

Figure 1. (a) Study Area Shown as Grid Cells. (B) Alluvial Aquifer Shown as Feet of Thickness in 2012.

Notes: Three eight-digit HUC watersheds define the outer boundary of the study area. An eight-digit HUC defines the drainage area of the sub-basin of a river. County lines overlay the study area. Public land and urban areas are excluded. Lighter shades indicate the groundwater resource is more abundant. The number by the side of the aquifer thickness map indicates the average.

spatial-dynamic model of groundwater use in an agricultural region with a spatially sensitive aquifer and with on-farm reservoirs that can recharge the aquifer.

The region for the application of our model encompasses three eight-digit hydrological unit code (HUC) watersheds (figure 1a) that span the farming region of the Arkansas Delta where the Mississippi River Valley alluvial aquifer (hereafter simply Alluvial aquifer) is most depleted (figure 1b). An eight-digit HUC defines the drainage area of the sub-basin of a river. Cropland irrigation accounted for 96% of Alluvial aquifer use in Arkansas in 2009 (Arkansas Natural Resources Commission, 2012). In 2007, the state supported 4.5 million acres of crop production under irrigation, making Arkansas the fourth largest user of groundwater nationally (Schaible and Aillery, 2012). Acreage under irrigation continues to expand because irrigation provides insurance against summer drought (Schaible and Aillery, 2012). A direct result of this growth in irrigated acreage is the depletion of the Alluvial aquifer. The current rate of pumping is unsustainable if not curtailed or if no recharge mechanism for the groundwater is created (Arkansas Natural Resources Commission, 2012). This puts water-intensive agricultural production at risk in the future; other adverse effects include land subsidence and reduced base flow to streams and wetlands.

As a result, state officials and policy makers are calling for water conservation by the agricultural community and for legislation encouraging alternate plans for water utilization (Hill et al., 2006). A portion of irrigation water can be recaptured and stored in reservoirs following release from flooded fields for future reuse, which is known as tail-water recovery. Rainfall runoff from fields is also stored in reservoirs as an irrigation source. These tail-water recovery and storage systems, however, occupy crop land, and cost-share assistance from EQIP funding and state income tax benefits have therefore been used to encourage the adoption of these water-saving practices (Wailes et al., 2004).

Earlier studies of on-farm reservoirs in the Delta use the Modified Arkansas Off-Stream Reservoir Analysis (MARORA) decision support software to evaluate whether individual farms should build reservoirs. MARORA estimates optimal reservoir size on a farm for different riceproducing locations with different saturated thickness levels and groundwater decline rates (Smartt et al., 2002). Young et al. (2004) report that the thirty-year net present value (NPV) of a farm with relatively inadequate groundwater increased by approximately \$2,000 per acre with the installation of a reservoir. Hill et al. (2006) examine how cost-share for reservoir construction and water diversion from the White river affects farm income and water use in the Grand Prairie of Arkansas. Farm size, crop mix, and groundwater conditions from eighty-four farms were entered into MARORA to identify the farms where building reservoirs would be economically advantageous.

Our model accounts for spatial variation in the saturated thickness of the aquifer, the yield of crops, and the costs of groundwater pumping. All of these factors influence the spatial-dynamic path of optimal management (Brozović, Sunding, and Zilberman, 2010). The central planner maximizes farm net returns over three decades by allocating acreage to crops and reservoirs subject to constraints on groundwater supplies, reservoir water availability, and land availability. Spatial groundwater flow occurs between sites in response to the distance from cones of depression formed by well pumping. The single-cell aquifer spreads the depletion of groundwater evenly over the study area. The boundaries of the single-cell aquifer and the study area do not exactly coincide, but the study area is large enough to reasonably assume that they are the same. With and without on-farm reservoirs, we compare model results for the spatial aquifer and the single-cell aquifer.

By allowing reservoirs to recharge the aquifer, we estimate the potential gains in farm net returns attainable from lower groundwater pumping costs as a function of the lower depth to the aquifer and therefore less energy to raise the water to the surface. The water table does not have to rise, only not fall as fast, to affect energy use over time. Even small reductions in pumping costs can translate to measureable gains over the long term. Groundwater provides farmers with a stable supply of water; the value of this certainty is called buffer value and is likely not considered by current farmers using groundwater, representing a loss to future generations. The social price of groundwater should then reflect its risk management or stabilization value. By adding the buffer value of groundwater to the objective, we estimate a social level of aquifer depletion and consider various policies to raise the water table.

We describe the models for the dynamics of land and water use and the optimal control problem faced by the planner in the next section. Data for the crop land and parameter values for the model are presented in the third section. Section four discusses the results and sensitivity analyses. We conclude with a summary of the major findings, relate the findings to prior work in a similar vein, and list future research needs.

Methods

Dynamics of Land and Water Use

Spatial dynamics of land and water use in the rice-soybean production region of the Delta focus on the supply of water available in the underlying aquifer. Our model follows from a map-grid representation of spatially symmetric cones of depression from groundwater pumping. The model consists of a grid of m cells (sites) and accounts for the amount of available groundwater by time period based on the pumping decisions of farms in and around the cell weighted by distance.

For each site i we track the acreage of land in use j for n land-cropping choices (rice, irrigated soybean, and non-irrigated soybean) at the end of period t with $L_{ij}(t)$. The decision to look only at rice and irrigated and non-irrigated soybeans keeps the focus on water use rather than relative crop prices. We assume that land can be converted to on-farm reservoirs $FR_{ij(t)}$ from existing land use j during period t, and the amount of land converted to reservoirs at the end of period t is $R_i(t)$ at each site. Farmers can choose to switch land out of water-intensive rice into irrigated soybeans in response to a growing water shortage, and this is tracked with the variable $RS_i(t)$. The declining groundwater availability may lead farmers to switch land out of irrigated crops into non-irrigated soybeans; the variable tracking the land switching to non-irrigated soybean is $DS_{ij}(t)$. Using these definitions, we model the dynamics of land use in each site as a system of difference equations:

$$\begin{aligned} L_{ij}(t) &= L_{ij}(t-1) - FR_{ij}(t) - DS_{ij}(t) - RS_i(t), \text{ for } j = \text{rice} \\ L_{ij}(t) &= L_{ij}(t-1) - FR_{ij}(t) - DS_{ij}(t) + RS_i(t), \text{ for } j = \text{irr. soybean} \\ L_{ij}(t) &= L_{ij}(t-1) - FR_{ij}(t) + \textstyle\sum_{j=1}^{n} DS_{ij}(t), \text{ for } j = \text{non-irr. soybean} \end{aligned} \tag{1}$$

$$R_i(t) = R_i(t-1) + \sum_{j=1}^{n} FR_{ij}(t) \tag{2}$$

In each period, the amount of irrigated land in use j is reduced by the amount of land converted to on-farm reservoirs or switched into non-irrigated soybean production. For cropland in rice (the most water-intensive irrigated crop), a switch to irrigated soybean (a less irrigation-intensive crop) can also occur where the decline in rice is offset by the increase in irrigated soybean. The amount of land in non-irrigated soybeans by the end of period t is the amount of land in non-irrigated soybeans in earlier periods and the sum of the land added to non-irrigated soybean from all land uses j less the land converted to on-farm reservoirs during period t (equation 1). The amount of land in on-farm reservoirs by the end of period t is the amount of land in reservoirs in earlier periods and the sum of the amount of land added to reservoirs from all land uses j during period t (equation 2). The total amount of land converted to a reservoir from land use j must be less than the amount of land in use j as of period $t-1$: $FR_{ij}(t) \leq L_{ij}(t-1)$.

Irrigation demand in acre-feet is given by wd_j, representing average annual irrigation needs for crop j. We assume the annual irrigation applied to crop j cannot be adjusted. Purcell, Edwards, and Brye (2007) find the yield response of soybeans to irrigation was linear, suggesting that an acre-inch applied, depending on weather and location, will yield a fixed extra yield regardless of the level of irrigation. A profit-maximizing producer would then either fully irrigate or go with rain-fed production depending on the marginal cost of pumping (MCP) and the marginal value product (MVP) of soybeans. The producer chooses to fully irrigate as long as MVP>MCP. The MVP increases for later season applications of water as late-season irrigation provides sufficient water to increase seed size and/or seed fill and hence yield. A producer that has started to irrigate would thus want to reap the final benefit. The variable $AQ_i(t)$ is the amount of groundwater (acre-feet) stored in the aquifer beneath site i at the end of the period t. The amount of water pumped from the ground is $GW_i(t)$ during period t, and the amount of water pumped from the on-farm reservoirs is $RW_i(t)$. The natural recharge (acre-feet) of groundwater at a site i from precipitation, streams, and underlying aquifers in a period is nr_i and is independent of crops grown on site i.

The reservoirs can be constructed to allow some ponded water to infiltrate through the soil and recharge the aquifer rather than lining the reservoir with a layer of clay to minimize seepage. The amount of water that an acre of reservoir can recharge the aquifer is r_i, which depends on the underlying soil present at site i. The runoff from site i is diverted to reservoirs through a tail-water recovery system. A reservoir, making up a small portion of acres available in site i, can be completely filled from the runoff collected from site i. A larger reservoir occupying a larger fraction of site i is only partly filled because the reservoir receives the same acre-feet of runoff.

Hence, the acre-feet of water an acre of reservoir can hold at full capacity from runoff throughout site i is $\omega_{\max}$. The water accumulated from rainfall into the reservoir is $\omega_{\min}$ per acre. The values for $\omega_{\max}$ and $\omega_{\min}$ are based on evaporation, rainfall, and the timing of rainfall during the season. We define the following function for the acre-feet of water stored in an acre of reservoir: $(\omega_{\max} + \omega_{\min}) - \frac{\omega_{\max}}{\sum_{j=1}^{n} L_{ij}(0)} R_i(t) - r_i$, which depends on the number acres of the reservoir $R_i(t)$, the total acreage of the farm field at site i, $\sum_j L_{ij}(0)$ with starting crop mix at $t=0$, and the seepage of water from the reservoir to recharge the aquifer r_i. Allowing the seepage of reservoir water appears counterproductive because the water is pumped from the ground later at greater expense, but the advantage of infiltration is the storage of water in the aquifer for use at a later date when water is more limited.

The function for the acre-feet of water stored in an acre of reservoir can be rewritten as $\omega_{\max}\left(1-\frac{R_i(t)}{\sum_{j=1}^{n}L_{ij}(0)}\right)+(\omega_{\min}-r_i)$. This shows that the acre-feet of water held by an acre of reservoir ranges from a low of $(\omega_{\min}-r_i)$, when the reservoir occupies the entire field and only annual rainfall fills the reservoir minus the infiltration, to a high of $\omega_{\max}+(\omega_{\min}-r_i)$ for a reservoir so small that tail-water recovery provides plenty of water to completely fill the reservoir. As the reservoir occupies more of the field, the term $\left(1-\frac{R_i(t)}{\sum_{j=1}^{n}L_{ij}(0)}\right)$ approaches zero because less recovered water is available to fill the reservoir to the maximum.

Further, we define p_{ik} as the expected proportion of the groundwater in the aquifer that flows underground out of site i into the aquifer of site k when an acre-foot of groundwater is pumped out of site k, where p_{ik} is based on the distance between sites i and k, the saturated thickness, and hydraulic conductivity of the aquifer. The amount of water leaving site i is then $\sum_{k=1}^{m} p_{ik}GW_k(t)$.

The cost of pumping an acre-foot of groundwater to the surface at site i during period t is $GC_i(t)$. Pumping costs depend on the cost to lift one acre-foot of water by one foot using a pump, c^p, the initial depth to the groundwater within the aquifer, dp_i, and the capital cost per acre-foot of constructing and maintaining the well, c^c. Note that we assume a producer drills a well deeper than the depth to the aquifer to allow for the eventual decline in the water table. Pumping costs vary by the energy required to lift water to the surface. The possibility of new well drilling in cases where the aquifer level drops below the initial drilled depth is captured in the capital cost per acre-foot. The dynamics of water use and pumping cost at each site is then represented by

$$\sum_{j=1}^{n} wd_j L_{ij}(t) \le GW_i(t) + RW_i(t); \tag{3}$$

$$RW_i(t) \le \left((\omega_{\max}+\omega_{\min}) - \frac{\omega_{\max}}{\sum_{j=1}^{n}L_{ij}(0)}R_i(t) - r_i\right)R_i(t); \tag{4}$$

$$AQ_i(t) = AQ_i(t-1) - \sum_{k=1}^{m} p_{ik}GW_k(t) + r_iR_i(t) + nr_i; \tag{5}$$

$$GC_i(t) = c^c + c^p\left(dp_i + \frac{(AQ_i(0)-AQ_i(t))}{\sum_{j=1}^{n}L_{ij}(0)}\right). \tag{6}$$

In each period, the total amount of water for irrigating crops grown at the site must be less than the water pumped from the aquifer and the reservoirs (equation 3), and the amount of water available from reservoirs must be less than the maximum amount of water that all the reservoirs built on the site can hold (equation 4). The cumulative amount of water in the aquifer by the end of period t is the amount of water in earlier periods plus the amount of recharge that occurs naturally and by the reservoirs less the amount of water pumped from surrounding sites weighted by the proximity to site i (equation 5).

Farm Net Benefits Objective

In the absence of available information on the location and size of individual farms under the direction of a particular farm manager and the location and size of existing wells, we make simplifying assumptions about the optimal construction of on-farm reservoirs subject to land- and water-use constraints. We set the size of each cell to 600 acres comprising $\sum_j L_{ij}(0)$ acres in field crops and the remainder in natural landscape, farmsteads, and public lands. The existing well capacity and pumping equipment only supports the current crop mix $L_{ij}(0)$ with ongoing payments made for this equipment. Investment in reservoirs and a tail-water recovery system includes additional pumping equipment for moving water from the tail-water recovery system into the reservoir and from the reservoir to the existing irrigation system at each site as well as annual maintenance costs. The overall objective is then to maximize the net benefits of farm production less the costs of reservoir construction and use over time.

Several economic parameters are needed to complete the formulation. The price per unit of the crop is pr_j and the cost to produce an acre of the crop is ca_j, which depends on the crop, j. The yield of crop j per acre is y_{ij} at site i. The net value per acre for crop j is then $pr_j y_j - ca_j$, excluding differential water pumping cost between well and reservoir water and the reservoir construction costs. The discount factor to make values consistent over time is δ_t. The annual per acre cost of constructing and maintaining a reservoir is c^r, and the cost of pumping an acre-foot of water from the tail-water recovery system into the reservoir and from the reservoir to the field plus the capital cost per acre-foot of constructing and maintaining the pump is c^{rw}.

The problem is to maximize net benefits of farm production:

(7)
$$\max_{\substack{FR_{ij}(t),\ GW_i(t),\\ DS_{ij}(t),\ RW_i(t)}} : \sum_{t=1}^{T} \delta_t \left(\sum_{i=1}^{m} \sum\nolimits_{,j=1}^{n} (pr_j y_{ij} - ca_j) L_{ij}(t) - c^r FR_{ij}(t) - c^{rw} RW_i(t) - GC_i(t) GW_i(t) \right)$$

subject to

$$L_{ij}(0) =: L_0^{ij},\ \ R_i(0) = 0,\ \ AQ_i(0) = AQ_0^i; \tag{8}$$

$$FR_{ij}(t) \geq 0,\ \ L_{ij}(t) \geq 0,\ \ AQ_i(t) \geq 0; \tag{9}$$

and the spatial dynamics of land and water use (equations 1–6). The objective (equation 7) is to determine $L_{ij}(t)$, $FR_{ij}(t)$, $RW_i(t)$, and $GW_i(t)$ (i.e., the number of acres of each crop, the number of acres of reservoirs, and water use) to maximize the present value of net benefits of farm production over the fixed time horizon T. Solving an infinite horizon problem with more than 2,900 sites i is not computationally feasible. Benefits accrue from crop production constrained by the water needed for the crops. Costs include the construction and maintenance of reservoirs/tail-water recovery, the capital costs and maintenance of the pumps, fuel for the pumping of water from the reservoirs or ground, and all other production costs. Equation (8) represents the initial conditions of the state variables, and equation (9) is the non-negativity constraint on land use and the aquifer as well as nonreversibility on reservoir construction. We solve this problem with Generalized Algebraic Modeling System (GAMS) 23.5.1 using the nonlinear programming solver CONOPT from AKRI Consulting and Development.

Optimality Conditions

The problem of the collective of farmers (equation 7) is used to examine how much land to convert to reservoirs for two cases. The first case assumes finite lateral flow of groundwater as described by equation (5), and the second case is of infinite lateral flow of groundwater (i.e., the single-cell aquifer). Based on the necessary conditions for equation (7) derived in the appendix, the amount of land converted to reservoir depends on the magnitude of groundwater pumping, which is affected by the degree of water flow underground. Lastly, the effect of allowing recharge of the aquifer through reservoirs on the land converted to reservoirs is evaluated.

The condition for the optimal acres to turn into on-farm reservoirs, $FR_{ij}(t)$, from crop j at site i is

$$\delta_t wd_j GC_i(t) + \delta_t GC_i(t)\frac{\partial RW_i(t)}{\partial FR_{ij}} + \lambda_R^i(t) = \delta_t(c^r + (pr_j y_{ij} - ca_j)) + \delta_t c^{rw}\frac{\partial RW_i(t)}{\partial FR_{ij}} + \lambda_L^{ij}(t). \quad (10)$$

The term $\frac{\partial RW_i(t)}{\partial FR_{ij}}$ is positive because more acres converted to a reservoir means more reservoir water available for the crops on the remaining land. Equation (10) indicates that the farm collective should convert land to reservoirs until the marginal benefit of more reservoirs, shown on the left-hand side of the equation (10), equals the marginal cost of more reservoirs, shown on the right-hand side of equation (10).

The marginal benefit of another acre in reservoir is the present value of the avoided groundwater pumping costs since crop j is not grown, $\delta_t wd_j GC_i(t)$, plus the present value of the avoided groundwater pumping costs from more reservoir water, $\delta_t GC_i(t)\frac{\partial RW_i(t)}{\partial FR_{ij}}$, plus the shadow value of reservoir land $\lambda_R^i(t)$. The marginal cost of another acre of reservoir is the present value of the construction cost c^r and the loss of crop j profit $pr_j y_{ij} - ca_j$, plus the present value of the pumping costs to use the reservoir water, $c^{rw}\frac{\partial RW_i(t)}{\partial FR_{ij}}$, and the shadow value of not having site i in crop j, $\lambda_L^{ij}(t)$).

Next, we show how the degree of lateral flow within the aquifer influences the optimal number of reservoirs. This is done by examining the necessary conditions of equation (7) for groundwater pumping, $GW_i(t)$, in the cases of finite and the single-cell flow of groundwater. With finite lateral flows, the condition for groundwater pumping can be solved as $\delta_t\left(\sum_{j=1}^n \frac{(pr_j y_{ij} - ca_j)}{wd_j} + c^{rw} - \sum_{k=1}^m \frac{c^p p_{ki}}{\sum_{j=1}^n L_{ij}(0)} GW_k\right) - \sum_{k=1}^m \lambda_{AQ}^k(t) p_{ki} = \delta_t GC_i(t)$. With single-cell lateral flows, the corresponding condition for groundwater pumping is $\delta_t\left(\sum_{j=1}^n \frac{(pr_j y_{ij} - ca_j)}{wd_j} + c^{rw}\right) - \lambda_{AQ}(t) = \delta_t GC_i(t)$. The marginal benefit of groundwater pumping can be less under the assumption of finite lateral flows aquifer because groundwater pumping at site i influences the pumping costs of nearby sites. As slower depletion of the aquifer occurs, the need for reservoirs is diminished. Looking at the left-hand side of equation (10), higher optimal groundwater pumping costs with the single-cell aquifer mean that more reservoirs are built.

The condition for the flow of the shadow value of the aquifer (i.e., the most the farm collective would pay for another acre-foot of water in the aquifer) with and without the spatial representation of the aquifer provides another perspective on why different acres of reservoirs are built for each type of aquifer. First, the condition for $AQ_i(t)$ when the aquifer is spatially segmented is $\dot{\lambda}_{AQ}^i = -H_{AQ_i} = \delta_t c^p \frac{GW_i(t)}{\sum_{j=1}^n L_{ij}(0)}$. Second, the condition for $AQ(t)$ when the aquifer is a single cell is $\dot{\lambda}_{AQ} = -H_{AQ} = \delta_t c^p \sum_{i=1}^m \frac{GW_i(t)}{\sum_{j=1}^n L_{ij}(0)}$. Greater groundwater pumping in the single-cell aquifer means that the shadow value of the single-cell aquifer is rising faster than the shadow value of an aquifer assumed to have finite lateral flow. The greater rise of shadow value in the single-cell aquifer means that farms turn faster to the construction of reservoirs for cheaper water.

When reservoirs can recharge the aquifer, equation (10) changes to

$$\delta_t wd_j GC_i(t) + \delta_t GC_i(t)\frac{\partial RW_i(t)}{\partial FR_{ij}} + \lambda_R^i(t) + \lambda_{AQ}^i(t) r_i = \delta_t(c^r + (pr_j y_{ij} - ca_j)) + \delta_t c^{rw}\frac{\partial RW_i(t)}{\partial FR_{ij}} + \lambda_L^{ij}(t). \quad (11)$$

The value of the additional aquifer water, $\lambda_{AQ}^i(t) r_i$, suggests that more land should be made into reservoirs, but the reduction in irrigation water held by reservoirs suggests that less land should be converted to reservoirs compared to the model in which reservoirs do not need to recharge because of the nonpermeable clay layer that prevents seepage.

Buffer Value Objective

Groundwater provides farmers with a stable supply of water that represents a value beyond that of a supplement to non-irrigated crop production. The economic value of this risk management or stabilization role is called buffer value. Tsur (1990) defines buffer value, BV, as the amount a grower facing an uncertain surface water supply would be willing to pay for groundwater above the corresponding amount the grower would be willing to pay had surface water supplies been certain (or certainty equivalent).

Let the uncertain supply of surface water, S, be distributed according to a cumulative distribution having mean μ and variance σ^2. In the absence of groundwater, growers use the surface water available and enjoy the operating profit per acre of $pF(S)$, where $F()$ represents per acre yield response to water and p is the net unit value of the crop. Tsur (1990) shows that buffer value is $BV(p,\mu) = pF(\mu) - pE\{F(S)\}$. By expanding $F(S)$ about μ, BV can then be approximated by $BV \cong 0.5p[-F''(\mu)]\sigma^2$. This indicates that BV depends on the value of marginal productivity of water at μ, the degree of concavity of F at μ, and the variance of surface water supply σ^2. We assume that BV remains constant over time for each acre-foot of water left in the ground.

We augment equation (7), our objective function, to include the buffer value of groundwater. The NPV of the buffer value of the groundwater is

$$BV \sum_{t=1}^{T} \delta_t \sum_{i=1}^{m} AQ_i(t). \qquad (12)$$

The buffer value objective is then equation (7) plus equation (12) and hence net benefits accrue from farm production as well as groundwater stocks.

Sensitivity Analyses

To evaluate the impact of tail-water recovery/reservoir systems, groundwater depletion, and the buffer value of the aquifer, model outcomes at different times are compared to the initial crop-acreage allocation for 2012. Model runs were performed by i) allowing the building of reservoirs or not by setting $\sum FR_{ij} = 0$; ii) allowing the spatially differentiated movement of groundwater or not by changing equation (5) to $AQ(t) = AQ(t-1) - \sum_{i=1}^{m}(GW_i(t) + nr_i)$ such that the pumped groundwater affects the pumping cost of all sites uniformly; iii) adding a buffer value to the objective function or not; iv) allowing the reservoirs to recharge the aquifer or not by setting r_i to zero in equations (4) and (5); and v) evaluating policy options for groundwater conservation that include cost share for reservoir construction by modifying c^r, subsidizing reservoir pumping cost c^{rw}, or taxing groundwater pumping cost GC.

Data

The study area includes three eight-digit HUC watersheds (L'anguille, Big, and the Lower White)[1] that represent the region of the Arkansas Delta where unsustainable groundwater use is occuring (figure 1a). The watersheds overlap eleven Arkansas counties: Arkansas, Craighead, Cross, Desha, Lee, Monroe, Phillips, Poinsett, Prairie, St. Francis, and Woodruff. The study area is divided into 2,973 sites to evaluate how farmers make decisions about crop allocation and water use in a spatially differentiated landscape. The 2010 Cropland Data Layer (Johnson and Mueller, 2010) determines

[1] The HUCs for L'anguille, Big, and the Lower White are 08020205, 08020304, and 08020303.

Table 1. Descriptive Statistics of the Model Data across Study Area Sites (n=2,973)

Variable	Definition	Mean	Std. Dev.	Sum (thousands)
$L_{i,rice}$	Initial acres of rice	123	116	366
$L_{i,irrsoy}$	Initial acres of irrigated soybean	184	103	548
$L_{i,non-irrsoy}$	Initial acres of dry land soybeans	59	63	174
$y_{i,rice}$	Annual rice yield (cwt per acre)	69	3	-
$Y_{i,irr-soy}$	Annual irrigated soybean yield (bushels per acre)	42	3	-
$y_{i,non-irrsoy}$	Annual non-irrigated soybean yield (bushels per acre)	26	4	-
dp_i	Depth to water (feet)	57	31	-
AQ_i	Initial aquifer size (acre-feet)	27,587	12,514	82,016
K	Hydraulic conductivity (feet per day)	226	92	-
r_i	Annual aquifer recharge from an acre of reservoir (acre-feet)	0.04	0.06	-
nr_i	Annual natural recharge of the aquifer per acre (acre-feet)	0.001	0.04	547

the initial acreage of rice and soybeans in each cell (table 1), and the irrigated versus non-irrigated soybean acreage is allocated on the basis of harvested acreage for 2010–2011 (Arkansas Field Office, 2011). The 2% real discount rate chosen for the analysis corresponds to the 5% average yield of the thirty-year Treasury bond over the last decade (U.S. Department of the Treasury, 2012) less a 3% inflation expectation. County crop-yield information for the past five years is used as a proxy for yields of each of the crops and not adjusted over time. The cost of production for all crops and the ownership and maintenance charges for reservoirs and wells are also held constant.

Groundwater Use and Recharge

The depth to the water table (from surface to the top of the water table) and initial saturated thickness (height of aquifer) of the Alluvial aquifer shown in table 1 come from the Arkansas Natural Resources Commission (Arkansas Natural Resources Commission, 2012). A thinner aquifer suggests greater depletion of the aquifer has occurred in that area (figure 1b). The initial size of the aquifer, $AQ_i(0)$, at site i is computed as the acreage, $\sum_j L_{ij}(0)$, times the saturated thickness of the aquifer. The natural recharge, nr_i, of the Alluvial aquifer is based on a calibrated model of recharge for the period 1994 to 1998 associated with precipitation, flow to or from streams, and groundwater flow to or from the underlying Sparta aquifer (Reed, 2003). Note that producers do not have access to the Sparta aquifer in this analysis because the greater depth to the Sparta aquifer makes pumping from the Sparta prohibitively expensive and municipalities use the Sparta for drinking water.

Groundwater pumping reduces the size of the aquifer for the grid cell with the pumped well and for the cells that surround the well. After pumping, some of the water in the aquifer flows from the surrounding cells into the cell with the pumped well. The size of the underground flow of water is based on the distance from the pump and the hydraulic diffusivity of the aquifer. Jenkins (1968) introduced a term that is widely applied in aquifer depletion problems called the "aquifer depletion factor" (ADF) to quantify the relationship between these two variables. The depletion factor for pumping at a particular location in an aquifer is defined as

$$ADF = \frac{D}{d^2}, \tag{13}$$

where d is the shortest distance between the pumped well and the nearby aquifer and D is the hydraulic diffusivity of the aquifer. The hydraulic diffusivity is the ratio of the transmissivity and the specific yield of the unconfined Alluvial aquifer (Barlow and Leake, 2012). Specific yield, which does not vary across cells in our study area, is a dimensionless ratio of water drainable by saturated aquifer material to the total volume of that material. The product of hydraulic conductivity and saturated thickness is the transmissivity, and hydraulic conductivity is the rate of groundwater flow per unit area under a hydraulic gradient (Barlow and Leake, 2012). The hydraulic conductivity in feet per day for the Mississippi River Valley alluvial aquifer comes from spatially coarse pilot points digitized in Clark, Westerman, and Fugitt (2013).

The depletion of the aquifer beneath the cell is greater (i.e, has a large ADF) if the grid cell is closer to the pumped well and the hydraulic diffusivity is bigger. We use *ADF* to determine the proportion (or spatial weight) of the acre-feet of water pumped from a well that reduces the aquifer beneath the surrounding cells. The distance from the well and hydraulic diffusivity (based on the saturated thickness and hydraulic conductivity) of the surrounding cells influence the p_{ik} used in the economic model. The table of the supplementary material indicates that incorporating hydraulic conductivity into spatial weights has minimal influence on the model results since the hydraulic conductivity does not vary much over the study area.

Water seepage from a reservoir into the underlying soil is estimated using soil-specific data assigned to each cell from the Soil Survey Geographic Database (SSURGO) (Soil Survey Staff, Natural Resources Conservation Service, and US Department of Agriculture, 2012). The average saturated hydraulic conductivity (Ksat) for the soil-mapping unit assigned to each cell is extracted from SSURGO. Saturated hydraulic conductivity values extracted from SSURGO are adjusted to approximate 10% of the estimated Ksat based on the soil surface texture of the soil mapping unit in each cell (Saxton et al., 1986). The adjusted Ksat values are assumed to reasonably represent the ability of unsaturated soil to transmit water over the course of one year. The hydraulic gradient is estimated based on an average of four acre-feet of constant ponded water at the soil surface in a reservoir that can be filled up to eleven feet high (the level of water in the reservoir fluctuates over the season) and the estimated depth to the groundwater table. Annual seepage from the reservoir into the underlying soil is then estimated as the product of the unsaturated hydraulic conductivity and the hydraulic gradient.

Farm Production

Table 2 indicates the costs of production by crop from the 2012 Crop Cost of Production estimates (Division of Agriculture, 2012). Variable irrigation costs regardless of water source include fuel, lubricant and oil, irrigation labor, and poly pipe for border irrigation plus the levee gates for the flood irrigation of rice (Hogan et al., 2007). Capital costs associated with wells, pumps, gearheads, and power units are charged on a per acre-foot basis and are incurred whether reservoirs are installed or not, as wells remain to cover potential reservoir shortfalls. The average water use over the course of the growing season (excluding natural rainfall) is about one acre-foot for soybeans and more than three acre-feet for rice (Powers, 2007). Crop prices are the five-year average of December futures prices for harvest time contracts for all crops (Great Pacific Trading Company, 2012). Cost of production, crop price, and yields do not vary over time.

The cost of pumping water from the ground and/or reservoir depends on the costs of the fuel, maintenance, and capital. The capital cost of the well, pump and gearhead, and power unit is amortized (Hogan et al., 2007) and divided by the acre-feet pumped from the well to calculate a capital cost per acre-foot applied. The reservoir and tail-water recovery system capital cost is also converted to periodic payments and depends on the reservoir acreage. The fuel cost per acre-foot of water from the aquifer depends on the depth to the water table and the corresponding fuel needed to raise water. Diesel use ranges from thirteen gallons of diesel per acre-foot for a 100-foot well to twenty-six gallons of diesel per acre-foot for a 200-foot well (Division of Agriculture, 2012). The diesel needed per acre-foot for pumping water to and from the reservoir is six gallons (Hogan et al., 2007). We use $3.77 per gallon of diesel fuel (Energy Information Administration, 2012) and add 10% to fuel cost to account for oil and lubricant for irrigation equipment (Hogan et al., 2007).

Table 2. Value of Model Parameters

Parameter	Definition	Value
pr_{rice}	Price of rice ($/cwt)	14.06
pr_{soy}	Price of soybeans ($/bushel)	11.56
ca_{rice}	Annual production cost of rice ($/acre)	692.3
ca_{irrsoy}	Annual production cost of irrigated soybeans ($/acre)	354.3
$ca_{non-irrsoy}$	Annual production cost of non-irrigated soybeans ($/acre)	299.1
wd_{rice}	Annual irrigation per acre rice (acre-feet)	3.34
$wd_{soybean}$	Annual irrigation per acre soybean (acre-feet)	1.00
T_{max}	Annual maximum capacity of a one acre reservoir (acre-feet)	11
T_{min}	Annual minimum holding of a one acre reservoir (acre-feet)	1.375
c^r	Estimated annual per acre cost of reservoir ($/acre)	96.7[a]
c^{rw}	Cost to re-lift an acre-foot to and from the reservoir ($/acre-foot)	22.62
c^p	Cost to raise an acre-foot of water by one foot ($/foot)	0.55
δ_t	Discount factor	0.98
BV	Buffer value of groundwater ($/acre-foot)	5.19

Notes: [a] This is the amortized cost to construct an additional acre of reservoir. The first acre of the reservoir constructed is more expensive and the last acre of reservoir constructed is less expensive.

Reservoir Use and Construction

Young et al. (2004) determined that 440 acre-feet is the maximum a reservoir can be filled using a tail-water recovery system from the average rainfall runoff on a 320-acre farm. This suggests that an acre of land can yield 16.5 acre-inches for holding at the reservoir. This is the minimum amount of water ($\omega_{\min}$) that we estimate an acre of reservoir can hold without the collection of runoff from a tail-water recovery system. The use of a tail-water recovery system allows a reservoir to fill to an estimated maximum capacity of eleven acre-feet per acre over the course of a year (Smartt et al., 2002). The reservoir's capacity is 1.5 times the storage height less what is lost to evaporation because runoff collected during the year refills the reservoir.

On-farm reservoir/tail-water recovery construction and maintenance costs for various reservoir sizes were estimated using MARORA (Smartt et al., 2002) for different size operations to obtain capital-cost estimates. Subsequently, total system cost was regressed against acres occupied by the reservoir to determine per acre investment cost for different reservoir sizes. Since a majority of the construction cost for a reservoir rests on the cost to move one cubic yard of soil, this cost was updated from $1 per cubic yard to $1.20 per cubic yard to reflect changes in fuel costs since 2002, when MARORA costs were last updated. The remainder of the investment and maintenance cost is based on estimates provided within MARORA and includes a pump for tail-water recovery and a pump for irrigation.

Note that while reservoirs already exist in the study region, we assume zero reservoirs in the baseline to highlight the potential for reservoirs. This is because of the scarcity of spatially explicit data on existing reservoirs as well as the objective to highlight how construction of surface water reservoirs for both irrigation use and aquifer recharge are important to farm profitability and potential groundwater maintenance.

Buffer Value

Using monthly rainfall data from June to September collected from the National Oceanic and Atmospheric Administration's (NOAA) weather station in Wynne, Arkansas, for thirteen years from 2000 to 2012, the average seasonal rainfall μ is 12 inches and the variance of the seasonal rainfall, σ^2, is 19.4 inches^2 (National Climatic Data Center, National Oceanic and Atmospheric Administration, 2014).

Table 3. Initial, 2022, and 2042 rr Conditions, and Farm Profits without Reservoirs for the Single-Cell and Spatial Aquifers

		Single Cell		Spatial	
Crop and Water Conditions	**Initial, 2012**	**2022**	**2042**	**2022**	**2042**
Rice (thousand acres)	366	157	150	132	93
Irrigated soybeans (thousand acres)	548	872	853	908	890
Non-irrigated soybeans (thousand acres)	174	59	84	48	106
Annual groundwater use (thousand acre-feet)	1,901	1,531	1,489	1,481	1,327
Aquifer (thousand acre-feet)	82,016	72,180	53,340	72,679	56,487
Average depth to aquifer (feet)	57.3	64.7	80.1	64.3	77.5
Annual farm net returns (millions in 2012$)	114	122	109	118	99
Thirty-year NPV farm net returns (millions in 2012$)	N/A	2,335		2,224	

Notes: All models have no buffer value for the groundwater.

Several functional forms are estimated for the response of soybean yield to water input, and the natural log form is chosen to determine the concavity of soybean yield response to water input at the average rainfall for the season, $[-F''(\mu)]$, roughly 0.15 bushels per acre inch^2. Based on a five-year average of December futures prices for harvest time contracts, the price of soybean is $11.56 per bushel (Great Pacific Trading Company, 2012) and the cost of production for a bushel of soybeans based on Arkansas production budgets is $7.99 (Division of Agriculture, 2012), making the net unit value of soybeans equal to $3.57.

The buffer value of an acre-foot of groundwater used to irrigate soybeans for an average season is then: $BV \cong 0.5p[-F''(\mu)]\sigma^2 = 0.5 \times 3.57 \times 0.15 \times 19.4 = \$5.19$. We base the buffer value of groundwater on soybean production, which is less profitable than rice, and hence this value is considered a conservative estimate.

Results

Using the single-cell aquifer and no reservoir construction, groundwater is used intensively to maintain as many profitable rice acres as possible. Rice acreage is diverted increasingly to soybeans (table 3). The irrigated crops allow farms to maintain greater net returns, but by 2042 the depth to the water table increases to eighty feet and only 65% of the aquifer remains (figure 2). The rice-intensive areas are less affected by groundwater depletion with the single cell than the spatial aquifer because the decline of the water table is spread evenly across the entire study area. In 2042, 42% of rice acres with the single-cell aquifer remain versus only 26% of rice acres with the spatial aquifer. Less groundwater use with the spatial aquifer comes at the cost of fewer rice acres and lower farm net returns (figures 3 and 4). With the spatial aquifer, farmers in the rice-intensive areas quickly increase the depth to the water table thereby accelerating the switch to soybeans (figure 2). The model using the single-cell aquifer neglects the spatially explicit changes in pumping cost and overestimates the thirty-year NPV by 5%.

The 15–30% higher annual farm net returns with reservoirs indicate reservoir construction is worthwhile to farmers (table 4). The total aquifer volume across the region still declines over time, as does rice production, resulting in more soybean production (figure 3). Nonetheless, the addition of reservoirs allows 91% of the aquifer and 89% of the rice acres to remain. The presence of reservoirs thus represents a cost-effective alternative to increasingly expensive groundwater use over time. Over thirty years, the use of reservoirs increases farm net returns more than 400 million compared to no construction of reservoirs.

The use of reservoirs reduces the difference in model outcomes between the single-cell and spatial aquifer from the above 5% to 2% as the presence of reservoir water reduces the reliance on increasingly expensive groundwater over time. Reservoirs are built sooner with the spatial aquifer because the pumping of groundwater in rice-intensive areas necessitates earlier reservoir

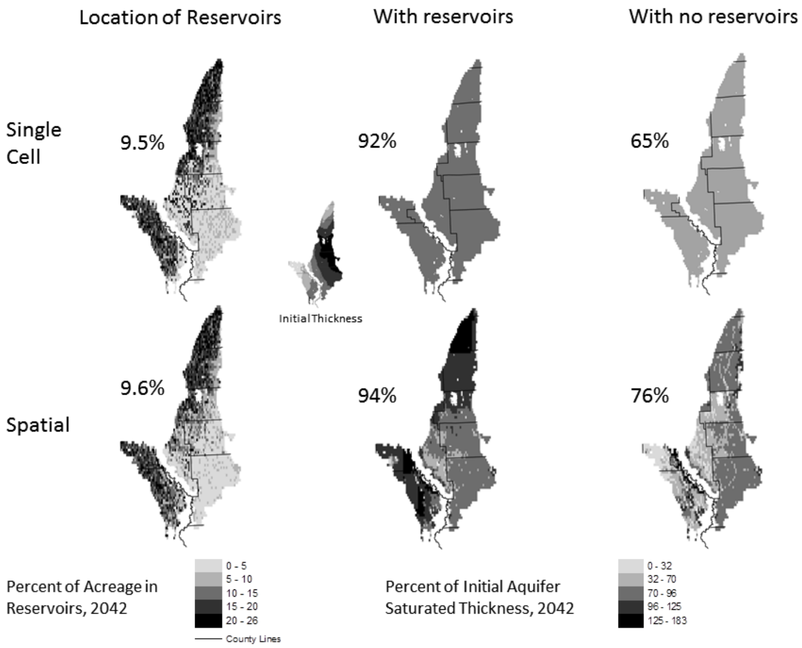

Figure 2. Reservoir Location under the Single-Cell and Spatial Aquifers along with Aquifer Decline with and without Reservoirs

Notes: Greater than 100% initial aquifer thickness is possible due to natural recharge. The numbers by the side of each map indicate study area averages. The smaller map in the middle is the initial aquifer thickness shown in detail in figure 1.

Table 4. Initial, 2022, and 2042 Crop Allocations, Water Conditions, Reservoir Adoption, and Farm Profits with Reservoirs for the Single-Cell and Spatial Aquifers

		Single Cell		Spatial	
Crop And Water Conditions	**Initial, 2012**	**2022**	**2042**	**2022**	**2042**
Rice (thousand acres)	366	316	315	316	315
Irrigated soybeans (thousand acres)	548	674	653	667	655
Non-irrigated soybeans (thousand acres)	174	6	6	6	6
Reservoirs (thousand acres)	0	93	114	100	112
Annual reservoir water use (thousand acre-feet)	0	1,000	1,214	1,101	1,220
Annual groundwater use (thousand acre-feet)	1,901	874	634	765	631
Aquifer (thousand acre-feet)	82,016	78,750	75,220	79,838	77,353
Average depth to aquifer (feet)	57.3	59.3	62.2	58.4	60.5
Annual farm net returns (millions in 2012$)	114	141	135	138	132
Thirty-year NPV farm net return (millions in 2012$)	N/A	2,772		2,706	

Notes: All models have no buffer value for the groundwater and allow no recharge of the aquifer from reservoirs.

construction. The availability of reservoir water allows similar acres of rice with and without the spatial aquifer. The fewer reservoirs built in 2022 with the single-cell aquifer leaves more land available for growing irrigated soybeans. Faster depletion of groundwater with the single-cell aquifer requires more reservoirs be built by 2042. Reservoirs are more evenly built across the study area with the single-cell aquifer than the spatial aquifer, but either way most reservoirs are built in rice-intensive areas (figure 2).

Table 5 shows how crop allocation, water use, and reservoir adoption change when the planner has the social objective to preserve the buffer value of the aquifer and reservoirs have the ability

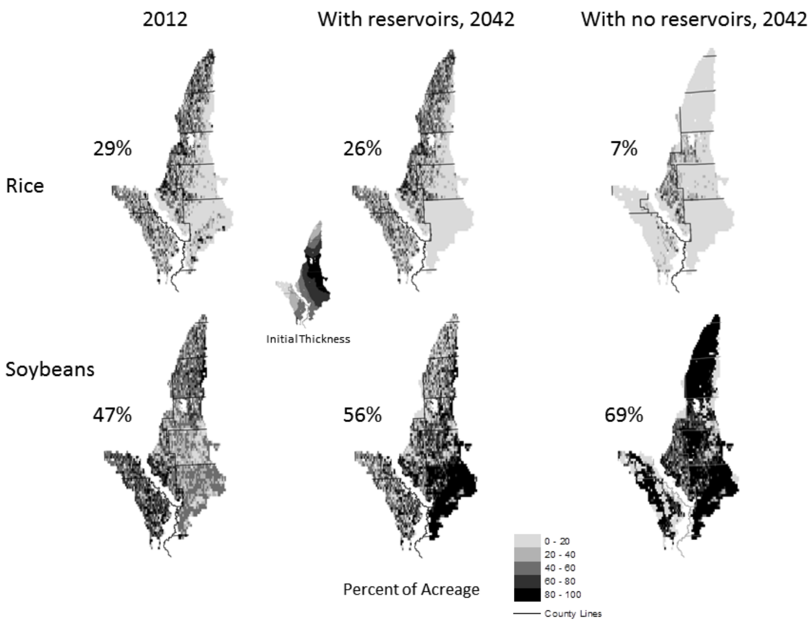

Figure 3. Crop Acreage Allocation Changes from 2012 to 2042 for Rice and Soybean with and without Reservoirs Using the Spatial Aquifer

Notes: Rice acreage can convert to irrigated or non-irrigated soybean or farm reservoirs. Irrigated soybean acreage can convert to non-irrigated soybean or farm reservoirs. The numbers by the side of each map indicate study area averages. The smaller map in the middle is the initial aquifer thickness shown in detail in figure 1.

Table 5. Initial and Final Crop Allocations, Water Conditions, Reservoir Adoption, and the Private and Social Net Returns with and without Buffer Values for the Groundwater and with and without Recharge from the Reservoirs

		Without Recharge		With Recharge	
Crop and Water Conditions	**Initial, 2012**	**2042, w/o Buffer Value (A)**	**2042, w/ Buffer Value (B)**	**2042, w/o Buffer Value (C)**	**2042, w/ Buffer Value (D)**
Rice (thousand acres)	366	315	315	315	315
Irrigated soybeans (thousand acres)	548	655	644	663	648
Non-irrigated soybeans (thousand acres)	174	6	6	6	6
Reservoirs (thousand acres)	0	112	124	105	120
Annual reservoir water use (thousand acre-feet)	0	1,220	1,353	1,145	1,310
Annual groundwater use (thousand acre-feet)	1901	631	485	716	533
Aquifer (thousand acre-feet)	82,016	77,353	82,717	76,455	82,278
Average depth to aquifer (feet)	57.3	60.5	56.5	61.2	56.4
Annual farm net returns (millions in 2012$)	114	132	132	133	132
Thirty-year NPV farm net return (millions in 2012$)	–	2,706	2,669	2,710	2,670
Thirty-year NPV social net returns (millions in 2012$)	–	4,567	4,606	4,568	4,607

Notes: All models use the spatial aquifer and allow on-farm reservoirs.
[a] The buffer value of the aquifer is not counted in the farm net returns.

to recharge the aquifer. Taking the buffer value of groundwater into account increases the volume of the aquifer to 89,000 acre-feet by 2042 (column B in table 5). This is done by building an extra 28,000 acres of reservoirs (column B vs column A), and farm net returns in 2042 only fall slightly since nearly all the rice acres remains. The thirty-year social net returns, where the objective is the farm net returns and the buffer value of the aquifer, are $89 million greater when preserving the groundwater, but the thirty-year farm net returns are $85 million lower.

Not lining the bottom of reservoirs allows the collected reservoir water to recharge the aquifer (columns C and D). Farms build 8,000 fewer acres of reservoirs (column C vs column A), since each reservoir is less efficient at providing water for the farm to use. The farmers thus substitute toward the use of more groundwater, and the aquifer declines faster. Farm net returns rise slightly when reservoirs recharge the aquifer since fewer reservoir acres mean more irrigated cropland, and pumping costs are slightly lower from the shallower pumping depth due to the aquifer recharge by the reservoirs. This suggests that farms can benefit when reservoirs allow recharge, although the aquifer is left more depleted. When the buffer value of the groundwater is accounted for, the opposite happens. One thousand more acres of reservoirs are built, (column D vs column B), using the reservoir recharge to raise the water table. By accounting for the buffer value of groundwater, the groundwater stock remains close to present levels, and thus sustainability of the aquifer is an outcome of internalizing the in situ value of groundwater. This lowers the thirty-year farm net returns, but the social net returns increase.

Given the positive effects of reservoirs on groundwater levels as well as on farm income, we explore policies to speed the adoption of reservoirs by farmers in table 6. The policies modify producer behavior to align profit maximization with the social goal of internalizing groundwater buffer value in the farm production decision. This includes varying the level of cost-share of reservoir construction, a subsidy on reservoir pumping costs, and a tax on groundwater pumping. The effectiveness of the policies is judged by the aquifer volume, farm net returns, redistribution of income between farmers and taxpayers, and conservation costs relative to the 2042 baseline outcome. Further, we assume no aquifer recharge from reservoirs, as most reservoirs in the past have prevented this, for the subsidy on reservoir water and tax on groundwater. For the cost-share program, we also evaluate the possibility of reservoir recharge.

Several observations are possible about the effect of the policies. One, the greater the level of the policy the faster is the transition to the 2042 reservoir acres. Two, cost share for reservoirs with recharge leads to fewer 2042 acres with the 65% cost share (128,000 acres) than the 30% cost share (134,000 acres) because there are different speeds of reservoir adoption and 2042 water table levels. Three, relatively high levels of policy intervention are needed to maintain the 2012 aquifer volume. Four, the tax option has the least income redistribution. Five, the low level subsidy and tax options result in greater social returns (net farm returns plus government revenue) than the baseline and hence the conservation cost per acre-foot is negative or highly cost effective. Comparing all options on the basis of conservation cost per acre-foot, the option to cost-share reservoirs with recharge can be eliminated. At the higher level of implementation, the cost share is the least effective at maintaining aquifer levels at the 2012 level.

Figure 4 shows income redistribution effects without policy intervention compared to the 2012 baseline as well as effects of the 30% tax, 40% reservoir pumping subsidy, and 65% reservoir cost share without reservoir recharge. Exploitation of the aquifer as would be evident without the construction of reservoirs shows the most bleak income picture concentrated in areas where initial aquifer saturated thickness was lowest. Adding reservoirs without government intervention raises income levels, not only in regions initially most depleted. Comparing the center top panel (no government intervention with reservoirs) with the bottom three panels, subsidies and cost share allow for greater income in regions where the water table was initially more depleted but at a cost to taxpayers.

Table 6. Water Conservation Policies Influence on Reservoir Adoption, Water Use, and Private and Public Returns

		Reservoir Acres (thousand)		Reservoir Water (thousand acre-feet)	Groundwater (thousand acre-feet)	Aquifer (thousand acre-feet)	Farm Net Returns[a] ($ millions)	Government Revenue ($ millions)	Conservation Cost[b] ($/acre-foot)
Policy	**Level**	**2022**	**2042**	**2042**	**2042**	**2042**	**30-Year NPV**	**30-Year NPV**	**2012**
Baseline		103	112	1,220	631	77,353	2,706	0	na
Cost-share reservoir construction costs, no reservoir recharge	30%	108	122	1,292	554	79,196	2,768	−65	1.37
	65%	115	124	1,315	528	80,659	2,847	−147	1.67
Cost-share reservoir construction costs, reservoir recharge	30%	108	134	1,396	437	80,376	2,762	−66	3.15
	65%	114	128	1,346	493	81,164	2,844	−148	2.50
Subsidize reservoir pumping fuel costs, no reservoir recharge	20%	112	115	1,221	633	79,067	2,816	−108	−1.44
	40%	120	122	1,295	552	81,655	2,929	−231	1.75
Tax on groundwater use, no reservoir recharge	10%	112	117	1,234	617	79,176	2,679	27	−0.04
	30%	122	123	1,308	536	82,147	2,634	61	2.20

Notes: All models have no buffer value for the groundwater and use the spatial aquifer.
[a] The farm net returns include the payments to or receipts from the government as a result of the policy.
[b] Conservation cost is calculated as the difference in combined farm net returns and government revenue relative to the baseline divided by the change in aquifer level between the policy option and the baseline. For example, subsidizing reservoir construction cost at 30% without reservoir recharge led to combined farm net returns and government revenue of $2,703 million or $3 million less than the baseline. That $3 million cost allowed 1.8 million extra acre-feet in the aquifer (79,196 âĂŞ 77,353) resulting in a cost of $1.37/acre-foot. Negative numbers imply greater combined farm net returns and government revenue compared to the baseline.

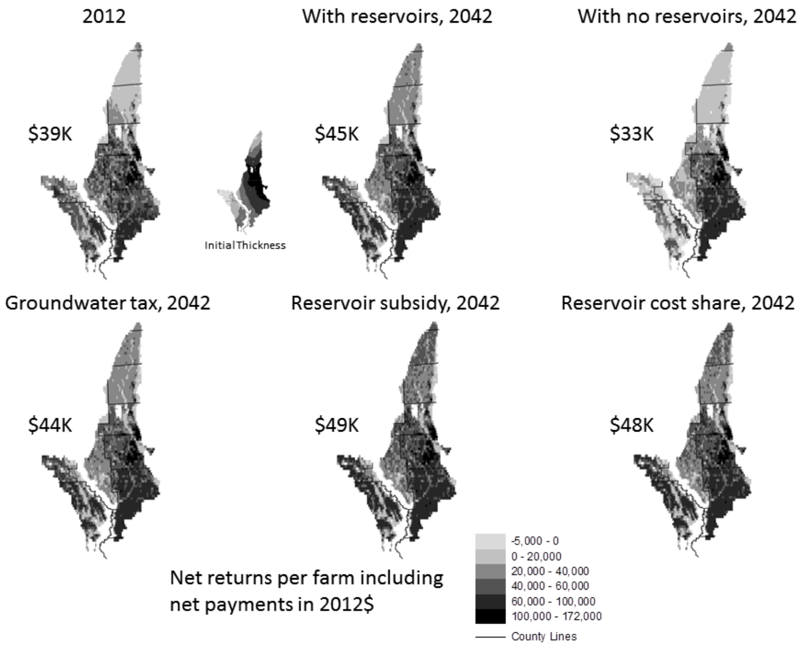

Figure 4. Spatial Income Redistribution in 2042 with and without Reservoirs and under Varying Policy Scenarios with Reservoirs Using the Spatial Aquifer

Notes: Corresponding to table 6, the tax is at 30%, subsidy at 40%, and cost share at 65% with no reservoir recharge. The numbers by the side of the map indicate study area averages. The smaller map in the middle is the initial aquifer thickness shown in detail in figure 1.

Conclusions

We consider two aspects to the sustainable management of groundwater: a spatially explicit representation of the aquifer and the potential of reservoirs to recharge the underlying aquifer. To evaluate these aspects, we develop a spatial-dynamic model of optimal water use in the rice-soybean production region of Arkansas. We focus on constructing reservoirs, which are valuable for collecting water for later use during the season but also occupy land that could be used for growing crops. A central planner determines the crop allocation and the number of reservoirs to build over time to maximize farm returns less the costs of production and reservoirs, subject to constraints on water and land availability. We use the model to explore how the spatial movement of groundwater, the capacity of reservoirs to recharge the aquifer, and the effectiveness of policies to obtain socially desirable aquifer levels affect net returns.

Since there is incomplete information about groundwater flow related to well pumping in the region modeled, there may be pockets of landscape where minimal flows occur and other areas where flow is seamless. Given the model assumptions, we find optimal management of a landscape with a single-cell aquifer depletes groundwater faster. Relative to the single-cell model, the spatial model led to the following estimates: i) the final thickness of the aquifer is 5% greater; ii) rice acreage in 2042 is 38% lower because the cones of depression hasten the switch to less water intensive crops; iii) thirty-year net returns are 5% lower. These findings are consistent with Brozović, Sunding, and Zilberman (2010), who show that the pumping externality of the single-cell aquifer is less than the pumping externality of a spatial aquifer by an order of magnitude. While seamless lateral flow may be suitable for the long run, the assumption of a single-cell aquifer appears to overestimate the optimal extraction in the gradual transition to the long run. This indicates that information about the spatial flow of groundwater in the aquifer is valuable to the resource planner for evaluating economic outcomes.

Constructing reservoirs improves farm net returns by 15% or more and lessens the rate of decline of the aquifer by more than 39%. In addition, reservoirs increase the rice acreage remaining in the final period by more than 180%. To increase the water table to the social (buffer value) level, the acres in reservoirs must rise by 24%, but the rice acreage declines by less than 1%. This means thirty-year net returns only decline by 3%. Allowing reservoirs to recharge the aquifer results in fewer reservoirs because each reservoir provides less water, but the recharge reduces the cost of well pumping, actually increasing farm net returns. With the buffer value of groundwater, the use of reservoirs with recharge increases social net returns because the water table level is the largest. Regulatory programs are needed for farms to internalize the buffer value of the aquifer for future generations.

We find that policies to enhance the aquifer resource differ in their effectiveness at supporting farm net returns while limiting government income redistribution. A tax on groundwater is the most effective at increasing the water table and lowers farm returns marginally with minimal income redistribution compared to a scenario without government intervention. Since a tax is likely an unpopular scenario with policy makers, the subsidy strategy for reservoir pumping at the 20% level provided cost-effective support for resource conservation but was not sufficient to maintain aquifer levels. Cost sharing with recharge, while effective at maintaining the aquifer, was not cost effective. The combination of a tax on groundwater use and potential subsidies may therefore prove worthy of consideration. On the basis of these results, however, it is unlikely that a single policy will lead to the socially optimal aquifer volume of 89.6 million acre-feet.

Prior empirical analyses that examined aspects of aquifer depletion, reservoir construction, and farm production have found different degrees of investment in reservoirs necessary to sustain the groundwater resource. For example, studies find the thickness of the aquifer has to fall to thirty feet before a reservoir is needed, though the optimal size depends on crop productivity and groundwater decline rate (Wailes et al., 2004; Hristovska et al., 2011). Conversely, our study observes reservoirs built where the aquifer is fifty feet thick. This discrepancy may be because of differences in the rate of groundwater pumping predicted over time and the fuel costs involved in the well pumping. Similar to Hill et al. (2006), we find fewer than half of the farms build reservoirs without cost-share or another incentive program. We also find that, on average, nearly 10% of a farm's acreage is optimally converted to a reservoir.

Other lines of research can investigate the relationships among groundwater use, a spatially explicit aquifer, and on-farm reservoirs. The assumption that farmers cannot adjust the irrigation applied to a crop within a season can be relaxed. Full information on the marginal product of water could permit an exploration of how farmers balance between less irrigation water and yield loss. Another line is how effectively reservoirs stabilize net returns under institutional arrangements other than optimal management. In a seminal paper, Gisser and Sánchez (1980) show that competitive pumping differs only slightly from optimal pumping in an application to the Pecos Basin in New Mexico. This result depends, among other things, on constant crop mix, constant crop requirement, constant energy costs, fixed irrigation technology, and constant hydrologic conditions (Koundouri, 2004). Later papers indicate that the inefficiency from competitive (myopic) pumping is only one of several externalities inherent in groundwater use. Negri (1989) develops a dynamic game-theoretic model of groundwater use in a common property setting to account for a strategic externality. Provencher and Burt (1993) consider risk-averse users in an environment of uncertain income returns where competitive groundwater use has a risk externality. These considerations are qualitative evidence that a divergence between competitive and optimal pumping can arise, and an exploration of these differences in the context of a spatial aquifer and on-farm reservoirs is worthwhile.

In addition to the use of on-farm reservoirs, other strategies have been proposed to stabilize declining water tables. These strategies include cost-share assistance for "water-saving" irrigation technologies (Peterson and Ding, 2005; Huffaker and Whittlesey, 1995), incentive payments to convert irrigated crop production to dryland crop production (Ding and Peterson, 2012; Wheeler et al., 2008), tradable quotas of groundwater stock (Provencher and Burt, 1993, 1994), and the planting of less water-intensive bioenergy crops such as switchgrass and sorghum (Popp, Nalley, and Vickery, 2010). The Arkansas Delta continues to utilize ever greater quantities of groundwater to maintain irrigated production, including rice, which is not grown as intensively elsewhere in the United States. As the Alluvial aquifer continues to decline, optimal groundwater management has the potential to sustain the aquifer and maintain farm profitability.

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Appendix

Spatial Hydraulic Conductivity

Incorporating the spatial variability of hydraulic conductivity into the spatial weights that influence the depletion of the aquifer from groundwater pumping has minimal influence on those weights. This means that the model results for crop allocations, reservoir creation, water use, and farm profitability are largely unchanged (table A1) when allowing for spatial hydraulic conductivity. The thirty-year NPV of farm profit for the case of no reservoir adoption is no different with or without spatial variability in hydraulic conductivity and less than 1% different for the model case that allows reservoir creation.

Table A1. Crop Allocations, Water Conditions, Reservoir Adoption, and Farm Profits in 2042 for Spatial Aquifers with and without Spatial Variability in the Hydraulic Conductivity and with and without Reservoirs

	Without Spatial Hydraulic Conductivity		**With Spatial Hydraulic Conductivity**	
Crop and Water Conditions	**w/o Reservoirs**	**w/ Reservoirs**	**w/o Reservoirs**	**w/ Reservoirs**
Rice (thousand acres)	96	325	95	325
Irrigated soybeans (thousand acres)	919	677	919	679
Non-irrigated soybeans (thousand acres)	109	6	110	6
Reservoirs (thousand acres)	–	116	–	114
Annual reservoir water use (thousand acre-feet)	–	1,220	–	1,204
Annual groundwater use (thousand acre-feet)	1,327	631	1,326	650
Aquifer (thousand acre-feet)	56,487	77,353	56,471	77,017
Average depth to aquifer (feet)	77.5	60.5	77.6	60.7
Annual farm net returns (millions in 2012$)	99	132	98	133
Thirty-year NPV farm net return (millions in 2012$)	2,224	2,706	2,224	2,711

Notes: All models have no buffer value for the groundwater and allow no recharge of the aquifer from reservoirs.

Optimality Conditions

The farm collective profit maximization problem is

(A1)

$$\max_{\substack{FR_{ij}(t),\, GW_i(t),\\ DS_{ij}(t),\, RW_i(t)}} : \sum_{t=1}^{T} \delta_t \left(\sum_{i=1}^{m} \sum_{j=1}^{n} (pr_j y_{ij} - ca_j) L_{ij}(t) - c^r FR_{ij}(t) - c^{rw} RW_i(t) - GC_i(t) GW_i(t) \right),$$

subject to:

(A2) $$L_{ij}(t) = L_{ij}(t-1) - FR_{ij}(t) + V_{ij}(t),$$

(A3) $$R_i(t) = R_i(t-1) + \sum_{j=1}^{n} FR_{ij}(t),$$

(A4) $$\sum_{j=1}^{n} wd_j L_{ij}(t) \le GW_i(t) + RW_i(t),$$

(A5) $$RW_i(t) \le \left((\omega_{\max} + \omega_{\min}) - \frac{\omega_{\max}}{\sum_{j=1}^{n} L_{ij}(0)} R_i(t) \right) R_i(t),$$

(A6) $$AQ_i(t) = AQ_i(t-1) - \sum_{k=1}^{m} p_{ik} GW_k(t) + nr_1,$$

(A7) $$GC_i(t) = c^c + c^p \left(dp_i + \frac{(AQ_i(0) - AQ_i(t))}{\sum_{j=1}^{n} L_{ij}(0)} \right),$$

(A8) $$L_{ij}(0) = L_0^{ij},$$

(A9) $$R_i(0) = 0,$$

(A10) $$AQ_i(0) = AQ_0^i,$$

(A11) $$FR_{ij}(t) \geq 0,$$

(A12) $$RW_i(t) \geq 0,$$

(A13) $$GW_i(t) \geq 0,$$

(A14) $$DS_i(t) \geq 0,$$

(A15) $$RS_i(t) \geq 0.$$

The Hamiltonian for this problem is

(A16) $$H(FR_{ij}, RW_i, GW_i, DS_{ij}, RS_i, \lambda_L^{ij}, \lambda_R^i, \lambda_{AQ}^i; pr_j, y_{ij}, ca_j, c^r, c^{rw}) =$$

$$\sum_{t,\, t+1} \delta_t \left(\sum_{i=1}^{m} \sum_{j=1}^{n} (pr_j y_{ij} - ca_j) L_{ij}(t) - c^r FR_{ij}(t) - c^{rw} RW_i(t) - GC_i(t) GW_i(t) \right) +$$

$$\sum_{t,\, t+1} \sum_{i=1}^{m} \sum_{j=1}^{n} \left(\lambda_L^{ij}(t)(-FR_{ij}(t) + V_{ij}(t)) + \lambda_R^i(t) \left(\sum_{j=1}^{n} FR_{ij}(t) \right) + \right.$$

$$\left. \lambda_{AQ}^i(t) \left(-\sum_{k=1}^{m} p_{ik} GW_k(t) + nr_i \right) \right)$$

$$\text{with } \sum_{j=1}^{n} wd_j L_{ij}(t) = GW_i(t) + RW_i(t),$$

$$RW_i(t) = \left((\omega_{\max} + \omega_{\min}) - \frac{\omega_{\max}}{\sum_{j=1}^{n} L_{ij}(0)} R_i(t) \right) R_i(t),$$

$$\text{and } GC_i(t) = c^c + c^p \left(dp_i + \frac{(AQ_i(0) - AQ_i(t))}{\sum_{j=1}^{n} L_{ij}(0)} \right).$$

Derivation of Necessary Conditions for Reservoir Land, Groundwater Pumping, and the Flow of the Shadow Value of the Aquifer

We can use a few of the necessary conditions of the collective farm problem to understand what influence the number of reservoirs created has on the landscape. In particular, the condition that determines how much productive agricultural land is converted to a reservoir is where we start.

The necessary condition for $FR_{ij}(t)$ is

$$
\text{(A17)} \quad \begin{aligned} H_{FR_{ij}} &= \delta_t \left((pr_j y_{ij} - ca_j) \frac{\partial L_{ij}(t)}{\partial FR_{ij}} - c^r - c^{rw} \frac{\partial RW_i(t)}{\partial FR_{ij}} - GC_i(t) \frac{\partial GW_i(t)}{\partial FR_{ij}} \right) - \\ & \lambda_L^{ij}(t) + \lambda_R^i(t) \quad = \quad 0. \end{aligned}
$$

Observing that $\frac{\partial L_{ij}(t)}{\partial FR_{ij}} = \frac{\partial}{\partial FR_{ij}}(L_{ij}(t-1) - FR_{ij}(t) + V_{ij}(t)) = -1$ and $\frac{\partial GW_i(t)}{\partial FR_{ij}} = \frac{\partial}{\partial FR_{ij}}(\sum_{j=1}^{n} wd_j L_{ij}(t) - RW_i(t)) = -\left(wd_j + \frac{\partial RW_i(t)}{\partial FR_{ij}} \right)$, then

$$
\text{(A18)} \quad \begin{aligned} H_{FR_{ij}} &= \delta_t (wd_j GC_i(t) - c^r - (pr_j y_{ij} - ca_j)) + \\ & \delta_t (GC_i(t) - c^{rw}) \frac{\partial RW_i(t)}{\partial FR_{ij}} - \lambda_L^{ij}(t) + \lambda_R^i(t) \quad = \quad 0. \end{aligned}
$$

Further, noting that since $\frac{\partial R_i(t)}{\partial FR_{ij}} = \frac{\partial}{\partial FR_{ij}}(R_i(t-1) + \sum_{j=1}^{n} FR_{ij}(t)) = 1$, then

$$
\text{(A19)} \quad \begin{aligned} \frac{\partial RW_i(t)}{\partial FR_{ij}} &= \left((\omega_{\max} + \omega_{\min}) - \frac{\omega_{\max}}{\sum_{j=1}^{n} L_{ij}(0)} R_i(t)) \right) - \left(\frac{\omega_{\max}}{\sum_{j=1}^{n} L_{ij}(0)} \right) R_i(t) \\ &= \left((\omega_{\max} + \omega_{\min}) - \frac{2\omega_{\max}}{\sum_{j=1}^{n} L_{ij}(0)} R_i(t) \right). \end{aligned}
$$

This means the necessary condition becomes

$$
\text{(A20)} \quad \begin{aligned} H_{FR_{ij}} &= \delta_t (wd_j GC_i(t) - c^r - (pr_j y_{ij} - ca_j)) + \\ & \delta_t (GC_i(t) - c^{rw}) \left((\omega_{\max} + \omega_{\min}) - \frac{2\omega_{\max}}{\sum_{j=1}^{n} L_{ij}(0)} R_i(t) \right) - \\ & \lambda_L^{ij}(t) + \lambda_R^i(t) \quad = \quad 0. \end{aligned}
$$

which shows that the net benefit of more land for reservoirs is the avoided groundwater pumping cost weighted by how much water the reservoir provides, plus the shadow value of having land in a reservoir, less four distinct losses: i) the cost of reservoir pumping weighted by how much water the reservoir provides, ii) the cost of reservoir creation, iii) the loss of profit from not having the land in crop j because the land went to a reservoir, and iv) the shadow value of not having the land in crop j.

Next, we consider how the degree of lateral flow in the aquifer affects how much land in reservoirs is optimally built. Begin by identifying groundwater pumping cost under each assumption of lateral flow in the aquifer.

First, the condition for $GW_i(t)$ with finite lateral flow is

$$
\text{(A21)} \quad \begin{aligned} H_{GW_i} &= \delta_t \left(\sum_{j=1}^{n} (pr_j y_{ij} - ca_j) \frac{\partial L_{ij}(t)}{\partial GW_i} - c^{rw} \frac{\partial RW_i(t)}{\partial GW_i} - GC_i(t) = \frac{\partial GC_i(t)}{\partial GW_i} GW_i(t) \right) \\ & - \delta_t \sum_{k \neq 1}^{m} \frac{\partial GC_k(t)}{\partial GW_i} GW_k(t) - \sum_{k=1}^{m} \lambda_{AQ}^k(t) p_{ki} \quad = \quad 0. \end{aligned}
$$

Since $\dfrac{\partial L_{ij}(t)}{\partial GW_i} = \dfrac{\partial}{\partial GW_i}\left(\dfrac{GW_i(t) + RW_i(t) - \sum_{k\neq j}^{n-1} wd_k L_{ik}(t)}{wd_j}\right) = \dfrac{1}{wd_j}$ and $\dfrac{\partial RW_i(t)}{\partial GW_i} = \dfrac{\partial}{\partial GW_i}\left(\sum_{j=1}^{n} wd_j L_{ij}(t) - GW_i(t)\right) = -1$, then

$$\begin{aligned} H_{GW_i} &= \delta_t\left(\sum_{j=1}^{n}\frac{(pr_j y_{ij} - ca_j)}{wd_j} + c^{rw} - GC_i(t) - \sum_{k=1}^{m}\frac{\partial GC_k(t)}{\partial GW_i}GW_k\right) - \sum_{k=1}^{m}\lambda_{AQ}^{k}(t)p_{ki} \\ &= 0. \end{aligned} \tag{A22}$$

Using

$$\begin{aligned} \frac{\partial GC_k(t)}{\partial GW_i} &= \frac{\partial}{\partial GW_i}\left(c^c + c^p\left(dp_i\frac{(AQ_i(0) - AQ_i(t))}{\sum_{j=1}^{n} L_{kj}(0)}\right)\right) \\ &= \left(\frac{-c^p}{\sum_{j=1}^{n} L_{kj}(0)}\right)\frac{\partial AQ_i(t)}{\partial GW_i} \\ &= \left(\frac{-c^p}{\sum_{j=1}^{n} L_{kj}(0)}\right)\frac{\partial}{\partial GW_i}\left(AQ_i(t-1) - \sum_{k=1}^{m} p_{ik}GW_k(t) + nr\right) \\ &= \left(\frac{-c^p}{\sum_{j=1}^{n} L_{kj}(0)}\right)(-p_{ki}) = \frac{c^p p_{ki}}{\sum_{j=1}^{n} L_{kj}(0)}, \end{aligned} \tag{A23}$$

then the necessary condition becomes

$$\begin{aligned} H_{GW_i} &= \delta_t\left(\sum_{j=1}^{n}\frac{(pr_j y_{ij} - ca_j)}{wd_j} + c^{rw} - GC_i(t) - \sum_{k=1}^{m}\frac{c^p p_{ki}}{\sum_{j=1}^{n} L_{kj}(0)}GW_k\right) - \\ &\quad \sum_{k=1}^{m}\lambda_{AQ}^{k}(t)p_{ki} = 0. \end{aligned} \tag{A24}$$

which states that the net benefit of additional groundwater pumping is the net value of production from groundwater plus the avoided reservoir water pumping cost less the cost of groundwater pumping in cell i and the higher cost of pumping in surrounding cells and less the shadow value of a diminished aquifer in cell i and the surrounding cells.

Second, the condition for $GW_i(t)$ when lateral flows are infinite as assumed in the single-cell aquifer is

$$H_{GW_i} = \delta_t\left(\sum_{j=1}^{n}\frac{(pr_j y_{ij} - ca_j)}{wd_j} + c^{rw} - GC_i(t)\right) - \lambda_{AQ}(t) = 0. \tag{A25}$$

which says that the net benefit of additional groundwater pumping is the net value of production from groundwater plus the avoided reservoir water pumping cost less the cost of groundwater pumping in cell i and the shadow value of the diminished single-cell aquifer.

Comparing equations (A24) and (A25), the net benefit of groundwater pumping is greater in the single-cell aquifer if the shadow values of the single-cell aquifer and the spatial aquifer are roughly equivalent because the single-cell aquifer has a less potent effect on groundwater pumping costs in the surrounding cells. Since pumping is greater in the single-cell aquifer, groundwater pumping costs rise more in the single-cell aquifer than the spatial aquifer. Looking at equation (A20), this suggests that we should expect a higher net benefit from reservoirs if the aquifer is assumed to be single-cell than if the aquifer has finite lateral flow.

Finally, the condition on the flow of the shadow value of the aquifer (i.e., the most the farm collective would pay for another acre-foot of water in the aquifer) with and without the spatial aquifer provides another perspective on why the different amounts of reservoirs are built for each representation of the aquifer.

First, the condition for $AQ_i(t)$ when the aquifer is spatially segmented:

$$
\begin{aligned}
\dot{\lambda}_{AQ}^{i} &= -H_{AQ_i} = -\delta_t GW_i(t)\frac{\partial GC_i(t)}{\partial AQ_i} - \\
&= \delta_t GW_i(t)\frac{\partial}{\partial AQ_i}\left(c^c + c^p\left(dp_i\frac{(AQ_i(0)-AQ_i(t))}{\sum_{j=1}^{n} L_{ij}(0)}\right)\right) \\
&= \delta_t c^p \frac{GW_i(t)}{\sum_{j=1}^{n} L_{ij}(0)},
\end{aligned}
\tag{A26}
$$

and, second, the condition for $AQ(t)$ when the aquifer is single-cell:

$$
\begin{aligned}
\dot{\lambda}_{AQ} &= -H_{AQ} = -\delta_t \sum_{i=1}^{m} GW_i(0)\frac{\partial GC_i(t)}{\partial AQ} \\
&= -\delta_t GW_i(t)\sum_{i=1}^{m}\frac{\partial}{\partial AQ}\left(c^c + c^p\left(dp_i\frac{(AQ(0)-AQ(t))}{\sum_{j=1}^{n} L_{ij}(0)}\right)\right) \\
&= \delta_t c^p \frac{GW_i(t)}{\sum_{j=1}^{n} L_{ij}(0)}.
\end{aligned}
\tag{A27}
$$

The greater groundwater pumping in the single-cell aquifer means the shadow value of the single-cell aquifer is rising faster than the shadow value of an aquifer assumed to have finite lateral flow. The greater rise of shadow value in the single-cell aquifer means that farms turn faster to the construction of reservoirs for cheaper water.

Derivation of Select Necessary First-Order Conditions for the Case that Reservoirs Recharge the Aquifer

The necessary condition for $FR_{ij}(t)$ is

$$
\begin{aligned}
H_{FR_{ij}} &= \delta_t(wd_jGC_i(t) - c^r - (pr_jy_{ij} - ca_j)) + \delta_t(GC_i(t) - c^{rw})\frac{\partial RW_i(t)}{\partial FR_{ij}} - \\
&\lambda_L^{ij}(t) + \lambda_R^{i}(t) + \lambda_{AQ}^{i}(t) r_i = 0.
\end{aligned}
\tag{A28}
$$

noting that

$$
\begin{aligned}
\frac{\partial RW_i(t)}{\partial FR_{ij}} &= \left((\omega_{\max}+\omega_{\min}) - \frac{\omega_{\max}}{\sum_{j=1}^{n} L_{ij}(0)}R_i(t) - r_i\right) - \left(\frac{\omega_{\max}}{\sum_{j=1}^{n} L_{ij}(0)}\right)R_i(t) \\
&= \left((\omega_{\max}+\omega_{\min}) - \frac{s\omega_{\max}}{\sum_{j=1}^{n} L_{ij}(0)}R_i(t) - r_i\right).
\end{aligned}
\tag{A29}
$$

This means the necessary condition becomes

$$
\begin{aligned}
H_{FR_{ij}} &= \delta_t(wd_j GC_i(t) - c^r - (pr_j y_{ij} - ca_j)) + \text{L} \\
\text{L} &+ \delta_t(GC_i(t) - c^{rw})\left((\omega_{\max} + \omega_{\min}) - \frac{2\omega_{\max}}{\sum_{j=1}^{n} L_{ij}(0)} R_i(t) - r_i\right) - \\
&\lambda_L^{ij}(t) + \lambda_R^i(t) + \lambda_{AQ}^i(t) r_i = 0.
\end{aligned}
\tag{A30}
$$

Comparing equations (A20) and (A30), the net benefit of land in reservoirs changes when reservoirs recharge the aquifer. The term $\lambda_{AQ}^i(t) r_i$ is the shadow value of more groundwater in the aquifer from the recharge times the recharge from a larger reservoir above. This term indicates more land should be converted to reservoirs. The recharge leads to less water in the reservoir for use by crops, and this must be replaced by groundwater pumping, which is costly for the farming cooperative. This suggests less land should be converted to reservoirs. This tradeoff can be assessed in the simulation analysis.