A THEORY OF PRODUCTION, INVESTMENT, AND DISINVESTMENT

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Rural change often involves altering the combination of durable assets owned by an economic unit. Rural change may involve the acquisition of additional durables, the disposal of current durables, or using retained durables in a different manner. The current theories of production, investment, and disinvestment in durable assets do not handle accurately the issues relating to using durable assets at varying rates, nor do they specify completely the related issue of the optimal length of life for durable assets. In this paper, we consider a production process which has both durable assets and the flow of services from the durables as inputs. We allow for a varying extraction rate and determine internally both the optimal amount of services to extract from the durable in each production period as well as the optimal life for the durable. We relate the optimal production activities associated with the durable to investments and disinvestments in the durable. The economic theory which guides decisions concerning these changes is important to decisionmakers at micro, regional, national, and supranational levels.

Theoretical Model

Our specification of the production, investment, and disinvestment process conceives the production process to be vertically integrated. The determination of the flow of services from durables will be specified at one level. This service flow is then fed into the production function to determine output. The expected future time pattern of utilization will in part determine the investment or disinvestment decision. A diagrammatic representation of this process for a production process using one durable is presented in figure 1.

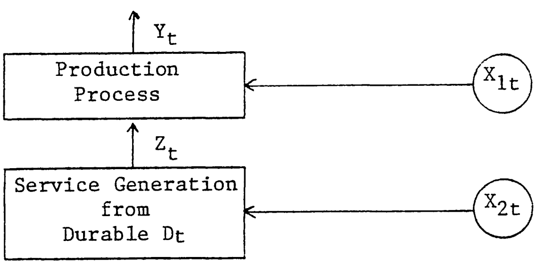

Figure 1. Two tiered vertically integrated production process

The mathematical characterization of the physical production process represented in figure 1 is:

(1) $Y_t = F(X_{1t}, Z_t)$, and

(2) $Z_t = G(X_{2t}/D_t)$,

where Y_t = quantity of product Y produced and sold in time period t,

X_{1t} = quantity of nondurable inputs X_1 used in production of Y_t in time period t,

Z_t = quantity of services generated from D_t used in production of Y in time period t, and

D_t = the stock of the durable asset D in time period t.

Equation (1) is a standard representation of a product process with flow variables as inputs. Equation (2) is a production relationship which indicates that service flows from a durable asset are generated or produced according to the function G by using one nondurable input (a flow variable) with a given stock of the durable asset. Thus, at this level of integration we need both stocks and flows in the production of services.

Specification of the production process in this manner allows us to vary the rate of use for durable assets. It also permits us to determine investment and disinvestment in durables simultaneously with the production activities associated with the durable. Finally, the optimal length of life for the durable is also determined internally.

The physical life of a durable asset is related to both the services extracted and the maintenance carried out during each year of its life. In our model, we express this physical relationship as:

$$(3)\quad T_D = h(Z_1, \ldots, Z_t, \ldots, Z_{TH}, X_{31}, \ldots, X_{3t}, \ldots, X_{3TH}),$$

where T_D = physical life of durable,

X_{3t} = aggregated maintenance variable in time period t, and

T_H = planning horizon for the firm.

T_H is chosen such that costs and returns beyond T_H would be discounted essentially at zero for any positive discount rate $T_H \geqq T_D$.

We assume that the firm operates in each time period to maximize current profits plus the change in the net present value of the durable asset. This objective function is consistent with the gain function used by Edwards and with Boulding's writings.

$$(4)\quad G_t = P_{yt}Y_t - P_{x1t}X_{1t} - P_{x2t}X_{2t} - P_{x3t}X_{3t} - TUC_N(Z_t)$$
$$- FC_t + \propto(D_t - D_t^o),$$

where P_{yt} = price received for Y in time period t,

P_{xjt} = price paid for nondurable X_j in time period t (j = 1, 2, 3),

$TUC_N(Z_t)$ = total user cost of extracting services Z_t in time period t,

FC_t^o = fixed cost associated with the durable in time period t (the "o" notation refers to initial levels while the "*", used below, refers to optimal levels), and

$\propto$ = gain in net present value of a unit of the durable.

For $D_t > D_t^o$, $\propto$ will equal the difference between the durable's value in use, NRD, and its acquisition price, P_{Dt}^a. For $D_t < D_t^o$, $\propto$ will equal the difference between the durable's value in use and its salvage price, P_{DT}^s. For $D_t = D_t^o$, $\propto$ will equal the durable's value in use.

The total user cost variable (TUC_N) in (4) deserves special explanation. The concept of user cost as recognized by Keynes and subsequently modified by Neal and Lewis considers the cost of using the asset as opposed to not using it. Equations (5) and (6) express the Neal and Lewis versions respectively.

$$(5) \quad TUC_N(Z_t) = S(t|Z_t = 0) - S(t, Z_t),$$

where $S(t|Z_t = 0)$ = salvage value at time t given no services are extracted, and

$S(t, Z_t)$ = salvage value at time t with Z_t services extracted.

$$(6) \quad TUC_L(Z_t) = NPV_{T+dt},$$

where $TUC_L(Z_t)$ = Lewis' formulation of user cost, and

NPV_{T+dt} = net present value of asset in time period T+dt (this is the time period excluded by current use of the asset).

The Neal version is an off-firm opportunity cost while the Lewis version is a within-firm opportunity cost. The former is important for service extraction decisions, while the latter is relevant for investment or disinvestment decisions.

Maximizing (4) subject to (1), (2), and (3) involves determining the optimal production, service generation, and investment or disinvestment activities. We separate the determination of the production and service generation activities from the investment or disinvestment activities for ease of presentation. Determining the optimal production and service generation activities involves maximizing the following Lagrangian expression:

$$\begin{aligned}(7) \quad L = {} & P_{yt}Y_t - P_{x1t}X_{1t} - P_{x2t}X_{2t} - P_{x3t}X_{3t} - TUC_N(Z_t) - FC \\ & - \lambda_{1t}[Y_t - F(X_{1t}, Z_t)] - \lambda_{2t}[Z_t - G(X_{2t}|D_t)] \\ & - \lambda_{3t}[T_D - h(Z_1, \ldots, Z_{TH}, X_{31}, \ldots, X_{3TH})].\end{aligned}$$

Upon taking the required partial derivatives, equating them with zero, and making appropriate substitutions, the following necessary conditions are derived:

$$(8) \quad P_{yt}(\partial Y_t/\partial X_{1t}) = P_{x1t},$$

$$\begin{aligned}(9) \quad P_{yt}(\partial Y_t/\partial Z_t)(\partial Z_t/\partial X_{2t}) = {} & P_{x2t} + MUC_N(Z_t)(\partial Z_t/\partial X_{2t}) \\ & - [P_{x3t}/(\partial h/\partial X_{3t})](\partial h/\partial Z_t)(\partial Z_t/\partial X_{2t}),\end{aligned}$$

$$\begin{aligned}(10) \quad & \{[MUC_N(Z_t) + P_{x2t}/(\partial Z_t/\partial X_{2t}) - P_{Yt}(\partial Y_t/\partial Z_t)]/(\partial h/\partial Z_t)\}(\partial h/\partial X_{3t}) \\ & = P_{x3t}, \text{ and}\end{aligned}$$

$$\begin{aligned}(11) \quad P_{yt}(\partial Y_t/\partial Z_t) = {} & MUC_N(Z_t) + [P_{x2t}/(\partial Z_t/\partial X_{2t})] \\ & - [P_{x3t}/(\partial h/\partial X_{3t})](\partial h/\partial Z_t).\end{aligned}$$

Equation (8) indicates that the optimal quantity of X_{1t} to use is determined by equating the value of its marginal product to its price. Equation (9) states that the optimal quantity of X_{2t} to use involves having the instrumental marginal value product equal to the marginal cost of using X_{2t}. The marginal cost of X_{2t} is the price of X_{2t} plus the marginal user cost of the services generated by using

X_{2t} plus the increased maintenance costs which must be incurred as a result of using the durable. For X_{3t}, equation (10) indicates that the net marginal value of maintenance should be equated to the marginal factor cost of maintenance. The net value of a unit of maintenance is given in the braces in (10). Equations (8) through (10) state the marginal conditions for the optimal levels of X_{1t}, X_{2t}, and X_{3t}, respectively. For services from the durable, equation (11) indicates that the value of the marginal product of services should be equated with the marginal cost of acquiring services. This marginal cost is composed of the marginal user cost, the weighted cost of acquiring X_{2t} and the weighted cost of increased maintenance.

The simultaneous solution of equations (8) through (11) for each t, t = 1, ..., T_H will yield the optimal production activities for the firm with its initial endowment of D_t. The following section specifies the optimality conditions for acquiring additional durables or disposing of currently held durables.

Investment and Disinvestment Decisions

In making adjustments to its initial quantities, the firm will want to acquire units of a durable when its value in use exceeds its acquisition price. It will want to dispose of units of an existing durable when its value in use is less than its salvage price. A durable's value in use is derived from the services generated over its lifetime. Both the services generated and the lifetime of the durable optimal quantity of services to generate in each time period was specified above. Determining the optimal lifetime for a durable, in essence, determines the point in time when the firm should disinvest in the durable.

The durable's value in use can be represented as:

$$(12)\quad NRD(Z^*, T_D) = PVS(Z^*, T_D) + [1/(1 + r)^{T_D}][S(Z^*, T_D)],$$

where $NRD(Z^*, T_D)$ = the net return to the durable as a function of the optimal services generated in each time period, Z^*, and the length of time the durable is used, T_D,

$PVS(Z^*, T_D)$ = present value of services generated which depends on Z^* and T_D,

r = discount rate, and

$S(Z^*, T_D)$ = salvage value of durable in time period T_D after Z^* services have been extracted.

With Z^* determined according to equations (8) through (11), T_D is determined so as to maximize $NRD(Z^*, T_D)$.

If we treated time as a continuous variable, we would differentiate (12) with respect to t and equate with zero. However, our model treats time as a discrete variable; thus, we cannot take derivatives. We can only state approximate marginal rules for determining T_D^*. Our approximate rule is to equate the additions to $PVS(T_D)$ with the reductions in $S(T_D)$. T_D^* is the point in time when the additions to $PVS(T_D^* + 1)$ are less than the reductions in $S(T_D^* + 1)$. In other words, $PVS(T_D^*) > S(T_D^*)$, but $PVS(T_D^* + 1) < S(T_D^* + 1)$. This procedure determines when to disinvest in a durable. It is based on comparing the durable's value in use with its salvage value.

As indicated above, the firm will acquire units of a durable when $NRD(Z^*, T_D^*)$ exceeds the acquisition price. Note that the investment decision requires the determination of both the optimal production activities and the disinvestment activities. The optimal quantity of a particular durable is determined in an iterative manner, since we consider durables to be available in discrete units only. For each unit the firm considers, the potential value in use is calculated and compared with the acquisition price. If the potential value in use exceeds the acquisition price, the firm acquires that unit and repeats the calculations for another unit. It continues until it finds the unit whose value in use does not cover its acquisition price. A similar process is followed for disinvesting. The firm disinvests in units of durables until either the value in use for a particular unit exceeds it salvage price or the initial endowment of durables is entirely disposed of.

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OPENER'S REMARKS--Richard A. King

Baquet has demonstrated that the decision concerning optimum replacement of a durable asset cannot be separated from decisions made in each period concerning the flow of services which the asset makes possible. An illustration may help to clarify the problem. Leontief was fond of using a taxi company to describe the interrelationship between flow of services and asset replacement. Suppose that a taxi has a life of five years. An individual operating a single taxi would provide new taxi services the first year, two year old taxi services the second year, and so on over each successive five year period. Only in the event that five taxis were in operation could the flow of taxi services be uniform form one period to the next. However, it is not necessary to drive a taxi the same distance every year.

A recent example comes from a conversation with the bus driver on our Conference farm tour. Asked the age of the vehicle, he responded that it was 18 years old. Rather than the 100,000 or 200,000 miles we thought it might have logged, he reported that it had traveled well over one million miles to date and was still going strong. Clearly, a sizeable amount of upkeep had been required to achieve such a flow of services.

In the general case, there are two dimensions to the issue of extraction of services: the services available in a given period, and the total units of services remaining to be extracted. Every input has an implicit price at the end of every production period. In the case of nondurables, this price is zero. In the case of durables, this price is larger than or equal to zero.

Baquet has provided a formulation in which a variable input X_2 such as the driver (flow) is added to the vehicle (stock) to provide the intermediate taxi input, Z. Transport services Y are produced when another variable input such as gasoline, X_1 is combined with the intermediate input Z. The cost of extracting a given level of services from a durable good can be thought of as the change in the end of period price (salvage value) from one period to the next or as the value of services in some future period that would be precluded by the planned level of extraction in the present period (opportunity cost).

The problem can be viewed as that of assembling a new bundle of inputs at the beginning of every production period. In making a decision concerning replacement time, a comparison must be made between three values: salvage value of the existing unit, the use value of the existing unit in future production periods, and the market price of new unit. Salvage value is determined by age and deterioration (salt on the road), maintenance (frequency of oil change), and services extracted (miles driven). Use value in future periods is the present value of alternative future streams of services. The price of a new unit must reflect the prices of replacements (new or used) and the cost of rental services as an alternative to ownership.

Each of these three measures must be considered simultaneously in investment or disinvestment decisions. Only the third measure is unaffected by the level of output selected in each production period. For this reason, output and durable asset holding decisions must be made simultaneously.

RAPPORTEUR'S REPORT--S. N. Kulshreshtha

The question of present use value of a durable input from current stock versus replaced stock has not been handled in this paper. Furthermore, optimal use of a durable input may also require variation in the degree of maintenance. Maintenance expenditures could be determined by conditions outside the system. For example, the road conditions affecting maintenance expenditures for a taxi company. The exclusion of risk and uncertainty in decisions regarding durable inputs was also noted.

Baquet replied that replacement and maintenance decisions are not included, but the theory presented would lend itself to these extensions. Risk and uncertainty are also not included, but work is under way in this regard. Empirical work is also under way. The inclusion of changes outside the farming system to make the model deal with a more or less general equilibrium type of situation is somewhat more complex, but Baquet felt that it should be possible.