A Linear Programming Model of a Multiplant Tomato Packing Firm

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ABSTRACT

A firm processes a perishable seasonal agricultural commodity into several finished products. The lots of raw commodity are obtained from many locations and have characteristics which vary with variety, location and time of season. These raw product characteristics affect the yield and quality of the various finished products differently but according to known relationships. These characteristics can be measured and estimates of their values for each lot can be obtained prior to harvest along with an expected harvest date.

The firm has geographically separated plants with different production capabilities at each plant as all finished products are not produced at all plants and production costs for a finished product may vary between plants. Production capacity is limited and must be considered. Transportation costs for a lot of raw products may vary between plants.

The expected prices of the finished products are known. There may be maximum or minimum quantities for some or all finished products and requirements for a quantity of a product to be produced in a given time period.

Management of the firm desires to know to which plant each lot of raw product should be delivered and to which finished product it should be converted subject to the finished product quanitity restrictions in order to maximize the profit accruing to the conversion process.

The problem as stated can be expressed as a linear programming problem but in actual practice the dimensions of the problem may be too

large or unwieldy to solve with standard linear programming computer codes. The decomposition principle of linear programming allows the solution of much larger problems if the problem can be formulated so it can be decomposed.

A linear programming model of an operating tomato processing firm with the above characteristics was developed. The decomposition principle of linear programming was applied to obtain the solution of the problem described above.

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CHAPTER I

INTRODUCTION

The Problem

The management of a firm processing a perishable agricultural commodity makes numerous decisions such as selecting the finished product forms, selecting the quantity of finished products, and scheduling the production of each form. The scheduling problem is complicated by the seasonal production pattern of raw products. They frequently have periods of low production at both the beginning and the end of the season and exhibit a marked production peak sometime during the middle of the season. The complexity of the situation facing the decision-maker is even greater if geographically separated plants and several supply areas must be considered.

If the raw product is essentially of a homogenous quality, the above problems, although complex, can frequently be solved by standard techniques. However, some raw commodities are quite variable, and thus it is necessary to consider several qualitative characteristics in addition to the transportation and production costs and product prices to obtain the production schedule and product mix required to maximize profits. These characteristics can be affected by such things as the plant variety, the region of origin, the number of previous pickings, the weather during the season prior to maturity, and the weather immediately prior to picking. The characteristics might also have a seasonal pattern of variation. The scope of the problem is further increased if each finished product can be packed in several styles and/or grades. Each product form can also be packed in more than one size container. Although the choice of container size is usually not affected by the raw product characteristics, it frequently is a factor to be considered in scheduling due to machinery or plant restrictions.

Consequently, the total number of variables, the number of product forms, and the number of possible production schedules which must be considered in order to maximize profits, is usually quite large and presents a very formidable, if not impossible, task for management to solve using standard techniques.

This study is an attempt to demonstrate the application of linear programming as a tool to aid management in solving all of the above problems. In particular, it is an attempt to develop and demonstrate procedures which will enable management to determine the profit maximizing schedule and product mix for a season by decomposing the problem into a series of smaller manageable problems for solution by existing linear programming techniques on computer facilities of reasonable size. A further objective is to examine the potential uses of decomposition techniques in terms of the information available and its flexibility as compared to conventional linear programming.

The Test Commodity - Tomatoes

Although any one of several perishable commodities might be appropriate for such a study, tomatoes were selected in this case.

Tomatoes are processed into a wide variety of finished products including juice, paste, catsup, sauces, and whole fruit. Each of these products is packed in several container sizes and has several specifications or grades.

Tomatoes have a number of qualitative characteristics including acceptability for peeling, size, solids content and insoluble solids content that are important in allocating the raw product to finished product. These qualitative characteristics vary, sometimes widely, as a result of such things as variety, weather, soil, and cultivation practices. Frequently, these qualitative characteristics exhibit seasonal patterns which should be considered in scheduling. For example, the percent of solids in raw tomatoes tends to increase as the season progresses.

In addition to the above attributes which make tomatoes an appropriate commodity to use in the study, a large multiplant tomato processor, Tri-Valley Growers, was willing to cooperate in providing both raw product and manufacturing data for use in constructing linear programming models of tomato processing operations. The availability of "live" data from an operating environment is of major value in this type of study.

In addition to the decisions required in the general problem described above, there are other decisions which relate more specifically to the commodity under study, in this case tomatoes. For example, is it possible for tomatoes for canning to be upgraded sufficiently by sorting before peeling to compensate for the increased labor costs? Should raw tomatoes be sold rather than processed, and if so, which ones? Similar types of decisions relate to the specific processing environment. How many plants should be utilized and when should each one start and stop operations? Should additional capacity be added to existing plants? These specific problems are included in this demonstration of the application of linear programming as a tool to aid management in their decisions.

CHAPTER II. DATA SOURCES AND INFORMATION FLOW

In the development of methods and procedures to aid the decision-making process of a firm, the first task is necessarily the identification of the decision areas under consideration. The next step is to determine the information required on which to base the decisions. At the same time, the available information and its path or flow through the firm from its source to the decision-maker should be determined and evaluated. A sizable discrepancy may exist between the already available information and the information required by a formalized decision-making process.

In the tomato problem, the decision areas are divided into four general groups:

1. Which finished product forms and how much of each to produce, i.e., the pack quantity problem.
2. Where and when to produce each product form, i.e., the scheduling problem.
3. The assignment of raw tomatoes to finished products, an allocation problem.
4. The assignment of raw tomatoes from grower to plant, a transportation problem.

It should be recognized that these are not isolated decision areas but are definitely linked together and that a change in any one area will probably affect the "best" solution to the others. However, if one doesn't have a way of obtaining the "best" overall solution, these are probably quite reasonable sub-areas in which to suboptimize.

For analytical purposes, we can look at the multiplant tomato processing firm as having the four information sources shown in figure 1. This does not represent the actual organization but is based on the type of information available at each point and the type of decisions which are made or recommended at that point. There are also related information (and decision) centers in the figure which, although generally outside the structure of the firm, definitely influence the firm and are influenced by it. Two of these are the producers of the raw product and the purchasers of the finished products.

The first information and decision center is the firm's headquarters or general management level. Detailed data about the firm's operations generally do not originate here.. Rather, the data originating here are more likely to be centered on such things as general economic conditions and long-range projections.

In addition, the firm's general management will generally have the responsibility for coordinating the collection and transfer of all information needed by the firm's decision-makers, especially if the information source and the decision-maker are in different parts of the firm. However, it should be obvious that it is to the advantage of the general management to avoid having to handle and inspect masses of information pertaining to routine decisions appropriately made elsewhere. Rather, most data should be collected, analyzed, and stored by routine operations, but should be readily available to aid management whenever necessary for evaluating either external or internal policies and procedures.

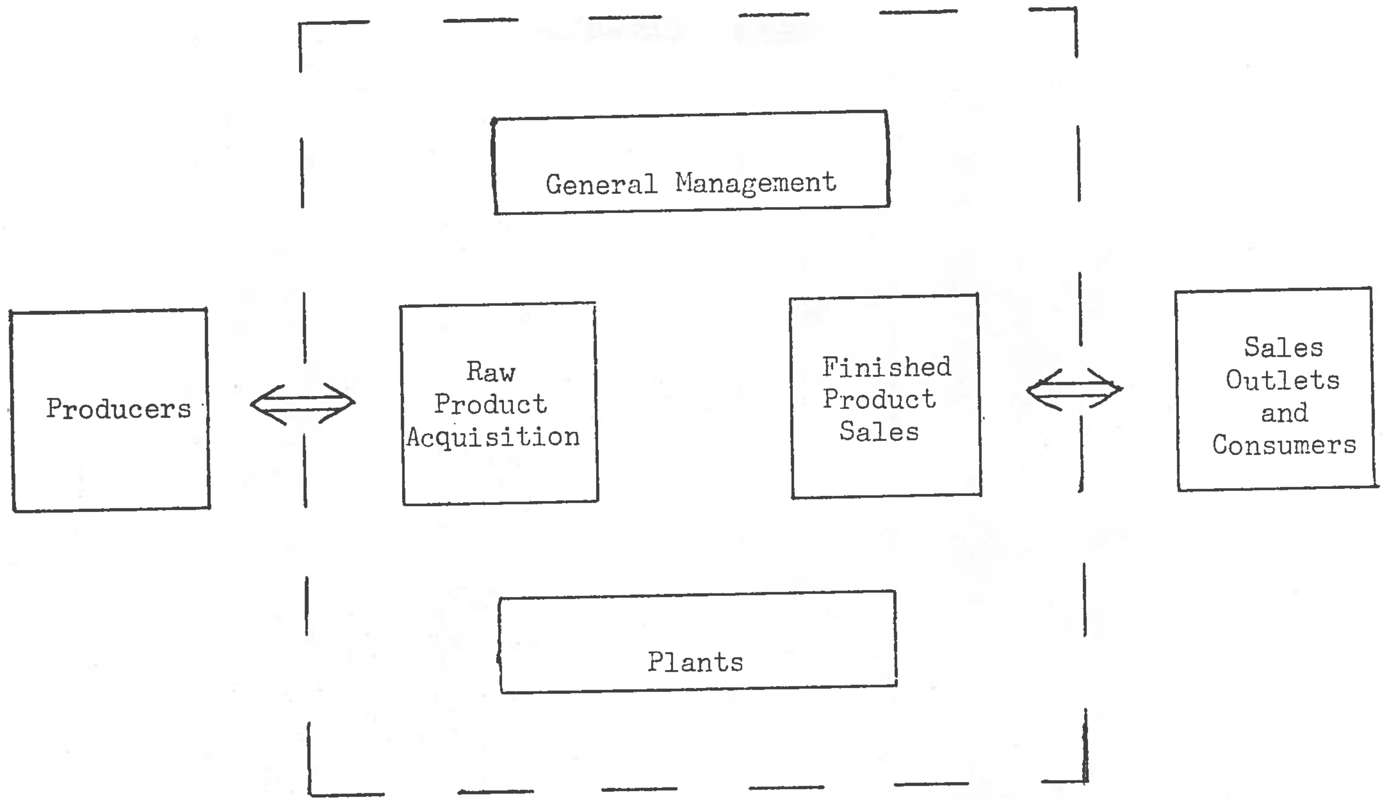

Figure 2-1

Information Sources

Another information and decision-area centers around the sales function. Here information is obtained about market conditions, estimates of future sales, and price trends. Information about the firm's inventory position would also be found in this area. Depending on the degree of centralization, decisions or proposals regarding price changes and promotional discounts are appropriately made here. Note that both the information and the decisions or proposals originating in this area are essentially outward looking and external to the processing functions of the firm.

A third information and decision area in Figure 1 is the raw product acquisition area. Here is where the firm obtains information on the availability of raw product and changes in the quantity or quality available. Decisions or proposals for changes in policy regarding raw product acquisition will originate here. Although not shown in Figure 1 because of the preeminence of raw product in the tomato problem, similar information and decision areas exist with respect to the purchase of materials other than raw products and the hiring of labor.

The last information and decision areas shown in Figure 1 are the plants. Each plant is generally unique and each probably should be considered as an individual information and decision center. The plants are the sources of data on processing costs and production restrictions.

The information required to base the types of decisions enumerated earlier necessarily comes from two or more of the information centers. These decisions normally affect more than one area of the firm. However, these decisions, although subject to some broad guidelines of policy, cannot be considered appropriately made at the headquarters level but rather at various operational levels. For example, decisions pertaining to the allocation of raw product to the production process of a commodity may sometimes be determined daily at the plant.

As a result of the necessity to make timely decisions in these areas and the time lags inherent in passing information through channels from one part of the organization to another (not to mention distortions caused by departmental self-images), decisions are frequently based on something less than full knowledge. For example, pack targets are necessarily based on the sales estimates which are obtained from knowledge of total industry supply, expected demand, consumer trends, and from information pertaining to the quantity of raw product which will be available for processing. However, after certain quotas have been obtained, the optimum breakdown of the production targets between individual products, can sizes, and grades will depend more on the quality of fruit available and on the manufacturing capabilities and costs than on the total tonnage available for processing. This information (and the means to analyze it quickly) is not usually available to the sales or marketing organization.

The linear programming model of the firm as developed here will formalize the information requirement so that all the appropriate information is included in the decision-making process. The solution to the linear programming problem will provide a basis for answering the questions in each of the four decision areas enumerated at the beginning of this chapter. In addition, if properly maintained, it can be used to furnish a basis for decisions pertaining to the related areas mentioned in Chapter 1.

The remainder of Chapter II discusses in detail the specific areas included in the model of the tomato packing firm and the types of information which must be considered in the decision process. Chapter III discusses the use of the decomposition principle of linear programming in the modeling of a multiplant firm.

Problems of Allocation and Scheduling

There are a large number of considerations involved in determining which commodity and to which plant a given lot of tomatoes should be allocated. We must consider the raw product characteristics of the tomatoes, the location of the field as reflected in the transportation cost, what plants are operating, what specialized equipment or processes are available at each plant, and the firm production targets by commodity.

We must simultaneously determine the commodity production targets for each plant. These are influenced by such things as the available machinery at each plant, the various plant costs and the raw product characteristics of the tomatoes which are available for shipping.

The raw product characteristics of most interests are:

1. Percent of choice peelers.
2. Percent standard peelers.
3. Percent peeling loss.
4. Percent total solids.
5. Percent insoluble solids.
6. Percent pulping and finishing loss.
7. Special problems such as mold and other defects.

The general classes of finished products are:

1. Whole peeled tomatoes (cored).
2. Whole peeled tomatoes (coreless).
3. Juice.
4. Puree.
5. Paste.
6. Sauce and catsup.

The effect of some of the raw product characteristics in allocating to the finished commodities should be obvious. A high percent of choice and standard peelers, and a low peeling loss are desired if the tomatoes are to be used for canning whole. These characteristics have no effect on the desirability of the lot for the other commodity classes. A high percentage of total solids is desired if the tomatoes are to be used for paste or puree since the total solids content of the product is the standard for paste and puree. Tomatoes with six percent solids will yield 120 percent as much paste or puree as tomatoes with five percent solids. In addition, less evaporation is necessary in the concentration process. Besides the lower processing cost, this means more tomatoes can be processed if evaporator capacity is limited. Similarly, high insoluble solids are desirable for sauce and catsup where the consistency is an important element of the standard. A low pulping and finishing loss is desirable for all the product classes but has no effect for whole peel tomatoes. Those tomato inputs falling in the category of special problems will usually result in higher processing costs, but their use in some types of products will normally yield better results than in others.

There are several considerations involved in determining to which plant site a particular lot of tomatoes should be sent. One of the most obvious is transportation cost, the major part of which is the cost of hauling. In the simplest case, the tomatoes are sent to the plant where the transportation cost is lowest. However, there are numerous circumstances where this might not be the most profitable action. For example, if the tomatoes are of extremely good peeling quality and the plant associated with the lowest transportation cost is only processing tomato products, the tomatoes might be sent to that plant producing whole peel tomatoes which has the lowest transportation cost. Conceivably, these tomatoes should be sent to a third plant if the second plant is utilizing its capacity fully.

In addition to the monetary cost of transportation, the perishable nature of the raw product is the source of an intangible cost while the tomatoes are being moved to the plant site. Although hard to evaluate, the deterioration of fruit between picking and processing can be a major cost to the firm.

If the equipment configuration varies between plants resulting in specialized equipment at one or more plants or the production costs vary widely between plants, it would be more profitable to send the raw tomatoes to the plant where the total of transportation and production costs is smallest, instead of basing the decision on transportation costs alone. Note that there might only be one commodity produced on the specialized equipment. If, for example, there are low costs of producing whole peeled tomatoes at a particular plant, higher transport costs only for peeling tomatoes might be justified.

There is also the possibility that it might be advantageous to incur extra transportation costs to ship tomatoes to a plant to balance raw product mix so that the equipment will be efficiently utilized.

In addition to determining allocation of raw product to plants, there are the problems of opening and closing plants and of maintaining a sufficient flow of raw material so that an efficient level of operation is maintained. It is desirable to have enough plants open so that transportation costs are reasonable and, at the same time, run those that are operating at an efficient level. These requirements are complicated by the fact that the cost of starting production or closing down a plant is sufficiently large so that it should usually be done only once during a season. Once a plant is opened, there should be sufficient raw material to enable it to operate at an efficient level until it is decided to close it down for the season. Note that the decision to open or close a plant will cause a reallocation of the tomatoes to plants. The destination of tomatoes on the margins of adjacent plants will usually be changed since the total supply of tomatoes available for the adjacent plants has been increased or reduced.

While we are allocating the raw product to the plants, we must simultaneously assign commodity targets to each plant (probably by grade and can size). These assignments must consider the machinery available at each plant, possible cost differences at the plants and any specific distribution consideration such as warehousing costs.

In scheduling the production of different products, one of the most important considerations is the distribution of tomato receipts during the season. Typically, the receipts are quite small initially, gradually build up to a peak which lasts a week or more and then fall off until the end of the season. During the seasonal peak, the receipts might be more than 20 percent greater than the immediately preceding and following weeks. These tomatoes are generally of high quality. Since the firm's capacity is based on some normal operating level, the large supply of tomatoes available during the peak may be greater than the plant can process unless production is scheduled to obtain the maximum flow-through of tomatoes. Their perishability effectively prohibits carry-overs of much more than one day. Even a short carry-over can cause marked deterioration in total yield and recovery. The use of load quotas or other limitations may reduce the quantity delivered, but damage due to overfilling boxes and the possibility that the tomatoes will be riper and not hold up well in the yard limit the overall effectiveness of such measures. At these times, judicious scheduling can be applied to increase the total quantity processed. For example, whole peel tomatoes might be packed in #10 cans rather than #303 cans to increase the flow through the canning lines and cookers. The key, under these conditions, is to avoid machinery or processing bottlenecks that limit the quantity which can be processed while other equipment is underutilized. An alternative possibility might be the sale of tomatoes to other firms to level off receipts. However, the nearby processing plants of other firms probably are also operating at capacity. If such an outlet is available, it should be considered. It might be best not to harvest some tomatoes. Total cost might be least if the poorest quality tomatoes were left in the field, eliminating picking and transport costs and overloading of the plant's facilities. Other considerations might preclude such action, but if possible it should be considered.

Another consideration in the scheduling of production runs is the fact that the raw product characteristics typically change during the course of the season. Peeling quality is generally lower during the beginning and ending weeks and total solids can be expected to increase throughout the season.

Other necessary considerations are those due to inventory and/or market conditions and the uncertainties of weather. Some commodities from last year's pack might be out of stock, so it is desirable to produce at least enough to meet current demands as soon as possible in the season. If an item is in short supply throughout the industry, it might be possible to advantageously market it early in the season. (However, these factors don't guarantee that the product should be packed early. Any increase in cost caused by producing the product early in the season should be compared to the increase in revenue from higher prices or balanced against the potential loss of good will if unable to fill orders.)

As a hedge against bad weather and other natural hazards, the firm might wish to attempt to meet certain minimum pack requirements as early as possible in the season or by some predetermined date. This might be an across-the-board requirement or it might apply to only selected products. Such a requirement might be considered as desirable, not mandatory, and the costs of obtaining it be considered as an insurance cost and be evaluated for each product or group of products.

It should be reiterated that the problems of scheduling production during the season, the problems of allocating tomatoes to plants and to commodities, and the assignment of commodity production targets to plants, are interrelated problems. The least-cost solutions to them cannot be determined independently. Rather, the solutions must be obtained simultaneously.

Assembly of Raw Product

The first step in the process of converting raw tomatoes into finished commodities is the picking of the raw fruit. The method used, the timing of the pick, and number of times the tomatoes are picked all affect the quality and characteristics of the fruit for processing. This is not to say that such things as the variety, cultural practices, and weather don't also affect the characteristics of the fruit being picked, but these latter factors cannot be influenced by the firm at the time of picking.

The fruit can be picked by hand or, in the case of some varies, by machine.[1/] The fields can be hand picked one, two, three or possibly more times. Machine picking can only be done once, but it does not have to be the first pick. The hand-picked fruit has less cracks and bruises and is generally better suited for use as whole peeled tomatoes. The mechanically harvested tomatoes are usually placed in pallet bins for

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[1/]The discussion of hand and machine picking in this section pertains to the conditions that existed during the 1965 crop year. In 1970 essentially all of the tomatoes processed in California are picked by machine including those used in whole peeled products. At the time the model was developed, the alternative of peeling mechanically harvested fruit was considered unsatisfactory.

transport to the plant, while the hand-picked tomatoes are customarily hauled in wooden lug boxes. Consequently, the two methods require different equipment for dumping the tomatoes at the plant. Also, the mechanically harvested tomatoes in the large containers will probably deteriorate more rapidly than fruit in the smaller wooden boxes unless special precautions are taken. In addition, there is a difference in the cost of hauling the bins and wooden boxes.

Once the picked tomatoes have been transported to the plant site and weighed, they will usually be held in the receiving yard after unloading for a few hours or more, depending on the backlog, and possibly on their condition. During this time, costs might be incurred for some of the lots if it is necessary to use insecticides.

At this stage of processing, there is a yard full of tomatoes of varying characteristics such as high quality for peeling or a high solids content. The determination of which commodity the tomatoes are to be allocated to generally has to be made at this point before they are dumped for washing and enter the plant.

Manufacturing Process (Whole Peel Tomatoes)

If the lot of tomatoes is to be used in canning whole tomatoes, some method must be used to separate fruit which is unsuitable for use as whole tomatoes from the rest. (This can be more than half of the tomatoes.) One method is to select out the choicest fruit for peeling and let all of the rest (except culls) go directly to the products operation without peeling. Another possibility is to peel all of the fruit except culls and send the fruit unsuitable for canning to the products operation. Peeled fruit used for products has a peeling loss which the fruit sent directly to products does not lose. The relative merits of the two procedures are dependent on a number of factors including the quality of the fruit, the cost of selecting out fruit, the desired grades of final product, and the peeling loss and the pulping and finishing losses of the tomatoes. A further consideration is whether there is sufficient peeler capacity within the existing plant configuration to peel tomatoes which will not be used for whole peel commodities. This is a short term consideration but it might preclude peeling without selection until additional peeler capacity could be obtained.

Ideally, we would compare the cost of preselecting tomatoes with the cost of the additional losses and operating expenses of peeling all the fruit and select the cheaper method. However, the choice of whether to select out peeling tomatoes before peeling or to peel all the fruit is complicated by three additional factors. First, the quality of the fruit for canning cannot be judged with complete accuracy before peeling. The color of peeled fruit sometimes differs markedly from the color of unpeeled fruit. Second, many good tomatoes may be missed in the selection process because of undermanning when the quality of tomatoes is high. If the quality is generally poor, there is a tendency to select poorer fruit in order to keep the peeler full. One solution might be to vary the manning of the selection line depending on the quality of the fruit, but this can result in an even more difficult labor scheduling problem. Another possibility would be to change the quantity of tomatoes as the quality changes and use a crew of fixed size. This also involves coordination problems since the flow of tomatoes to products will vary and some of the products operations require a fairly constant input of tomatoes for the most efficient operation.

A third factor to consider is the greater processing loss on the fruit which is peeled but not canned. This will vary between lots of tomatoes but would normally be in the area of ten percent. However, if the fruit has many cracks or is over-ripe, the loss can be much higher. Finally, the characteristics of the peeling equipment can influence whether the tomatoes should be selected before peeling.

After the tomatoes have passed through the peeler, they must be separated by quality (essentially color and wholeness) for packing into cans. Two or more different grades are usually packed at once so that the higher quality tomatoes can be used in the top grade or grades, and the lower quality can be used in the lower grades. The tomatoes of a quality too low to be used in any grade being packed are sent to a chopper for use in the products operation.

After the tomatoes are sorted by grades, they are placed in cans. After filling, cans are sent through a syruper where juice and condiments are added, then to a seamer and from there to a cooker. Cooking is essentially the last step of the production process. After being sufficiently cooled, they are removed to the warehouse for storage from which they will eventually be labelled and shipped.

Manufacturing Process (Products)

If the lot of tomatoes is allocated to products, immediately after dumping and washing, it is visually inspected and the fruit unsuitable for processing is removed. The tomatoes are then crushed in a chopper and pass through a hot break into a holding tank if appropriate. The crushed tomatoes are then reduced to pulp in a pulper. The tomato pulp eventually passes through a finisher. The texture of the finished tomato pulp can be varied as desired for various products by changing the screen sizes used in the pulpers and finishers. Ideally, there are two or more sets of equipment operating in parallel so that the tomato pulp can be segregated by its desired end use which is based on the raw product characteristics.

After finishing, the tomato pulp which will be used for juice goes to a holding tank where it can be sent to the whole peel syrupers for use in filling cans of whole peel tomatoes or to a product filling line where it is canned as tomato juice. Tomato pulp which will be used for paste or puree passes to a holding tank from which the pulp will be drawn into preheaters and then into the evaporation equipment. After sufficient evaporation has occurred so that the pulp has the desired level of solids, the concentrated pulp goes to the appropriate product canning line.

The tomato pulp utilized for sauce or catsup is handled in a similar fashion. The pulp is finished to various levels depending on the product. The pulp is evaporated until the desired consistency is obtained and removed to mixing tanks where other ingredients are added, and is then moved to the appropriate canning lines.

Blending of the Raw Product

In one of the previous sections, the question of whether to select the tomatoes for canning before peeling or whether to peel the entire lot was discussed. A related question is whether lots of tomatoes with different raw product characteristics should be blended. (The characteristics of interest are primarily the percent of choice peelers, percent of standard peelers and percent of peeling loss.) Although the blending of two or more lots of tomatoes can be carried out regardless of whether the tomatoes are selected for canning before or after peeling, it can also affect the relative advantages of selection and non-selection.

By blending tomatoes of various qualities together, a more uniform raw material can be maintained from which to select tomatoes for peeling (and thereby maintain a stable crew size and a more stable quantity of tomatoes going directly to products). Similarly, if the tomatoes are sent straight to peeling without selection, the quality of tomatoes which are being peeled is more uniform and the proportion in each grade available for canning will be more stable. This should have the effect of generally stabilizing production rates for all grades rather than spurts on one line or another because of changes in fruit quality.

Blending could also be used to level out the quality of the fruit throughout the day. Even if enough fruit of desired quality was available to run for several hours, it might be desirable to blend it with other fruit either of a desired, or less than desired, quality so that the fruit would be uniform throughout a shift, or entire day, or possibly for several days to maintain a more stable production rate and avoid changing labor requirements.

Similarly, it might be desirable to blend tomatoes going into the products operation on the basis of total solids or insoluble solids or other characteristics. The required solids levels of the various products vary from less than nine percent tomato solids for sauces to 32 percent or higher for paste. The efficiency with which water is evaporated from the pulp will usually vary inversely with the concentration of the pulp although the characteristics of evaporators vary significantly. Generally, it is desirable that the low solids tomatoes go to the less concentrated products although this will not necessarily be true if the total evaporator capacity is plentiful compared to the capacity of other equipment.

CHAPTER III

DECOMPOSITION AND THE FIRM

Allocation of Raw Products to Plants as a Decomposition Problem

In the previous chapter some of the problems of allocating perishable raw products with different characteristics from a large number of sources to several geographically separated plants with different machinery configurations were discussed. It has also been stated that linear programming is a useful tool to maintain control of raw product allocation and assignment of production targets while obtaining the maximum profit product mix and production schedule. Detailed knowledge of the plant capacities, the raw product information, finished product requirements and sales information would be collected and maintained at operating levels of the firm for inclusion in the linear programming (LP) model. Such a programming model might be of extremely large size, perhaps even too large to solve with standard LP codes currently available. The use of the decomposition principle of linear programming would enable the solution of such a large-scale problem. In addition to this aspect, there are also interesting possibilities of organizational decentralization to consider.

For instance, the decomposition of a linear programming model of a multiplant firm might be undertaken to maintain centralized control of raw product allocation and production quotas while leaving the operating decisions completely to the management of each plant. By the use of the decomposition technique, furthermore, the maximum profit allocation and schedules (within the framework of the L P model) can be determined.

The firm can maintain control of the plants by the prices it charges for the different raw products, and the prices it is willing to pay for the finished products. Each plant manager maximizes profits based on these prices and his available facilities by the use of an L P model for his plant. For instance, the raw product acquisition group must give each plant manager full information about the characteristics of the raw products available. The transportation costs for each grower should either be given to the plant manager or maintained by him.

When the plant model and raw product information is complete, the firm has the sales department give each plant manager a detailed list of product specifications and the sales price and sales target for each finished product. On the basis of this information, i.e., the characteristics of the tomatoes available, the transportation costs, desired product mix and expected sales prices, each plant management having detailed knowledge of their plant machinery and strong points, computes their maximum profit product mix from the products in the sales targets and submits it to the firm's headquarters. At the same time, each plant offers to purchase the raw product which they desire.

In decomposition terminology, each plant problem is called a subprogram. Note that there are no restrictions on each plant's product mix or the quantity of raw product desired except that both must be within the total quantity to be sold or available for purchase to the firm. (Even this restriction is unnecessary, but tends to simplify the problem.)

The firm now has the various profit maximizing product mixes from the plants and in addition has the quantities of raw product desired by each plant. However, the total of the proposed product mixes will probably not be equal to the sales target of the firm and the quantity of tomatoes desired will probably not be equal to the quantity available. In particular, some products will probably be significantly over the sales targets and some under. Some products will probably not be included at all. Similarly, some tomatoes might be requested by every plant and some not bid for at all. The firm now computes a revised price list for the products, offering a bonus for those products where the sales target is not met and reducing the price of those commodities where there was an oversupply offered. In fact, it might even assign a negative price. At the same time, a charge or penalty cost is assigned to tomatoes which are over-utilized by the plants and a bonus or subsidy is assigned to those tomatoes which were not completely utilized by the plants. These charges and bonuses can be obtained from the dual solution to a linear programming formulation of the firm problem. In decomposition terminology, the firm program is called the master program.

The revised product prices and the penalties and subsidies for the use of the lots of tomatoes are then forwarded to the plants and the plant is asked to recompute their most profitable product mix based on these prices. This new group of prices is all the additional information the plants are given or need to obtain. The plant's new maximum profit product mix and tomato requirements are then sent to the firm's headquarters. These plans are then rechecked to see if they meet the sales target requirements and the available raw product limitations. If they do not, another set of bonus and penalty prices is determined and the process repeated. Although this is an iterative process and requires several solutions for the firm and each plant, it can be shown that an overall profit maximizing solution can be reached in a finite number of iterations.[1/]

In this manner, the management of the firm can maintain centralized control of the production targets for its plants and the allocation of raw product to the plants knowing it will obtain the maximum overall profit while being relieved of the necessity of maintaining and reviewing all the detailed plant data.

Scheduling as a Decomposition Problem

The previous section was concerned with the problems of allocating raw product and/or production targets to the various plants of a multiplant firm. However, this discussion did not consider the complications caused by changes in the characteristics of tomatoes from a given source as the season progresses. In addition, there may be requirements for scheduling some products during a particular time period. In this section, we will consider the problem of determining

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1/George Dantzig, (Linear Programming and Extensions) (Princeton: Princeton University Press, 1963), p. 452.

the maximum profit production schedule for the season which extends over several time periods. The decomposition technique can be used to determine the maximum profit schedule of production.

Consider, for simplicity, a single plant firm in the process of scheduling production for the coming season. The firm can break the season up into a series of periods of some reasonable length for planning purposes. Each time period becomes a subproblem. In the tomato problem, calendar weeks were used. The firm must determine fixed production requirements and raw product availability by time periods. It will also want to consider any other special characteristics such as an anticipated labor shortage or machinery overhauls which may change from period to period.

The firm then obtains the maximum profit product mix for each time period (subproblem) through the use of linear programming using the expected product selling prices and quantities of raw product available in each period. These product mixes for each time period are considered a tentative schedule for the season and then evaluated via a master program. Since the total of the products for the season will undoubtedly be unsatisfactory for the season's output, it will be necessary to change the tentative schedule somewhat. This is done by increasing the prices of those commodities which would be underproduced if the tentative schedule were followed and decreasing the price of those products of which too much would be produced. These prices are obtained from the dual solution to the master program.

The maximum profit product mix is then obtained for each time period using the new prices. It should be noted that the new prices may not be the same for every week. If there was a requirement that production of one product be completed early in the season, the price for the product obtained from the master program could be higher earlier in the season. These new weekly production figures are taken as a tentative schedule and evaluated for consistency with seasonal targets. If this schedule is not satisfactory, new prices are computed and the process repeated as often as necessary. As in the allocation problem, the maximum profit solution will be obtained in a finite number of iterations. In actual practice, it would be unnecessary to obtain the solution for every time period with every set of prices. Rather, only one time period solution need be obtained before a new set of prices is generated. This will considerably reduce the computational effort at least in the early stages of the scheduling process.

Allocation and Scheduling as a Decomposition Problem

In the first section of this chapter, the use of decomposition procedures for allocating raw product inputs and production quotas to plants was discussed. In the second section, the use of decomposition for temporally scheduling the production of a plant was discussed. The combined problems of allocation and scheduling for a multiplant firm can be approached simultaneously.

One approach would be to give each plant manager the same information as they were given for the allocation problem except that raw product inputs would be by time period and not for the entire season. In addition, the plant managers would be informed of any special production requirements or special circumstances pertaining to the time periods.[2] Each plant would then submit its tentative maximum profit production plans for each week. The firm then has a plan from each plant for each time period to evaluate for consistency. The prices that the firm would offer to pay plants for products that were under target would then be increased and prices for products which the plants wanted to produce in surplus would be decreased. In addition, if the plants failed to offer a sufficient quantity of a product to meet a time period requirement, the price for that product would be increased during that time period. These new prices would then be submitted to the plant managers with instructions to recompute new production plans.

In addition to controlling production targets for the plants through the use of offer prices for the commodities, it is also necessary to allocate the raw material to the plants by time period through the use of subsidies and penalty charges for the raw product inputs.

This procedure requires that the firm know how much of each input is available during each week and be informed of the quantity of each input the individual plants require each week.

Extensions

The previous sections of this chapter have been concerned with the use of the decomposition technique to aid the firm in solving the

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2/This is not a requirement for the solution. If management did not want to divulge the special requirements for security or other reasons, it would not be necessary.

allocation and scheduling problems pertaining to a single type of raw product. It was implicitly assumed that the machinery and labor in each plant were devoted exclusively to the product considered and there were no competing products produced at that plant. This need not be the case. Two or more products might compete within a plant for machinery or labor available at that plant. In the California tomato industry, there is frequently an overlapping of the cling peach and tomato seasons with a period of time when both compete for the facilities of the firm. A general example of a potentially scarce resource throughout the year which would affect all products is warehouse space. Decomposition might prove useful in this case if the scarce resources used by more than one product were included in the master program and a subprogram was developed for each commodity group. Then, during the periods when there were no competing products, the solutions to the subprograms would give the maximum profit utilization of the resources and facilities. If there are several plants, decomposition might be done in more than two stages, or the plant restrictions encompassing more than one product might be put in the overall master.

The multiplant firm which does not have any of the circumstances outlined above might still find the decomposition technique useful. For instance, consider capital budgeting. Rather than attempt to build one large overall budget model of the entire firm, if linear programming models of the plants (or divisions) were available with capital requirements as elements of the plant (or division) solution values, the decomposition principle would allow the plant models to be combined into a capital budgeting model. The available plant L P models would be the subprograms and the master would consist of only one constraint row for each plant, an objective function, and as many rows as were necessary to write the budget restrictions.

The addition of the decomposition principle should increase the effectiveness of the linear programming tools already available to the firm. L P models of the individual plants, parts of plants or groups of plants, can be used to aid in scheduling for day-to-day operations and/or seasonal scheduling. The relatively small plant models could be solved as often as necessary to adjust day-to-day operations to changing conditions while decomposition can be used on those occasions when it is necessary to evaluate the whole operation for investment decisions, seasonal scheduling, or price changes.

CHAPTER IV

THEORY OF DECOMPOSITION OF LINEAR PROGRAMMING

Theory of Decomposition

The general linear programming problem[1/] is to maximize or minimize a linear function subject to a set of linear constraints.[2/]

For example:

$$\text{Maximize } c_1X_1 + c_2X_2 + \ldots\ldots + c_jX_j + \ldots\ldots c_nX_n = Z$$

subject to

$$\begin{array}{l}
a_{11}X_1 + a_{12}X_2 + \ldots\ldots a_{1j}X_j + \ldots\ldots a_{1n}X_n = b_1 \\
a_{21}X_1 + a_{22}X_2 + \ldots\ldots a_{2j}X_j + \ldots\ldots a_{2n}X_n = b_2 \\
\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \\
a_{i1}X_i + a_{i2}X_2 + \ldots\ldots a_{ij}X_j + \ldots\ldots a_{in}X_n = b_i \\
\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \\
a_{m1}X_1 + a_{m2}X_2 + \ldots\ldots a_{mj}X_j + \ldots\ldots a_{mn}X_n = b_m
\end{array}$$

and $X_j \geq 0$ for all j $\qquad m < n$

1/For a more extended treatment of the linear programming problem see Gass, Linear Programming (New York: McGraw-Hill, 1964), p. 45-47.

2/The constraints in the standard problem are strict equalities. Sets of linear constraints which include inequalities can be written in equation form by adding or subtracting nonnegative slack variables. The c_j associated with these slack variables are assigned values of zero.

The a_{ij}, b_i and c_j are all known constants and the X_j are the variables. Where necessary the equations are multiplied by -1 to make all the $b_i \geq 0$.

In economic terms, the b_i are frequently the total quantity available of a scarce resource (e.g., the quantity of available tomatoes or the capacity of a machine) or the minimum or maximum quantity of a finished product that it is desired to produce. The X_j are frequently the quantities of raw materials used, the quantities of finished product manufactured, or the levels of particular processes. The a_{ij} are the yield or requirement coefficients of activity j on restriction i.

The c_j are the costs or revenues attributed to a unit of each activity. If the c_j are expressed as revenue or profit, then the objective function $\Sigma c_j X_j$ is typically maximized, i.e., total profit or revenue is maximized. (If the c_j are expressed as costs, then $\Sigma c_j X_j$ is minimized.)

In matrix notation, this can be expressed as:

$$\text{Maximize } C'X = Z$$

$$\text{subject to } X \geq 0$$

$$\text{and} \qquad AX = b$$

A is the m x n matrix of coefficients a_{ij}, X is a vector of n elements corresponding to the X_j and b is a vector of m elements corresponding to the m restrictions. C' is a vector of n elements corresponding to the c_j's. Z is the total profit or cost.

If the size of the problem is not too large and if a solution exists, the linear programming problem can be solved by any of several algorithms on digital computers. However, if m and n are quite large, the problem may exceed the capacity of the computer program or it may be quite unwieldy for data handling, parameter manipulation and/or interpretation of the results. Under certain conditions, problems such as these can be solved using the decomposition principle of linear programming.3/ For example, if A, X, b and C are defined as follows:

$$A = \begin{bmatrix} \underset{(m_1 \times n_1)}{G} & 0 \\ 0 & \underset{(m_2 \times n_2)}{H} \end{bmatrix} \qquad X = \begin{bmatrix} \underset{(m_1 \times 1)}{X_1} \\ \underset{(m_2 \times 1)}{X_2} \end{bmatrix} \qquad b = \begin{bmatrix} \underset{(m_1 \times 1)}{b_1} \\ \underset{(m_2 \times 1)}{b_2} \end{bmatrix}$$

$$C' = \begin{bmatrix} \underset{1 \times n_1}{C_1'} & \underset{1 \times n_2}{C_2'} \end{bmatrix} ,$$

Then the problem

$$[A] \quad [X] \quad = \quad [b]$$

$$X \geq 0$$

$$C'X = Z \max$$

can be rewritten

$$\begin{bmatrix} G & 0 \\ 0 & H \end{bmatrix} \begin{bmatrix} X_1 \\ X_2 \end{bmatrix} \begin{matrix} = \\ = \end{matrix} \begin{bmatrix} b_1 \\ b_2 \end{bmatrix}$$

$$X_1, X_2 \geq 0$$

$$\begin{bmatrix} C_1' & C_2' \end{bmatrix} \begin{bmatrix} X_1 \\ X_2 \end{bmatrix} = Z \max$$

3/George Dantzig and Philip Wolfe, "Decomposition Principle for Linear Programs", Operations Research, Vol. 129, No. 1 (January 1960), pp. 101-111.

and the solution of the two separate parts or subproblems

1) $GX_1 = b_1 \qquad X_1 \geq 0$

$C_1' X_1 = Z_1$ max

2) $HX_2 = b_2 \qquad X_2 \geq 0$

$C_2' X_2 = Z_2$ max

is identical to the solution of $AX = b$ with Z max $= Z_1$ max $+ Z_2$ max. It therefore makes no difference whether we solve the original problem or the two subproblems. An example might be a firm with two plants with completely different product lines. However, in general this is a trivial problem and is of little interest from either conceptual or computational viewpoint. However, a more general problem exists if we define A, X, b and C as:

$$A = \begin{bmatrix} F_1 & F_2 \\ G & 0 \\ 0 & H \end{bmatrix} \qquad X = \begin{bmatrix} X_1 \\ X_2 \end{bmatrix} \qquad b = \begin{bmatrix} b_1 \\ b_2 \\ b_3 \end{bmatrix}$$

$$C' = \begin{bmatrix} C_1', & C_2' \end{bmatrix}$$

Now the requirement that $AX = b$ cannot be expressed in independent parts as before. Consider expressing $AX = b$ as

1) $\overline{F} X = b_1$

2) $G X_1 = b_2$ or $\overline{G} X = b_2$

3) $H X_2 = b_3$ or $\overline{H} X = b_3$

where

$$\bar{F} = [F_1 : F_z]$$

$$\bar{G} = [G : 0]$$

$$\bar{H} = [0 : H]$$

Now each relationship contains variables included in another part. This is a more typical case and is schematically represented in figure 4-1. We can obtain the solution to this problem by solving the linear programming problem[4]

$$\text{a)} \quad \begin{aligned} F_1 X_1 + F_2 X_2 &= b_1 \\ C'_1 X_1 + C'_2 X_2 &= Z \max \end{aligned}$$

subject to the conditions

$$\text{b)} \quad G X_1 = b_2 \,, \quad X_1 \geq 0$$

$$\text{c)} \quad H X_2 = b_3 \,, \quad X_2 \geq 0$$

In the usual case, there are a very large number of vectors X_1 and X_2 that satisfy the latter conditions (i.e., any feasible solution to (b) or (c)). However, any vector X_1 in the set of vectors satisfying $G X_1 = b_2$ can be expressed as a convex combination of the P different solutions for (b).[5] Similarly any vector X_2 satisfying (c) can be

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4/In economic terms, the equations of (a) might be the overall constraints on the firm while the equations of (b) and (c) are the constraints on two essentially independent parts of the firm, i.e., manufacturing plants or divisions. In practice, the corporate constraints or restrictions of (a) might include limitations on the total raw materials or capital available to the firm and/or the total quantities of finished goods that can be marketed by the firm. The restrictions of (b) and (c) might correspond to the existing technology, manufacturing capacities, labor availability and other resources available to the individual manufacturing plants or divisions.

5/Dantzig, Linear Programming and Extensions (Princeton: Princeton University Press, 1963), p. 449.

expressed as a convex combination of the Q basic feasible solutions for (c).

Hence, problem (a) can be rewritten as:

$$\text{a}')\quad \begin{aligned} &F_1\left[v_1\,\overline{X}_{11} + \ldots + v_p\,\overline{X}_{1p} + \ldots + v_p\,\overline{X}_{1p}\right] \\ &+ F_2\left[u_1\,\overline{X}_{21} + \ldots + u_q\,\overline{X}_{2q} + \ldots + \;_q\,\overline{X}_{2q}\right] = b_1 \\ &C_1\left[v_1\,\overline{X}_{11} + \ldots + v_p\,\overline{X}_{1p} + \ldots + v_p\,\overline{X}_{1p}\right] \\ &+ C_2\left[u_1\,\overline{X}_{21} + \ldots + u_q\,\overline{X}_{2q} + \ldots + u_q\,\overline{X}_{2q}\right] = Z \max \end{aligned}$$

$$\sum_{p=1}^{P} v_p = 1 \qquad v_p,\, u_q \geq 0$$

$$\sum_{q=1}^{Q} u_q = 1$$

where

$$X_{1p} \quad (p = 1, \ldots\ldots, P)$$

$$X_{2q} \quad (q = 1, \ldots\ldots, Q)$$

represent all the basic feasible solutions for the subproblems

b) $G\,X_1 = b_2\,,\quad X_1 \geq 0$

c) $H\,X_2 = b_3\,,\quad X_2 \geq 0$

(a') is now a linear programming problem with two additional constraints, i.e., $\sum_{p=1}^{P} v_p = 1$ and $\sum_{q=1}^{Q} u_q = 1$. In decomposition terminology (a') is called the master program or extended master program.[6] If all the basis vectors $\overline{X}_{1p}$ and $\overline{X}_{2q}$ for the subproblems were known, this problem could be solved directly. The number of columns in (a') although

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[6] It is also referred to as the extremal program or executive program.

finite could be quite large. However, the $\overline{X}_{ij}$ are not known and would require a considerable effort to obtain them. Instead, in actual computation only a few of the vectors are obtained.

Typically, a basic feasible solution to each of the subproblems, $\overline{X}_{11}$ and $\overline{X}_{21}$, are obtained by solving

$$\text{b}') \quad \begin{aligned} G\,X_1 &\leq b_2 \\ C_1\,X_1 &= Z_1 \max \\ X_1 &\geq 0 \end{aligned} \qquad\qquad \text{c}') \quad \begin{aligned} H\,X_2 &\leq b_3 \\ C_2\,X_2 &= Z_2 \max \\ X_2 &\geq 0 \end{aligned}$$

$\overline{X}_{11}$ and $\overline{X}_{21}$ are placed in (a') and (a') is solved; (b') and (c') are then solved again with revised objective functions based on the results of the solution to (a'). The two new vectors, $\overline{X}_{12}$ and $\overline{X}_{22}$, are placed in (a') which is then resolved with the additional vectors. This iterative procedure is then repeated until the overall optimum is obtained.

In decomposition terminology, the solution of the subproblems and the subsequent solution of the master program is called a major iteration. In practice it is not necessary to solve all of the subproblems during each major iteration. Any $\overline{X}_{ij}$ which enters the solution to the master program will improve the solution value of the master and allow the iterative process to continue.

This decomposition principle can be generalized to any linear programming problem which has the structure[7/]

$$\begin{bmatrix} A_1 & A_2 & A_3 & \cdots & A_r \\ B_1 & 0 & 0 & \cdots & 0 \\ 0 & B_2 & 0 & \cdots & 0 \\ 0 & 0 & B_3 & \cdots & 0 \\ \cdot & \cdot & \cdot & \cdots & \cdot \\ \cdot & \cdot & \cdot & \cdots & \cdot \\ \cdot & \cdot & \cdot & \cdots & \cdot \\ 0 & 0 & 0 & \cdots & B_r \end{bmatrix} \begin{bmatrix} X_1 \\ X_2 \\ X_3 \\ X_4 \\ \cdot \\ \cdot \\ \cdot \\ X_r \end{bmatrix} = \begin{bmatrix} b_o \\ b_1 \\ b_2 \\ b_3 \\ \cdot \\ \cdot \\ \cdot \\ b_r \end{bmatrix}$$

7/*Ibid*., pp. 448-455.

$X_j \geq 0$ for $j = 1, . . . , r.$

$$\sum_{j=1}^{r} C_j' X_j = Z \max$$

Usually, each $B_j X_j = b_j$ is an independent subset of equations referring to the same production facility, time period or similar grouping and $\sum_{j=1}^{r} A_j X_j = b_0$ is a set of equations tying the subsets (or subproblems) together. However, any linear programming problem can be decomposed to take advantage of its structure if decomposition would present computational advantages. There is no requirement that the subproblems have an economic or physical relationship but only that the problem can be decomposed into independent subparts.

The Dual and Its Economic Interpretation

The discussion at the beginning of this chapter defined the usual linear programming problem as finding the column vector

$$X = (x_1, x_z, ..., x_n)$$

which maximizes the linear function

$$C' X = Z$$

subject to the conditions

$$AX = b$$

$$X \geq 0 \qquad m < n$$

Associated with every linear programming problem of this type is a corresponding problem called the dual.[8] The original problem is called

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8/Gass, op. cit., pp. 83-94.

the primal. The dual problem associated with the primal problem stated above is known as the unsymmetric dual. The dual problem is to find a row vector

$$W = (w_1, w_2, \ldots\ldots\ldots, w_m)$$

which minimizes the linear function

$$Wb = Y$$

subject to the conditions

$$WA \geq C'$$

Note that in the unsymmetric dual the variables w_i are not restricted to be non negative.

Feasible solutions to the primal and the dual may appear to have little relation to one another; however, their optimum basic feasible solutions are such that it is possible to use one to obtain the other readily.

This relationship between solutions to the primal and the dual is stated in the Duality Theorem.

> If either the primal or the dual has a finite optimum solution, then the other problem has a finite optimum solution and the extremes of the linear functions are equal, i.e., max Z = min Y.
>
> If either problem has an unbounded optimum solution, then the other problem has no feasible solution.[9]

When the primal problem is formulated in equations as above, the variables of the dual problem are unrestricted in sign. Specifically, if the ith constraint in the primal is an equation, then the ith dual

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9/Gass, op. cit., p. 84.

variable is unrestricted in sign.[10/] However, when the primal problem is formulated as strict inequalities, the variables of the dual are restricted to be non-negative. This is the summetric dual problem. In this case, the primal problem is

Maximize $C'X = Z$

subject to

$AX \leq b$

$X \geq o$

The associated dual is

Minimize $WB = Y$

subject to

$WA \geq C'$

$W \geq 0$

When the linear programming problem is stated in this way we can use two important theorems which state the properties that are frequently referred to as complementary slackness.[11/] (The theorems are stated in terms of the primal but conversely hold for the dual as the dual of the dual is the primal.)

1. If a slack or surplus variable x_{m+i} which has been added to the ith primal constraint appears in an optimal basic solution, then for the corresponding optimal solution to the dual, the ith dual variable is zero, that is $w_i = o$.

[10/]Conversely if some variable X_i in the primal is unrestricted in sign, then the ith constraint of the dual will be a strict equality.

[11/]Hadley, Linear Programming (Reading, Massachusetts: Addison-Wesley, 1962), p. 259.

2. If the variable x_j appears in an optimal basic solution to the primal problem, then in the corresponding optimal solution to the dual, the jth constraint holds as a strict equality, that is, the dual slack or surplus variable $W_{n+j} = o$.

The usual economic interpretation of the primal problem can be stated:

> For a given value per unit of output (c_j) and an upper limit on the availability of each resource (b_i), how much of each output (X_j) should be produced in order to maximize the value of total output ($\sum_{\text{all } j} C_j X_j$)?

In this framework the physical dimensions of the variables X_j are units of goods being produced. The dimensions of the b_i are the units of resources being consumed. The a_{ij} then have the dimensions of units of resource i per unit of good j.

To be consistent, $w_i a_{ij}$ of the dual must have the dimensions of units of value per unit of good j. Since the dimensions of a_{ij} are units of resource i per unit of good j, the dimensions of w_i must be units of value per unit of resource i. That is w_i is the value of a unit of resource i.

The optimal values of the structural variables of the dual problem (w_1, w_2, ..., w_m) can be interpreted as the shadow prices or marginal profit values of the variables. That is, w_i tells how much the objective function value would be increased if the quantity of input i available to the firm were increased by one unit without changing the solution to the dual or

$$w_i = \delta z / \delta b_i$$

In practice if we change b_i to $b_i + 1$, the profit will not generally increase by w_i because the entire optimum solution changes. These dual variables w_i are also referred to as dual prices or shadow prices. Note, however, that they have nothing to do with the actual costs of the resources (which are not stated in the problem and may not even be known.) If there is a positive slack variable in constraint i so that not all of resource i is used, then $w_i = 0$, and the resource i is a free good.

Similarly, the w_{m+j}, the optimal values of the slack variables, indicate opportunity costs, i.e., how much the objective function would be reduced if one unit of commodity j which is not in the optimal solution were produced by the firm.[12]

The Solution of the Decomposed Problem

The solution of the decomposed problem is obtained by the iterative procedure described briefly earlier in this chapter. This section will describe the solution procedure including how the changes are made to the objective function of the subprograms during each major iteration and the necessary conditions for an optimal solution to the original problem. The shadow prices or dual variables of the solution to the master program are used in both.

The function of the master program is to determine the weight of each proposed subproblem solution in the solution to the original problem. The total of the weights to be assigned to solution vectors from any subproblem must be ≤ 1 if the weighted combination of solutions

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[12] William Baumol and Tibor Fabian, "Decomposition, Pricing for Decentralization and External Economics", Management Science, Vol. 9, No. 4 (September 1964) pp. 5-6.

to the subproblem is to be a feasible solution (to both the subproblem and the original problem). These weights are the variables in the master problem.

The restrictions of the master program consist of the corporate or overall constraints which limit such things as the total use of a resource by all parts of the firm to the total quantity available to the firm and the total production of a good to that quantity which can be marketed by the firm. In addition there is one constraint for each subproblem of the form

$$\sum_{\text{all } q} v_{rq} \leq 1$$

where v_{rq} is the weight assigned to the qth solution vector from subproblem r.

The coefficient matrix consists of the subproblem solution vectors[13/] augmented by a one in the corresponding subproblem weight constraint and zeros in all other subproblem constraint rows. Consequently, the elements of X_r which were the primal variables in subproblem r are constants in the master program, and the weights v_{qr} to be assigned to the subproblem solution vectors are the primal variables in the master. Note that the number of columns or augmented solution vectors from the subproblems will increase during each major iteration[14/] as new subproblem solutions are obtained.

13/Actually only those elements of the solution vector that are subject to the corporate constraints.

14/In practice, this does not cause a dimension problem in the master as vectors which do not enter the solution to the master can be dropped from future iterations.

The objective function values of the master program are the solution values or max $Z = C_r' X_r$ associated with the subproblem solution vectors.[15/] The primal master problem at each major iteration then is to find the weights for the subproblem solution vectors which will maximize the total value of the weighted subproblem solutions subject to the corporate constraints. This total value[16/] is then increased during each major iteration. However, the solution values of the variables in the master are disregarded until the final iteration which gives the optimal solution to the original problem. Rather the dual variables or shadow prices of the solutions to the master are the basis for making changes to the objective function prices ($C_r'^s$) of the subprograms and also for determining when the overall solution is obtained.

Once we have a basic feasible solution to the master, we can obtain the vector of dual variables or prices, which we shall denote as π (or π^s during the sth major iteration. Note that the vector π is equivalent to the vector W in the preceding section.)

15/Computational Note: As will be shown later, solutions to the subprograms are obtained using different objective function values ($C_r'^s$) in each major iteration. However, the C_r' of $C_r'X_r$ used in optimizing the master program are always the objective function values of the original problem. This does not present a problem as long as the computer code used allows multiple objective function rows. One objective function row in the subproblem is revised after each major iteration. A second objective function row has the original values and is left unchanged. The solution to the subproblem is obtained using the revised prices in the first row, but the right-hand side value of the second objective function row is used in the master program. The value of the revised objective function is only used to determine when the optimum solution to the original problem is obtained and in the upper bound computations.

16/This corresponds to total corporate profit.

The vector π consists of one simplex multiplier or dual variable for each restriction of the master. For example, a master with m corporate restrictions and n subproblems would have

$$\Pi = \pi_1, \pi_2, \ldots \pi_i \ldots, \pi_m, \bar{\pi}_1, \bar{\pi}_2, \ldots, \bar{\pi}_r \ldots \bar{\pi}_i$$

where π_i is the dual variable for corporate restriction i, and $\bar{\pi}_r$ is the dual variable corresponding to the constraints on the weights of the rth subproblem,

$$\sum_{\text{all } q} v_{rq} \leq 1.$$

If the original objective function vector of the rth subproblem is C'_r, then at any stage, say after s - 1 major iterations, the revised unit prices (or c_{rj}) for that subproblem for major iteration s can be calculated from

$$c^s_{rj} = c_{rj} - (\pi_1 a_{1j} + \pi_2 a_{2j} + \ldots . + \pi_m a_{mj})$$

where the a_{ij}'s are the appropriate coefficients from the jth column in the original (undecomposed) problem. Note that there are m coefficients, a_{ij} in this expression or one for each corporate constraint. In practice, this expression frequently reduces to

$$c^s_{rj} = c_{rj} - \pi_i a_{ij}$$

or

$$c^s_{rj} = c_{rj} - \pi_i$$

if a_{ij} equals one for those products or resources that have only a single corporate constraint in the master.

Consequently, the revised unit prices to be used in the solution to the subproblem during major iteration s have been adjusted by subtracting the shadow price for that restriction on that good or resource in the master program. This is equivalent to applying a set of penalties or bonuses to the original unit prices.

If the shadow price or dual variable π_i is positive, then unit price for the corresponding good will be reduced or penalized as there is already sufficient production (or resource utilization) entering the solution to the master to fulfill the corporate constraint. If there is slack in restriction i then $\pi_i = 0$, and the original unit price or c_{rj} is used. If the master is formulated with equations rather than inequalities, the π_i are unrestricted in sign. If π_i is less than zero, the product is in short supply and subtracting a negative number is equivalent to adding a bonus to the original unit price.

Those products or resources which have more than one restriction in the master have a corresponding number of dual prices to be subtracted from the unit price in the original objective function.

The vectors of revised unit prices C'^s_r for $r = 1, \ldots R$ are then used as the objective functions to solve the R subproblems[17/] for major iteration s

$$\max Z^s_r = C'^s_r X_r \qquad r = 1 \ldots . R$$

The solutions X^s_r are now available for inclusion in the master program.

The test for optimality of the overall solution is applied at this time. It is in this test that the last R shadow prices π_r are employed.

17/In practice frequently less than R subproblems are solved in a major iteration to reduce the total computation time. This is discussed in Chapter VIII.

The solution value with the revised prices for each of the R sub-problems

$$\max Z_r^s = C_r'^s X_r ,$$

is compared with its respective $\overline{\pi}_r$ from the previous solution to the master. The current solution to the original problem is optimal if no subproblem solution using the revised prices is greater than its corresponding dual variable from the last solution to the master.[18/] That is, the current solution is optimal if for all r

$$Z_r^s \max = \overline{\pi}_r^{s-1}$$

If any Z_r^s max is greater than the corresponding $\overline{\pi}_r$, then the new subproblem solution[19/] can increase the solution value of the master. Z_r^s is a measure of the net profit contribution from the new solution to subproblem r while $\overline{\pi}_r$ is a measure of the profit contribution offered by the previous solutions from subproblem r. If Z_r exceeds $\overline{\pi}_r$, the new solution offers a higher contribution to profit than the optimally weighted average of the old solutions, as it can be shown that every previous subproblem solution X_r^{s-i}, $i > 0$ which has a weight greater than zero in the last solution to the master has a profit contribution[20/]

$$C_r'^s X_r^{s-i} = \overline{\pi}_r$$

thus if any Z_r^s max $> \overline{\pi}_r$, it will pay to introduce the solution vector X_r^s

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18/ Baumol, op. cit., p. 11.

19/ If two or more subproblems have solution values greater than their corresponding $\overline{\pi}_r$, all will not necessarily improve the master solution during the next major iteration.

20/ Baumol, op. cit., p. 14.

into the master program. If all $Z_r^s \max = \overline{\pi}_r$, no improvement can be made and the current solution is optimal for the original problem.

An upper bound on the solution to the master program (and therefore the undecomposed problem) can be obtained from the $\overline{\pi}_r$ and the solution values of the subproblems.[21] Let M be the solution value of the master program. Then the upper bound to the master is computed as

$$\max M \leq M^{s-1} + \sum_{\text{all } r} (Z_r^s - \overline{\pi}_r^{s-1})$$

that is, the upper bound is equal to the last solution value of the master plus the sum of the differences between the new subproblem solutions with revised prices and their corresponding dual prices from the last solution to the master. Note that the computation of the upper bound requires the solution of all of the subproblems during a major iteration. However, the solution of all the subproblems is usually not necessary to improve the solution value of the master.

After the overall optimum to the master program is obtained, an optimal set of weights v_{rq} is available to construct the optimal weighted solution to each subproblem r from its Q previous solutions. These optimally averaged solutions to the subproblems along with the solution to the master program furnish the solution to the original problem.

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[21] Dantzig, op. cit., p. 452.

In the tomato problem, the A portion of figure 4-2 has one restriction for every finished product of the form:

beginning inventory + production	=	minimum final inventory + deliveries during the season.

This portion of the program assures that the production targets for the season will be met (if possible). These restrictions were frequently changed to ranges to allow more flexibility.

The B portion of figure 4-2 has restrictions to assure that inventory and/or current production is sufficient to meet any shipments required in a given week. B_i contains a restriction for every product with a requirement of the form:

beginning inventory + production during the first i weeks	=	deliveries required during the first i weeks.

It would also be possible to write restrictions to keep production of given commodities out of the first weeks, or to cause production to occur in any given week or group of weeks. The A and B restrictions were combined to form the master program in the tomato problem.

The blocks W_1 through W_{10} represent the ten subprograms, one for each week of the tomato season. As discussed in Chapter VII, the subproblems contain the restrictions on the quantity of raw product available and the capacity restrictions in the plants for that week.

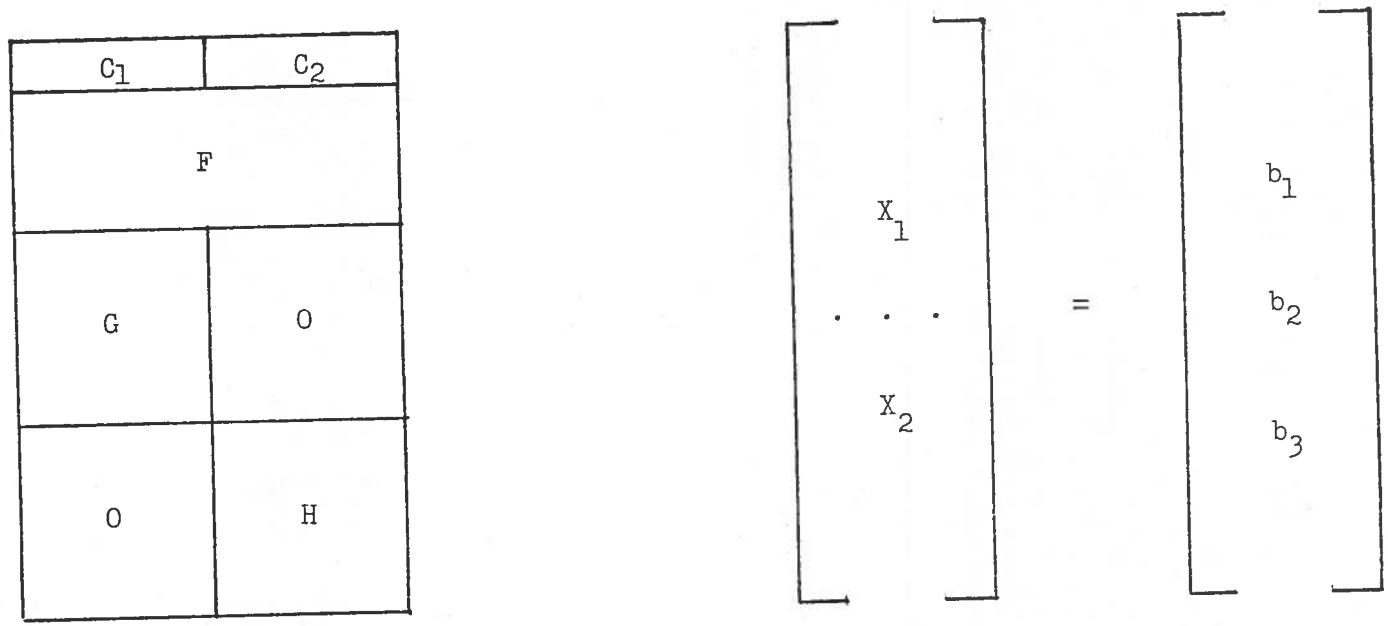
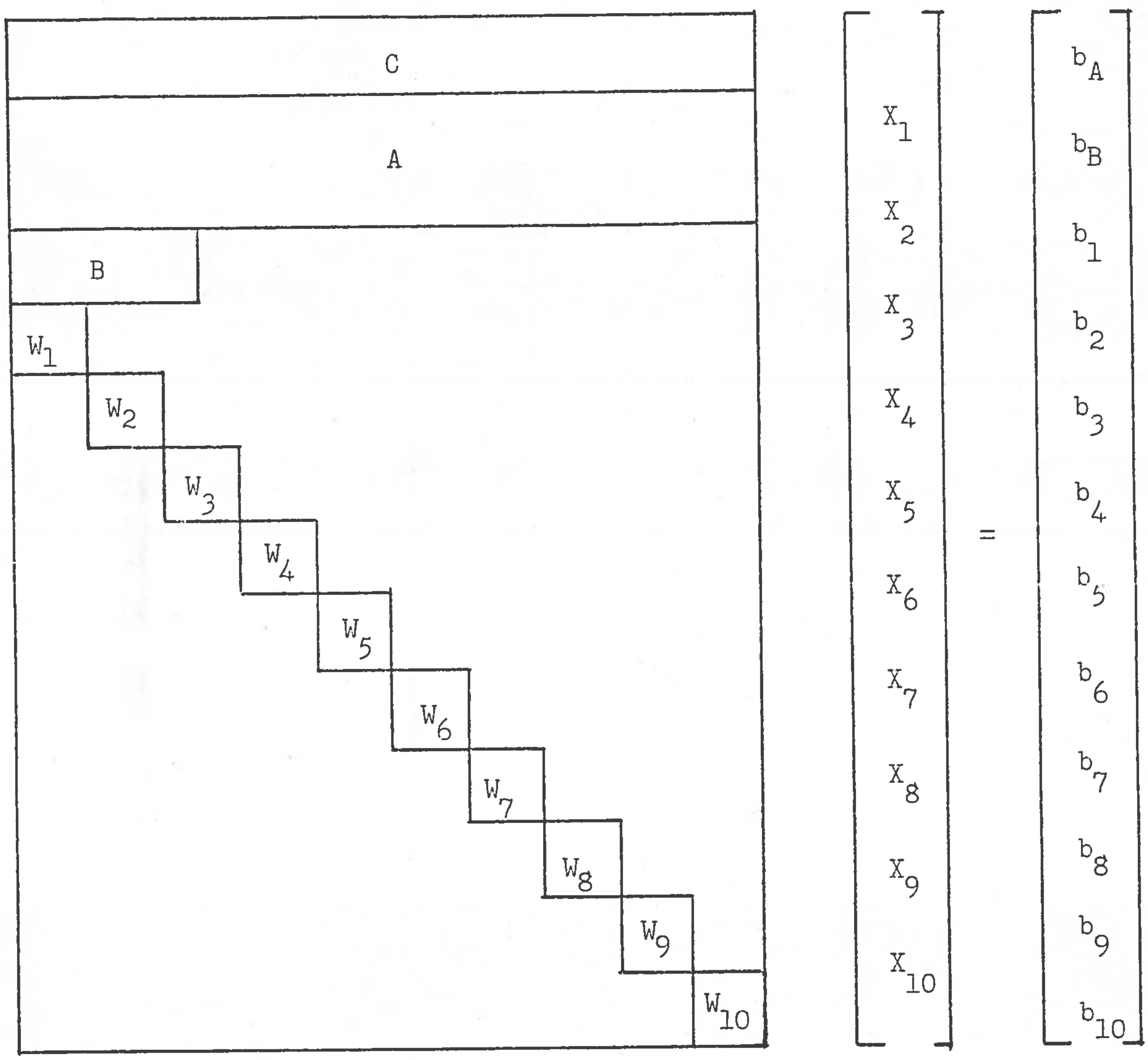


Figure 4-1

Schematic of Test Problem

Figure 4-2

Schematic of Full-Scale Tomato Problem



(lacking in numbering only)

APPENDIX TO CHAPTER IV

DECOMPOSITION OF THE TOMATO PROBLEM

The test problem to be discussed in Chapter V is decomposed into the form

$$A = \begin{bmatrix} F_1 & F_2 \\ G & 0 \\ 0 & H \end{bmatrix} \qquad X = \begin{bmatrix} X_1 \\ X_2 \end{bmatrix} \qquad b = \begin{bmatrix} b_1 \\ b_2 \\ b_3 \end{bmatrix}$$

$$C' = \begin{bmatrix} C_1' & : & C_2' \end{bmatrix}$$

which was discussed in the first section of this chapter. The test problem is represented schematically in figure 4-1. The matrices, G and H, represent the raw product and plant restrictions for each of two weeks, while F represents the output restrictions for the two week season. C_1 and C_2 represent the costs of inputs and product prices for the first and second week respectively.

Figure 4-2 represents the structure used in constructing the full-scale model of the tomato firm discussed in Chapter VII. The first, or firm level, consists of the A and B portion of the coefficient matrix. The restrictions on this level are developed from decisions or goals of the firm which occur over more than one time period but not necessarily over the entire season.

CHAPTER V

THE TEST PROBLEM

Purpose and Scope of the Test Problem

Before attempting to develop a full-scale model of a multiplant firm, a test problem involving only a single plant was formulated. This test problem was designed to test the feasibility of some of the internal operations which seemed desirable to include in the model and to evaluate the accuracy and/or adequacy of the coefficients which were being developed for the full-scale model. In addition, it provided valuable experience both in the application of the particular linear programming code used and in decomposition procedures.

The plant formulated for the test problem was given the capability of producing any grade of whole peeled tomatoes in #303, #2½, or #10 cans, as well as most common tomato products. In the development of the internal structure of the test plant, machinery limitations and capacity restrictions were inserted where such bottlenecks usually occur or where they could reasonably be expected to occur. In general, the sequence of operations was that described in Chapter II.

In the receiving and dumping operations, there were three such bottleneck possibilities, each caused by physical limitations of dumping capacities. It was assumed that there was a limitation on the physical quantity of tomatoes which could be dumped into the coreless[1] whole peel operation, that there was a physical limit on the amount of tomatoes which could be dumped into the round peeling operation and that there was a physical limit on the quantity of tomatoes (coreless or round varieties combined) which could be dumped mechanically into the product operations. The last was alleviated somewhat by allowing quantities greater than this physical limit to be dumped by hand at an increase in cost. There was no limit put on the quantity that could be dumped by hand when the model was tested, although provision was made to set an upper bound if it was deemed necessary or desirable. There were no limits placed on the capacity of the yard to receive and hold tomatoes.

Another group of machinery restrictions were those caused by the tomato peelers. The test plant had two types of peelers, a "Dole" peeler which could be used to peel coreless varieties without selection or to peel round varieties with or without selection, and an FMC peeler which was used only with round varieties of tomatoes selected before peeling. It was assumed that two Dole peelers and one FMC peeler were available for three 7-hour shifts, six days a week.

The next set of capacity or machinery restrictions which might impede the flow of tomatoes through the model plant were the can fillers. There were five possible fillers which could be used to can round tomatoes

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[1]/The coreless varieties are varieties of tomatoes which are suitable for mechanical harvesting. Due to their small core size, they are not cored in the peeling operation.

and three additional fillers which could be used on coreless varieties. This set of restrictions was fairly elaborate since each filler could handle two or more can sizes, and the fillers operated at different speeds. In addition, it was assumed that due to space limitations, only four of the fillers for round tomatoes could be used at any one time. Each filler could be used for three 7-hour shifts, six days a week.

The next set of capacity restrictions pertaining to the production of whole peel tomatoes were those caused by the syrupers. It was assumed that there was one syruper for each of the three can sizes, each of which could be run for the same period of time as the fillers.

The next set of restrictions were caused by the cooker capacities. There was one cooker each for #10 and #2½ cans, and two cookers for #303 can sizes. In addition, there was additional cooking capacity in the form of retorts. The retorts could cook any can size, although at additional cost.

It should be noted that these capacity restrictions were not likely to all be binding at once. However, as the product mix changes, the identity of the machines operating at capacity is also likely to change. For example, if there was not the possibility of using retort capacity, the #10 syruper capacity and #10 cooker capacity should not both be considered bottlenecks. Rather, only the one with the smallest capacity would be limiting since the cans of tomatoes pass through them in sequence. If the #10 syruper has greater capacity than the #10 cooker, it is impossible to tell if #10 cooking capacity is limited until after the entire whole peel product mix and total retort utilization is known.

There were no further limitations in the whole peel tomato capacities as it was assumed that there were no handling or warehouse limitations.

In the products portion of the plant, there were several machinery and capacity restrictions. These restrictions were generally based on a 24-hour day, six day week operating schedule. They included a capacity limitation on the tonnage of tomatoes which could be processed through the pulpers, a limitation on the quantity of tomatoes which could be returned to the products operation after having been peeled but found to be unacceptable for use in whole peel cans, and a restriction on the amount of evaporation which could be done by the plant's evaporators.

The last set of physical limitations was imposed by four product canning lines. One line was used only for #10 paste. Another line was devoted to other products in #10 cans. A third line was used for eight-ounce and #300 tomato sauce while the last line was capable of being used for either 46-ounce juice or eight-ounce and #300 sauce. These capacity restrictions are listed in table 5-5.

After the machinery configuration and capacities had been developed for the test plant, a LP model of the plant was developed. Although the configuration of the test plant as described would have permitted a full line of whole peeled tomatoes and an extensive line of products, the product line was limited to the items listed in table 5-1. The first column of table 5-1 gives the number of cans per case and the can size of the commodities. The second column has the commodity nomenclature. Columns three and four give the maximum and minimum production levels allowed in the test problem in 100 case units. The last column is the objective function value for the activities producing the commodity. This figure is in dollars per 100 case unit. It is obtained by adjusting the expected price by some of the direct costs of production. Although there are only 13 items listed in table 5-1, there were 21 production activities in the LP model. It was possible to can either size of sauce on two product lines, and each of the whole peel commodities could be made from either round or coreless tomatoes.

For test purposes, a group of tomato inputs with varying raw product characteristics was developed. There were ten lots of tomatoes representing round varieties and five lots representing coreless varieties. These 15 tomato inputs and the raw product characteristics ascribed to each are listed in table 5-2. The first column of table 5-2 lists the grower number assigned to each lot. The next five columns give the raw product characteristics in percentages. (The sixth characteristic, pulping loss, was not included in the test problem.) The last two columns give the percentages of tomatoes suitable for use as whole peeled fruit if they go through the selection process prior to being peeled. (There are no entries in the last two columns for the coreless tomatoes because preselection was not allowed as an alternative use for the coreless tomatoes.) The mean and range of each raw product characteristic is approximately equal to the expected mean and range for the season.

In the test problem, there were no price or transportation cost differences among the lots of tomatoes. All lots were assigned a price of $25 per ton and transportation costs of $5 per ton. The cost before processing was, therefore, $30 per ton. In addition, there was a selection cost of $3.50 per ton for those round tomatoes which were selected prior to peeling.

The Structure of the LP Test Model

The linear programming model of the test plant was composed of 94 rows, 92 restrictions and two objective functions. There were 345 columns of which 280 were included in the plant matrix and 65 were tomato input activities. The first 15 rows were devoted to input restrictions requiring that the total tomatoes used from each lot are less than or equal to the quantity available. This is one restriction for each of the 15 lots having the raw product characteristics listed in table 5-2.

The next 28 rows were related to capacity or machinery limitations. These restrictions are listed in table 5-5 along with the capacity levels which were assumed for the test plant.

There were eight requirements that the amount of finished product be equal to or greater than a given quantity. If a specific product could be made by only one activity, then the finished product requirement could be met by using the bounded variable option of the LP code and no row was necessary. However, as previously mentioned, some of the products could be made on more than one line (sauce) or from two different materials (round or coreless peeled tomatoes) and were represented by more than one activity.

The remainder of the rows or restrictions were transfer equations necessary to transfer the raw or intermediate product from one operation to another (or to dispose of waste). The general nature of the LP model is illustrated in table 5-7, which is an abbreviated version of the test problem. The transfer rows will be covered in more detail in both the appendix and in the explanation of the activities.

Activities

Each one of the ten groups of round tomatoes could be brought into the program in one or more of five input activities. The five activities were the FMC peeler (select) activity, the "Dole" peeler (select) activity, the Dole peeler (non-select activity), the paste, puree, or juice activity, and the sauce or catsup activity. Each of the five lots of coreless tomatoes could be brought into the program in one or more of three activities. These were the same activities as for round varieties except that the FMC peeler and the "Dole" (select) peeler activities were omitted. A total of 65 columns is devoted to input activities. The input activities for three growers are shown in table 5-7. A detailed explanation of the coefficients will be given in the appendix.

The next group of activities are those for blending lots of tomatoes which have different percentages of choice quality peelers. This group of activities allowed the model plant to blend lots of tomatoes in order to run at the most efficient level. The raw tomatoes were assigned to one of seven categories based on the percent of choice peelers in the input section. These categories ranged from 20 percent to 80 percent choice at ten percent intervals. Each of the blending activities in effect took tomatoes from two of the categories in appropriate quantities and deposited them in an intermediate category.

For example, 20 and 40 percent choice peelers could be blended into 30 percent choice peelers. Using the ten percent increments, there were 35 blending activities for each category of whole peel tomatoes, i.e., select rounds, non-select rounds and coreless varieties.

The following group of activities were used to transfer the choice peeled tomatoes out of the seven percentage categories into the row for either choice round or choice coreless tomatoes as appropriate. There are seven of these activities for each of the three classes of peeled tomatoes (select round, non-select rounds and coreless).

Another group of activities performs the function of blending tomato inputs of various solid levels to that level at which the plant will operate most efficiently. Tomatoes were divided into categories based on their solids content in levels differing by .2 percent from 4.7 percent to 6.3 percent. This group included 85 activities which made it possible to blend lots of tomatoes of any two solids levels to obtain any intermediate solids level (on .2 percent intervals).

Further activities were used to transfer tomatoes from the solid level rows into the juice row or the concentrated pulp rows. The evaporation process occurred at this point in the LP model. There were sets of nine activities (one for each solids row) to transfer pulp to juice, to evaporate pulp to 10.8 percent solids pulp (for 1.045 puree) to evaporate pulp to 14.2 percent solids (for 1.06 puree) and to evaporate pulp to 25.5 percent pulp (for paste).

There are three remaining groups of activities. The first of these consists of activities to transfer production capacity between rows. For example, some of the whole peel fillers could be used to fill more than one can size. A transfer activity was used here to give the LP model the same flexibility as the test plant.

Another type of transfer activity was used to allow higher quality raw product to be substituted for a lower quality if this would be the most economical course of action. For example, this allowed choice tomatoes to be used in place of standard tomatoes. These activities also allowed raw material to be disposed of as waste if this would be the most economical course of action.

The final group of activities consists of the 21 production activities, one for each of the products listed in table 5-4, and the waste disposal activity. By using a waste disposal activity, the cost of waste disposal can be included in the objective function. If the cost of waste disposal shifted upward at various levels, it might be desirable to use several such activities with appropriate prices and upper bounds. On the other hand, if the cost of waste disposal was fixed or unimportant, an explicit activity might not be necessary.

Solutions

After the construction of the LP model based on the test plant and the development of the raw product characteristics and the finished product requirements listed previously in this chapter, the model was tested using four different raw product input levels. These input levels were designed to range from definite under-utilization of the plant capacity to providing a definite surplus of raw material. The quantities of tomatoes available from each of the 15 growers were kept equal at each input level. The solutions from two of the levels are shown in table 5-3 through 5-6.

Table 5-3 shows the utilization of each lot of tomatoes for each of the two weeks. In addition, the shadow prices and the reduced costs are listed as well as the values of the alternative ways to utilize that lot of tomatoes. There are several significant differences in the allocation of the lots of tomatoes, even though the only difference in initial conditions was that the input quantity for the second test was greater. Also, note that the shadow prices are lower in all lots in the second week.

Table 5-4 gives the levels of each of the 21 production activities during the two weeks and the value of one additional case of product. There are several commodities which have been forced into the solution at their lower bound in both solutions and in the higher level week, production of extra standard catsup is at its upper bound. There are some changes in the production pattern for which the reason is not obvious. For example, choice coreless tomatoes in #10 cans are produced at the lower level of raw product but not at the higher.

Table 5-5 shows the machinery and capacity restrictions. The columns on the right show the value of one additional period of time to the plant as obtained from cost ranging. The values for additional capacity do not increase uniformly as the input level increases. In fact, some remain the same and some even decrease as input increased.

Table 5-6 lists the other activities which were included in the optimal solution for the two levels of inputs. The first activities in the table are the transfer of peeled tomatoes from each of the input rows. (There were no blending activities of tomatoes for peeling in the solutions to the test problem.) Note that there are fewer levels of peelers used in the higher week as the quantity of tomatoes in each lot increased.

The function of the next set of activities in the table is to blend tomato inputs of different solids levels. There were no changes in the blending activities in the optimal basis between the two levels although the quantities differ. The total quantity of pulp which was blended in the test problem is insignificant relative to the total quantity processed. The next group of activities transfers tomato pulp to the juice row. These activities are relatively low in tomato solids. In the 520 ton week, only the two lowest solids levels are used for juice. This is reasonable since the higher solids tomatoes will yield more paste. (There were no 4.9 percent solids tomatoes in the test problem.) Immediately following are the evaporation activities. The only change of interest between the two weeks is the drop in the level of solids used for tomato paste from over 5.7 percent to less than 5.3 percent. Although not obvious from casual inspection, part of the shift was the result of an increase in the catsup and sauce production which utilized the higher solids tomato pulp. Also, the puree and paste were all forced into the solution at their lower bound in the 260 ton test. The production of these commodities during the 260 ton week was at a higher level than if the solution were unbounded and required a major part of all solids not used in the whole peel operation.

The last group includes the transfer activities which augment machine capacities. In the 520 ton test, it was necessary to dump tomatoes by hand for the products operation.

In both weeks, the capacity of the two straight line can fillers was completely utilized to augment the #2½ and #303 can fillers. In addition, the retort was used in both weeks to cook #2½ cans. The last activity is the waste disposal activity which gives the quantity of waste in 1,000 pound units.

APPENDIX TO CHAPTER V

EXPLANATION OF THE LINEAR PROGRAMMING MATRIX FOR THE TEST PROBLEM

Table 5-7 is a portion of the linear programming matrix used in the test problem. The rows and columns have been selected so that all of the types of activities and restrictions used in the test problem are included. The table in conjunction with this appendix should illustrate the method of construction and the structure of the test program. Some sections of the test problem have been omitted if they are similar to those included. For example, there are no coreless tomato activities included in the sample matrix since these are quite similar to the round tomato activities. The first row of the table contains the column number which is a number assigned to the column for identification throughout this appendix. The next row contains the column name as used in the program. These usually have some consistent characters throughout a group of activities. For example, all of the tomato input activities begin with the letter G (for grower) followed by the two-digit grower number 01, 02, etc. The last digit indicates the type of input activity. For example, FMC whole peel is one, Dole select whole peel is two, nonselect whole peel is three, paste is four, and sauce is five.

The next two rows are the objective function rows. All of the tomato plant problems were handled as profit or revenue maximization problems. Consequently, these activities which caused net costs have negative values in the objective function rows and those representing revenue-producing activities are positive. (The second objective function is used for computational convenience in the decomposition procedure.) The entries in the left-hand column are the row numbers assigned for describing the matrix in this appendix. The next column has the row names used in the test problem. Whenever possible, they are mnemonic. The rest of the columns except number 64 are the matrix entries. Column 64 contains the right-hand side values. If there is no entry in the table, the value of that matrix element is zero.

In this abbreviated version of the matrix, the first five columns represent the five different ways in which the first lot of round tomatoes could be used in the plant. Each unit of the activity is equivalent to one ton of tomatoes. The first column represents tomatoes which are peeled in the FMC peeler. The objective function value represents the cost of the tomatoes plus the cost of selecting for the FMC peeler in hundreds of dollars. Note that the cost is greater for this activity than for the next activity where there is no expense of selection. Other handling expenses could be added if desired. In the full-scale model, transportation costs from the farm to the plant and yard handling costs were added to the objective function cost.

The next entry in the first column is in row three, R01110, which limits the amount of tomatoes which can be obtained from the grower one. The entries in this row are necessary to insure that the total tons of tomatoes input through the five activities for each grower are less than or equal to the tonnage available from the grower.

The next entry is in row six. It represents the quantity of choice tomatoes (in 1,000 pound units) that can be obtained after selection and peeling in the FMC peeler, a quantity of 204 pounds in this instance. The entry is made in this row because "grower one" had 20 percent choice peelers. 30 percent peelers would be entered in RSEL30.

The entry in row 18 represents the amount of whole peel box dumping capacity used by one ton of tomatoes. This particular row insures that the dumping capacity required will not exceed the capacity available in the plant. Row 22 represents the amount of FMC peeler capacity used by one ton of tomatoes. The numbers .007874 and .024316 represent the fractions of a three and one-half hour time period (one-half shift) required to process one ton of tomatoes on that equipment. The FMC peeler, therefore, takes substantially more time per ton than the box dumper. However, there are other activities, the "Dole" peeling activities, which utilize the box dumper.

The entry in row 23 represents the amount of pulper capacity that is utilized by one ton of tomatoes in this input process. This is the total amount of pulper capacity required and includes both that used to pulp tomatoes not selected for peeling and for those that are peeled but returned to the products operation.

The entry in row 15, RSOL53, gives the number of 1,000 pound units of tomato solids obtained when the tomatoes are processed through the FMC select activity. In this case, the entry .068 in the RSOL53 row means that there are .068 thousand pound units of tomato solids or 68 pounds of solids available for use in the juice or products operation. This is the amount of solids which can be obtained from the tomatoes which are either not selected for peeling or returned from the canning lines to the product operation. This entry was in RSOL53, the 5.3 percent solids row, because this lot of tomatoes had 5.3 percent solids.

The row 25 entry represents the amount of tomato waste from one ton of tomatoes used in this activity. This figure is .216 thousand pound units or 216 pounds. This represents both peeling loss from tomatoes going through the peeler, and pulping and finishing loss from tomatoes used in the products operation. The last entry in this column, row 29, represents the 306 pounds of standard peeled tomatoes obtained from a ton of this lot of tomatoes processed through the FMC peeler activity.

In short, this column of coefficients tells us that from one ton of tomatoes with these characteristics processed through the FMC peeler, we will obtain:

204 pounds of choice peeled fruit
206 pounds of standard peeled fruit
1,274 pounds of tomatoes for juice or paste
(68 pounds of solids is equal to 5.3 percent of 1,274 pounds of juice)
216 pounds of waste

These figures are based on the assumption that 40 percent of the tomatoes (or 800 pounds) are selected for peeling with the percent of choice and standard after selection as given for grower one in table 5-2. Of the 800 pounds of tomatoes peeled, 240 pounds are choice, 360 are standard, and 200 pounds can not be used as whole peeled fruit. After a peeling loss of 15 percent (also obtained from table 5-2), 204 pounds of choice fruit and 306 pounds of standard fruit remain. The 200 pounds of fruit not suited for canning yield 170 pounds of fruit which is sent to the products operation.

There are 1,370 pounds of tomatoes which go to the products operation, 1,200 of which were not selected for peeling and 170 pounds from the peelers. Assuming a seven percent pulping loss, the pulping loss is 96 pounds. (Seven percent was used for all lots of tomatoes in the test problem, but a ten percent average pulping loss was used in the full-scale model.) The 1,274 pounds of 5.3 percent solid tomatoes yield 68 pounds of tomato solids.

Column two represents the activity of peeling all the tomatoes without selection in the "Dole" peeler. One difference between activity one and activity two is that the objective function cost is less in column two. The smaller cost is due to the fact that there is no cost due to selecting tomatoes. The meaning of the entry in the third row is the same, since this activity also used tomatoes from the first grower.

The next entry is in row 10, the RNON20 row. This row is different from RSEL20 which had the comparable entry for the first activity even though both contain choice tomatoes from 20 percent choice stock. This is to maintain the identity of the assigned peeler while blending. Blending must occur before peeling in the actual plant operation, but in order to reduce the number of restrictions in the LP model, the peeling operation appears here before blending. The .34 represents 340 pounds of choice peeled tomatoes (the 400 pounds available in one tone less a 15 percent peeling loss.

The machinery restrictions are essentially the same as in the first activity. These tomatoes are peeled in the "Dole" peeler, rather than the FMC, and the coefficient is entered in row 21 which is labeled RDOLPL. Since fewer tomatoes go to the products operation, the púlping requirement in row 23 is less than for the first activity.

The .033 in RSOL53 (row 24) indicates that 33 pounds of tomato solids are obtained from a ton of tomatoes in this activity. This quantity of solids is computed by reducing the 800 pounds of non-peeling stock by 15 percent to correct for peeling loss, which means that 680 pounds of tomatoes are sent to the pulper. This quantity is reduced by the seven percent pulping loss to give 632 pounds of tomatoes. At 5.3 percent solids, solids available is 33 pounds. The last two entries in the second activity indicate that there are 348 pounds of waste to be disposed of and 680 pounds of standard peeled tomatoes available for canning (800 pounds of standard tomatoes in one ton less 120 pounds peeling loss). The waste is greater than in the first activity since all the tomatoes (rather than selected ones) are peeled with a 15 percent peeling loss.

Column three, the Dole select activity, is identical to column one except that the peeler capacity entry is in the "Dole" peeler row rather than the FMC peeler row. In this case, it was assumed that peeling loss, selection costs, and recovery rates for the two peelers were equal. If they are not, the appropriate coefficients would be different.

Column four is the paste, puree and juice activity for grower one. Since the tomatoes go directly to products and there is no selection cost, the objective function is 0.3 or a cost of $30.00 a ton. Tomatoes entering the plant and going directly to products are dumped by box dumper two until the capacity constraint is reached. This box dumping requirement is represented by the entry in row 19, RBOX02. The solids entry in this column is .099, indicating that 99 pounds of tomato solids will be obtained from one ton of lot one. This is 5.3 percent of 1860 pounds (2,000 pounds reduced by the seven percent pulping loss). The waste is 140 pounds as indicated by .14 entered in row 25. The only other entry in the column represents the pulper capacity utilized by one ton of tomatoes.

The last activity for the first lot of tomatoes is the sauce and catsup activity. It has the same entries as the preceding activity except that there is an entry in row 28, the RSAUCE raw, rather than in the solids row, and there is an entry in the REVAPR (evaporator) row. This lot of tomatoes had only eight percent insoluble solids as a percent of total tomato solids. Because of this, more total solids are needed to give the same consistency in sauce and catsup as tomatoes with a higher percentage of insoluble solids. For eight percent insoluble solids, a case yield factor of 80 percent is used, i.e., only 80 percent as much product can be obtained from this lot of tomatoes as a similar lot with ten percent insoluble solids. The equivalent yield in terms of ten percent insoluble solids can be obtained by multiplying the available solids by .8. This gives the entry of .079 in the RSAUCE row (.8 x .099). There is an evaporator capacity requirement for sauce production entered in row 22. In the full-scale model, the evaporator utilization for sauce was entered in transfer columns in the same manner as the evaporator utilization for paste. (See the explanation for columns 40-43 in this appendix.)

The next five columns are for another lot of round tomatoes. Many entries are the same as those for the first lot. In the test problem, the objective function costs are the same for all lots. In the full-scale model, the costs varied by differences in transportation costs. The costs entered here might vary if some lots required special handling. The next entries are in the grower row which, of course, is different. The following entries are in the choice rows. The second lot of tomatoes has 30 percent peelers so, for example, the entry for FMC select (column six) is in row seven, RSEL 30 rather than RSEL 20 as in the first lot. The peeling loss is 20 percent, so there are 256 pounds of choice peeled tomatoes and an entry of .256 in the first column. As before, 40 percent of the tomatoes are selected. Of these, 40 percent are choice (320 pounds), and 20 percent pooling loss amounts to 60 pounds, leaving 260 pounds of choice tomatoes.

Many of the machinery coefficients are the same. These are generally the coefficients determined by the physical handling of one ton of tomatoes. Any differences in machinery coefficients between the first and second lot are caused by a different proportion of the tomatoes utilizing that particular machine.

The insoluble solids for this lot of tomatoes is ten percent or average. In column ten the sauce entry of .095 in row 26 is, therefore, the same as the solids entry in column nine.

The set of columns 11-15 is quite similar to the first two sets. One point of interest is that the insoluble solids are 12 percent of the total solids. In this case, the sauce coefficient is greater than the entry in the solids row because a higher case yield is possible. There are only three sets of grower inputs included in the sample matrix, all of which were round varieties. There were 15 sets of inputs in the test problem, one for each of the sets of raw product characteristics listed in table 5-2. These included five lots of coreless varieties.

The next group of activities, columns 16 through 23, are the whole peel blending activities. Column 16, BSE 243, for example, is the activity to blend raw tomatoes with 20 percent choice peelers and raw tomatoes with 40 percent choice peelers to obtain raw tomatoes with 30 percent choice peelers. In the actual operation, blending would occur as the tomatoes were being physically dumped, but in the linear programming model it occurs after peeling. This was done in order to reduce the size of the matrix. Peeling loss and percent of standards for each lot are entered into the matrix via the input activities. If the blending was done in the dumping activities, each lot would have to be blended with many other lots with a large increase in the number of activities required. Some errors enter the program using this more compact procedure because of lots having greater peeling losses or a different percent of standard peelers than the average, but the errors should be relatively small. In the full-scale model, correction rows were used to reduce the possible errors. In this example, the columns with labels starting BSE are the blending activities for the selected tomatoes, and those starting with BNS are the blending activities for the nonselected tomatoes. In the test problem, there was also a similar set of activities for the coreless tomatoes. All of the blending activities for the four levels of choice tomatoes included in the example are illustrated. In the test problem there were seven levels of choice tomatoes and, consequently, more possible blends.

The interpretation of the coefficients in column 16 (BSE 243) is that .3333 units of choice tomatoes (333.3 pounds) are taken from the 20 percent choice row and .6667 units of 40 percent choice tomatoes are taken from the 40 percent choice row. The total of one unit, or 1,000 pounds of choice tomatoes, is then put in the 30 percent choice row. The 333 pounds of choice tomatoes in the 20 percent row actually requires a total input of 1,667 pounds of raw tomatoes via the grower input activities. The 667 pounds of choice tomatoes in the 40 percent choice row also represent a total input of 1,667 pounds of raw tomatoes before selection. The net result of blending is 3,333 pounds of raw tomatoes with 30 percent choice peelers before selection. These 3,333 pounds of raw tomatoes contain one unit or 1,000 pounds of choice tomatoes.

The next group of columns, 24 through 31, are activities which transfer the choice peeled tomatoes from the rows in which they have been entered or blended into one row for all choice round tomatoes. This row is an equality row with the RHS set equal to zero. Every pound of tomatoes which is entered here must be taken out by one of the whole peel canning activities or be downgraded to standard. The "-1" in the particular row of choice peeled tomatoes indicates that 1,000 pounds are being withdrawn from that row; the "1's" in row 30, the choice tomato row, indicate that 1,000 pounds of choice tomatoes are being placed in the choice round row. Note that there are two sets of transfer activities, one for the selected peeler rows and one for the non-selected peeler rows. In the test problem there was another set of transfer activities for the coreless tomatoes. (There was also a separate choice row for the coreless tomatoes.) The group of columns, 32 through 35, are activities for blending lots of tomatoes which are to be utilized in products. Column 32, the 51553 column blends tomatoes with 5.1 percent solids with 5.5 percent solids and yields 5.3 percent solids. The coefficients in the matrix are in units of 1,000 pounds of tomato solids. The columns 36-39 are juice transfer activities. Here, the effect is to convert tomato solids back into their juice equivalent. There is, of course, no such activity in a tomato plant. However, by handling juice in this way, the total number of columns in the LP matrix is considerably reduced. In column 36, the -.051 in row 14 means that the total amount of solids for processing is reduced by 51 pounds and the 1 in row 27, the juice row, stands for 1,000 pounds of juice. Columns 40 - 43 are evaporation activities. In addition to the evaporation activity, the tomato solids are transferred to the solids row for 1.045 puree. (The 1.045 puree contains 14.2 percent tomato solids.) The physical interpretation of this set of activities is that 142 pounds (.142 units) of tomato solids are removed from a solids row in juice form. An appropriate amount of water is evaporated, and 1,000 pounds of 14.2 percent tomato puree is put into row 28, the 14.2 percent solids row. The amount of evaporator capacity utilized in the concentration is represented by the entry in the evaporator row. The entry in the objective function row represents the cost of evaporation in this model. In the test model there were also sets of evaporation activities for 1.06 puree and for paste.

Column 44 provides for the use of choice tomatoes in place of standard tomatoes when the supply of fruit and the desired product mix is such that it is economical to do so.

Column 45 provides for the pulping of peeled standard tomatoes into juice if juice is a more economical use than whole peel commodities. To accomplish this, one thousand pounds of standard tomatoes are removed from row 29, and 930 pounds (.93) are entered in the juice row. This is 1,000 pounds less a seven percent pulping loss. There are also entries required in the pulper capacity and waste rows.

Column 46 is the waste disposal activity. Since the corresponding row is an equality, the quantity of waste that entered in the input and processing activities must be equal to the activity level of this column. A cost of $7.50 a thousand is entered in the objective function rows. In the case of multiple plants, differing waste disposal costs could be entered. By using the bounded variable technique, the total quantity of waste can be restricted to an upper limit. Also, if the cost of waste disposal increases and the quantity increases, this can be handled by using two or more disposal activities with upper bounds to give an exact or approximate answer depending on the nature of the cost function.

Column 47 allows the dumping of product tomatoes by hand if the mechanical box dumper is utilized to capacity. There is a negative value in the objective function because hand dumping is an added operating expense. Because of the cost, the activity will enter the solution only after the mechanical dumper capacity is completely utilized. The negative coefficient in row 19, the mechanical box dumper row, and the positive coefficient in row 20, the hand dumping row, are physically analagous to the hand dumping of fruit. In this example, there is a RHS value for the hand dumping row to limit the total amount of fruit which could be hand dumped.

The next three columns, 48-50, are activities to utilize 'straight line' can fillers after the capacity of the more efficient 'hand pack' fillers are utilized. There is one hand pack filler for each of the two whole peel can sizes. There are also two straight line fillers, but one could be used only for #303 cans while the other could be used for both #303's and #2½ cans. These transfer activities are constructed similarly to the one for the hand dumping activity. Row 31, RCANL, the straight line filler which can be used for either #303 or #2½ cans, has to have two entries to allow both additional #2½ and #303 can filling, and cannot be removed in this manner. The negative objective function values are the differences in the costs of running the straight line fillers and the hand pack fillers for 100 cases of product.

Activities 51 and 52 allow the use of retorts when the cookers are operating at capacity. These activities are quite similar in construction to the two preceding groups.

The remaining activities are production activities. These are the only revenue producing activities in the matrix. Column 53, headed CHR 303, is the production activity for choice, round tomatoes in #303 cans. The unit here is 100 cases and the objective function is scaled by $100. The prices used here are approximate wholesale prices in 1964 and are not corrected for direct supplies and labor. In the full-scale model both of these factors were subtracted from the price. The -.525

in the juice row indicates that 525 pounds of juice are used in 100 cases of choice 24/303. The next entry, .22857 in RCANL 4, indicates that packing 100 cases takes 22.857 percent of a three and one-half hour time period on filler line four. The last two entries indicate that 100 cases of #303's takes only .06079 of a three and one-half hour period on the #303 syruper and only .082237 of a three and one-half hour period on the #303 cooker. Since the same number of units (36) is the restriction for each piece of machinery, the last two rows would have been unnecessary for they would never be restrictive if not for the ability to utilize the hand fillers and the retorts. However, if these possibilities did not exist, these machinery restrictions would not be left out if it was desired to investigate the effects of changing the right-hand side values, i.e., investigate the possibility of increasing machine capacity or adding more equipment.

The remainder of the production activities are constructed and interpreted similarly. However, there are two separate activities, numbers 60 and 62, which can produce eight-ounce sauce and two activities, numbers 61 and 63, which can produce #303 sauce as these products can be produced on two different product lines. Rows 43 and 44 are inequalities which insure that the combined production of one of these products on both lines meets the desired restrictions. In the test model, which included the coreless tomatoes, there were several other rows of this nature since whole peeled tomatoes could be canned from either coreless or rounds, each of which required a canning activity. In the full-scale model some of the whole peel commodities could be made by three activities. This type of row restriction was not used in the full-scale model because of the size limitations on the matrix. The activity levels were summed before being inserted in the master program.

Table 5-1. Production Limits and Objective Function Values for Test Problems

Case Size	Commodity	Maximum Level (100 Cases)	Minimum Level (100 Cases)	Objective Function Value[1]
24/303	Choice Whole Peeled[2]	600	200	$115
24/303	Standard Whole Peeled[2]	250	0	105
24/2½	Choice Whole Peeled[2]	200	0	195
24/2½	Standard Whole Peeled[2]	112	0	170
6/10	Choice Whole Peeled[2]	330	100	160
6/10	Standard Whole Peeled[2]	120	0	145
6/10	25% Paste	500	100	430
6/10	1.06 Puree	250	20	250
6/10	1.045 Puree	75	30	190
6/10	X Standard Catsup	250	10	265
48/8 oz.	Sauce[3]	700	0	85
12/46	Juice	450	50	120
24/303	Sauce[3]	500	0	80

1/Expected price per 100 cases adjusted for direct costs of production.

2/Can be either round or coreless tomatoes.

3/Can be canned on either of two lines.

Table 5-2. Raw Product Characteristics for Test Problem

Grower Number	Percent Choice	Percent Standard	Percent Peeling Loss	Percent Total Solids	Insoluble Solids as % of Total Solids	Percent Choice After Selection	Percent Standard After Selection
Rounds							
1	20	40	15	5.3	8	30	45
2	20	40	20	5.7	11	30	45
3	30	40	15	5.5	9	40	45
4	30	30	20	5.1	10	40	35
5	30	40	25	4.7	12	40	45
6	40	40	15	5.0	11	50	40
7	40	20	20	5.3	8	50	30
8	50	30	15	5.7	12	55	35
9	60	30	20	6.0	9	65	30
1070	70	20	15	5.3	10	75	20
Average Range	20-70	20-40	15-25	4.7-6.0	8-12	30-75	20-45
Coreless							
11	20	50	15	5.0	9		
12	30	60	20	4.5	11		
13	40	30	20	6.0	10		
14	60	20	15	5.0	8		
15	70	10	20	6.2	12		
Average Range	20-70	10-60	15-20	4.5-6.2	8-12		

Table 5-3. Comparison of Raw Product Utilization at 260 Tons per Week and 520 Tons per Week

Grower			(1)	(2)	(3)	(4)	(5)
			Basic Cost per Ton				
			$33.50	$30.00	$33.50	$30.00	$30.00
			FMC	Dole Non-select	Dole Select	Paste, puree and juice	Sauce and catsup
<u>Grower #1</u>							
20 % Choice	260 ton/wk	Quantity used	260T				
15 % Peeling loss		Shadow price[1]	23.20				
5.3% Total solids		Use[2]	Juice				
8 % Insoluble solids		Reduced cost[3]		5.90	4.70	1.70	17.70
		Alternative value[4]		17.30	18.50	21.50	5.50
	520 ton/wk	Quantity used				520T	
		Shadow price				5.11	
		Use				Paste	
		Reduced cost	.10	11.20	4.10		9.80
		Alternative value	5.01	-6.90	1.01		-4.69
<u>Grower #2</u>							
20 % Choice	260 ton/wk	Quantity used				260T	
20 % Peeling loss		Shadow price				25.50	
5.7% Total solids		Use				Paste	
11 % Insoluble solids		Reduced cost	1.60	9.50	6.30		2.30
		Alternative value	23.90	16.00	19.20		23.20
	520 ton/wk	Quantity used					520T
		Shadow price					8.47
		Use					sauce & catsup
		Reduced cost	3.40	16.10	7.40	.30	
		Alternative value	5.07	-7.63	1.07	8.17	

(continued)

Table 5-3. (continued)

			(1)	(2)	(3)	(4)	(5)
Grower #3							
30 % Choice	260 ton/wk	Quantity used	260T				
15 % Peeling loss		Shadow price	26.00				
5.5% Total solids		Use	Paste				
9 % Insoluble solids		Reduced cost		4.30	4.70	2.70	14.40
		Alternative value		21.70	21.30	23.30	11.60
	520 ton/wk	Quantity used	520T				
		Shadow price	8.11				
		Use	puree				
		Reduced cost		8.30	4.00	1.70	8.30
		Alternative value		-.19	4.11	6.41	-.29
Grower #4							
30 % Choice	260 ton/wk	Quantity used	260T				
20 % Peeling loss		Shadow price	21.70				
5.1% Total solids		Use	Juice				
10% % Insoluble solids		Reduced cost		7.90	4.70	.30	8.70
		Alternative value		13.80	17.00	21.40	13.00
	520 ton/wk	Quantity used				520T	
		Shadow price				3.46	
		Use				Paste	
		Reduced cost	1.20	13.70	5.20		2.60
		Alternative value	2.26	-10.24	-1.74		.86
Grower #5							
30 % Choice	260 ton/wk	Quantity used				260T	
25 % Peeling loss		Shadow price				21.10	
4.7% Total solids		Use				Juice	
12 % Insoluble solids		Reduced cost	.80	8.90	5.50		3.90
		Alternative value	20.30	12.20	15.60		17.20

(continued)

Table 5-3. (continued)

			(1)	(2)	(3)	(4)	(5)
	520 ton/wk	Quantity used					520T
		Shadow price					3.97
		Use					Sauce % catsup
		Reduced cost	2.20	13.90	6.20	.70	
		Alternative value	1.77	-9.93	-2.23	2.27	
<u>Grower #6</u>							
40 % Choice	260 ton/wk	Quantity used	24T	236T			
15 % Peeling loss		Shadow price	24.60	24.60			
5.0% Total solids		Use	Juice	Juice			
11 % Insoluble solids		Reduced cost			4.70	4.40	8.40
		Alternative value			19.90	20.20	16.20
	520 ton/wk	Quantity used	520T				
		Shadow price	6.70				
		Use	Juice				
		Reduced cost		2.30	4.00	4.00	3.40
		Alternative value		4.40	2.70	2.70	3.30
<u>Grower #7</u>							
40 % Choice	260 ton/wk	Quantity used	260T				
20 % Peeling loss		Shadow price	22.00				
5.3% Total solids		Use	Juice				
8 % Insoluble solids		Reduced cost		6.90	4.70	.50	16.40
		Alternative value		15.10	17.30	21.50	5.60
	520 ton/wk	Quantity used				520T	
		Shadow price				5.11	
		Use				Paste	
		Reduced cost	1.00	13.90	5.00		8.80
		Alternative value	4.11	-8.79	.11		-3.69

(continued)

Table 5-3. (continued)

			(1)	(2)	(3)	(4)	(5)
Grower #8							
50 % Choice	260 ton/wk	Quantity used	260T				
15 % Peeling loss		Shadow price	28.00				
5.7% Total solids		Use	Paste				
12 % Insoluble solids		Reduced cost		1.70	3.00	2.50	.20
		Alternative value		26.30	25.00	25.50	27.80
	520 ton/wk	Quantity used					520T
		Shadow price					11.93
		Use					Sauce & catsup
		Reduced cost	1.70	6.00	.90	3.80	
		Alternative value	10.23	5.93	11.03	8.13	
Grower #9							
60 % Choice	260 ton/wk	Quantity used	108T			152T	
20 % Peeling loss		Shadow price	29.00			29.00	
6.0% Total solids		Use	Puree			Puree	
9 % Insoluble solids		Reduced cost		4.30	4.70		13.20
		Alternative value		24.70	24.30		15.80
	520 ton/wk	Quantity used	440T			80T	
		Shadow price	10.93			10.93	
		Use	Puree			Puree	
		Reduced cost		5.00	4.00		8.00
		Alternative value		5.93	6.93		2.93
Grower #10							
70 % Choice	260 ton/wk	Quantity used		260T			
15 % Peeling loss		Shadow price		29.70			
5.3% Total solids		Use		Juice			
10 % Insoluble solids		Reduced cost	3.00		7.70	8.20	14.90
		Alternative value		26.70	22.00	21.50	14.80

(continued)

Table 5-3. (continued)

			(1)	(2)	(3)	(4)	(5)
	520 ton/wk	Quantity used		520T			
		Shadow price		11.13			
		Use		Paste			
		Reduced cost	1.40		5.40	6.00	8.90
		Alternative value	9.73		5.73	5.13	8.23
<u>CORELESS VARIETIES</u>							
<u>Grower #11</u>							
20 % Choice	260 ton/wk	Quantity used				260T	
15 % Peeling loss		Shadow price				19.40	
5.0% Total solids		Use				Juice	
9 % Insoluble solids		Reduced cost		1.70			10.60
		Alternative value		17.70			8.80
	520 ton/wk	Quantity used				520T	
		Shadow price				3.60	
		Use				Juice	
		Reduced cost		8.10			5.90
		Alternative value		-4.50			-2.30
<u>Grower #12</u>							
30 % Choice	260 ton/wk	Quantity used				260T	
20 % Peeling loss		Shadow price				19.30	
4.5% Total solids		Use				Juice	
11 % Insoluble solids		Reduced cost		.60			7.70
		Alternative value		18.70			11.60
	520 ton/wk	Quantity used				520T	
		Shadow price				2.02	
		Use				Juice	
		Reduced cost		1.90			2.20
		Alternative value		.12			-.18

(continued)

Table 5-3. (continued)

			(1)	(2)	(3)	(4)	(5)
<u>Grower #13</u>							
40 % Choice	260 ton/wk	Quantity used				260T	
20 % Peeling loss		Shadow price				29.00	
6 % Total solids		Use				Puree	
10 % Insoluble solids		Reduced cost		12.00			8.10
		Alternative value		17.00			20.90
	520 ton/wk	Quantity used				520T	
		Shadow price				10.93	
		Use				Puree	
		Reduced cost		16.60			4.20
		Alternative value		-5.67			6.73
<u>Grower #14</u>							
60 % Choice	260 ton/wk	Quantity used		260T			
15 % Peeling loss		Shadow price		23.60			
5 % Total solids		Use		Juice			
8 % Insoluble solids		Reduced cost				3.30	20.40
		Alternative value				20.30	3.20
	520 ton/wk	Quantity used		236T		284T	
		Shadow price		2.71		2.71	
		Use		Juice		Juice	
		Reduced cost					9.10
		Alternative value					-6.39
<u>Grower #15</u>							
70 % Choice	260 ton/wk	Quantity used					260T
20 % Peeling loss		Shadow price					32.90
6.2% Total solids		Use				1.045 Puree	X Std. Cat.
12 % Insoluble solids		Reduced cost		10.10		-	
		Alternative value		22.90		32.90	

(continued)

Table 5-3. (continued)

		(1)	(2)	(3)	(4)	(5)
520 ton/wk	Quantity used					520T
	Shadow price					15.74
	Use				Puree	Sauce and catsup
	Reduced cost		12.10		-	
	Alternative value		3.64		15.74	

1/Shadow price is the value which would be added by one additional ton of tomatoes from that grower.

2/Use for columns 1-3 are for those tomatoes not used for whole peel.

3/The reduction in total profit if one ton was used in that activity rather than the selected activity. (Alternatively, the amount costs have to be reduced for that activity to enter the solution at a positive level.)

4/The value of one additional ton of tomatoes if used in the alternative activity. (Equal to the shadow price minus the reduced cost.

Table 5-4. Commodity Production Levels in Test Problem

Product/ can size	Price per case[1]	260 T/wk # Cases	260 T/wk Value of one additional case	520 T/wk # Cases	520 T/wk Value of one additional case
Choice round 303	$1.15	5,496[2]	-.016	7,642[2]	-.039
Standard round 303	1.05	23,776	.003	20,381	.001
Choice coreless 303	1.15	14,505[2]	-.016	12,358[2]	-.039
Standard coreless 303	1.05	-	-.003	3,395	.054
Choice round ~~303~~ 2½	1.95	17,536	.000	18,534	.009
Standard round 2½	1.70	-	-.105	-	-.104
Choice coreless 2½	1.95	-	-.006	-	-.026
Standard coreless 2½	1.70	-	-.105	-	-.133
Choice round 10	1.60	10,282	.005	19,328	.001
Standard round 10	1.45	7,265	.000	5,837	.013
Choice coreless 10	1.60	525	.020	-	-.075
Standard coreless 10	1.45	2,473	.005	-	-.077
Juice 46 oz.	1.20	43,680	.149	43,680	.379
Pur 1.045 10	1.90	3,000[2]	-.535	3,000[2]	-.085
Pur 1.06 10	2.50	2,000[2]	-.723	22,165	.085
Paste 25 10	4.30	10,000[2]	-1.652	35,280	.403
X std cat 10	2.65	6,273	.082	25,000[3]	.566
Line 2 sauce 8 oz	.85	-	-.373	-	-.070
Line 2 sauce 300	.80	-	-.270	88,200	-
Line 3 sauce 8 oz	.85	-	-.462	-	-.297
Line 3 sauce 300	.80	-	-.344	-	-.188

1/Expected price per case adjusted for direct costs of production.

2/Commodity was forced into solution at lower bound.

3/Production was restricted by upper bound on commodity.

Table 5-5. Machinery Restrictions and Simplex Multipliers for Test Problem

Restriction	Number of periods[1]	Simplex Multipliers 260 Tons/wk	Simplex Multipliers 520 Tons/wk
Dole peelers (2)	72	$122.66	352.50
FMC peeler	36	-	387.45
Box dumper 1	36	-	-
Box dumper 2	42	-	127.00
Coreless hand dump	36	-	-
Products hand dump	42	-	-
Canning line chopper	36	-	-
Pulper	42	-	-
Evaporator	150[2]	-	-
Canning line 1	36	18.06	22.47
Canning line 2	36	18.06	22.47
Canning line 3	36	46.82	57.83
Canning line 4	36	44.08	52.89
Canning line 5	36	-	-
Total canning #10	24	-	-
Total canning #2½	48	-	-
Total canning #303	72	30.08	35.02
#10 Syruper	36	-	-
#2½ Syruper	36	-	-
#303 Syruper	36	-	-
#10 Cooker	36	-	-
#2½ Cooker	36	37.98	37.98
#303 Cooker	36	66.13	58.58
Retort	108	-	-
Product line 2	42	-	-
Product line 3	42	154.60	507.45
Product line 4	42	-	-
Product line 5	42	-	-

1/Whole peel activity periods are three and one-half hours of operation. Product activity periods are four hours of operation.

2/1,000 lbs. of water.

Table 5-6. Miscellaneous Activities in Optimal Solutions to Test Problems

Activity	Level in 260T week	Level in 520T week	Units
Transfer selected 20% choice rounds	53.0	-	1,000 lbs. after peeling
Transfer selected 30% choice rounds	137.3	141.4	1,000 lbs. after peeling
Transfer selected 40% choice rounds	91.4	176.8	1,000 lbs. after peeling
Transfer selected 50% choice rounds	97.2	-	1,000 lbs. after peeling
Transfer selected 60% choice rounds	45.0	183.3	1,000 lbs. after peeling
Transfer non-select 40% choice rounds	160.5	-	1,000 lbs. after peeling
Transfer non-select 70% choice rounds	309.4	618.8	1,000 lbs. after peeling
Transfer coreless 60% choice rounds	265.2	240.7	1,000 lbs. after peeling
Blend 53% solids pulp with 5.9% to yield 5.5%	21.21	7.58	1,000 lbs. of solids
Blend 5.5% solids pulp with 6.1% to yield 5.7%	57.94	63.63	1,000 lbs. of solids
Blend 5.7% solids pulp with 6.1% to yield 5.9%	20.58	127.26	1,000 lbs. of solids
Transfer 4.7% solids pulp to juice	945.96	929.36	1,000 lbs. of solids
Transfer 5.1% solids pulp to juice	514.74	1,460.05	1,000 lbs. of solids
Transfer 5.3% solids pulp to juice	890.96	-	1,000 lbs. of solids
Evaporate 6.1% solids pulp to 10.8% for 1.045 puree	117.56	117.56	1,000 lbs. of 1.045 puree

(continued)

Table 5-6. (continued)

Activity	Level in 260T week	Level in 520T week	Units
Evaporate 5.3% solids pulp to 14.2% for 1.06 puree	-	2.64	1,000 lbs. of 1.06 puree
Evaporate 5.9% solids pulp to 14.2% for 1.06 puree	-	878.42	1,000 lbs. of 1.06 puree
Evaporate 6.1% solids pulp to 14.2% for 1.06 puree	79.50	-	1,000 lbs. of 1.06 puree
Evaporate 5.1% solids pulp to 25.5% (paste)	-	141.41	1,000 lbs. paste
Evaporate 5.3% solids pulp to 25.5%	-	592.52	1,000 lbs. paste
Evaporate 5.7% solids pulp to 25.5%	363.27	-	1,000 lbs. paste
Evaporate 5.9% solids pulp to 25.5%	52.98	-	1,000 lbs. paste
Use Hand Pump	-	229.5	Manhours
Use canning line 2 for $2\frac{1}{2}$ cans	17.9	22.7	100 case units
Use canning line 1 for 303 cans	78.8	78.7	100 case units
Use canning line 2 for 303 cans	56.4	50.4	100 case units
Use retort to cook $2\frac{1}{2}$ cans	65.1	70.0	100 case units
Waste disposal	810.26	1,343.84	1,000 lbs. of waste

Table 5-7. Sample LP Matrix

		1 G01111	2 G01112	3 G01113	4 G01114	5 G01115	6 G04111	7 G041112
1	OBJEC1	-.335	-.3	-.335	-.3	-.3	-.335	-.3
2	OBJEC2	-.335	-.3	-.335	-.3	-.3	-.335	-.3
3	R01110	1.	1.	1.	1.	1.		
4	R04110						1.	1.
5	R08110							
6	RSEL20	.204		.204				
7	RSEL30						.256	
8	RSEL40							
9	RSEL50							
10	RNON20		.34					
11	RNON30							.48
12	RNON40							
13	RNON50							
14	RSOL51						.065	.03
15	RSOL53	.068	.033	.068	.099			
16	RSOL55							
17	RSOL57							
18	RBOXD1	.007874	.007874	.007874			.007874	.007874
19	RBOXD2				.007874	.007874		
20	RHANDP							
21	RDOLPL		.095238	.038095				.095238
22	RFMCPL	.024316					.024316	
23	RPULPR	.000266	.001063	.000266	.003125	.003125	.00025	.001
24	REVAPR					.02		
25	RWASTE	.216	.348	.216	.14	.14	.255	.455
26	RSAUCE					.079		
27	RJUICE							
28	RSS142							
29	RSDRDS	.306	.68	.306			.224	.48
30	RCHRDS							
31	RCANL1							
32	RCANL2							
33	RCANL3							

(continued)

Table 5-7. (continued

	8 G04113	9 G04114	10 G04115	11 G08111	12 G08112	13 G08113	14 G08114	15 G08115
1	-.335	-.3	-.3	-.335	-.3	-.335	-.3	-.3
2	-.335	-.3	-.3	-.335	-.3	-.335	-.3	-.3
3								
4	1.	1.	1.					
5				1.	1.	1.	1.	1.
6								
7	.256							
8								
9				.374		.374		
10								
11								
12								
13					.85			
14	.005	.095						
15								
16								
17				.067	.018	.067	.106	
18	.007874			.007874	.007874	.007874		
19		.007874	.007874				.007874	.007874
20								
21	.038095				.095238	.038095		
22				.024316				
23	.00025	.003125	.003125	.000106	.000531	.000106	.003125	.003125
24			.02152					.01729
25	.255	.14	.14	.208	.324	.208	.14	.14
26			.095					.127
27								
28								
29	.224			.238	.51	.238		
30								
31								
32								
33								

(continued)

Table 5-7. (continued)

	16 BSE243	17 BSE253	18 BSE254	19 BSE354	20 BNS243	21 BNS253	22 BNS254	23 BNS354
1								
2								
3								
4								
5								
6	-.3333	-.4444	-.1667					
7	1.	1.		-.375				
8	-.6667		1.	1.				
9		-.5556	-.8333	-.625				
10					-.3333	-.4444	-.1667	
11					1.	1.		-.375
12					-.6667		1.	1.
13						-.5556	-.8333	-.625
14								
15								
16								
17								
18								
19								
20								
21								
22								
23								
24								
25								
26								
27								
28								
29								
30								
31								
32								
33								

(continued)

Table 5-7. (continued

	24 TRSE20	25 TRSE30	26 TRSE40	27 TRSE50	28 TRNS20	29 TRNS30	30 TRNS40	31 TRNS50
1								
2								
3								
4								
5								
6	-1.							
7		-1.						
8			-1.					
9				-1.				
10					-1.			
11						-1.		
12							-1.	
13								-1.
14								
15								
16								
17								
18								
19								
20								
21								
22								
23								
24								
25								
26								
27								
28								
29								
30	1.	1.	1.	1.	1.	1.	1.	1.
31								
32								
33								

(continued)

Table 5-7. (continued)

	32 515553	33 515753	34 515755	35 535755	36 TRJU51	37 TRJU53	38 TRJU55	39 TRJU57
1								
2								
3								
4								
5								
6								
7								
8								
9								
10								
11								
12								
13								
14	-.5	-.667	-.3333		-.051			
15	1.	1.		-.5		-.053		
16	-.5		1.	1.			-.055	
17		-.3333	-.6667	-.5				-.057
18								
19								
20								
21								
22								
23								
24								
25								
26								
27					1.	1.	1.	1.
28								
29								
30								
31								
32								
33								

(continued)

Table 5-7. (continued)

	40 E51142	41 E53142	42 E55142	43 E57142	44 TRCHSD	45 TRSDJU	46 WASTED	47 HANDDP	48 T212L2
1	-.0666	-.0616	-.0592	-.0559			-.075	-.01	-.0038
2	-.0666	-.0616	-.0592	-.0559			-.075	-.01	-.0038
3									
4									
5									
6									
7									
8									
9									
10									
11									
12									
13									
14	-.142								
15		-.142							
16			-.142						
17				-.142					
18									
19								-.007874	
20								.009524	
21									
22									
23						.001562			
24	.01784	.01645	.01582	.01491					
25						.07	-1.		
26									
27						.93			
28	1.	1.	1.	1.					
29					1.	-1.			
30					-1.				
31									
32									.57142
33									-.22857

(continued)

Table 5-7. (continued)

		49 T303L1	50 T303L2	51 T212RE	52 T303RE	53 CHR303	54 SDR303	55 CHR212	56 SDR212
1	OBJEC1	-.018	-.018	-.124	-.0545	1.15	1.05	1.95	1.70
2	OBJEC2	-.018	-.018	-.124	-.0545	1.15	1.05	1.95	1.70
3	R01110								
4	R04110								
5	RSEL20								
.									
.									
.									
18	RBOXD1								
19	RBOXD2								
20	RHANDP								
21	RDOLPL								
22	RFMCPL								
23	RPULPR								
24	REVAPR								
25	RWASTE								
26	RSAUCE								
27	RJUICE					-.525	-.825	-1.2	-1.425
28	RSS142								
29	RSDRDS					-.1875	-1.4175	-.3	-2.4975
30	RCHRDS					-1.6875	-.1575	-2.7	-.2775
31	RCANL1	.45704							
32	RCANL2		.45704						
33	RCANL3							.22857	.22857
34	RCANL4	-.22857	-.22857			.22857	.22857		
35	RSY212							.0907	.0907
36	RSY303					.06079	.06079		
37	RC0212			-.32653				.32635	.32653
38	RC0303				-.082237	.082237	.082237		
39	RETORT			.34482	.151515				
40	RPRDL2								
41	RPRDL3								
42	RPRDL5								
43	RQSA30								
44	RQSA8Z								

(continued)

Table 5-7. (continued)

	57 JUIC46	58 P10610	59 YSDK10	60 2SAU8Z	61 2SA300	62 3SAU8Z	63 3SA300	64 RHS
1	1.2	2.5	2.65	.85	.8	.85	.8	
2	1.2	2.5	2.65	.85	.8	.85	.8	
3								≤ 520
4								≤ 520
5								≤ 520
.								
.								
.								
18								≤ 36
19								≤ 42
20								≤ 42
21								≤ 72
22								≤ 36
23								≤ 42
24			.00963					≤ 150
25								
26			-.572	-.264	-.231	-.264	-.231	
27	-3.735							
28		-3.975						
29								
30								
31								≤ 36
32								≤ 36
33								≤ 36
34								≤ 36
35								≤ 36
36								≤ 36
37								≤ 36
38								≤ 36
39								≤ 108
40				.057471	.047619			≤ 42
41	.096154					.057471	.047619	≤ 42
42		.089286	.078125					≤ 42
43								≥ 1250
44								≥ 3500

CHAPTER VI

THE SOLUTION OF A TEST PROBLEM BY DECOMPOSITION

After the test problem described in Chapter III had been solved and the results evaluated, the test model was used as the basis for a pilot decomposition study. The problem had two subprograms representing weeks and a master for the season, so the basic configuration was similar to figure 4-1. The two input levels of 260 tons per grower (week one) and 520 tons per grower (week two) were selected to be used as the two "weeks". The raw product characteristics and the plant's configuration and capacity were those outlined in Chapter V. A master program was constructed that had the "seasonal" quantity restrictions on products listed in table 6-1. Most of the restrictions are the maximum amounts to be produced during the season, but there are six that require a minimum amount of a product to be produced in a given week. These restrictions are less than the weekly restrictions listed in table 5-1 which allow production at a level of 1200 tons a week. However, they are consistent with the initial solutions obtained at the 260 and 520 ton level.

The master program initially was composed of a single objective function, the 13 seasonal restrictions limiting total production, and the two restrictions requiring that the sum of the weekly solutions for each week be less than or equal to one in the solution to the master.

The six restrictions requiring minimum weekly production were not included until the fifth major iteration. The master was first solved using only the optimal solutions to the two weeks that were previously obtained in the trial runs.

Since there were no minimum restrictions, the master program was immediately feasible although neither of the weekly solutions was included in the master solution with a weight of one. The simplex multipliers were used to obtain the revised objective functions for the subprograms. The procedure throughout the test problem was to obtain optimum solutions to both weeks in each major iteration. (This procedure was altered in the full-scale problem.)

After the first major iteration the optimal solutions to the subproblems were in the solution to the master with values of .04 and .99. The objective function value of the master was $1567.413. This is not much larger than the objective function value for the solution to week two and substantially less than the eventual solution to the test problem of $2294.732. The maximum production restrictions for each week in the subprograms was equal to the maximum production restrictions for the season in the master program. Since some of the products were produced at their maximum level in week two and also produced in week one, week two with its higher objective function value blocked week one out of the solution to the first major iteration. If the solutions to the subproblems had not been restricted, their initial optimal solutions would have been larger, but the objective function value of the master after the first major iteration would have been even less since a smaller fraction of the week two solution would have filled at least one seasonal production restriction. It appears that unrestricted solutions would have required more computations to approach an approximate solution to the test problem, but this is an area which requires more investigation. A great deal depends on the nature of the subproblems and on the properties of the particular linear programming computer code.

Even though the subproblems had bounds so that the minimum weekly quantities required in the master would be produced each week, these minimum production restrictions were obvious possible sources of infeasibility in the master as long as both weeks were not in the solution with summed weights equal to 1.0. For instance, the 25 percent paste was relatively uneconomical to produce and was in the optimal solution to week one at the lower bound and at an intermediate level in week two. Both weeks could not come in with a weight large enough to meet both the lower limit on paste and be less than the seasonal upper bounds simultaneously. In this test problem, no great difficulties were encountered by the simultaneous requirement of maximum and minimum restrictions because the master program was simple enough to be inspected visually and the minimum restrictions were not added to the master tableau until it was possible to get a feasible solution. In any case, meeting these requirements can take considerable computing time. Five sets of solutions to the subproblems were required before a feasible solution was obtained.

There are several ways to handle the problem of initial infeasibilities in the master.

1. Ignore either the maximum or minimum requirements until a sufficient number of product vectors are available so the infeasibilities are eliminated as was done in the test problem. This process might be

speeded up by restraining some activity levels in the subprograms either by bounds or artificial prices.

Using this approach, it might also be advantageous not to bring in all the restrictions at once as was done in the test problem but to phase them in singly or in small groups.

2. Bounds and constraints could be added to the restrictions used for the initial solutions to the subprograms so that the master program will be feasible after one set of solutions to the subprograms is completed. For a large master program, however, this would require considerable bookkeeping. It might not be practical or possible to put all of the needed constraints into the subprogram without significant changes in its formulation.

3. Artificial variables could be inserted in the initial tableau of the master for all the restrictions which might cause infeasibilities. This has the advantage of having the full set of restrictions in the master from the very beginning without continuously changing the master program except to drop out dummy variables. These artificial variables could be given objective function values which would make them relatively uneconomical activities. This will also influence the simplex multipliers on the applicable rows and thereby adjust the objective function values of the activities in the subproblems.

4. Use artificial variables in the master and solve with a general "two-phase technique",[1] in which Phase I consists of maximizing an

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[1] G. Hadley, Linear Programming (Reading, Massachusetts: Addison-Wesley Publishing Co., Inc., 1962), pp. 149-151.

objective function with a cost of -1 assigned to each artificial variable and a cost of zero assigned to every other variable. When Z max equals zero, all of the artificial variables have been driven out of the solution and Phase II is accomplished by maximizing the original objective function. If the optimality conditions have been satisfied in Phase I and Z max is less than zero, there is no feasible solution to the overall problem.[2] There is no similar computational proof of infeasibility available from the other methods described, so that computations could continue indefinitely if the overall problem was infeasible. On the other hand, if the problem is feasible, it would appear that obtaining feasibility via Phase I would frequently add to total computational time.

A combination of two and three above was used for the full-scale model with dummy variables being used to eliminate the infeasibilities that it was impractical to avoid by using bounded variables or constraints. Due to the nature of the IBM 1620 LP code (but not 1130-LP-Moss), considerable computational time could have been saved if the subprogram solutions had been manipulated only by objective function changes. Solving a problem with the IBM 1620 on which either the bounds or restrictions had been changed took essentially as long as solving a new problem, while solving a problem with a revised objective function typically took from five to 50 percent as long to solve.

The solution to the master after each of the first seven and the 10th major iterations is given in the upper portion of table 6-3. The bottom portion

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[2] _Ibid_., p. 406.

of the table gives the levels of the individual products. Table 6-4 gives the optimal solution to the master and the final product levels. (This was after 40 major iterations.)

The first row of table 6-3, labeled objective function, gives the value of the objective function of the master after each iteration. The next row gives the objective function as a percent of the maximum value for the season of $2,294.732. (This value is scaled by 100.) The decrease in the objective function value from iteration four to iteration five was due to the addition of the three minimum production restrictions in the master for each week. As was noted earlier, this was the first time that the master would yield a feasible solution with these restrictions included. The objective function value (and solution) did not change from iteration five to iteration six because the sixth solution to the subprograms used prices that were computed before the additional restrictions were added to the master. As a result, these subprogram solutions did not enter the solution to the master at that time. However, they did enter after an additional set of subprogram solutions was obtained using the prices obtained with the minimum restrictions added to the master.

The next two rows in table 6-3 are the values of the simplex multipliers associated with the rows restricting the sum of the weights for each week to be less than to or equal to one. $\overline{\pi}_1$ is the multiplier for week one and $\overline{\pi}_2$ is the multiplier for week two. These two multipliers combined with the objective function value of the subprogram can be used to determine the upper bound to the overall solution.[3/] (They are

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[3/]Dantzig, <u>Linear Programming and Extensions</u>, p. 452.

also used in determining when an overall optimal solution is obtained.)[4] The following two rows are the values of the slacks for the restrictions on the weeks. Following the slacks are the weights for each week in each of the ten solutions and the product levels obtained from each solution.

The master for the first major iteration had 13 rows and only two non-slack columns. As might be expected since the subprograms had relatively large bounds on the products, two products, juice and catsup, were restrictive and there were slacks for both weeks. Week two had a value of .99, however, since its objective function value was much larger than week one (see table 6-2) as it had more raw tomato inputs than week one. Both produced the same amount of juice, the most over-produced commodity in total for the two weeks.

The solution to the master after the second major iteration is shown in the next column of table 6-3. There were four input vectors or columns in the master during this major iteration of which all four entered the basis. Three of the commodities restrictions were effective. The fourth active restriction was the requirement that the sum of week two solutions be ≤ 1. The third major iteration produced similar results. Although there were six input vectors, only four entered the basis. Profit for the two week season did not increase appreciably during the first three iterations especially when it is realized that the optimal solution to week two had an objective function value of $1541 and, after three major iterations, the objective function for the season was on

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[4] Baumol and Fabian, _op_. _cit_., p. 14.

$1642. However, there was a significant improvement after the fourth major iteration, although there were slacks in both weekly restrictions. The value of 2065.916 was over 90 percent of the optimal objective value for the season although the weekly minimum production requirements were not yet included in the master.

The fifth and sixth iterations were the first to require the minimum quantities for each week. Consequently, the objective function value was somewhat smaller, although it was still 85 percent of the season maximum. The seventh major iteration was the first to have no slacks in the restrictions that the sum of each week's weights equal one. The objective function value was over $2,237 or 97.5 percent of its optimal value for the season (which was not obtained until after 40 major iterations). The increase in the objective function from the seventh to the tenth major iteration is less than one percent. On an actual dollar basis, (unscaled), this was a difference of slightly less than $2,000. The value at the seventh iteration is less than $6,000 less than the overall maximum, and the value after the tenth iteration is less than $4,000 from the maximum of $229,473.

There are few differences in the product levels for the seventh through the tenth iterations. Nine of the products are already at their maximum levels after seven major iterations. The production levels for the final solution, which are shown in table 6-4 are quite similar. The six whole peel products are all at their upper limits in both the tenth and the final iteration. Paste production has shifted in the final iteration between weeks one and two until only the minimum is produced in week one. (However, the shadow price on this restraint is only 1.4¢ per case, so this is not an expensive limitation.) Shadow prices for the final solution are shown in table 6-4. Catsup and juice production also was shifted between weeks and the production of eight-ounce sauce has been eliminated from the final solution. Eight-ounce sauce was dropped from the solution in the 13th iteration and never entered again after the 20th.

The greatest amount of physical production did not coincide with the largest profit solution. Rather, the greatest amount of production occured after the 13th major iteration. The number of units of paste produced was 379. All the other products, except for eight-ounce sauce, were at their maximum. The objective function value was $2,276,186 or 99.2 percent of the final value. After that, total production decreased slightly until the final solution was reached. (there was one occasion when #300 sauce dropped below the maximum and paste production increased to 383 units, but the pattern of decreasing total production returned in the next major iteration.)

Any solution after the completion of the seventh iteration would have been very effective. Unfortunately, the maximum value of the objective function is unknown at this point. We know that it cannot exceed $2,575.239, which is the sum of the objective functions when week one and week two were solved independently, but upper bound calculations give a result larger than this value until after the tenth major iteration. The value of the upper bound computed after the tenth iteration was $2,473. This meant that the current solution was between 93 and 94 percent of the lowest known upper bound at that time. Actually, the solution value was 98.4 percent of the final value at that point.

After the 11th major iteration, the computed upper bound decreased to \$2,348. The corresponding solution value was then greater than 96 percent of this value. The upper bounds computed after each of the next three major iterations were all greater than \$2,348. After the 15th major iteration, the computed upper bound was \$2,313. The solution value at this time was \$2,276 or over 98.3 percent of the bound and over 99.2 percent of the final solution value. This bound of \$2,313 was the lowest bound obtained until after the 28th major iteration. These bounds ranged from \$2,316 to \$2,395 with six of the first seven being greater than \$2,330. After the new lower bound was obtained after the 28th major iteration, the actual solution of \$2,292 was 99.5 percent of the bound (and 99.8 percent of the final value).

Although upper bounds can be used to determine when to stop if it is not desired to do all the computing necessary to obtain the optimal overall solution, they might not be particularly efficient. In fact, if it was desired to minimize or limit the amount of computational expense, this information would not have been available, since it involves completely solving all the subprograms using the objective function values computed after the last master. In the test problem, there were only two subprograms so the price paid for the information about the bound would have been at most a doubling of computer time (if the optimal solutions to both subproblems were always obtained). In the early stages of the problem, both solutions frequently entered in the solutions to the master so the effort was not wasted. Only two of the 20 subproblem solutions generated had not entered into a solution by the end of the tenth major iteration. (Thirteen of these were in solution to the master at that time.) Both new vectors did not enter the solution in only four of the first ten major iterations. After the first ten major iterations, however, the entering of both vectors was more unusual. It only occurred three times in the last 20 major iterations. Only 60 of the 79 subprogram solutions generated actually were used in a solution to the master, and many of these entered for just a few iterations at a low level.

In the test problem, only optimal solutions to the subprograms (and masters) were used. It would speed convergence up if non-optimal solutions had been used, either obtaining several feasible non-optimal solutions while iterating to an optimal solution or entering another feasible solution into master after a fixed number of iterations[5/] and then proceeding with the revised prices and not bothering with obtaining optimality of the subproblems. Under such operating conditions, one is effectively barred from obtaining the upper bound unless it is explicitly sought. However, the upper bound is only an estimate and does not necessarily decrease. It can have wide fluctuations if calculated only at intervals and not at every major iteration. It would be possible not to get a good estimate of the upper bound until long after the solutions to the master were converging on the maximum.

A possible approach to obtaining a satisfactory but not necessarily optimal solution to the program would be to stop computing after the

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[5/]Eli Hellerman, Large Scale Linear Programs: Theory and Computation. Paper presented at Technical Association of the Pulp and Paper Industry Symposium, Philadelphia, Pennsylvania, March 28-30, 1966.

incremental improvements to the objective function of the master become less than a predetermined amount. It might be possible to determine this quantity by solving the problem once and inspecting the sequence of objective function values of the master after each major iteration. In the test problem, there was a definite tendency for the increases in the objective function of the master to get smaller and smaller as the solution converged to the maximum. There was an increase of almost six at each major iteration number ten to number 20. After the 20th iteration, the average increase was less than one. No iteration improved the solution more than one after the 23rd iteration. The last 19 iterations improved the solution in total less than eight.

Since the problem converged rapidly to over 99 percent of its final value once a feasible solution to the master was obtained, an experiment was performed to determine how difficult it was to obtain a feasible solution for a starting point. Ten sets of vectors from the 12th through the 21st major iteration were introduced by pairs into the master. (Only three of these vectors were in the final solution.) The results are shown in table 6-5. A feasible solution to the master was obtained after what was equivalent to the third major iteration.

After the tenth solution, the objective function was 93.4 percent of the maximum. Even at this stage, however, there is still a slack in the constraint for week one. If at this point (or earlier) further solutions to the subproblem for week one were obtained using objective functions computed from the last solution to the master, the solution value for the master would be considerably higher. This procedure would be directly applicable to the solution of a decomposed problem with multiple objective functions. After solving the problem for the first objective function, there will be a number of solutions to each subprogram. These solutions can then be used in the master when maximizing the next objective function. If there are no changes to the matrix or the right-hand side of the master, it will be feasible and with a fairly high value for the objective function. When several objective functions are to be used, this process can be done in sequence. In these circumstances, if vectors which don't enter the solution to the master are dropped from the files, it might be wise to retain them and reintroduce them when starting work on a new objective function. Week U-1, for instance never entered the solution to the master in the test problem although it had a value of .192 in the last solution to the master in table 6-5.

In the same manner, the repeated use of vectors will provide an advanced start when using multiple right-hand sides in the master program or making changes to the matrix of the master program. Even if there are substantial changes to the right-hand side of the master, the previous subproblem solutions might provide a feasible basis for the master or at least reduce the number of infeasibilities that would have to be worked out if starting with all new subproblem solutions.

Table 6-1. Restrictions for the Test Master Program

Commodity Case Size	Grade	Maximum Level In 100 Case Units	Minimum Level In 100 Case Units
24/303	Choice	600	
24/303	Standard	250	
24/2½	Choice	200	
24/2½	Standard	112	
6/10	Choice	330	
6/10	Standard	120	
6/10	25% Paste	400	
6/10	25% Paste (Week 1)	-	100
6/10	25% Paste (Week 2)	-	100
6/10	1.06 Puree	250	
6/10	1.045 Puree	75	
6/10	Extra Standard Catsup	250	
6/10	Extra Standard Catsup (Week 1)	-	10
6/10	Extra Standard Catsup (Week 2)	-	10
48/8 oz.	Sauce	700	
12/46	Juice	450	
12/46	Juice (Week 1)	-	50
12/46	Juice (Week 2)	-	50
24/303	Sauce	500	

Table 6-2. Objective Function Values of Subproblems

Iteration	260 Ton/Week	520 Ton/Week
1	1033.9	1541.3
2	842.8	1215.9
3	931.4	1316.6
4	976.8	1374.6
5	930.1	1303.3
6	817.9	1170.7
7	493.1	1186.3
8	737.4	1085.9
9	857.7	1207.3
10	738.3	1217.9

Table 6-3. Solutions to Master Program of Test Model

	Iteration Number						
	1	2	3	4	5 & 6[1]	7	10
Objective Function Value	1567.413	1588.537	1642.781	2065.916	1949.803	2237.626	2257.692
Objective Function as a % of Final Value	.683	.692	.716	.900	.850	.975	.984
$\bar{\pi}1$	–	–	–	–	–	294.209	335.540
$\bar{\pi}2$	–	373.102	229.655	–	–	342.731	588.127
SLACK1	.96	.949	.876	.046	.000	–	–
SLACK2	.01	–	–	.170	.305	–	–
WEEK.1	.040	.038	.124	.129	–	.324	.261
WEEK.2	.990	.990	.895	.311	.538	.222	.255
WEEKA1		.014	–	.264	.262	.012	.185
WEEKA2		.010	.032	–	–	.084	–
WEEKB1			–	.006	.205	.056	–
WEEKB2			.073	.519	–	.088	.120
WEEKC1				.555	.458	.469	.468
WEEKC2				–	–	–	–
WEEKD1					.076	–	–
WEEKD2					.157	.307	.255
WEEKE1					–	–	.045
WEEKE2					–	.057	.016
WEEKF1						.139	.042
WEEKF2						.241	.288
WEEKG1							–
WEEKG2							.045
WEEKH1							–
WEEKH2							.006
WEEKI1							–
WEEKI2							.015
CH.303	206	210	232	600	485	600	600
SD.303	245	250	250	167	250	250	250

(continued)

Table 6-3. (continued)

	Iteration Number						
	1	2	3	4	5 & 6	7	10
CH.212	185	189	200	200	156	179	200
SD.212	0	0	1	90	112	112	112
CH..10	185	190	186	222	302	323	330
SD..10	69	70	75	120	120	120	120
P25.10	368	371	194	284	248	377	368
P25.10 WEEK1	na	na	na	na	100	126	108
P25.10 WEEK2	na	na	na	na	148	251	260
P10610	220	224	226	220	191	250	250
P10410	31	34	39	75	75	75	75
XSDK10	250[2]	250	250	222	240	250	250
XSDK10 WEEK1	na	na	na	na	66	34	20
XSDK10 WEEK2	na	na	na	na	174	216	230
JUIC46	450	450	447	450	450	450	450
JUIC46 WEEK1	na	na	na	na	214	347	328
JUIC46 WEEK2	na	na	na	na	236	103	122
.SAU8Z	0	0	0	0	0	0	11
.SAU30	470	483	492	500	341	500	500

1/Major iteration 5 was the first time the master program included minimum production restrictions.

2/Underlined values indicate production is restricted at upper limit.

(lacking in numbering only)

Table 6-4. Optimal Solution to Test Problem (After 40 Major Iterations)

Row	Value	
Objective Function		2294.732
Week 1	1.0	513.2212
Week 2	1.0	854.9285
Choice #303	600	.197
Standard #303	250	.480
Choice #$2\frac{1}{2}$	200	.266
Standard #$2\frac{1}{2}$	112	.610
Choice #10	330	.136
Standard #10	120	.569
25.5% Paste	372	-
Paste - Week 1	100	-
Paste - Week 2	272	-
1.06 Puree	250	.151
1.045 Puree	75	.292
X Standard Catsup	250	.685
X Standard Catsup - Week 1	99	-
X Standard Catsup - Week 2	151	-
46 oz. Juice	450	.480
Juice - Week 1	149	-
Juice - Week 2	301	-
8 oz. Sauce	0	-
Sauce #303	500	.017

Table 6-5. Experimental Master Solutions

	Major Iteration							
	3rd[1]	4th	5th	6th	7th	8th	9th	10th
Objective Function Value	1510.582	1581.722	1675.014	1912.818	1921.314	1997.648	2135.31	2142.647[2]
$\bar{\pi}1$			15.670		43.187	138.595		
$\bar{\pi}2$								35.915
Slack 1	.364	.470		.116			.016	.021
Slack 2	.211	.096	.295	.059	.114	.108	.003	
Week L1	.064							
Week L2	.088	.084		.064				
Week M1	.571	.440	.066			.004		
Week M2				.035	.024		.262	.269
Week N1			.352					
Week N2	.701	.701	.108					
Week P1		.091	.091		.080		.128	
Week P2		.119	.596	.528	.429	.423	.029	.027
Week Q1			.492	.728	.764	.841	.544	.589
Week Q2								.004
Week R1				.156	.156	.154	.128	
Week R2				.314	.169		.255	.330
Week S1							.149	.197
Week S2					.264			
Week T1								
Week T2						.469		
Week U1							.034	.192
Week U2							.450	
Week V1								.199
Week V2								.171
Choice #303								600
Standard #303								175
Choice #2½								200

(continued)

Table 6-5 (continued)

	Major Iteration							
	3rd	4th	5th	6th	7th	8th	9th	10th
Standard #2½								56
Choice #10								304
Standard #10								81
25% Paste								400
Paste Week 1								105
Paste Week 2								296
1.06 Puree								250
1.045 Puree								59
X Standard Catsup								250
X Standard Catsup Week 1								68
X Standard Catsup Week 2								172
Juice 46 oz.								450
Juice Week 1								50
Juice Week 2								400
8 oz. Sauce								73
#300 Sauce								500

1/First feasible solution.

2/93.4 percent of optimal solution to test problem.

CHAPTER VII

THE FULL-SCALE LINEAR PROGRAMMING MODEL

The full-scale model was a model of the TriValley Growers tomato packing operation for the 1965 season. Consequently, it was considerably larger than the test problem previously described. There were approximately 80 growers supplying tomatoes. The total number of different raw product inputs was even larger since some growers grow more than one variety or pick tomatoes both by hand and by machine. Instead of a two-week season as in the test problem, the season may last as long as 14 weeks. TVG had packed tomatoes in three separate plants in previous years, but the model included only the two plants which were expected to be used in the 1965 season.

There were 32 possible machinery or capacity limitations considered in the construction of the model of the larger plant and 15 potential capacity limitations in the smaller plant. Upon investigation, it was determined that some of these potential restrictions were redundant based on the current machinery configurations. These redundant restrictions were not included after the initial stages of development although it was recognized that these might have to be re-entered if extensive post optimal analysis was desired.

The number of restrictions was reduced to 26 for the larger plant and five for the smaller plant. Most of these remaining restrictions were actually binding in one or more of the various solutions to the subprograms.

The total number of unique products considered for the model was larger than might be expected. Within each general category, such as whole peel, juice, paste, and puree, and sauce and catsup, there are numerous finished products due to differences in can size, grade, and percent of solids. Initially, over 100 different products were identified from the price lists and data on previous packs. Some were of such a small volume that they were consolidated with a similar product reducing the number to be considered. Conversely, it was found that some products were produced to such distinct specifications within a standard that two or more unique products for modeling purposes existed under a single standard. The final list contained 60 products.

The full-scale problem was reduced to 96 different tomato inputs (one tomato input for each grower, variety, and method of picking combination) and ten weekly time periods before solution was attempted. (See Appendix A.) If the problem was to be solved by a standard formulation of linear programming even at this point, the total number of rows would have exceeded 1,800 and the number of columns would have been approximately 9,000.

Although it would be possible to further reduce the number of rows required, this was thought undesirable because it would entail a loss of information. In any event, it would be impossible to reduce the number of rows down to the capacity of most large LP codes at the time the model was formulated and still have a realistic and detailed weekly model of the tomato packing operation. In 1965 the ALPAC code of the Service Bureau Corporation would handle a maximum of 1,100 rows on an IBM 7094.[1] The Advanced Linear Programming System for the Honeywell 800/1800 had solved low density problems with up to 850 rows with 16K of memory. Memory size was not a limiting constraint on these problems, but digital accuracy was.[2] The LP codes for the Burroughs B 5500 computer ALPS-I and ALPS-III would solve problems with up to 1022 rows.[3]

In the decomposed formulation, there were ten subproblems with dimensions varying from about 160 to 200 rows and 700 to 900 columns. The difference in dimensions was due to the varying number of raw product inputs. Each subproblem represented a week of the season and included both processing plants and all the grower-variety-pick-combinations with production during the week. In the initial formulation, any product could be produced in any week. This was later changed so that no whole peel products could be produced during the first and last week of the season. Only one processing plant was assumed to be open during these two weeks. These changes were made because it was assumed that there would not be enough fruit of good peeling quality to support the peeling operation during these weeks nor would there be sufficient total fruit to justify keeping both plants open during those weeks.

1/ALPAC Users Manual, The Service Bureau Corporation, Computing Sciences Division, October 1965, p. I-1.

2/Honeywell EDP Software Manual, H 800/1800 Advanced Linear Programming System (ALPS) File No. 123.8300.000B. 0-171, April 15, 1966, p. 2-1.

3/Burroughs B 5500 Algol Linear Programming System ALPS-III, Burroughs Corporation, Detroit, Michigan, 1967, p. I-1.

The master program was composed of 78 rows and the objective function. The number of columns varied with the continual addition of new weekly product vectors. There were 66 rows which were restrictions on the amount of product that could be produced. Many of these were ranges rather than simple upper or lower limits. Artificial upper or lower limits were included for the other products in case it was desired to change their restrictions from a single limit to a range. There were eight requirements that part of the production of a product be accomplished early in the season because of a short or out-of-stock inventory position. There were ten rows devoted to the restrictions that the sum of the weights for each week be less than or equal to one.

The first time the master was run, there were 62 dummy variables entered as columns in the master so that the initial solution to the master would be feasible. This was more than twice as many as necessary, but rather than attempt to determine which dummy variables were essential beforehand, dummy variables were included corresponding to all the $>$ restrictions and to all the $<$ restrictions which had significant production in the subprogram solutions included in the first run of the master.

APPENDIX A TO CHAPTER VII

DATA COLLECTION AND CONSTRUCTION OF THE FULL-SCALE MODEL

The collection of data for a model of this size was a task of considerable magnitude. It required the use of several sources including some outside of the firm, and, in a few instances, no accurate data was available at all. In these instances, what appeared to be reasonable estimates were generated and data collection programs initiated if feasible. In other cases, although some data was available, further or more refined data collection procedures were initiated.

Data sources within TVG included the Field Department, the Industrial Engineering Department, the Quality Control Department, and the Sales Department. Standard cost data for such items as cans and labels were obtained from the public accounting firm of Touche, Ross, Bailey and Smart. Background information on items such as the evaporation characteristics of tomato pulp and the effect of insoluble solids on the consistency of tomato products were obtained from outside sources.1/

TVG Field Department was the source for the estimate of the quantity of tomatoes which would be available from each grower. Data sheets were obtained from the Field Department listing each grower and his

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1/These sources included a conversation with Walter Krenz of the firm of Oscar Krenz, Inc., manufacturer of evaporating equipment, and unpublished reports on Factors Affecting the Consistency of Tomato Concentrates of the Department of Food and Science Technology, University of California, Davis.

acreage of tomatoes by field and variety. The expected yields of each field by pick, the expected dates of maturity, and the expected method of picking were included. This information was converted into an expected schedule of receipts by week. Each weekly estimate was further broken down into estimates of the quantity expected from each grower by variety and picking method. This information formed the basic raw product quantity estimates used in the model. The basic transportation costs (i.e., trucking) from each grower to each plant were also obtained from this source.

Besides the estimates of the quantities of raw tomatoes, the raw product characteristics were needed. TVG had a continuing program of obtaining raw product data by sampling loads of tomatoes upon receipt. The information recorded for each sample included the grower's name, the tomato variety, the sample weight (since the number of tomatoes in the sample was constant, this is a measure of size), the percent solids, the pH, and the percent of choice canning tomatoes. In addition, the percent of mold and percent of total defects were obtained from the state inspection report.

Although this information was adequate in areas covered, further information was needed. There was no continuing collection of data on peeling losses. The percent of choice canning tomatoes was an estimate based on the appearance of the tomatoes when they entered the yard. In some cases, the appearance was markedly different after peeling several hours later. Because of these shortcomings in the raw product data, especially as it pertained to peeled tomatoes, a sampling procedure was instituted during the 1964 season to obtain extensive information about peeling quality and peeling losses.

The general procedure was for a given lot of tomatoes to be sampled at several locations throughout the plant. Although the locations of the sampling points varied somewhat depending on the type of peeling machine and whether the tomatoes were being selected for peeling or were all being peeled, the basic procedure was to collect a sample of 50 tomatoes from the same truckload of fruit at three locations. The flow of tomatoes through the plant was carefully timed to enable the samplers to obtain samples of just one grower's fruit. The first location was before any selection (or rejection) of fruit occurred; the next was just before entering the peeler; and the last location was after peeling but before the tomatoes got to the canning tables. Some additional sampling points were included on some occasions for comparisons and for special purposes.

The 50 tomato samples were sorted by quality, counted and weighed. The basic quality categories at the first location were:

1. choice peeling tomatoes,
2. tomatoes which could be canned without trimming although not of choice quality,
3. tomatoes which would require trimming before they could be canned, and
4. tomatoes which had defects or were of such small size that they could not be canned.

At the next sampling station, just before peeling, the tomatoes were separated on the basis of:

1. choice peeling tomatoes,
2. broken and cracked tomatoes,
3. standard tomatoes, and
4. those not suited for canning.

At the station after peeling the categories were:

1. choice peeling tomatoes,
2. standard peeling tomatoes,
3. tomatoes which should be returned for processing into products, and
4. those which were unfit for use.

During the course of the season, about 290 truckloads of round tomatoes and 235 loads of hand-picked coreless varieties were sampled.

In addition to providing basic information on peeling losses and the percent choice and standard canning tomatoes by grower and variety, some of the other uses of these data were to determine peeling loss by week, the correlation between broken and cracked tomatoes and peeling loss, and the effect of selection on the quality of the peeled tomatoes.

Another investigation was attempted using the general sampling method described above. It was desired to determine the effect on peeling loss and general quality from leaving tomatoes in the receiving yard for more than 24 hours. It was thought that an estimate of the loss due to deterioration before processing could be obtained. After entering the yard, one-half of a truck load would be dumped and peeled immediately and the other half of the load left approximately 24 hours before peeling. The sampling and evaluation procedure was the same as above. Unfortunately, the results were inconclusive, perhaps because the loads selected were not from the same growers or of the same variety and/or no attempt was made to select loads of the same general degree of ripeness. Also, since only one load a day was designated for sampling, weather conditions varied. As a result, some tomatoes improved in peeling quality while standing, while others remained about the same or deteriorated in various degrees. One load, in fact, was virtually unfit for peeling the second day, although they were of fair peeling quality the first day. This area of deterioration and losses (or as the test showed, possible improvement in peeling quality) over time requires more investigation since these are important considerations for the optimal utilization of the raw fruit.

Another concurrent investigation was the evaluation of two methods of handling tomatoes after peeling but before canning. By using the sampling procedures described above immediately after the peeler and immediately before the canning table, it was determined that unnecessary losses were occurring with one method.

An analysis was made to determine if it was possible to estimate the difference in the quality of peeled tomatoes caused by selecting the tomatoes before peeling rather than peeling all the tomatoes except the culls. However, there was little, if any, difference in the percent by count of choice peeling tomatoes before and after selection. The samples of 50 selected tomatoes were significantly heavier than the corresponding samples of tomatoes before selection. Although the broken and cracked tomatoes were not counted in the before selection samples, spot checks indicated that selection reduced the amount of damaged fruit and the severity of the damage. The general conclusions were that selection did not improve the peeling quality of the fruit with regard to appearance except when the general level of choice peelers was low. The tomatoes selected were of a more uniform size, somewhat larger than the average of the load and less likely to contain cracks or breaks. The latter would tend to reduce peeling loss and the former would be indicative of possible savings in the coring and canning operations. Unfortunately, these considerations are intangible and very difficult to evaluate from a cost or operational standpoint. The percents of tomatoes suitable for canning before and after peeling used in the model are listed in table 7-3.

The question of how much effect the selection of tomatoes for peeling has is compounded by the fact that it is possible to change the rate of flow of tomatoes past a selection crew of fixed size. This could cause extensive changes in the net effect of the selection.

At the time the model was constructed, data was not available by grower or variety for two important raw product characteristics which were incorporated in the model with the expectation that they would be obtained by a continuing data collection system. These are the pulping and finishing losses for product tomatoes and the insoluble solids content, both of which can play an important part in the allocation of raw material to commodity class. Similarly, peeling loss data was not available for the pipe peelers from the 1964 data. Peeling losses in the pipe peeler for the coreless varieties were assumed to be the same as the peeling loss in the "Dole" peeler, and peeling and coring losses (from the use of the auto corers) for round varieties were assumed to be the same as those of the FMC peeler. These assumptions should be confirmed or modified by actual data.

The plant layout, machine capacities, labor standards, etc. were required to develop the processing capacity restrictions and matrix coefficients. This type of information was obtained from the TVG Industrial Engineering Department. Development of this portion of the model started with construction of diagrams from the equipment descriptions and floor plans. At the same time, a list of equipment capacities was developed for both maximum rated capacity and full operating capacity. After the various possible product flows from the receiving yard to the warehouse entry were outlined in Scematic form, the capacity of each equipment in the product flow path was evaluated. The equipment with the smallest capacity in each flow was obviously a capacity restriction and required a restriction in the plant LP matrix. In addition, any equipment that could be used in two product flows concurrently, either because of the divisibility of the equipment or because two commodities had a common product flow path through part of the plant also required the use of a capacity restriction. An additional group of machinery restrictions outlined at this time were those which were definitely not restricting but were of such a nature that if other capacities were enlarged, the machines would be restrictive. An example of this is the syrupers which were not restrictive since they had more capacity than the continuous flow cookers which they preceded. However, by adding cooker capacity until it exceeded the syruper capacity, the syruper would become the effective restriction. This type of potential restriction was recorded and placed in the LP plant model. (They were removed prior to the computer runs to reduce the size of the plant matrix and computer running time.)

In developing the restrictions, one problem area was defining the different possible product flows where there were several pieces of equipment for the same task and several raw materials or finished products being processed. This is serious if the equipment units vary in size and the rates of product flow vary. If the product flow paths are such that the alternatives are sharply limited, i.e., by physical location, piping, or fixtures, it might be possible to represent all the distinct alternatives in the model. This was done for the round whole peel operation at plant four which is discussed in Appendix B. On the other hand, if the number of combinations is large or if some of the possible layouts are quite complex, it might be more desirable or even mandatory that the number and complexity of the product flows be somewhat simplified and the LP solutions be inspected to see if any impossible combinations are required. If they are, either the solution or the model should be corrected. It was necessary to take an approach of this type in handling the evaporators at the larger plant. There were six evaporators which could be used on various products either in parallel or series. The model did not attempt to define all the possibilities; rather, evaporation was handled with two machinery restrictions. Although this was an oversimplification of the present system, in this case the plant could be modified to handle many additional combinations if desired.

A somewhat similar problem was posed by the fact that for some product changeovers a line could be stopped for a number of hours for equipment adjustments. Although it would have been possible to include down time in the model, this would have increased the complexity of the matrix considerably, so it was deemed more efficient to leave line changeovers out of the model and rely on inspection of the solution to discover if the required number of changeovers would take too much time.

Labor standards were obtained from the TVG Industrial Engineering Department. These were used in conjunction with the hourly wage rates to obtain variable labor costs per unit for use in the objective function. In a few cases, these standards were used in determining the product flow rate, for example, the coring rate on the autocorers.

Using the rated capacity of the evaporators as restrictions required adjusting the evaporation coefficients in the matrix. The efficiency of the evaporators depend on a number of factors, one of the most important being the density of the pulp. Tomato pulp evaporator capacity is typically rated in pounds of water removed at a density of 13 percent tomato solids. However, as a general rule, more water will be evaporated per hour at a lower percent solids and less at a higher percent of solids. Hence, the total amount of evaporator capacity utilized can vary when an identical amount of water is evaporated when making different products. Sauce has a content of 11 percent solids, so the evaporator is always removing water faster than the stated rate for 13 percent solids. Paste has about one and one-half times as much water removed after the pulp reaches a density of 13 percent solids than before so the total capacity when the evaporator is used for paste is less than the rated capacity. These differences were accounted for by adjusting the coefficient in the matrix representing the amount of water to be removed for each product rather than adjusting the evaporator capacities. Cost of evaporation was then considered a constant value per unit of evaporator capacity.

The information on the required fill weights and quality standards was obtained from the TVG Quality Control Department. Target fill weights and actual fill weights were inspected and compared. The model was constructed using the target weights as the basis for the matrix coefficients. However, during some parts of the season, the targets are more difficult to meet due to the quality or mix of available raw product. Under these conditions, it might be necessary to either change the coefficients to bring them more in line with what is practical or possible to attain or to increase the costs to account for the increased labor required to reach the standards.

Production target quantities for the 1965 pack were obtained from the TVG Sales Department. In addition to the target number, it was determined whether the targets were considered minimum desirable amounts or maximum desirable amounts. Target ranges rather than single target quantities are desirable. Inventory position as such was not obtained, but this had been considered in the formulation of the production target. However, it would be possible to obtain only the sales targets for the coming year and adjust them for inventory position either internally in the master program or in the data development and maintenance phase of the LP model. The out-of-stock items were also obtained from the Sales Department and formed the basis for the production requirements in the early weeks. If there had been contracts with specific delivery dates, this information would also be obtained from sales and included in the model at this stage.

The expected sales prices of the commodities were obtained from the TVG Sales Department. The gross sales price was adjusted for direct variable costs for use in the model. Standard cost data for materials such as tin cans and labels were obtained from industry cost estimates made by the accounting firm of Touche, Ross, Bailey and Smart.

Estimates of other expenses such as brokerage, cash discounts, and selling expense were based on the percentages used by the same firm in determining standard costs. A few items, such as drums for tomato paste, were not available in the industry cost estimates. These were obtained from the TVG Purchasing Department.

A continuing program to obtain the costs of material would improve the cost figures in the model since the standard cost figures are based on industry averages, and there are undoubtedly differences in most material costs between firms and in some cases between plant locations of a single firm.

APPENDIX B TO CHAPTER VII

DETERMINATION OF MATRIX COEFFICIENTS

When the information on the available raw product was analyzed, it was found that fruit was expected from nearly 80 growers with 106 distinguishable grower-variety combinations. The total number of combinations was somewhat higher than this originally but was reduced by eliminating strains of tomatoes for which little data on the raw product characteristics existed and could not be obtained. Allowance was made for this production by assigning it to that grower's dominant variety. Similarly, a few growers with very small amounts of production were combined with other growers with the same varieties in the same area.

Using 106 grower-variety combinations would have resulted in 71 tomato inputs during one week of the season. This was larger than the maximum of 60 inputs which was originally intended. Further consolidation was accomplished by pooling the production of small growers of popular varieties of round tomatoes with the same transportation costs and similar picking schedules. Upon completion, there were 96 grower-variety combinations. The largest number of tomato inputs in any week was 64. The number of grower-variety combinations and the expected tonnage of raw product in each week is shown in table 7-1.

Next the raw product characteristics of each grower were determined. The basis for estimating the characteristics was the information obtained by sampling in 1964 as explained in Appendix A. However, the sample size was not large enough to make estimates by individual grower. To obtain a reasonable range and mix of raw product characteristics, a random number generator was used and sets of random numbers assigned to each grower. Table 7-2 has the means, standard deviations, and ranges of each characteristic as well as deviations from the means due to variety and/or week if applicable. Values falling beyond the allowed range were set equal to the value of the range limit. Seasonal values are found in the first three lines of the table. The next portion of the table has the varietal deviations for the percent of solids and fruit size. The bottom portion has the deviations from the seasonal averages of the percent solids, percent choice tomatoes and percent peeling loss by week. The percent of solids is below the average at the beginning of the season and rises until it is above average at the end of the season. The percent of choice peelers is lower at the beginning and end of the season while the peeling loss is higher at the beginning and end of the season.

Raw Product Input Vectors

Figure 7-1 is a general schematic of the LP models used for weeks two through nine. The first section consists of the raw product input vectors.

It was originally intended to generate a set of input vectors for each week including all the grower-variety combinations produced during that week. However, since weeks four through seven all had identical coefficients, only one set of vectors was generated for each grower-variety combination for those weeks.

Since the IBM 1620 LP code allowed the use of column identification files to select the vectors for a given problem from a larger set of vectors on disk storage, this procedure was used to furnish the raw product input vector portion of the matrix from the single set of vectors for week four through seven. In addition to the reduction in the necessary data handling, the problem loading time and the computer storage area required were reduced.

As indicated in table 7-2, the other six weeks had different average percent of canning tomatoes and solids so the input vectors would not be identical. However, rather than generate six distinct sets of vectors, one group of vectors was generated for the first three weeks of the season and another group for the last three weeks. The raw product characteristics for the grower-variety combinations used in developing the input vectors were those for the week with the largest expected production of that particular grower-variety combination.

Each grower with round tomatoes had six possible input vectors. Round tomatoes could enter the program through the FMC peeling operation at plant four (which always required selection before peeling), through the pipe peeler at plant four with selection before peeling, through the pipe peeler at plant four with no selection before peeling, into the sauce and catsup operation at plant four, or into the paste, puree, or juice manufacture at either plant three or plant four. If the percent of choice peelers was less than 20 percent, the three whole peel input vectors were not used. There were three input vectors for each grower of mechanically harvested tomatoes. These were for the sauce and catsup operation at plant four, and the paste, puree, and juiceooperation at either plant three or plant four. Those coreless varieties which were picked by hand had each six input vectors. In addition to the three product vectors listed above, they could be peeled at plant four (without selection) or peeled at the plant three with or without selection. Hand picked coreless varieties with less than 20 percent choice peelers did not have the three peeling input vectors. The total number of tomato input vectors for individual weeks ranged from 105 in the first week to 306 in the fifth week.

Table 7-4 has a set of input vectors developed for a lot of round tomatoes using the characteristics for week five, and table 7-5 has the set of input vectors developed for the same tomatoes for using the characteristics for week two. Table 7-6 has a set of input vectors developed for a lot of handpicked coreless varieties during week five. The raw product characteristics for each lot are found at the bottom of the appropriate table.

These vectors and the coefficients are similar to those outlined in the appendix to Chapter V, although a number of refinements have been added. In this case, one unit of a vector is 1,000 pounds. In table 7-4, the first two rows are the objective function values. (Two objective functions were needed because of the manner in which the decomposition technique was applied.) The differences among the six objective function values are due to yard costs and transportation.

The development of the objective function rows are covered in greater detail in Appendix C.

The next row, titled Grower Restriction, is the restriction limiting the amount of tomatoes which can be obtained from a grower.

The next three rows have entries only in the first input vector, the one for tomatoes selected for the pipe peeler. The entries (as are most of the entries pertaining to quantities of raw products) are in 1,000 pound units. The .404 in the select choice row represents 404 pounds of choice tomatoes, the adjusted quantity available from 1,000 pounds of this lot of tomatoes. The .3131 in the standard row represents 313.1 pounds of standard tomatoes available. The .0101 in the correction row represents 10.1 additional pounds of standard peeler tomatoes available. This entry and the last two entries are correction factors to correct for the difference between this lot of tomatoes and the "average" 40 percent peelers. The other two entries (.0028 in the juice row and -.01 in the waste row) indicate that there will be 2.8 pounds more of tomato juice obtained from this 1,000 pounds of tomatoes than the average and that there will be ten pounds less waste to dispose of than the average.

The coefficients were obtained by the following procedure. The raw product percent choice was rounded to the nearest ten percent value between 20 and 60 percent. In this case, the raw product had 35 percent choice and was rounded to 40 percent. The basic value used for tomatoes entering the 40 percent row was .4 or 400 pounds. The regular percentage of standard tomatoes from a grower with 40 percent choice tomatoes was 30 percent or 300 pounds which gave a coefficient of .3. However, these tomatoes had a peeling loss of 15 percent instead of the season's average of 16 percent and a percent of standard tomatoes of 31 percent.

Tomatoes with a lower peeling loss than average will have a higher weight after peeling. Since the identity of the tomatoes is lost in the model before peeling because of the blending activities, an adjustment for differences in peeling loss is made in the input vector. In this case, the coefficients in the choice 40 percent and the select standard rows are increased by the appropriate amount, one percent, to .404 and .3131. The select standard correction row sums across all the select input vectors (FMC and pipe peeler selection activities) for the week, to find the total adjustment for standard tomatoes. This is the quantity greater or less than the amount of standard tomatoes available if only the average was used. The entry in the juice row represents the increase in the amount of juice available due to the increase in tomatoes which are peeled but not canned. The -.01 in the waste row adjusted the total waste to allow for the below average peeling loss. All 1,000 pounds of tomatoes are not represented here, i.e., the sum of the coefficients is not equal to one. The remainder is accounted for in the transfer activities to be discussed later.

The next two vectors are very similar to the first one. In fact, the second vector is identical to the first except that entries are in the non-select 40 percent row, the non-select standard row and the non-select correction to standard row rather than the select rows of the same names. Each peeling operation has a 20 percent choice row, 30 percent choice row, etc., through a 60 percent choice row, but there is only one standard row and one correction to standard row for each type of peeling operation.

There are different coefficient values in the juice row and the waste row for the FMC peeler input vector. This is because the FMC peeler had a selection rate significantly different than the other peeler. It was also necessary to make further adjustments for the FMC peeler input. The most notable is the entry in the solids row. This indicates that for every 1,000 pounds of tomatoes going to the selection process for the FMC peeler, there were 35.11 pounds of tomato solids available for products. (In the other peeling operations, it was assumed that those tomatoes not selected or canned were converted to juice. However, using this assumption with regard to the FMC peeler would yield too much juice.) The juice and waste row coefficients are also adjusted accordingly for the FMC peeler inputs.

The next input vector in the table is the paste, puree, or juice input activity at plant four. The first three entries have the usual interpretation. The entry of .0504 in the RSOL55 row (5.5 percent tomato solids at plant four) indicates that 50.4 pounds of tomato solids are obtained from 1,000 pounds of this lot of tomatoes. These tomatoes have 5.6 percent solids and are entered in the corresponding (5.5 percent) row. Since there is a ten percent pulping and finishing loss, the amount of tomato solids available is 50.4 pounds per 1,000 pound input of tomatoes. The .10 entry in the waste disposal transfer row represents the 100 pounds of waste obtained from a ton of tomatoes used in this manner. The reason that this coefficient in the waste row is larger than the corresponding waste coefficients in the whole peel input columns is the fact that all of the waste from products is entered at this point, while the entries in the whole peel inputs are only correction factors.

The next two entries are in machinery restriction rows. The right-hand side value of most of the machinery restrictions is expressed in units of three and one-half hours corresponding to one-half of a normal shift. (For those machines which operate continuously, a four hour unit was used.) The usual right-hand side value for the machinery restrictions was 36 corresponding to operating three shifts a day for six days during the week. The .003937 is derived from the box dumper's capacity of 36 tons per hour or 252 1,000 pound units per period. Each 1,000 pounds of tomatoes dumped in the box dumper utilizes .003927 of one period. Similarly, the pulpers have a total capacity of 80 tons per hour. Since they run continuously, one time period is four hours. A total of 640 1,000 pound units can be handled in a four hour period so the resulting coefficient is .001562.

The sauce and catsup input vector is identical to the plant four paste vector with two exceptions. There is an entry of .05544 in the RSAU55 row. This corresponds to the entry in the RSOL55 row in the paste input vector. The difference in the two coefficients results from the fact that the insoluble solids as a percent of the total tomato solids are higher than average. This results in higher than average case yields when used in catsup or sauce. Tomatoes with 11 percent insoluble solids were estimated to yield 110 percent as much sauce or catsup as tomatoes with average insoluble solids. Because the identity of the tomatoes is lost in the blending activities portion of the matrix, the increased yield is accounted for by increasing the coefficient in the sauce row. Instead of using the actual tomato solids available from a 1,000 pound lot of tomatoes, the figure used is the amount of tomato solids with the average percentage of insoluble solids necessary to yield the same amount of sauce or catsup as this lot of tomatoes.

The entry in the sauce evaporation row is a correction factor. The total quantity of water which would be evaporated from this lot differs from the amount required in the evaporation activities for sauce because of the difference in insoluble solids. More water has to be removed even though the yield is higher because more sugar solids are added to the high yielding lots.

The last vector is for the input of tomatoes for paste and puree at plant three. The first three entries have the usual interpretation while the entries in the plant three solids row and waste row are identical to the corresponding entries in the paste vector for plant four. There are no machinery restriction entries necessary here because the capacity bottlenecks at plant three are in a different part of the matrix.

Table 7-5 contains the input vectors for the same grower's tomatoes during week two. The format of the table is the same as for table 7-4, and many of the coefficients are the same. However, since the raw product quality of the tomatoes changes during the season, those coeffients directly related to the raw product quality characteristics are changed.

In the first input vector corresponding to the entry in the select choice 40 percent row, in table 7-4 is an entry in the select choice 30 percent row. (The percent of choice tomatoes was reduced by five percent in week two). This entry is .297 which is slightly less than the .3 which is the basic entry. There is a larger peeling loss in week two than week five, resulting in less tomatoes being available after peeling.

The entry in the correction to standard row is based on the difference between the actual level of 31 percent standards and the computed value of 35 percent standards. The -.0396 is the difference adjusted for the 17 percent peeling loss in week two. The juice entry is based on the increase in the quantity of tomatoes going directly to products without being peeled because the total quantity of choice and standard tomatoes is less than average. The .01 in the waste row is the additional waste due to the above-average peeling loss.

The entries in the column for non-selected tomatoes being peeled in the pipe peeler are analagous to those just discussed. The entries in the FMC column are also similar. However, the solids entry is in the 5.3 percent solids row and is smaller than the entry in table 7-4. The solids level of this lot of tomatoes was lower in week two than in week five, which accounted for most of the change. The remainder of the difference is due to the difference in the quantity of standard tomatoes available for selection.

The only changes in the last three columns are the coefficient values in the 5.3 solids and 5.3 sauce solid rows. These differences are due to the tomatoes having 5.4 solids in week two instead of the 5.6 solids of week five.

Table 7-6 has a set of input vectors for a lot of hand picked coreless tomatoes. (The vectors of mechanically harvested lots were similar except that there were no input vectors for whole peel activities. Other differences included a slightly changed objective function formula and no dumper restriction as the bin dumper for mechanically harvested tomatoes had surplus capacity.)

The first three rows of all the columns are the usual objective function rows and grower restriction. The next entries in the choice rows are developed in the same fashion as for the rounds. In this case, the .515 entries rather than .5 are due to the tomatoes having a 13 percent peeling loss. The additional yield of peeled tomatoes is approximately equivalent to .515 units of 50 percent choice peelers. The other entries in the column are obtained in the same manner as the values in the whole peeled vectors. The coefficients in the next two columns are identical to those in the first column although appropriately entered in other rows. The entries in the three product input vectors are obtained in the same way as the entries for the rounds.

Blending Activities

The next area on figure 7-1 is the blending activity area. There were six sets of blending activities for the whole peel operation and three for the products operation. These were:

1. Round select pipe peeler.
2. Round non-select pipe peeler.
3. Round select FMC peeler.
4. Coreless non-select at plant four.
5. Coreless non-select at plant three.
6. Coreless select at plant three.
7. Paste, puree, juice at plant four (rounds and 145's were blended together).
8. Sauce and catsup (rounds and 145's were blended together).
9. Paste and puree at plant three (rounds and 145's were blended together).

The blending was handled in the same manner as in the small test problem. For numerical examples, see the appendix to Chapter V describing the test problem which covers blending in detail. In the full-scale problem more types of material were blended and there were more rows for each type of material. There were five choice rows for each type of whole peel input. There were only ten possible blends of two percentages which would give another included even percentage level, i.e., 20, 30, 40, 50, or 60 percent. These ten blends were all included as activities. However, there were 11 solids levels on .2 percent intervals between 4.5 percent and 6.5 percent, inclusive, for each of the three product categories. This meant that there would be 169 possible blends for each product category. Originally, all the possible blends were developed and placed in the matrix. However, machine storage capacity limitations forced a reduction to about one-third of this number. This was done by removing every second and third blending vector except that certain "key" vectors which would be difficult or impossible to duplicate by using a combination of two or more blends were left in. It has not been determined whether this had any particular effect on the solutions. However, in retrospect, it seems reasonable that generally if matrix size presents a problem, it would probably be more appropriate to reduce the number of solids categories and include all the appropriate blending activities. It would seem that unless there were many distinct alternative product lines, five general levels of solids content would have been sufficient (although perhaps the absolute level of these categories should be changed if the average level of solids changes during the season.)

Peeling, Evaporation and Transfer Activities

The next area in the matrix illustrated by figure 1 includes the peeling, evaporation, and transfer activities. There is a set of five peeling activities and two transfer activities which peel the fruit, consolidate it as raw material for canning and complete the corrections of each of the six types of whole peel inputs. Table 7-7 contains the set of peeling and correction activities for the non-select rounds.

Each unit of an activity represents the peeling of 1,000 pounds of tomatoes. All the coefficients in the table are also in units of 1,000 pounds of material. Choice and standard tomatoes, juice, and waste are placed in the appropriate rows for transfer to the production activities. In addition, the appropriate adjustments are made to yields based on the previous entries made in the correction rows.

The objective function entries for this type of whole peel operation are based on the labor costs of operating the autocorers. Note that the cost increases as the percent of choice tomatoes increases. This is because there are more tomatoes suitable for canning, and therefore more which must be cored per 1,000 pounds of input.

The next entries are in the various non-select rows. The entries here, for example, -.2 in the 20 percent row, have the effect of removing 1,000 pounds of 20 percent choice tomatoes from this input row and peeling them. As a result of peeling and coring, there are 168 pounds of peeled choice tomatoes to be transferred to the canning operation. This is represented by the .168 entry in the row labeled RCHRDS. The .168 pounds is the expected yield of peeled choice tomatoes from

1,000 pounds of 20 percent choice, i.e., 200 pounds less the expected 16 percent peeling loss. The next two entries in the first column represent the amount of standard tomatoes which are expected to be peeled (.4 or 400 pounds) and the amount of peeled standard tomatoes to be transferred to the canning operation. The entry in the juice row represents the quantity of tomatoes which will be sent to the products operation rather than the canning lines. The quantity of tomatoes which were not of choice or standard quality is 400 pounds in the 20 percent choice peeling activity. However, in addition to the 16 percent peeling loss reduction, this quantity must also be reduced by the pulping and finishing losses. A combined loss factor of 20 percent was used. This resulted in a net amount of 320 pounds of tomatoes available for transfer to the juice (products) operation. (For all types of peeling activities except the FMC operation, it was assumed that it would be adequate to put the non-cannable tomatoes directly in the juice row. The FMC selection rate was low; hence, the quantity of non-selected tomatoes would have resulted in more juice than required to support the whole peel operation. The non-selected tomatoes in the FMC input activities were converted to solids which could be used in the paste activities.

The next entry is in the waste row. This is based on a 16 percent peeling loss, plus an additional 4 percent pulping and finishing loss for those tomatoes which were peeled but not canned. In the first column, the waste was 176 pounds, 160 peeling loss and an additional 16 pounds of loss in the pulping and finishing process.

The last entries are in the machinery restriction rows. Note that the ripe peeler (RPIPPL) row entries are the same for all the peeling activities. This number is the portion of one three and one-half hour time period that is required to peel 1,000 pounds of tomatoes so the quality of the tomatoes has no bearing on the coefficient. However, the other two restrictions have different values for each quality level of tomatoes peeled. The corer coefficients increase as the number of tomatoes to be peeled increases, and the pulper coefficient decreases as the number of tomatoes sent to the products operation decreases. Round tomatoes were considered to average 3.1 tomatoes per pound. In the 20 percent choice peeling activity, there are 600 pounds of tomatoes to be cored or 1,860 tomatoes. One autocorer operator can core about 8,400 tomatoes in a three and one-half hour period so 1,000 pounds of these 20 percent choice will utilize .22143 of a period. For those tomatoes which were of a different size than 3.1 per pound, a correction to the autocorer capacity row was included in the input vector. The tomatoes used for an example in table 7-4 were of average size. If, for example, they were heavier and were only three to a pound, there would have been an entry of -.008331 in the RCORER row in the non-select pipe peeler input vector in table 7-4 since a smaller number of tomatoes would be cored per 1,000 pounds. The pulper coefficient is the amount of pulper capacity utilized in processing the 336 pounds of tomatoes which go to the juice operation.

The two columns on the right side of table 7-7 are the correction columns which connect the "correction to standard" row with the "standard" peeled transfer row. In table 7-4 there was an entry of .0101 in the "correction to standard" rows since the quantity of standard quality tomatoes in the lot was higher than the usual quantity for 40 percent choice. In table 7-5 there was a negative entry in this row since the quantity of standard quality tomatoes was less than usual. If the total positive and negative quantities entries cancel out in the final solution, there is no problem; but since this is unlikely, the quantity of standards available for canning has to be adjusted. If the net quantity of standard tomatoes available is less than that for which the input activities have been computed, the NOSTCM vector will be forced into the solution at a positive level. The positive level will be equal to the total quantity of standard tomatoes of which the input vectors are deficient. The 1.0 in the RNONSD row is necessary so the row can meet the condition of equality but has no special significance. The next entry, however, -.84 in the standard peeled tomato row, has the effect of reducing the peeled standard tomatoes available for canning. This coefficient is -.84 and not -1.0 because the yield of peeled standard tomatoes averages 84 percent because of the 16 percent peeling loss. The next entry increases the quantity of tomato juice by 800 pounds since the non-standard tomatoes are available to the juice operation. The quantity of waste increases by 40 pounds since the tomatoes sent to juice have a further pulping and finishing loss. The last entry is the amount of pulping capacity utilized by 840 pounds of tomatoes. The next column NOSTCP is for the situation when there are excess standard tomatoes. This column has the effect of increasing the quantity of standard tomatoes available for peeling. The only difference in these columns is in the signs of the coefficients.

Table 7-8 contains the activities for peeling the selected rounds in the pipe peeler. In determining the coefficients, it was assumed that 15 tons of tomatoes were sent to the selection process per hour and that a selection crew large enough to keep the autocorers operating at capacity was maintained. This meant selecting about 7.74 tons per hour or that 1939 pounds of tomatoes were processed to obtain 1,000 pounds of selected tomatoes.

The -.388 in the select 20 percent peeling activity represents 388 pounds of choice tomatoes. This is the quantity of choice tomatoes in 1,939 pounds of 20 percent choice tomatoes. The other entries in the choice input rows are obtained in the same manner. The entries in the select standard (RSELSD) row are obtained similarly. The quantity of choice tomatoes in 1,000 pounds of fruit selected from 20 percent choice is assumed to be 320 pounds (see table 7-3). After adjusting for a 16 percent peeling loss, the quantity of choice tomatoes remaining is 268.8 pounds which has a coefficient value of .2688. Similarly, from table 7-3, the percent of standard tomatoes selected is assumed to be 40 percent or a quantity of 400 pounds. Deducting 64 pounds peeling loss gives a coefficient value of .336.

The value of the juice coefficient is determined from two factors: the quantity of tomatoes going directly to the peeling operation (which has a ten percent pulping and finishing loss) and the quantity of tomatoes which are peeled but not canned (which have a combined peeling and pulping and finishing loss of 20 percent. The quantity going directly to the peeler is 939 pounds for all levels of choice tomatoes. These two quantities are combined to give the quantity of juice available.

Similarly, the waste row coefficients are composed of three parts: the portion from peeling 1,000 pounds of tomatoes, the portion from the pulping and finishing of the non-selected and the peeling and finishing losses on the noncannable tomatoes.

The pipe peeler row coefficients are the same as in table 7-7 for non-select tomatoes since 1,000 pounds of tomatoes are peeled. However, the autocorer coefficient is larger because more tomatoes are suitable for canning and cored. The pulper coefficient includes the capacity required by the 939 pounds of tomatoes sent directly to the products operation and the quantity of noncannable tomatoes sent to products.

The correction columns (SESTCM and SESTCP) to adjust for an above or below average percent of standard tomatoes are similar to those in the table 7-7. However, since all the tomatoes were not peeled, the net effect of the correction is not as large. It was assumed that the reduction or increase in the quantity of standards going to the peeler was proportional to the quantity of tomatoes selected. In this case, 51.6 percent of the tomatoes were selected, so the adjustment in the rows was based on 51.6 percent of the difference in quantity. The coefficient in the standard rounds row of .433 is 84 percent of .516, i.e., the correction less the peeling loss.

Table 7-9 has the peeling and transfer activities for tomatoes peeled by the FMC peeler. The most important difference between the vectors representing this peeling operation and the one just discussed is in the selection rate. In determining the coefficients it was assumed that the tomatoes were going to the selection table at a constant rate of 15 tons per hour and that the FMC peeler had an average of 90 percent of its flights utilized. This meant that 4.55 tons of tomatoes were selected and peeled per hour when the tomatoes averaged 3.1 per pound. The selection rate was considerably less than for the previous set of peeling activities. In this case, 3,296 pounds of tomatoes were input for every 1,000 pounds peeled. Using these factors, the coefficients are determined in the same manner except for the juice row entry. As was mentioned previously, to send all the tomatoes not canned to juice would have resulted in more production of juice than could be utilized in the whole peel operation. Consequently, the only entry in the juice row was for those tomatoes which were peeled but not canned. The FMC input vectors contained an entry in the appropriate solids row to account for the tomatoes which were not selected and sent directly to products.

It should be noted that both of these sets of peeling activities are based on fixed selection rates and the expected results of selection. These rates are assumed to be close to the actual operation, but it must be recognized that there are many possibilities for deviation from these rates in practice. An investigation to determine how sensitive the program is to changes in these rates could be accomplished by running the problem with the peeling activities changed to allow for different selection rates and comparing the results.

The peeling and transfer activities for the hand picked coreless varieties are developed in the same manner as for the rounds. The selected coreless peeling activities have the same general selection rates as the rounds selected for the pipe peeler with the exception that the selection rates and costs are based on an average of 4.2 tomatoes per pound. Because of the similarity, a detailed explanation of the coreless peeling vectors is not included here.

The next group of transfer activities are the juice activities. There is one set of activities for each plant. The juice transfer activities are of the same form as those in the test problem matrix illustrated in the appendix to Chapter V. As was noted there, these columns are used to reduce the total number of activities required in the overall plant matrix, and there is no objective function value. One change (in addition to the increase in number to 11 categories) was necessary. The quantity of juice obtained from the tomatoes which were sent to products after having been peeled was sometimes substantial. In order to maintain the juice row as an equality and not be faced with the possibility of disposing of juice either as waste or a free good under some conditions, an activity was included to transfer juice to the solids rows. This was done by merely making an entry of -1 in the juice row and entering .045 in the 4.5 solids row. (It was necessary to use a low solids content to eliminate the possibility of tomatoes with low solids being put into the solids rows, transferred to the juice row, and then transferred back to products, but into a higher solids row, effectively increasing the solids content of the tomatoes.)

The next activities are the evaporation activities which are also very similar to those in the test problem discussed in Chapter V. There were 12 sets of evaporation activities. These included sets for 1.045

puree, 1.06 puree and 25.5 percent paste at plant four and a set of activities for 25.5 percent paste at plant three. In addition, there were separate evaporation activities for the three grades of catsup, for three levels of sauce, and for chili-sauce and for pizza sauce.

Miscellaneous Activities

The next activities are grouped under the heading of "miscellaneous" in figure 7-1. These included several machinery transfer activities similar to those in the test problem described in the appendix to Chapter V. There are also activities to downgrade choice tomatoes to standard and standard tomatoes to juice. In addition, there are activities to dispose of juice as waste. This activity is not necessary to obtain a realistic solution and probably would not be in an optimal solution if the product prices are realistic. However, it was included under the assumption that it would allow major changes in the product mix to occur very soon after the product prices were changed in the subproblem. However, there was no practical way to test this hypothesis and the actual effect is unknown, although these vectors were included in some non-optimal solutions.

Another activity in the miscellaneous category was used to produce puree at plant three. This vector activity took material out of the 25.5 percent paste row and put it in a 1.045 puree row. There were positive values in the objective function since the cost of evaporating to puree would be less than the cost to evaporate to paste. In addition, there was a negative entry in the evaporator row since the evaporator capacity used would be less. This column was used in place of a set of activities to evaporate solids to 1.045 puree since it was not expected that puree would be produced in large quantities at plant three, but there was a requirement for using puree in at least one product that could only be produced at plant three. The objective function values and the evaporation row entry in this activity were based on a solids level of the raw product of 5.5 percent. This is, of course, an approximation and could be misleading, especially if the solids content of tomatoes going to plant three was considerably lower. (For the products that required 1.06 puree at plant three, an appropriate entry was made in the 1.045 puree row and asjustments were made to the objective functions and to the evaporator row in the production vectors.

The last portion of the model as diagrammed in figure 7-1 included the production activities. There were a total of 89 production activities for 58 different products. These activities were very similar to those described in the test problem. Upper bounds equal to the seasonal restriction were placed on any activities that could produce the pack target for the season in one week. However, no rows were used to restrict products that could be produced by two or more activities because of the size limitations of the LP codes used.

APPENDIX C TO CHAPTER VII

DETERMINATION OF THE OBJECTIVE FUNCTION VALUES

The objective function values can be divided in the same general categories as the overall matrix in figure 7-1. In the model the actual values were scaled by $100 before being placed in the LP matrix. The objective function values for the grower inputs in the problem were based on the following costs:

1. The raw product cost.
2. The transportation cost to the plant.
3. The cost of handling in the yard and any special handling charges for boxes, bins, etc.

The determination of the objective function values for the input vectors illustrated in tables 7-4 and 7-5 is shown in table 7-10. The raw product cost is the actual cost that is paid for the product. In the problem, this was always $35 per ton. However, this price would also reflect adjustments for different lots of tomatoes if these were known ahead of time. The transportation cost is the charge for hauling the tomatoes to the plant. In the problem, the two plants were located in the same general vicinity and although the trucking cost varied between growers, the costs to each plant were equal from any given grower. Yard costs were the variable costs for unloading trucks and dumping tomatoes. It was impossible to determine these variable costs exactly since minimum crew sizes are generally necessary. Some labor costs, like the scale men and foremen, were considered fixed and not included. The remaining labor costs required at the level of operations assumed for the processing activities were divided by the total tonnage to obtain the yard and receiving costs per ton. In addition, if there were any special treatments or handling of the tomatoes in the yard, the costs would be included here.

The next group of objective function values are the peeling and evaporation activities values. (The blending activities have no costs associated with them in the model.) The objective function costs associated with the peeling activities are composed of the direct labor costs of selecting and handling the tomatoes. It would also be possible to include any direct costs due to peeler operation or differences in the costs of operation of two peelers if sufficient detailed information was available.

The objective function value for the non-select rounds peeling activities are determined by the number of tomatoes per 1,000 pounds which will be cored on the autocorer. For example, a lot of 20 percent choice tomatoes will have about 1,860 tomatoes to be cored. The autocorer operator will core 2,400 tomatoes per hour. At this rate, it will take .775 hours. At an average cost of $2.58 per hour, the cost of this activity will be $2.00 per 1,000 pounds. Similarly, if the lot is of 60 percent choice tomatoes there will be about 2,480 tomatoes per 1,000 pounds to core. This will take about 1.033 hours or a cost of $2.665 per 1,000 pounds peeled. The cost for peeling the selected rounds in the pipe peeler is based on the costs of coring and also on the cost of selecting. The coring costs are determined in the same way as for the non-selected tomatoes. It was estimated on the basis of crew size that it would cost about $2.153 to select 1,000 pounds of tomatoes if they were 40 percent choice or better. The cost is higher if they are of poorer quality since it takes a larger crew size to select out the same quantity of tomatoes. The cost of peeling tomatoes selected from 40 percent rounds is $4.736 of which $2.583 is coring cost and $2.153 is selection cost. The FMC costs are figured similarly. There is no coring cost since the tomatoes are cored by the peeler, but there is additional cost besides selection since the tomatoes must be positioned stem end down before the peeling and coring operation.

There are no objective function costs for the non-select coreless tomatoes since there is no coring involved. The cost of the peeling activities for the coreless select tomatoes was based only on the costs of selection which were somewhat higher per 1,000 pounds than for the rounds. The total number of tomatoes which must be selected to obtain an equal weight of the coreless variety is higher.

There are no costs associated with the juice transfer activities. The costs associated with the evaporation activities are based only on the cost of evaporation. It was computed on the basis of $1.06 for every thousand pounds of water (or the computed water equivalent for the sauce and catsup evaporation activities) to be removed.

The most complex objective function values were those of the finished products. These included the following items:

1. Expected sale price.
2. Variable labor cost associated with the production activities.

3. Warehousing, freight and delivery costs.

4. Material costs for cans, cases, labels, condiments, etc.

5. Allowances for brokerage, discounts, swells, etc.

The expected sales price was obtained from the Sales Department. The last three items were obtained from the industry cost estimates of Touche, Ross, Bailey and Smart for tomato products for all the products included in these reports. Those items which were not available from that source were obtained from Purchasing Department records. Since an individual firm's costs will vary from the industry average, this is an area where further work is needed, either to develop the firm's own costs from purchasing records and accounting data or to determine whether the firm's costs vary sufficiently from the average to make the cost data collection worthwhile.

The most difficult factor to determine was the variable labor cost. This included all the labor that could be isolated by product after the coring operation (or after peeling for coreless tomatoes) and before warehousing for whole peel products. For the products operation, it included all the labor expended after the dumping operation until warehousing. When a worker might be involved in two operations at once, (for example, running a cooker processing two grades at once,) the costs were prorated on the basis of effective capacities. (Maintenance and repairs were regarded as fixed costs and not considered in this analysis.)

For example, the direct labor costs of canning #10 cans at plant four on line five were determined in the following manner:

> The joint labor cost of preparation of tomatoes and handling empty cans, etc., per hour for peeled tomatoes is $56.90. The capacity of line five (#10 cans) is 7.5 tons per hour. The total capacity

of the whole peel operation is about 2 tons per hour. The direct labor cost associated with line five is $11.53 per hour. The production rate is 300 cases per hour.

#10 share of joint cost 7.5T x 56.90	$20.32
Direct labor cost	11.53
Total cost per hour	$31.85

Labor cost per case = hourly labor cost ÷ 300 $ .10617

This figure was increased by 15 percent to include the costs of fringe benefits which averaged 15 percent of the hourly wage costs so the final direct labor cost per case was $.1221.

The direct labor costs for products are generally not as involved as for whole peel tomatoes. Joint cost did not enter the computations since the only labor cost considered was the cost of manning the product lines which accounted for the major portion of the labor costs. The total labor cost per hour for a product line was divided by the number of cases of expected production per hour to obtain the cost per case.

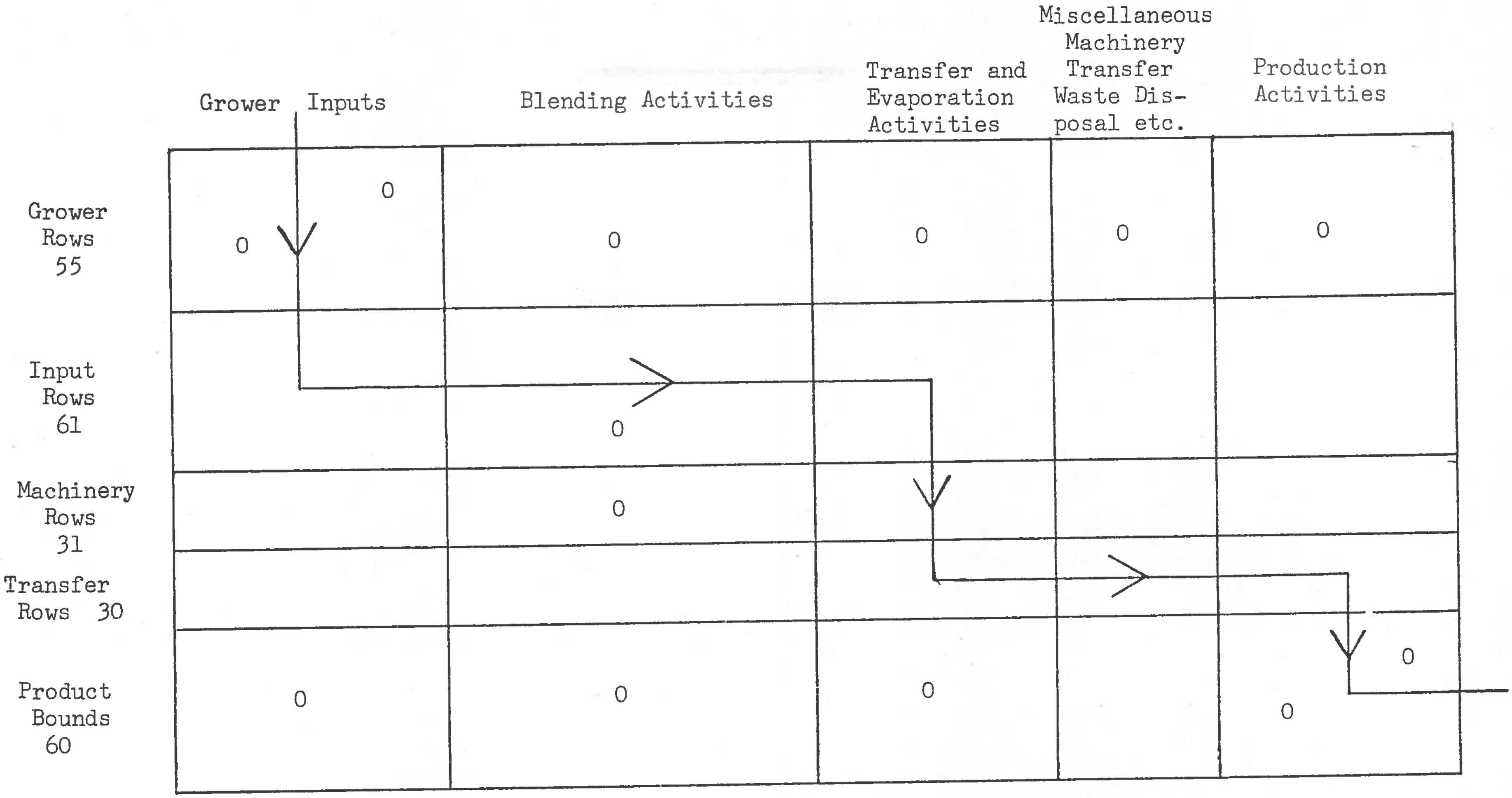

Figure 7-1

Flow of Tomatoes Through LP Plant Model

Table 7-1. Available Raw Product by Week

	Week									
	1	2	3	4	5	6	7	8	9	10
Number of Grower Variety Combinations	20	35	47	56	64	58	55	44	55	37
1,000 lb. Units of Tomatoes	10171	32477	33047	33266	47398	38063	29696	28757	30151	18222

Table 7-2. Raw Product Characteristic Distributions

		Percent Solids	Percent Insoluble Solids	Tomatoes per Pound	Percent Choice Peelers	Percent Peeling Loss	Percent Pulping Loss	Percent Standard Peelers
Season Average		5.5	1.0		35	16	10	[1]
Standard Deviation		.4	.1		10	3	1	5
Allowed Range		4.5-6.5	.8-1.2		To 60[2]			
Deviation by Variety								
(Rounds)	A	.2(.18)[3]		3.1				
	B	.51		4.1				
	C	.2(.26)[3]		3.1				
	I	.0		4.2				
	K	-.29		2.9				
	L	.13		3.1				
	N	.47		3.0				
	O	.2(.19)[3]		3.1				
	P (pear-shaped)	.0	1.2(All Lots)	7.7				
(145's)	D	-.15		4.2				
	F	.46		4.6				
	G	-.05		4.5				
	H	-.32		4.2				
Deviation by Week								
	1	-.2			-10	+4		
	2	-.2			-5	+2		
	3	-.1			-5	+1		
	44	0			0	0		
	5	0			0	0		
	6	0			0	0		
	7	0			0	0		
	8	+.1			-5	+1		
	m 9	+.2			-5	+2		
	10	+.2			-5	+3		

(footnotes on next page)

Table 7-2. (continued)

1/Percent standards equals 50 minus ½ of the percent choice.

2/Tomatoes with less than 20 percent choice were not considered for whole peel products.

3/Varieties A, C, and O were combined. Actual deviation is shown in parentheses.

Table 7-3. Percent of Cannable Tomatoes

Before Selection		After Selection	
Choice	Standard	Choice	Standard
20	40	32	40
30	35	40	35
40	30	48	30
50	25	56	25
60	20	64	20

Table 7-4. Input Vectors for Lot G10A - Week Five (Rounds)

Restrictions	Row ID	Selected for Pipe Peeler	Not Selected for Pipe Peeler	FMC Peeler	Plant 4 Paste Puree Juice	Sauce Catsup	Plant 3 Paste Puree
Objective Function 1	OBJEC1	-.200165	-.200165	-.200165	-.199780	-.199780	-.204610
Objective Function 2	OBJEC2	-.200165	-.200165	-.200165	-.199780	-.199780	-.204610
Grower Restriction	G-10A	1.0	1.0	1.0	1.0	1.0	1.0
Select 40% Rounds	RSEL40	.4040					
Select Standard Rounds	RSELSD	.3131					
Select Standard Correction	RSESTC	.0101					
Non-Select 40% Rounds	RNON40		.4040				
Non-Select Standard Rounds	RNONSD		.3131				
Non-Select Correction to Standards	RNOSTC		.0101				
FMC Peeler 40% Rounds	RFMC40			.4040			
FMC Peeler Standards	RFMCSD			.3131			
FMC Standards Correction	RFMCST			.0101			
Juice Row	RJUICE	.0028	.0028	.00085			
5.5% Solids Row at Plant 4	RSOL55			.03511	.0504		
5.5% Sauce Row	RSAU55					.05544	
Waste Disposal Row at Plant 4	RWAST4	-.01	-.01	-.003034	.10	.10	
Box Dumper Restriction	RBOXD2				.003937	.003937	
Pulper Restriction	RPULPR				.001562	.001562	
Sauce Evaporation Restriction	REVAPS					.0009	
5.5% Solids Row at Plant 3	RYOL55						.0504
Waste Disposal Row at Plant 3	RWAST3						.1-

RAW PRODUCT CHARACTERISTICS OF LOT G10A IN WEEK FIVE

Percent Choice	= 35
Percent Solids	= 5.6
Insoluble Solids as a Percent of Total Solids	= 11
Percent Standards	= 31
Percent Peeling Loss	= 15
Pulping Loss	= 10

Table 7-5. Input Vectors for Lot G10A - Week Two (Rounds)

Restrictions	Row ID	Selected for Pipe Peeler	Not Selected for Pipe Peeler	FMC Peeler	Plant 4 Paste Puree Juice	Sauce Catsup	Plant 3 Paste Puree
Objective Function 1	OBJEC1	-.200165	-.200165	-.200165	-.199780	-.199780	-.204610
Objective Function 2	OBJEC2	-.200165	-.200165	-.200165	-.199780	-.199780	-.204610
Grower Restriction	G-10A	1.0	1.0	1.0	1.0	1.0	1.0
Select 30% Choice Rounds	RSEL30	.297000					
Select Standard Rounds	RSELSD	.3069					
Correction to Select Standard Rounds	RSESTC	-.0396					
Non-Select 30% Choice Rounds	RNON30		.2970				
Non-Select Standard Rounds	RNONSD		.3069				
Correction to Non-Select Standard Rounds	RNOSTC		-.0396				
FMC Peeler 30% Choice	RFMC30			.2970			
FMC Standard Rounds	RFMCSD			.3069			
Correction to FMC Standard Rounds	RFMCST			-.0396			
Juice Row Plant 4	RJUICE	.043	.043	.001305			
5.3% Solids Row at Plant 4	RSOL53			.033855	.0486		
5.3% Sauce Row	RSAU53					.05346	
Waste Disposal Row at Plant 4	RWAST4	.01	.01	.003034	.10	.10	
Box Dumper Restriction	RBOXD2				.003937	.003937	
Pulper Restriction	RPULPR				.001562	.001562	
Sauce Evaporation Restriction	REVAPS					.0009	
5.3% Solids Row at Plant 3	RYOL53						.0486
Waste Disposal Row at Plant 3	RWAST3						.10

RAW PRODUCT CHARACTERISTICS (ADJUSTED FOR WEEK TWO)

Percent Choice	= 30
Percent Solids	= 5.4
Insoluble Solids as a Percent of Total Solids	= 11
Percent Standards	= 31
Percent Peeling Loss	= 17
Pulping Loss	= 10

Table 7-6. Input Vectors for Lot G16FA Week Five (Hand Picked Coreless)

Restrictions	Row ID	Non-Select Peelers Plant 4	Non Select Peelers Plant 3	Select Peelers Plant 3	Paste Puree Juice Plant 4	Sauce Catsup	Paste Puree Plant 3
Objective Function 1	OBJEC1	-.202135	-.205885	-.205885	-.199780	-.199780	-.204610
Objective Function 2	OBJEC2	-.202135	-.205885	-.205885	-.199780	-.199780	-.204610
Grower Restriction	G16FA	1.0	1.0	1.0	1.0	1.0	1.0
50% Choice, Plant 4	R14550	.5150					
Non-Select Standards	R145SD	.2472					
Correction to Select Standards, Plant 4	R145SC	.0103					
50% Choice, Plant 3	RY4550		.515				
Non-Select Standards	RY45SD		.2472				
Correction to Non-Select Standards, Plant 3	RY45SC		.0103				
50% Choice Selected	RYS550			.515			
Selected Standards	RYSESD			.2472			
Correction to Selected Standards, Plant 3	RYSESC			.0103			
Juice Row, Plant 4	RJUICE	.0081					
Juice Row, Plant 3	RYJUIC		.0081	.0081			
Waste Row, Plant 4	RWAST4	-.03			.09	.09	
Waste Row, Plant 3	RWAST3		-.03	-.03			.09
5.3% Solids, Plant 4	RSOL53				.04823		
5.3% Solids, Plant 3	RYOL53						.04823
5.3% Sauce Row	RSAU53					.043407	
Pulper Restriction	RPULPR				.001562	.001562	
Sauce Evaporator Restriction	REVAPS					.000910	
Box Dumper Restriction	RBOXD2				.003937	.003937	

RAW PRODUCT CHARACTERISTICS

Percent Choice	= 49	Percent Standards	= 24
Percent Solids	= 5.3	Percent Peeling Loss	= 13
Insoluble Solids as a Percent of Total Solids	= 9	Percent Pulping Loss	= 9

Table 7-7. Peeling and Transfer Activities for Non-Select Rounds for Pipe Peelers at Plant 4

Restriction	Row ID	Peel Selected 20% Choice	30% Choice	40% Choice	50% Choice	60% Choice	NOSTCM	NOSTCP
Objective Function 1	OBJEC1	-.02	-.02167	-.02332	-.025	-.02665		
Objective Function 2	OBJEC2	-.02	-.02167	-.02332	-.025	-.02665		
20% Non-Select Choice Rounds	RNON20	-.2						
30% Non-Select Choice Rounds	RNON30		-.3					
40% Non-Select Choice Rounds	RNON40			-.4				
50% Non-Select Choice Rounds	RNON50				-.5			
60% Non-Select Choice Rounds	RNON60					-.6		
Choice Rounds Transfer Row	RCHRDS	.168	.252	.336	.42	.504		
Non-Select Standard Row	RNONSD	-.4	-.35	-.3	-.25	-.2	1.0	-1.0
Standard Rounds Transfer Row	RSDRDS	.336	.294	.252	.21	.168	-.84	.84
Juice Transfer Row Plant 4	RJUICE	.32	.28	.24	.2	.16	.8	-.8
Waste Disposal Plant 4	RWAST4	.176	.174	.172	.17	.168	.04	-.04
Correction to Non-Select Standards	RNOSTC						1.0	-1.0
Pipe Peeler Restriction	RPIPPL	.007143	.007143	.007143	.007143	.007143		
Corer Restriction	RCORER	.22143	.24	.25829	.27686	.29514		
Pulper Restriction	RPULPR	.000525	.000475	.000394	.000328	.000263	.00131	-.00131

Table 7-8. Peeling and Transfer Activities for Select Rounds for Pipe Peelers at Plant 4

Restriction	Row ID	Peel Selected 20% Choice	30% Choice	40% Choice	50% Choice	60% Choice	SESTCM	SESTCP
Objective Function 1	OBJEC1	-.0563	-.05192	-.04736	-.04852	-.04952	0.	0.
Objective Function 2	OBJEC2	-.0563	-.05192	-.04736	-.04852	-.04952	0.	0.
20% Choice Rounds	RSEL20	-.388						
30% Choice Rounds	RSEL30		-.582					
40% Choice Rounds	RSEL40			-.776				
50% Choice Rounds	RSEL50				-.97			
60% Choice Rounds	RSEL60					-1.163		
Choice Rounds Transfer Row	RCHRDS	.2688	.336	.4032	.4704	.5376		
Standard Rounds Transfer Row	RSDRDS	.336	.294	.252	.210	.168	.433	-.433
Select Standard Row	RSELSD	-.776	-.679	-.582	-.485	-.388	-1.0	1.0
Correction to Standard Row	RSESTC						1.0	-1.0
Juice Transfer Row	RJUICE	1.10666	1.08266	1.05866	1.03466	1.01066	.4157	-.4157
Waste Disposal	RWAST4	.22754	.22634	.22514	.22394	.22274	.0173	-.0173
Pipe Peeler Restriction	RPIPPL	.007143	.007143	.007143	.007143	.007143		
Corer Restriction	RCORER	.26	.276857	.286	.298857	.31		
Pulper Restriction	RPULPR	.001817	.001795	.001742	.001705	.001667	.000677	-.000677

Table 7-9. Peeling and Transfer Activities for FMC Peeled Selected Rounds At Plant

Restriction	Row ID	Peel Selected 20% Choice	Peel Selected 30% Choice	Peel Selected 40% Choice	Peel Selected 50% Choice	Peel Selected 60% Choice	FMSTCM	FMSTCP
Objective Function 1	OBJEC1	-.04334	-.03796	-.03257	-.03257	-.03257		
Objective Function 2	OBJEC2	-.04334	-.03796	-.03257	-.03257	-.03257		
20% Choice Rounds	RFMC20	-.659						
30% Choice Rounds	RFMC30		-.989					
40% Choice Rounds	RFMC40			-1.319				
50% Choice Rounds	RFMC50				-1.649			
60% Choice Rounds	RFMC60					-1.978		
Choice Rounds Transfer Row	RCHRDS	.2688	.336	.4032	.4704	.5376		
Standard Rounds Transfer Row	RSDRDS	.336	.294	.252	.210	.168	.2548	-.2548
FMC Select Standard Row	RFMCSD	-1.319	-1.154	-.989	-.824	-.659	-1.0	1.0
Correction to FMC Select Standard	RFMSTC						1.0	-1.0
Juice Transfer	RJUICE	.224	.2	.176	.152	.128	.2446	-.2446
Waste Disposal	RWAST4	.3032	.3020	.3008	.2996	.2984	.0102	-.0102
FMC Peeler Restriction	RFMCPL	.03144	.03144	.03144	.03144	.03144		
Pulper Restriction	RPULPR	.003783	.003745	.003708	.00376	.003633	.000398	-.000398

Table 7-10. Computation of Objective Function Values of Input Vectors from Lots 10A and 16FA

	10A (Rounds)			16FA (Handpicked 145's)			
	Whole Peel Plant 4	Products Plant 4	Products Plant 3	Whole Peel Plant 4	Whole Peel Plant 3	Products Plant 4	Products Plant 3
Cost of Raw Product Per Ton	$35.00	$35.00	$35.00	$35.00	$35.00	$35.00	$35.00
Transportation Cost	3.90	3.90	3.90	3.90	3.90	3.90	3.90
Yard Cost	1.133	1.056	2.022	1.527	2.277	1.056	2.022
Total Cost Per Ton	40.033	39.956	40.922	40.427	41.177	39.956	40.922
Total Cost Per 1,000 Lbs.	20.0165	19.978	20.461	20.2135	20.5885	19.978	20.461

CHAPTER VIII

SOLUTION OF THE FULL-SCALE TOMATO PROBLEM

Introduction

As stated in previous chapters, the full-scale model for the season was composed of ten LP models (subproblems) which contained the plant and raw product restrictions for each of ten weeks and an additional LP model (the master program). The master program contained the seasonal limitations on the production (maximum, minimum, and when required) of individual products. It also contained the requirements that the sum of the weights for each subproblem be less than or equal to one.

Subproblems B through I (representing the second to the ninth week) has 150 to 180 rows and 700 to 1,000 columns each. Subproblems A & J representing the first and tenth week had 66 and 83 rows and 445 and 522 columns. These two models were substantially smaller because their production activities were limited to a single packing plant and did not allow production of peeled tomato products. The number of growers was also substantially less than for most weeks. The subproblem dimensions are listed in table 8-1.

The total size of the equivalent LP problem in non-decomposed form would have been approximately 1,550 rows and 7,750 columns. (The initial model was about one-third larger, and was reduced in size after results indicated that various activities and constraints were redundant for practical purposes).

The linear programs were initially solved on an IBM 1620 computer. This equipment was converted to an IBM 1130 prior to the completion of computations and later solutions were obtained on that machine. Both configurations were disc-oriented and equipped with single disc drives. The standard IBM linear programming programs 1620 LP and 1130 LP-Moss were used. Fortran programs were used in conjunction with the LP 1620 output (punched cards) to construct revised objective functions for the subproblems and to convert subproblem output to the proper format for input to the master program. These operations could have been programmed to be performed automatically without operator intervention, but some of the problem files were off line on another disc-pack at all times. In a situation where sufficient on-line storage exists, there would be no need for operator intervention between major iterations other than to communicate solution strategy.

The Solution Process

The first requirement for an overall solution is a feasible solution to the master. Although there would normally be a feasible solution to a properly constructed problem of this nature, infeasibilities could occur under various conditions. For example, the minimum production restrictions may be too large to obtain from the available raw product or the available machinery capacity inadequate to meet specific pack targets. In order to determine if there is a solution to the overall problem and to furnish simplex multipliers to revise the objective functions of the subproblems, it is desirable to obtain a feasible solution to the master at an early stage.

This master program had many minimum production requirements and restrictions requiring a portion of the season's production to be produced during the first three weeks of the season. Consequently, it was necessary to decide on a course of action to obtain the first feasible solution to the master.

The procedure used in the test problem was to generate a series of subproblem solution vectors by changing the relative prices in the subproblems according to plan. This would have been virtually impossible for the full-scale problem because of the large number of restrictions and the large number of subproblems. The development of a plan was hampered by the lack of knowledge about the interrelationships prior to the start of computation. For example, it was not known what price differences are required if choice grades of commodities are desired rather than standard grades or if more catsup is desired with a decrease in paste. In fact, throughout the computations it was difficult, if not impossible, to predict the products which would be in a new subproblem solution from knowledge of the changes in the price vector.

Another approach to obtaining a feasible solution to the master is through the insertion of dummy variables in those rows which might cause infeasibilities. In this master program only the "greater than" restrictions could cause infeasibilities. These dummy variables are given negative objective function values (in the revenue maximization case) so they are not considered a free good and will be driven out of the basis by revenue producing activities. (The use of dummy variables with negative prices with zero prices assigned to all legitimate variables in the master will guarantee a feasible solution the the master or prove that one does not exist.[1] This precaution was not taken as it was assumed that there was a feasible solution to the problem and that such a Phase I-Phase II procedure would be computationally wasteful.)

A third approach used was to develop vectors which are "favorable" to the master through the elimination of products in long supply in previous subproblem solutions by deleting the activity, applying an upper bound of zero or by changing its price. Conversely, products in short supply can be brought into the subproblem solutions by using positive lower bounds or increasing the objective function values. Although increasing or decreasing the level of production activities may seem to imply that some other production activities will decrease or increase, this is not always true. For example, the tomato model did not require that all of the raw product be utilized. Consequently, some products may never have been produced regardless of the production level of the other products if their production cost was greater than the objective function value. In any event, it is quite conceivable that it might be advisable to limit production of some products while encouraging production of others.

Both dummy variables and the development of favorable vectors were used in the full-scale problem although the dummy variable method was used more extensively.

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[1]Hadley, *op. cit.*, p. 406.

The first subproblem solution was obtained for the fifth week of the season. This subproblem was selected to be solved first because it had the largest input of raw tomatoes. The initial solution had no upper or lower bounds on the quantities of finished products to be produced. All other bounds including those on machinery and raw products were included. The prices used were the expected product prices for the season. This procedure determined if the plant models as formulated had sufficient machinery capacity to handle the expected crop. This was more than a simple check on the model, as the cooperating firm had utilized three plants in the past but was planning to only operate two in the crop year under investigation. A feasible solution indicated that the plant capacity was adequate.

The next solution obtained was for the same subproblem but included the seasonal production restrictions as upper bounds for the individual products. This was a further test to determine if the plants had sufficient capacity to handle the peak week raw product and stay within the season pack target for the firm. A feasible solution again indicated this would be possible. (This solution utilized all the raw product available, although there were slack activities for the raw product input restrictions.)

Next, four additional subproblems (weeks) were solved using the expected finished tomato product prices for the season. Prior to solving each new subproblem upper bounds of zero were placed on products which had entered a previous solution at the seasonal upper bound or were in two or more subproblem solutions and had a total production of more than the seasonal upper bound. This was an attempt to develop "favorable solutions". This procedure was successful as all four solutions were in the final solution to the master 96 major iterations later. Three of the four were in every solution to the master.

By the time the solution was obtained for the fourth subproblem, more than two-thirds of the products had been given upper bounds of zero. One more subproblem was solved (without any upper bounds of zero although upper bounds equal to the seasonal restrictions were used). The master program was then run for the first time with these seven solutions to six subproblems. Both maximum and minimum season production restrictions were included in the master. The early season production restrictions were not included at this time because subproblems for two of the first three weeks had not yet been solved. Dummy variables were used in the master to insure feasibility. There were 24 dummy variables included in the first solution. When using this procedure those dummy variables not in the basis to the master should be removed each time the master is run. If not, they can reenter the basis. At this point in the solution process a feasible solution to the master is desired more than an increase in the objective function value of the master. The objective function value of the master is not applicable as long as there are large numbers of dummy variables in the solution.

The simplex multipliers from this solution to the master were used to compute a new price vector for the subproblems. Another subproblem was solved and the new product vector was added to the master. (The subproblem solution was feasible but not optimal with respect to the revised set of prices. Feasible, non-optimal solutions were used frequently throughout the solution process.) This new vector was then added to the master, and the master was reoptimized. (The usual procedure was to solve just one or a few of the subproblems rather than all of the subproblems. This is discussed in Appendix 8-A.)

At that point in the solution process, all seven weeks were in the solution at the 1.0 level along with 21 dummy variables. A new price vector for the subproblems was developed using the simplex multipliers from the master. After an inspection of the product vectors in the master, several prices were not changed from their previous revised level. The corresponding products were at an intermediate level in the basis but were very near their upper or lower bounds. Consequently, their status had really changed very little from the previous solution. This procedure was not followed during most of the solution process because the price revisions were done routinely by computer programs and the revised prices were examined quite infrequently. During the fifth major iteration , the early season production restrictions were added to the master program. Additional dummy variables were added at this time to maintain the feasibility of the master.

After nine major iterations (with a total of 16 different solutions to seven subproblems), there were still a relatively large number (29) of dummy variables in the solution to the master. Many of these were in place of early season production. The subproblems for the second and third week were optimized using revised prices, but when the product vectors were placed in the master program they did not enter the solution.[2/]

2/This was one of the few times that there was no change in the master after the inclusion of a new subproblem vector. Considering the early stage of solution and the apparent need for those products in those weeks, it is surprising, but probably indicates that the early season restrictions were relatively unprofitable.

At this point the value of the objective function for these dummy variables was changed from -5 to -20. Normal prices ranged from +1 to +8. This caused several of the dummy variables to drop from the solution to the master.

After the tenth major iteration there were 14 dummy variables left in the basis to the master. These fell into two categories. One category consisted of dummy variables for products which exceeded the season production target. These were generally products that were produced in abundance even at low prices. The other dummy variables were for products which were not in solutions to the model until their assigned prices were substantially higher than the actual expected prices. These products were relatively uneconomical to produce, possibly even in an absolute sense.

To remove the group of dummy variables with excess production from the master, several feasible subproblem vectors were constructed by modifying previous subproblem solutions. Selected products which were exceeding the seasonal restriction in the master were removed from the previous solution vectors. The objective function value of the subproblem solution was reduced by the full amount contributed by that product; i.e., the product quantity times its objective function value. (This is a larger reduction in the objective function than would actually occur, as no allowance was made for the production of alternative products or the reduction in raw material costs because of a reduced production level.) After four major iterations which included 16 of the modified vectors and six new subproblem solutions, there were only three dummy variables remaining. All three were required to meet minimum production requirements. The remaining three dummy variables were eliminated after three more major iterations. (These three products, along with a few others were never in any subproblem solution unless their objective function value in the subproblem was greater than the expected price for the commodity indicating their relative unprofitability in the model). The modified subproblem solution vectors eventually dropped out of the master as more profitable solutions were obtained. However, since they were feasible (although interior) solutions, they might have legitimately stayed in the solution.

Obtaining the feasible solution to the master required 28 major iterations. There were 63 solution vectors from the subproblems available (including 17 which were constructed from other feasible solutions) when the first feasible master was obtained without the use of dummy variables. 31 of the subproblem vectors were in the solution to the master. The objective function value of the master at this time was $35,165, 73.1 percent of the final value of $48,137.6 and 57.8 percent of the computed Dantzig-Wolfe upper bound of $60,910.5.

The feasible solution might have been obtained sooner if the master had been solved using dummy variables after the solution to the first subproblem. This would have reduced the problems caused by products with production greater than the seasonal restrictions. Another possible aid in obtaining a feasible master would have been the limiting of the total production of each product in the subproblems to the seasonal restriction when a product is produced in two plants or on two or more lines. This was not done because of the size limitations of the LP code. Once a feasible master is obtained, however, there are undoubtedly times when it would be advantageous not to bound the subproblem production activities at all so that the new subproblem solutions will have the activities most favorable to the master.

After a feasible master was obtained, 68 major iterations were performed which included 160 new subproblem vectors. A majority of the major iterations were performed with the addition of only one subproblem vector. Most of the subproblem vectors were feasible but non-optimal. The procedure was terminated when the solution value was 48,137.6 and the computed Dantzig-Wolfe upper bound was 60,910.5. The solution is exhibited in table 8-2.

Suggested Solution Procedures

After considering the total number of major iterations needed, the difficulties in obtaining a feasible solution to the master (in spite of efforts to avoid them) and the problems of finding a realistic upper bound and determining when to stop iterating the suggested solution procedures are somewhat different from those just described.

For a problem similar to the tomato problem, it is suggested that the procedure be initialized by optimizing each subproblem using the expected prices and the seasonal production restrictions as upper bounds. In addition to furnishing an initial basis for each subproblem, an upper bound for the overall solution can be obtained. In a problem such as the tomato problem, the sum of the solutions of the subproblems optimized independently with each subproblem subject to the seasonal restrictions will provide an upper bound.[3/] The upper bound is needed in determining when it is economically desirable to stop computing. The Dantzig-Wolfe procedure for determining an upper bound described in Chapter IV requires a considerable amount of computation which is wasted as far as the overall solution is concerned and is of no value to the firm's management. It appears that the upper bound that would have been obtained by optimizing all the subproblems would have been lower than any computed Dantzig-Wolfe upper bound until after more than 90 major iterations. (See Appendix 8-B.) It would appear that problems of similar structure would also take a large number of major iterations. (In the test problem with only 92 restrictions and two subproblems, it required 40 major iterations to obtain the overall optimum.) The solution of all the subproblems may also be of interest for investigating plant capacities and determining which, if any, activities dominate throughout the season or whether shifts occur in production, allocation, or machinery utilization as the season progresses. Such information can then be utilized in forming solution strategies by selecting the subproblems whose original optimal solution most closely corresponds to a favorable input to the master.

Finally, the optimum solution to a given subproblem should be of interest to management for the consideration of questions such as:

> How does the proposed plan or schedule (final overall solution) differ from the "most efficient" way I can operate during that time period? Is the difference significant and if it is,

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3/In the case where the subproblems are not independent, this will not be true; i.e., where the finished product of one time period or plant becomes the raw material for another.

> can action be taken via sales promotion, machinery changes, or a revision of pack targets to operate more nearly like the optimal solutions?4/

The master program should then be run immediately using dummy variables to obtain a feasible solution. The number of dummy variables should be kept to the minimum possible. In the tomato problem, this would mean running the master as soon as all of the ten subproblems had been solved but without including any dummy variables for the "less than" restrictions. If it is not desired to solve all the subproblems with the original prices, then the master should be solved using dummy variables as soon as it is desired to stop using the original prices.

The procedure of running only a small proportion of the subproblems in any major iteration should be followed. Experience on the tomato problem indicated that initially vectors from more than one subproblem will enter the master solution on each major iteration. However, unless the number of subproblems is small, they will not all enter. The average number that will enter each time should be determined from observation, and that number of subproblems or less run for each major iteration. It was observed that vectors that don't enter the solution to the master when first generated seldom enter the master later. Of those that do enter later, few have a significant impact on the progress of the solution. The number of subproblems that will have vectors enter

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4/The tomato pack target or production objective is based on the sales objective which is derived from knowledge or past history and current and expected market conditions. If the firm has unique production advantages for a product or group of products, it should consider pricing, promotion, or sales efforts to change market conditions.

the master solution, on each major iteration will probably decrease as the solution progresses.[5] The number of subproblems solved for each major iteration should be correspondingly reduced. There was some evidence in the tomato problem that vectors from several subproblems increased the solution value of the master more than vectors from a single subproblem. It is not clear at what point additional subproblem solutions in a major iteration are warranted. It will depend to a large extent on the relative time spent in solving the subproblems and the master.[6]

There are advantages to using prices other than those obtained using the simplex multipliers in the early stages of solution. Prices of products at upper or lower bounds should be changed, but products that were at a bound in the previous master solution and are still near the bound, but with a simplex multiplier of zero, can be left with the same price as they previously had or be given an arbitrary price. The usefulness of this procedure in the tomato problem was limited to the earlier stages when most of the products were not yet restricted in the master. As the solution progressed, few products which were bounded on one iteration were not bounded on the next.

5/There was not sufficient opportunity to observe this in the large problem. However, it was apparent in the test problem. See Chapter VI.

6/As the overall optimum is approached, some vectors will not enter the solution to the master at all, and non-optimal solutions will be less likely to enter the master. A solution strategy at this point depends to a large extent on the relative running times of subproblems and master. Solution of the tomato problem was terminated before this became a consideration and no observations were obtained.

There is no need to completely optimize the subproblems each time.[7/] More than one non-optimal solution can be obtained from the same subproblem with the same revised objective function. In a problem like the tomato problem, this has the advantage of having a greater variety of products in the new vectors since the first vector with new prices will have some products from the previous solutions. In the full-scale problem, the least optimal solution frequently came in at the exclusion of more optimal vectors.

There is no need to obtain new vectors from a different subproblem each major iteration. The same subproblem was used for two or more major iterations in succession with no apparent difficulties. The ability to use the same subproblem will allow savings of computer time in some operating environments. A procedure for determining which subproblem or subproblems will improve the solution to the master the most would be of benefit, but no such procedure was developed. One procedure that was used throughout the tomato problem was the bounding of the activities in the subproblems to be less than or equal to the restriction on the season's product. These bounds were originally included to aid in obtaining a feasible master but were not changed when a feasible solution to the master was obtained. However, it may be that unbounded solutions cause the solution to the master to increase more rapidly than bounded ones. There will be larger quantities of fewer products produced if the production activities are not constrained.

[7/]Assuming that the number of iterations required to go from a good solution to an optimal solution is significant.

In that case the products that are produced are generally not at the upper bound in the master or at least have low simplex multipliers.

Termination of Computation

The decision whether to obtain an optimal solution to the overall problem or whether to terminate prior to that point will depend on the circumstances, but in an applied problem such as the tomato problem, it would be unrealistic to attempt to obtain an overall optimum. There will be a point where the cost of computation exceeds the average increase in the solution to the master. However, computations should generally be terminated before that point is reached.

One reason is that the day-to-day variations in raw product quantities and characteristics and in the performance of labor and machines, as well as the imprecise measurements and approximations incorporated in any model will negate the value of a statement that any one solution is the optimal solution.

Another reason for not attempting to obtain an overall optimum to a very large problem is that there are subjective considerations that are extremely difficult if not impossible to incorporate in a model. A solution must be acceptable to management. An unacceptable solution cannot be optimum. The labor supply, company employment policies, and union regulations have to be considered in detail in the final schedules. Similarly, the sequencing of products and practical limitations on changeovers have to be considered before drawing up final production schedules.

Criteria for estimating how "good" the current solution is are discussed in Appendix 8-B. None of these can really be considered satisfactory. More empirical analysis is required to determine which are valid. Lacking an adequate theoretical criteria, the decision to terminate must be based on the knowledge and judgment of the analyst and management.

Once the decision has been made to stop iterating, the weekly production quantities are obtained by applying the weights from the master program to the subproblem solutions. These weekly production quantities should be used as lower bounds in obtaining new solutions to the subproblems to determine the allocation of raw product and machinery requirement for each week's pack.[8/] Although a feasible solution for the week could be obtained by multiplying the entire subproblem solutions by the appropriate weights, the product yields will be increased by optimizing with the lower bounds.[9/] (This procedure also relieves the firm of maintaining a large number of complete solutions either on hard copy or in computer storage. Some decomposition computer codes complete this procedure automatically.)

The final step in developing allocations and schedules is to determine the sequencing of products on multiproduct lines and making adjustments to meet run length or labor criteria. (This might be done in two steps with large adjustments being done before solving the

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8/Some of the product levels would be equalities or free variables if circumstances warrant.

9/This is true only if the decomposition solution process is terminated before reaching the overall optimum.

subproblems with lower bounds and making the final adjustments to the schedule afterwards).

Computations in the tomato problem were terminated after 97 major iterations. During the last major iteration, all of the subprograms were solved, and the Dantzig-Wolfe upper bound was computed. The final solution value and the value of the Dantzig-Wolfe upper bound are discussed in Appendix B to Chapter 8, along with other methods of obtaining an estimate of the upper bound.

At that time, the solution value was 77.9 percent of the computed value of the Dantzig-Wolfe upper bound and 88.3 percent of an estimate of the upper bound obtained by one of the other methods. One of the reasons it was decided to terminate computation at that point was that virtually all of the products were being produced at the upper limit of the seasonal pack restrictions. It appeared that improvement in the solution value achieved by further computations would be due to minor shifts in the production of commodities between weeks. It appeared that sufficient information had been obtained to allocate the lots of tomatoes to the proper plant and to the appropriate finished products. Information was also available to schedule the plants as accurately as possible within their limitations imposed by institutional factors and the day-to-day changes in product quality and availability.

After the computations were terminated, a production schedule was developed for the season. This production schedule is displayed in table 8-3. The method by which the schedule was developed is discussed in Appendix C to this chapter.

APPENDIX A TO CHAPTER VIII

COMPUTATIONAL CONSIDERATIONS

Subproblem Solutions

After obtaining a feasible solution to the master program of a large decomposed problem, it is desirable to obtain a satisfactory overall solution with the least expenditure of resources. This is not equivalent to minimizing computer expenditures, as there may be human resources involved in making decisions such as:

1. Selecting the number of subproblems to be run in each major iteration and the selection of the individual subproblems to be included in that major iteration.

2. Determining whether individual subproblems should be optimized using the revised objective function or whether they should be terminated at some point less than the optimum. Similarly, should more than one solution be obtained from a subproblem for a given set of prices?

3. Determining when a satisfactory overall solution has been obtained.

One approach is to optimize each subproblem during each major iteration. This is what was done in some of the initial computer decomposition algorithms. However, this is generally inefficient, as all of the subproblem solutions do not enter the solution to the master. An improvement will be made in the master when a solution vector from any one of the subproblems is added to the master if the revised objective function value of the subproblem (whether optimal or not) is greater than the value of the simplex multiplier corresponding to the restriction on that subproblem in the master. Some computer codes now allow the selection of one or more subproblems for use in the next major iteration. A method is needed to determine how many and which subproblems to solve in the next major iteration to get the largest improvement in the master.

An analysis was made of the solutions to 23 major iterations which included solutions from more than one subproblem. Solutions to the master included new vectors from two subproblems in just over half of those major iterations with new vectors from two or three subproblems. In the limited number of cases where there were more than three subproblems with new vectors in a major iteration, the largest number of new subproblem solutions that entered the solution to the master was three.

In general, those major iterations with new vectors from more than one subproblem had larger increases in the solution value of the master than those with new vectors.

The usual procedure for the generation of new subprogram solution vectors was to use a revised objective function until the solution was substantially changed rather than optimal. The structure of the subproblems was such that 25 percent or more of the computing time could be devoted to increasing by only five percent the value of the objective function. (For example, one problem was solved in 389 iterations. The last 100 iterations increased the objective function from 3679.611 to 3690.058. The problem was feasible after 36 iterations with an objective function value of 2315.289.)

In the tomato problem there was no advantage to be gained by solving subproblems to their optimum value. A non-optimal solution from one subproblem was just as likely to enter the master as an optimal one from another. A subproblem solution further from optimum was just as likely to enter the master as one from nearer optimum with the same prices.

This observation should be modified by the following considerations:

1. The distribution of the optimal solutions was heavily weighted with the smaller problems since they took less time to solve. These subproblems were less profitable and so were at a disadvantage when compared to the subproblems with more raw tomato inputs.

2. The problem was not near its optimum overall solution when some of the solutions were obtained. It is to be expected (as was indicated in the test problem) that fewer subproblem vectors will enter the solution to the master as the overall solution is approached. In addition, it might be necessary to bring the subproblem solutions closer to optimal to get them to enter the solution to the master.

Computer Size

During the solution of the full-scale problem, there were many difficulties encountered because of the small size of the computers used. It is true that using the decomposition principle of linear programming allows the solution of problems that normally would be too large for a particular computer configuration because only a portion of the model, i.e., a subprogram, is being solved at any given time. However, the computer configuration must be powerful enough to solve that particular subprogram efficiently if decomposition is to be effective, since each subprogram must be solved numerous times. Several of the subprograms which represented weeks of the season in the full-scale tomato model, had dimensions very close to the upper limit of the computer equipment and LP code. Consequently, solutions frequently required a large amount of computer time along with computational difficulties because all the computer resources were being used. This would not have been a major problem if each subprogram was solved once or a very limited number of times. The difficulties came because each subprogram has to be solved many times before obtaining an overall optimum.

There are alternatives which may avoid problems of this kind. One is to insure that all the subproblems are small enough that they can be handled efficiently within the size and scope of the computer equipment and LP codes available. In some cases, if the indicated size of the matrix is large but it can be determined through exploratory studies that there are a relatively small number of restrictions, it might not be necessary to model the entire plant but just the effective machinery restrictions. Another possibility is to reduce the total number of columns. For example, if there is a lot of tomatoes which from past experience or preliminary study is demonstrated to always be assigned to a given end product, it could be left out of the matrix with a reduction in the number of input columns.

Another alternative would be to use a larger computer system. The feasibility of this alternative depends, of course, on a relative cost and availability to the firm of various computer systems. Some firms have a hierarchy of computer systems which include small computers in plants and larger systems in division or corporate headquarters. Some have access to service bureaus at a cost. An alternative would be a compromise between these two methods whereby the problem or at least those portions of the problem which are too large for a small computer would be solved on a larger computer. The results would then be studied and the subproblems tailored so that they contain all the meaningful restrictions but are of reasonable size for the smaller equipment.

In some cases, it might be advisable to go to a large computer, solve the entire program and use the results of this solution for a tentative schedule for the entire year. Then, portions of the problem could be resolved when prices change, raw product characteristics change, or it was desired to evaluate machinery or other capacities. Many computer LP codes include methods to construct submodels from large models stored in mass storage.

In general, the increasing power of computer systems along with improved computational methods and file handling techniques make these considerations outdated for a problem the size of the full-scale tomato problem. However, as data pertaining to product characteristics is developed and refinement in plant and machinery modeling occur, it is probable that such models will increase in size and continue to exceed computer or code limits.

One specific area which caused computation problems was the lack of flexibility in LP codes available when the problem was first formulated. Problem setup required so much time that it was necessary to include all possible restrictions and activities at the time of initial formulation of the problem. This, of course, added to the size of the LP matrix. The increased flexibility of codes available now allows the addition of rows, columns, bounds, and in some cases, individual elements, etc., essentially at will. Consequently, one can start with a much smaller matrix and make additions as questions arise or if there is a need to investigate a given area further.

APPENDIX B TO CHAPTER VIII

UPPER BOUNDS

The solution process was terminated after 97 major iterations. At that time, the objective function value of the master was $48,137.6. The Dantzig-Wolfe upper bound[1/] was computed during this major iteration and was found to be $60,910.5. The solution value at this time was 77.9 percent of this upper bound.

Optimal solutions to all ten subproblems are required to obtain the Dantzig-Wolfe upper bound, so this was the first time that it had been computed. However, the simplex multipliers for the restrictions on the weights of the subproblem vectors and the values of the revised objective functions were compared frequently to make a rough estimate of the value of the upper bound. With the possible exceptions of two of the last major iterations, this computed Dantzig-Wolfe upper bound is lower than any that could have been computed previously. (Even these two major iterations are estimated to give higher Dantzig-Wolfe upper bounds.)

After computing the upper bound, it should be possible to make relatively good estimates of the upper bound for several iterations without solving all the subproblems. In the tomato problem, the difference

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1/The Dantzig-Wolfe upper bound is described in Chapter IV.

in the optimal solution values of two subproblems solved with the same objective function was relatively constant. An estimate of the upper bound can be determined by estimating the value of the revised objective function of each subprogram. This is computed by adding or subtracting the difference between the subprogram objective function value for the last solution and its solution value when solved with the same objective function as the subprogram being estimated. This value is an estimate of the subproblem solution using the current prices. We can then proceed with a pseudo Dantzig-Wolfe upper bound computation. If it appears that a substantial reduction in the upper bound can be made, the decision whether or not to solve all the subproblems to compute a new upper bound is made at that time.

The Dantzig-Wolfe upper bound computation method did not provide a satisfactory tool for determining when to stop computation. The final computed value of the master was probably much closer to the actual optimum solution value than indicated by its being 77.9 percent of the Dantzig-Wolfe upper bound.

In a completely decomposed problem like the tomato problem, the solution for the season with no finished product quantity restrictions is equal to the sum of the solutions of the individual weeks. This is an upper bound. Similarly, a solution based on the sum of the individual weeks solved with the seasonal pack restrictions applied to each week is an upper bound.

The optimal solutions to all of the subproblems using the seasonal bounds on production and the expected season prices are not available. All of the weeks were not solved with the expected season prices.

However, the sum of the largest objective function values obtained for each week is available. This is \$60,232.6. Most of these objective function values are for solutions obtained early in the solution process before relatively unprofitable commodities were being forced into the solution. They should approximate the optimal solution values for those weeks. These values are substantially higher than those obtained later in the solution process and are from solutions which are generally not in the solution to the final master.

The sum of the largest objective function value for each subproblem solution included in the last solution to the master was \$54,598.3. In the ten previous iterations, this sum ranged from \$58,123.9 to \$54,863.3 and was generally decreasing. Considering the structure of the tomato problem and since many commodities were being produced at their upper bounds, each increase in the value to the master solution meant that more of the relatively unprofitable products were being included in the solution. This indicates that the overall optimum solution will be less than the \$60,232.6 and probably less than the sum of the highest subproblem solution values in the last solution to the master.

Most of the subproblems had one or two solutions which had high weights in the final solution and several previous solutions to the master program. These weeks with high weights do not in general have relatively high objective function values. Two have the lowest objective function value of vectors for that week in the solution to the master. Three have the highest objective function value of the vectors that are in the solution to the master for that week, and five have neither the highest or lowest value. In no case was the vector with the highest objective function value obtained for that week during the course of the study in the final solution to the master. In the case of week three, the dominant vector in the solution to the master had a weight of .536 or over half of that week's weight in the master. That vector had the lowest objective function value of all the feasible solutions which had been included for that week. In week two, cost ranging incicates that a very small change in the objective function of the heaviest weighted vector in the solution with a weight of .28 would have increased the vector's weight to 1.0. This particular solution vector had one of the lowest objective function values for week two.

The computed Dantzig-Wolfe upper bound almost 70 major iterations after the master became feasible was still greater than the sum of the largest solution values to the subproblems and approximately 10 percent larger than the sum of the values of the largest solution vectors which entered the last solution to the master. In this particular application, the sum of the largest solution values to the subprogram is a more practical way to compute an upper bound than the Dantzig-Wolfe computations.

The final computed objective function value of the master was $48,137.6. This was 88.3 percent of the sum of the largest objective function values of each subproblem in the last solution to the master.

Solution values of the master during the last ten major iterations increased from $46,842.4 to $47,997.8, a total of only $1150 or an average of $115 per iteration. The average gain is decreasing, but since the objective function is scaled by 100, more iterations might be economically feasible if one felt there was sufficient accuracy in the models and that the institutional factors would allow such gains to be obtained. In terms of the tomato problem, it is highly doubtful that either of these requirements is present.

APPENDIX C TO CHAPTER VIII

THE DEVELOPMENT OF A PRODUCTION SCHEDULE BY WEEKS

Linear programming computations were terminated after 97 major iterations. At that time, the final solution to the master had 46 of the 58 products at their upper bounds for the season. There were no products at their lower bound for the season. One of the eight products which had an early season production requirement was at the lower bound for early production. The product levels in the final solution to the master are found in the next to the last column of table 8-2.

There were 55 subproblem solution vectors in the final solution with weights ranging from .002[1/] to .835. The number of subproblem vectors for each week in the solution ranged from three to eight.

The tentative weekly production schedule displayed in table 8-2 was developed by multiplying the weights from the solution to the master by the appropriate subproblem solution vectors and summing to form a composite vector for each week. The composite vectors are reproduced without any adjustments in table 8-2. The first column of the table gives the product name and description and plant location for production if the product is produced at more than one plant. The next

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[1/] The vector with the .002 was not used in forming the schedule. The next smallest weight was .007 which was used.

columns labeled one to ten are the composite vectors for the respective weeks of the season. The following column labeled ten week total is the total of the composite vectors for all ten weeks. This is followed by the level of that product in the last solution to the master program. The last column is the maximum pack target or the seasonal restriction on production for that product or group of products. The weekly and ten week total have more entries since the same production is produced on different lines or different plants and, consequently, have overall production restriction in the master program. The total of the composite vectors over all weeks in the table does not equal the maximum pack target for those products at their upper bounds because the computer output (i.e., the weights obtained from the solution to the master program) were truncated rather than rounded.

The usual procedure at this point would be use the quantities from these composite product vectors as lower bounds on the product activities for their respective weeks and optimize each subproblem subject to these lower bounds with the expected prices for the season. This procedure will provide the optimal solution to each subproblem that meets the lower bound conditions. The total yield should increase and/or other production efficiencies should result because some of the vectors included in the composite were feasible but not optimal.[2/]

[2/] It would also be possible to construct an entire composite solution vector in the same way as the composite product vector. However, this is not normally done as all the solution vectors to the subproblems are not always available.

However, the tentative production schedule has a large number of items listed for production each week. The usual subproblem solution vectors have from 15 to 30 production activities indicating that to be the optimum number of production activities for any particular set of prices. However, because of the composite nature of the vectors, the tentative schedule has up to 60 production activities in one week.[3/] The largest number of production activities were generally in the peak weeks of the season when product changeovers are least desirable.

Considering the tangible and intangible costs of changeovers and short run lengths, too many products were scheduled in each week. Fortunately, many activity levels in a given vector were quite low. It was determined that if one could combine or drop activities that had a production level of less than 20 units or 2,000 cases in any given week, the number of entries in the table would be reduced to 247.

The 2,000 case cut-off was chosen arbitrarily, but for most products it was reasonable as it represented less than one shift of production on most lines. Consequently, these activities were dropped from the schedule for that week. In some cases compensating adjustments were made in the level of similar products. This procedure had little impact on those products with a season target of more than 200 units or 20,000 cases. After dropping small entries for these products, their activity level was still substantially more than 90 percent of the ten week total in

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3/There are 800 production activities by week positions in table 8-2 as there are 90 production activities possible in weeks two through nine and 40 production activities possible for weeks one and ten. There are 422 entries in the table, a density of over 50 percent.

most cases and in all cases above the lower bound for the season for those items. Those with production targets of 200 units or less were examined individually as the production remaining after the small entries were dropped varied from zero to 100 percent of the original ten week total. Those with less than 80 percent of the master solution remaining were annotated to be adjusted later.

Many small production quantities that are impractical to schedule would appear to be a problem in decomposition models (as well as any LP allocation and scheduling model of this type). It can not be determined whether the overall optimum solution would have had a reduction in the number of production activities in the composite vectors,[4/] but since it isn't practical to obtain the optimum, some procedure such as the one above is necessary to reduce and/or consolidate to a reasonable level for final scheduling.

An alternative to inspection would be to limit the number of subproblem vectors allowed in the composite vector for each week. For example, rerun the master with just the vectors with a weight of ten percent or more. This would have reduced the number of solution vectors to 28 or half the number of those included in the final solution to the master. Another alternative would be to rerun the master program with only the two or three most heavily weighted vectors for each week from the last master solution. The first alternative would have had at most 298 entries in the tentative schedule. The use of only three vectors

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4/Further "optimization" will tend to increase the number of production activities as each individual lot of tomatoes is assigned to that product for which it is best suited.

for each week in the solution to the master would have resulted in at most 294 entries in the tentative schedule. The use of vectors with weights of at least .1 but no more than the three most heavily weighted vectors would have resulted in at most 266 entries in the tentative schedule.

The next step in the development of the production schedule is to examine the remaining entries in the tentative production schedule in detail. Table 8-3 is the resulting schedule after modifications were made based on this examination. The following changes were made to the schedule (the numbers in parentheses in table 8-3 are the original values from table 8-2 if the quantity has been increased due to a combination of activities).

1. Production of all grades of coreless tomatoes in #2½ cans at plant four should be discontinued and that production consolidated at plant three. There is very little volume in the tentative schedule for coreless production in #2½ cans indicating larger costs or lack of capacity at plant four. The volume in the tentative schedule is too low to justify production at two locations.

2. Production of labeled round tomatoes in #2½ cans should be discontinued and the volume transferred to coreless #2½ cans at plant three. Only six percent of the pack target was allocated to rounds in the tentative schedule indicating the round #2½ cans are relatively uneconomic packs compared to the coreless. (The season pack target did not distinguish between round and coreless for this particular product.)

3. Production of labeled #303's (round and coreless) should be consolidated with choice #303's and produced in weeks two, five, and six at management discretion. The total volume in labeled #303's is too low to justify separate scheduling in several different weeks.

4. Several minor consolidations of choice #303's were made. These are high volume items and the cut-off of 2,000 cases per week was too low.

5. Production of coreless standard #303's should be consolidated at plant four. The volume is not sufficient to justify lines running at both plants.

6. Production of standard coreless in #10 cans should be dropped at plant four and consolidated at plant three.

7. Diced tomatoes should be produced only during weeks two and five. The week eight volume of 2,100 cases exceeds the arbitrary cut-off of 2,000 but is too small to justify a production run in week eight.

8. Production of labeled stew tomatoes in #303 cans should be consolidated with fancy stew tomatoes and scheduling left to the discretion of management. However, round stew tomatoes and coreless stew tomatoes can be scheduled separately.

9. Production of stew tomatoes in #10 cans should be consolidated with the production in #303 cans and scheduling left to the discretion of management. This change is not reflected in table 8-3. The nature of the canning line makes this particular change of relatively small consequence in the scheduling procedure.

10. Production of all grades of whole tomatoes packed in paste or puree should be consolidated. Rounds should be canned at plant four and the coreless varieties canned at plant three.

11. Paste production in #10 cans is necessary at plant three only during week five when the availability of raw product is at its peak. There is no reason to produce more than one grade of paste at plant three for #10 cans.

12. Production of less than 2,000 cases a week of #10 paste products at plant four, catsup products and #303 - eight-ounce sauces were consolidated with other grades frequently rather than completely dropping that capacity requirement from the schedule. These are minor changes that can be adjusted throughout the season by management.

13. The production of appropriate quantities of 1.045 puree in picnic cans, #10 sauce, #10 pizza sauce, and various sizes of juice and tomato cocktail were assigned to week one. (Week one had plenty of raw product and capacity as the sum of the weights on its vectors in the final solution of the master was only .138.) These products were in the week one composite vector but at less than the 2,000 case cut-off for scheduling. Several did not meet the pack target in the tentative schedule because they were in too many weeks at a level of less than 2,000 cases and were dropped

in those weeks. Consequently, to insure that the minimum production targets were met, these items were included in the production schedule for week one.

The adjusted production schedule in table 8-3 can be used to furnish lower bounds to the subprograms just as the original composite vectors in table 8-2. The subproblems should be optimized with these lower bounds using the expected prices for the season. This will furnish the allocation of raw product to finished product and the allocation of raw product to the appropriate plant as well as indicating the appropriate blending activities. It will then be necessary for management to exercise its discretion in scheduling those products as indicated above. In some instances, it might be necessary to reduce the production of a commodity if the total that will be produced during the season is greater than the seasonal pack target. This final optimization was not carried out for the full-scale tomato problem because of the lack of time and computer facilities.

Table 8-1. Final Subproblem Dimensions

Week	Number of Rows	Number of Columns	Number of Grower Inputs	Number of Input Columns
1	66	445	20	90
2	150	786	35	155
3	162	834	47	203
4	171	877	56	246
5	179	899	64	268
6	173	877	58	246
7	170	866	55	235
8	159	821	44	190
9	170	842	55	199
10	83	522	37	167

Table 8-2. Tentative Production Schedule by Week

Product Name	Week 1	2	3	4	5	6	7	8	9	10	Ten week total[1]	Master Solution Total	Maximum Pack Target
1. Choice Round #2½		30.0	146.9	129.7	28.2	42.9	140.4		154.5		672.5	675	675
2. Choice Coreless #2½ Plant 4		1.2			7.7	11.2					20.1		
3. Choice Coreless #2½ Plant 3		10.1	80.4	39.7	48.0	67.4	31.5	1.4			278.6	300	300
4. Labeled Round #2½		17.3			8.4	4.3					30.0		
5. Labeled Coreless #2½ Plant 4		14.8			8.2	2.2			55.7		81.0	525	525
6. Labeled Coreless #2½ Plant 3		140.5	28.8		29.9	19.0	46.3		148.1		412.7		
7. Standard Rounds #2½		147.7	39.8	87.0	139.4	141.6	75.9	187.3	3.9		822.7	825	825
8. Standard Coreless #2½ Plant 4			17.6		33.1						50.8		
9. Standard Coreless Coreless #2½ Plant 3		6.3	42.0		78.8	21.5	24.3	1.0			173.8	225	225
10. Choice Rounds #303		82.6	101.4	123.5	332.4	116.9	51.9	49.6	294.0		1152.3	1155	1155
11. Choice Coreless #303 Plant 4		83.7	81.4	39.4	102.3	23.4	29.6		59.3		419.1		

(continued)

Table 8-2. (continued)

Product Name	Week 1	2	3	4	5	6	7	8	9	10	Ten week total	Master Solution Total	Maximum Pack Target
12. Choice Coreless #303 Plant 3		118.6	116.7	9.3	129.7	7.4			21.1		(419.1) 402.8	825	825
13. Labeled Rounds#303		26.4			10.8	8.1					45.3		
14. Labeled Coreless #303 Plant 4		26.5			13.5	10.6	1.5	2.6	.3		55.0	150	150
15. Labeled Coreless#303 Plant 3		24.8			13.5	10.6					48.9		
16. Standard Round #303		63.6	111.1	115.0		84.5	58.6		15.0		448.7	450	450
17. Standard Coreless #303 Plant 4				6.3		6.3		1.2	66.4		80.2	150	150
18. Standard Coreless #303 Plant 3						4.9			64.6		69.5		
19. Choice Round #10		181.4	69.2	168.8	230.1	213.3	165.4	328.8	4.3		1361.2	1367	1380
20. Choice Coreless #10 Plant 4		74.7	101.3		24.6	7.3	1.0				209.0	1041	1050
21. Choice Coreless #10 Plant 3		41.5	206.0	67.8		175.7	107.4	212.6	14.6		825.6		
22. Standard Rounds #10		16.8	185.7	113.1	107.8	81.2	127.9	39.2	359.2		1031.0	1033	1050

(continued)

Table 8-2. (continued)

Product Name	1	2	3	4	5	6	7	8	9	10	Ten week total	Master Solution Total	Maximum Pack Target
23. Standard Coreless #10 Plant 4				5.8	21.8			.6			28.1		
24. Standard Coreless #10 Plant 3		11.0		51.8	16.7	10.6	29.7	1.5			121.3	150	150
25. Round Stew #303			10.5		107.1	121.5	34.4	21.4	112.3		407.2	675	675
26. Coreless Stew #303		162.2	41.4		30.2	16.6	2.4		12.0		264.8		
27. Labeled Round Stew #303					32.8		7.2				40.0	75	75
28. Labeled Coreless Stew #303					32.8		1.1		.9		34.8		
29. Round Stew #10				23.9		9.4		8.1	.9		42.2	75	75
30. Coreless Stew #10				16.4		9.4		6.8			32.5		
31. Rounds -1.06 Puree		8.6	12.8			12.3	34.5				61.2		
32. Coreless -1.06 Puree Plant 4		45.2									45.2	385	525
33. Coreless 1.06 Puree Plant 3				41.6	5.7	12.2			211.2		270.5		
34. Rounds 1.045 Puree						14.4	16.5		2.5		33.4		

(continued)

Table 8-2. (continued)

Product Name	Week 1	2	3	4	5	6	7	8	9	10	Ten week total	Master Solution Total	Maximum Pack Target
35. Coreless 1.045 Puree Plant 4						14.4			2.5		(33.4) 16.9	75	75
36. Coreless 1.045 Puree Plant 3		7.6				14.4			2.5		24.5		
37. Heavy Pack Rounds		4.2		24.0			1.1				29.3		
38. Heavy Pack Coreless Plant 4		4.2					1.1				5.3	70	75
39. Heavy Pack Coreless Plant 3		.5		24.0	3.7		7.2				35.4		
40. Coreless Diced #10		104.4			119.6		15.2	20.6			259.8	261	300
41. Paste 6oz.		227.7	403.2	332.9	349.6	261.6	266.6	338.3	375.6	290.6	2846.0	2850	2850
42. Paste #303	91.4	10.2		382.0	154.6	113.7	119.6	109.8	13.5		994.3	1000	1000
43. Paste 25% #10 Plant 4		73.3	18.1		167.3	72.3			12.6	3.5	347.1	400	400
44. Paste 25% #10 Plant 3					52.4						52.4		
45. Paste 26% Plant 4		10.3		116.5	36.3	39.7	26.8		47.1	111.3	388.0	400	400
46. Paste 26% Plant 3							10.6				10.6		
47. Paste 30% Plant 4		46.4	23.6	24.9	71.9	34.5	.3	3.9		122.1	327.6	350	350
48. Paste 30% Plant 3				11.1	5.4	2.5		2.0			20.9		

(continued)

Table 8-2. (continued)

Product Name	Week 1	2	3	4	5	6	7	8	9	10	Ten Week Total	Master Solution Total	Maximum Pack Target
49. Paste 32% Plant 4		34.7	16.1		23.2		29.8			31.6	135.3	166	200
50. Paste 32% Plant 3					21.5	8.4					29.9		
51. Paste Drums 26%							516		8		524	527	3,000
52. Paste Drums 32%		3165	3304	1874	3498	2909	2320	3116	3390		23,576	23,635	25,000
53. Paste Drums 36%				1572	330	537	432		54		2,985	3,000	3,000
54. Puree 1.06 #303			26.8		1.4		31.2			4.9	64.3	64	75
55. Puree 1.045 #2½					10.0	213.6					223.6	225	225
56. Puree 1.06 #2½	1.7	86.0	107.2		5.4	74.8	12.4		10.0	1.4	298.9	300	300
57. Puree Picnic		5.5	9.2	19.1	1.4	10.1	6.1			23.6	74.8	75	75
58. Puree 1.045 #10 Plant 4	5.6	57.5	37.6	5.7	33.1	1.9	49.5	16.4	7.2	188.4	402.9	600	600
59. Puree 1.045 #10 Plant 3				13.9	40.1	73.2	49.4	16.9			193.5		
60. Puree 1.06 #10 Plant 4		89.0			63.4	54.2	1.8	338.3		55.0	601.6	825	825
61. Puree 1.06 #10 Plant 3					119.0		94.6	3.2			216.9		
62. Puree 1.07		24.6	50.3		29.9						104.8	105	105
63. Fancy Catsup		137.1		8.2	77.8	9.1	103.5	40.5		71.7	447.8	450	450

(continued)

Table 8-2. (continued)

Product Name	Week 1	2	3	4	5	6	7	8	9	10	Ten Week Total	Master Solution Total	Maximum Pack Target
64. Extra Standard Catsup			26.7	28.8	209.7	78.7	26.6	4.5	15.0	58.3	449.3	450	450
65. Standard Catsup		4.2	217.8	18.4		143.0	140.5		224.2		748.3	750	750
66. Sauce #10	15.9				23.7				96.9	13.2	149.7	150	150
67. Chili Sauce		7.0		307.1	17.6	147.3	19.4				498.3	500	500
68. Pizza Sauce	8.1	49.6		.7	19.2	17.3	103.9			.6	199.4	200	200
69. Concentrate					49.8	24.9			5.0	8.1	87.8	88	100
70. Sauce A Line 2 8oz.		22.7					8.5	291.9			319.3	850	850
71. Sauce A Line 3 8oz.						1.7	12.0	517.0			530.6		
72. Sauce A Line 2 #303					59.3		45.8				105.0	375	375
73. Sauce A Line 3 #303		7.5			8.2	175.5	60.8			16.1	268.1		
74. Sauce B Line 2 8oz.		5.6	175.0		60.0	.7	142.2			98.8	482.2	850	850
75. Sauce B Line 3 8oz.		13.3	63.5		73.6	60.1	82.1			72.8	365.3		
76. Sauce B Line 2 #303					27.0			282.0			309.0	375	375
77. Sauce B Line 3 #303					27.0	39.8					66.8		
78. Sauce C Line 2 8oz.	7.1	152.8	206.3								366.2	625	625
79. Sauce C Line 3 8oz.	8.2	152.7				26.1		68.6			255.5		

(continued)

Table 8-2. (continued)

Product Name	Week 1	2	3	4	5	6	7	8	9	10	Ten Week Total	Master Solution Total	Maximum Pack Target
80. Sauce C Line 2 #303		163.2		29.6	8.2					2.6	203.7	250	250
81. Sauce C Line 3 #303		39.4								5.3	44.7		
82. Juice $5\frac{1}{2}$oz.				21.9		48.0	88.2			141.3	299.4	300	300
83. Juice #12 [2]		13.3		2.0	10.9	4.9	5.1	.4		13.2	49.7	50	50
84. Juice #303	3.3	9.1				16.5	16.7			24.1	69.6	70	70
85. Juice 2T		27.3		6.0	4.1	78.5			7.5	26.1	149.4	150	150
86. Juice 46oz.		51.7	405.2		162.2	6.4	4.0	10.3	658.3		1298.2	1300	1300
87. Juice #10	4.8	17.0						6.8		20.0	48.7	49	50
88. Tomato Cocktail	.7	21.5			13.4	12.0	16.5		34.8	.9	99.7	100	100
89. Tomato Cocktail 46oz.			36.6	8.0	19.2	17.2	66.9			51.2	199.1	200	200

1/Schedule totals are less than pack target maximum because of truncation in LP code.

2/Product was at intermediate level in final solution to master.

Table 8-3. Final Production Schedule by Week

Name	Week 1	2	3	4	5	6	7	8	9	10	Adjusted Schedule Total	Master Solution Total
Whole Peel Tomatoes												
Choice Round $2\frac{1}{2}$		30	147	130	29	43	141		155		675	675
Choice Coreless $2\frac{1}{2}$ Plant 3			81	40	56 (48)	79 (68)	32				288	300
Labeled Coreless $2\frac{1}{2}$ Plant 3		155 (141)	29		39 (30)	22 (19)	47		204 (149)		496	525
Standard Round $2\frac{1}{2}$		148	40	88	140	142	76	188			822	825
Standard Coreless $2\frac{1}{2}$ Plant 3			60 (42)	112 (79)	22	25					219	225
Choice Rounds #303		110 (83)	102	124	344 (333)	125 (117)	52	50	315 (294)		1,201 2/	1115
Choice Coreless #303 Plant 4		110 (84)	82	49 (40)	116 (103)	53 (24)	32 (30)		81 (60)		523	975
Choice Coreless #303 Plant 3		144 (119)	117		145 (130)						406	
Standard Round #303		64	112	116		85	59				436	450
Standard Coreless #303 Plant 4									131 (67)		131	150

(continued)

Table 8-3. (continued)

Name	Week 1	2	3	4	5	6	7	8	9	10	Adjusted Schedule Total	Master Solution Total
Choice Round #10		182	70	169	230	213	165	328			1,357	1,366
Choice Coreless #10 Plant 4		75	102		0 (25)						177	1,042
Choice Coreless #10 Plant 3		42	206	68		176	108	213			813	
Standard Rounds #10			186	114	108	82	128	40	360		1,018	1,033
Standard Coreless #10 Plant 3				58 (52)	38 (16)		30				126	150
Stew Round #303 (Produce #10's in Week 4)					141 (107)	122	42 (35)	22	113		418	750[3]
Stew Coreless #303 (Produce #10's in Week 4)		163	42		64 (31)						269	
Diced #10		105			120						235	261
Rounds Packed in Paste or Puree #10				24		27	53				104	134
Coreless Packed in Paste or Puree #10 Plant 3		58 (9)	55			42 (27)			217 (214)		372	398

(continued)

Table 8-3. (continued)

Name	Week 1	2	3	4	5	6	7	8	9	10	Adjusted Schedule Total	Master Solution Total
<u>Products</u>												
Paste 6oz.		228	403	333	350	262	267	339	376	291	2,849	2,850
Paste #303	92			382	155	114	120	110			973	1,000
Paste 25% #10 Plant 4		84 (74)		167	73						324	400
Paste 25% #10 Plant 3					74 (53)						74	
Paste 26% #10				117	37	40	38 (27)		48	112	392	400
Paste 30% #10		47	58 (24)	25	72	35				122	359	350
Paste 32% #10		35			45 (24)		30			32	142	166
Paste Drums 26%							516				516	527
Paste Drums 32%		3,166	3,304	1,874	3.498	2,908	2,320	3,116	3,391		23,577	23,635
Paste Drums 36%				1,572	330	537	432				2,871	3,000
Puree 1.06 #303			27				32				59	64
Puree 1.045 #2½						214					214	225

(continued)

Table 8-3. (continued)

Name	Week 1	2	3	4	5	6	7	8	9	10	Adjusted Schedule Total	Master Solution Total
Puree 1.06 #2½		86	108			75					269	300
Puree 1.045 PIC	51[1/]									24	75	75
Puree 1.045 #10 Plant 4		58	38		33		52 (50)			189	370	600
Puree 1.045 #10 Plant 3					40	74	50				164	
Puree 1.06 #10 Plant 4		90			64	55		355 (339)		55	619	825
Puree 1.06 #10 Plant 3					120		95				215	
Puree 1.07 #10		25	50	30							105	105
Fancy Catsup #10		142 (138)			78		104	46 (41)		72	442	450
Extra Standard Catsup #10				56 (29)	210	88 (79)	27			59	440	450
Standard Catsup #10			217			143	141		240 (225)		741	750
Sauce #10	53[1/]				0 (24)				97		150	150

(continued)

Table 8-3. (continued)

Name	Week 1	2	3	4	5	6	7	8	9	10	Adjusted Schedule Total	Master Solution Total
Chili Sauce #10				308		148					456	500
Pizza Sauce #10	46[1]	50					104				200	200
Juice 5½oz.				22		48	88			142	300	300
Juice #12	50[1]										50	50
Juice #300	45[1]									25	70	70
Juice 2T		28				78				27	133	150
Juice 46oz.		52	405		163				658		1,278	1,300
Juice #10	50[1]										50	49
Cocktail #12	65[1]										100	100
Cocktail #46	44[1]		37				67		35	52	200	200
Concentrate					50	25					75	88
Sauce A Line 2 8oz.								292			292	850
Sauce A Line 3 8oz.								517			517	

(continued)

Table 8-3. (continued)

Name	Week 1	2	3	4	5	6	7	8	9	10	Adjusted Schedule Total	Master Solution Total
Sauce A Line 2 #303					68 (60)		46				114	375
Sauce A Line 3 #303						179 (176)	61				240	
Sauce B.Line 2 8oz.			175		60		151 (143)			99	485	850
Sauce B Line 3 8oz.			64		73	60	95 (83)			73	365	
Sauce B Line 2 #303					36 (27)			282			318	375
Sauce B Line 3 #303					27	40					67	
Sauce C Line 2 8oz.		153	207								350	625
Sauce C Line 3 8oz.		166 (153)				26		69			261	
Sauce C Line 2 #303		164		30							194	250
Sauce C Line 3 #303		48 (40)									48	

(footnotes next page)

Table 8-3. (continued)

1/Production arbitrarily assigned to week 1.

2/Includes production of labeled rounds #303.

3/Includes production of labeled stew #303.

4/Includes all products packed in paste or puree.

CHAPTER IX

CONCLUSIONS

Decomposition

The decomposition principle of linear programming has substantial usefulness as a management tool under a variety of situations. Its first and most obvious use is in obtaining solutions to linear programming problems that are too large to solve with available computer equipment and codes. The nature of the program determines whether decomposition is suitable as a means of solution for such problems.

In addition, some problems can be solved as one large linear program, but can be solved more efficiently if decomposed. One set of observations indicated that the running time for large decomposed problems goes up linearly (plus a fixed amount per problem) rather than as the cube of the number of equations expected from the standard (undecomposed) linear program.[1]

In addition, some special structures of problems such as those which can be handled as a set of transportation or weighted transportation problems subject to a set of overall restrictions are very amenable to decomposition and frequently offer computational advantages even if the problem could be solved in a regular LP format. This is also true of

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[1] Helleman, _op_. _cit_., p. 20.

some problems where the parts are essentially independent of each other except for a few overall restrictions. The structure of the problem and its actual formulation are quite important as it is a general rule of decomposition that the fewer restrictions in the master, the fewer major iterations necessary.

An important conceptual application of the decomposition principle is as a management control tool for the centralized control of decentralized operations. The most obvious example is a multiplant firm where each plant management has developed a linear programming model of its own operation. Plant management uses the linear programming model of their plant to operate at maximum efficiency as they are responsible for the least-cost operation of the plant. Planning and goal setting are the functions of the firm. The firm provides a master program to find optimal solutions to the problem of utilizing the resources available to all their plants. Firm restrictions could take a variety of forms, including total capital availability, geographic limits, and raw product restrictions.

Another use of the decomposition principle as a management control tool is the utilization of facilities, capital or materials over time. The subproblems could be independent (the simplest case) or related, for example, the output of one period is used in the next.

A potential advantage of the decomposition approach is that the discipline of modeling can be installed and maintained at various levels. This approach will insure that appropriate models are available for the plants or other organizational units at all times. The models will be available for the plant's own use as well as for the use of the firm.

It is not necessary to use the plant models or other subproblems only in conjunction with the full model of the entire firm. Plant management and analysts can use the models at any time to investigate proposed changes in technology and resource utilization to improve plant operations. This includes the extensive application of parametric programming and cost ranging on the individual models if necessary. These tools cannot generally be applied to the solution of the decomposed problem. This capability coupled with the knowledge of the analyst of specialized technology of the plant and other local conditions should result in improvements to operations at individual plants that would be missed at higher levels.

There can be drawbacks to the use of decomposition rather than ordinary linear programming or other types of solution methods to large-scale problems. It is not a panacea for solving all large LP problems as it can take longer to solve a poorly formulated decomposition program than it would by a standard linear program. Problems that are to be solved by decomposition should be formulated with the computation procedure in mind. This requires consideration of both the problem and the characteristics and capacities of the particular computer programs to be used.

Post-optional analysis of the solution to the original problem is generally not available (although the master and each subproblem can be analyzed independently). In some applications, the information obtained from cost ranging and right-hand side ranging is used extensively.

Problem Formulation and Computation Procedures

Decomposition provides a workable method for solving large linear programming problems. Because of the size and probable complexity of problems for which decomposition is appropriate, preliminary analysis of the problem followed by careful model building is required. Knowledge of both the problem and the computer algorithms by the analyst is a necessity as each problem has unique characteristics and the current state of the art is such that manual intervention (in computerized procedures) will frequently speed up the solution process.

In general, the analyst should attempt to hold the number of constraints in the master to a minimum even if it means adding to the number of rows in the subproblems as the number of major iterations increases with the number of master constraints.

When formulating subproblems, consideration should be given to the computational efficiencies of the computer programs with respect to such things as scaling and specialized algorithms available.

Knowledge about the problem or previous solutions to all or part of the problem will frequently allow the analyst to submit trial proposals (subproblem solutions) initially or at intermediate stages, which will reduce the total computation time required.

Knowledge of the problem and experience with the problem will aid the analyst in answering such questions as whether the subproblems should be optimized at each major iteration or whether computation should be terminated after a fixed number of iterations. Should solutions from one, all, or several subproblems be obtained during each major iteration?

Should more than one solution be obtained from a subproblem during a major iteration? Should the decomposed problem be optimized or should computations terminate at a given percentage of the upper bound? Each problem is different, and comprehensive rules are not yet available for the above and other procedural questions. Consequently the judgement of the analyst is required.

Applications

Linear programming including the decomposition principle provides great promise for applications in fruit and vegetable processing. Assignment of raw product to the optimum finished product is a very important application that should result in major increases in total yields of finished products. Increases in yields through linear programming have been demonstrated in the past in areas such as meat packing, flour milling and in the similar application area of oil refining. However, the relative increase in finished product yields should be greater in fruit and vegetable processing. The finished product is not perishable like meat or even flour, so that relatively uneconomic assignments do not have to be made in response to short term or seasonal supply-demand imbalances. For example, the tomato pack is for a one year period and takes place over about ten weeks at a given location. If the optimal raw products for making a particular finished good comes from a single farm during a one week period, with sufficient plant capacity that raw product can all be assigned to that specific finished good and produced for distribution over the coming year. The maximum yield possible for the year is attained. This is, of course, impossible with a perishable product such as meat.

In addition, the perishability of the raw product forces the processing firm and the grower into a closer relationship than, for example, the wheat grower and the flour miller. The processor can consequently exert greater influence on the expected characteristics of the raw product. Linear programming models provide a tool for the processor to determine what characteristics are needed throughout the season to optimally utilize the plant while fulfilling the pack target. He can then attempt to obtain raw products with these characteristics.

Linear programming can also provide a tool to aid in scheduling a plant to get the maximum throughput or the most profitable level of throughput during the seasonal peak in availability of raw product. The latter was demonstrated in the tomato study. If the most profitable level of throughput is less than the quantity of raw product available, then the technique of linear programming can aid management in the selection of which available lots of raw product to purchase or if the raw product has been contracted in advance, which lots to process and which lots should be disposed of because they are uneconomical to process.

Another application is the evaluation of different methods of processing with the same or with different kinds of equipment. An example in the tomato study was the determination of whether tomatoes should be selected for canning before peeling or if all tomatoes should be peeled. A similar example was the inclusion of two types of peeling equipment in the model to determine which should be used, and when both were used which lots of tomatoes to peel on each type.

Linear programming can determine the proper blends of lots of raw product such as peeling tomatoes or other types of raw product to minimize preparation costs. One specific application in this study is the increase in yields through the blending of solids for the best operating conditions and best total yields.

A fifth application of linear programming for fruit and vegetable processing firms is the determination of equipment requirements. In particular, it can be used to find bottlenecks within the plant and to evaluate what costs are being incurred by operating with the existing equipment. It can be used to evaluate proposed capital expenditures for new equipment by changing the restrictions on the plant capacity or the elements within the model which relate to the productivity of a given piece of equipment.

Linear programming can be used to evaluate the type and quantity of raw products in terms of plant capacity and in terms of the sales target. It can be used to evaluate expected changes in raw product quality or expected price changes of raw products to aid in changing production targets for the year. It can be used to aid in evaluating proposed bids for raw product and to aid in determining what the characteristics of the raw products are that would be most desirable to get into the firm's mix of raw product. That is, it can be used to answer the questions: How does our raw product supply fit in terms of our sales target and our plant capacity? Should we change our sales targets or should we attempt to change the amount or type of raw product that we buy? Should we change the price we pay for raw products or should we change our plant capacity?

The Tomato Model

A large number of activities and restrictions were included in this model for testing and demonstrating specific applications. On analysis it was determined that a large number of blending activities were not needed for allocation or scheduling in either the whole peel operation or for product operations. There were many activities included originally in the model to maintain the identity of the tomatoes and their characteristics. However, there were not a large number of blending activities that actually had a major impact on the solution. Unless one can direct disposition of all the specific lots of tomatoes (which is very difficult considering actual yard operations), it is probably sufficient to have just enough blending activities to give general guide lines of what the optimum production schedule would be. Lots of tomatoes could generally be classified based on their characteristics for input to groups of finished products each week. This is particularly true in the case of peeling tomatoes. In the configuration of the model, blending of lots of different percentages of choice peelers did not occur except to meet the restrictions of the model.

There were too many finished products included in the model. More products should have been combined so that the minimum pack target of any one product grouping would have been equal to a larger quantity such as one week's production on its line. A particular problem in using linear programming to schedule by week develops if the volume of any given commodity is too small. This causes the model to schedule uneconomic runs in each of several weeks. It would be advantageous to have enough volume in each group of finished products so that the model could allocate the raw product to the group. Each week one or two of the finished products are scheduled to be produced instead of having several very short production runs. This means that some other method is used to do the scheduling of the finished product within each group. The linear programming model should be used to allocate the lots of raw product to the products from that particular group of the finished goods each week.

In general, the use of the tomato plant model for planning and operations purposes requires that it be solved at least two and possibly three times a year. It should be solved once prior to finalization of acreage and variety to determine what types of raw tomatoes are desired. It should be solved once during the growing season to develop a production plan and a tentative schedule after the size and nature of the crop becomes evident. It should be solved a third time during the packing season if the expected characteristics of the fruit or expected selling prices change substantially.

In addition to the full solutions outlined above, the solutions of subprograms or parts of the matrix might be necessary in case of machine breakdown during the middle of the packing season. Other reasons for resolving the subproblems would include the raw product characteristics being changed drastically because of a damaging rain storm or a long delay in getting crops out of the fields. A large order for specific commodity obtained during the packing season could be cause for resolving the problem. It would be advantageous in some instances to resolve the model to investigate how to bid on a sale of a large amount of a specific commodity not in the original pack target if the sale can be made during the packing season.

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