A BAYESIAN APPROACH TO OPTIMAL CROSS-HEDGING OF COTTONSEED PRODUCTS USING SOYBEAN COMPLEX FUTURES

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Abstract

Cottonseed crushers face substantial risk in terms of input and output price variability and they are limited in their planning by the lack of viable futures markets for cottonseed or cottonseed products. This study examines the feasibility of cross-hedging cottonseed products using soybean complex futures. Bayesian tests for market efficiency are performed on the cash and futures prices. The test results reject the presence of nonstationary roots, leading to the conclusion that the markets are not efficient. Different cross-hedging strategies are designed and analyzed for eight different hedging horizons in order to maximize the expected profit and utility of the crusher. A Bayesian approach is employed to estimate the parameters, which is consistent with expected utility maximization in the presence of estimation risk. The investigation reveals that both whole cottonseed and cottonseed products can be successfully cross-hedged using soybean complex futures. The profitability of cross-hedging cottonseed products depends not only on the appropriate size of the contract but also on the optimal choice of strategy consistent with the time of placing and lifting hedge and the appropriate hedging horizon.

Keywords: Cross-hedging, Bayesian decision science, nonstationarity, estimation risk, hedging horizon.

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Introduction

With each hundred pounds of fiber, the cotton plant produces approximately 155 pounds of cottonseed. At present production levels the national average is around 990 pounds of cottonseed produced per acre of cotton grown (National Cottonseed Products Association). Less than five percent of the seed must be set aside to plant the following year's crop. The remaining seed is used as raw material for the cottonseed processing industry or is fed to cattle, while a small amount is exported. When raw cottonseed moves from the gin to a cottonseed oil mill, it is made up of three parts: linters, which are short fibers still clinging to the seed; hulls, a tough, protective coating for the kernel; and the protein and oil rich kernel itself. In recent years, industry-wide yields of products per ton of cottonseed have averaged about 320 pounds of oil, 900 pounds of meal, 540 pounds of hulls, and 160 pounds of linters, with manufacturing loss of 80 pounds per ton (NCPA). Thus, the value of cottonseed is determined by the value of the products produced.

Of the four primary products produced by cottonseed processing plants, oil is the most valuable. On the average, it accounts for about 40 to 50 percent of the total value of all four products. Approximately 1.3 billion pounds of cottonseed oil are produced annually, making cottonseed oil the third leading vegetable oil in the United States (NCPA). Cottonseed meal is the second most valuable product of cottonseed, usually accounting for over one-third of total product value. It may be sold in the form of meal, cake, flakes, or pellets. Cottonseed meal is used principally as feed for livestock and is usually sold at a 41 percent protein level. Its major value is as a protein concentrate. Cottonseed hulls are used primarily as feed for livestock. Hulls differ from meal, however, in that it is roughage rather than a protein supplement. In feeding value, hulls are comparable to good quality grass hay and can serve as a practical supplement to pastures. Cottonseed linters, the short fibers removed from seed as the first step in processing, are sometimes referred to as "the fabulous fuzz". Through mechanical and chemical conversion, they enter a wider variety of end use products than any of the other products of cottonseed.

However, cottonseed products enter markets that are highly competitive. Soybean oil, corn oil, peanut oil, sunflower and safflower oil, and some of the animal fats are competitors of cottonseed oil. Cottonseed meal encounters a similar degree of competition from other protein concentrates, like peanut meal and sunflower meal, but especially soybean meal. Cottonseed hulls meet competition from various types of hay and from such feed as corn and sorghum silage. Moreover, the market for hulls is restricted geographically, and the value of hulls is determined largely by the supply and demand for hay and other roughages produced in the area. Like the other cottonseed products, linters meet a great many competitors in their struggle for markets. Cotton waste, a by-product of the textile industry, is linters' major competitor in the bedding, automotive, and furniture industries. Wood pulp is the principal competitor of linters in the chemical products market. As a result, cottonseed crushers face a substantial price risk, similar to other feed ingredients processors in terms of input and commodity price variability. They are limited in their planning because no viable futures market currently exists for cottonseed oil, meal, hulls and linters.

The main objectives of this research are to explore alternative marketing strategies for cottonseed products, examine their potential use to minimize input and price risk, and improve profitability of cottonseed processing. Specifically, the possibilities of cross-hedging the whole cottonseed, cottonseed meal and cottonseed oil using soybean, soybean meal and soybean oil futures contracts, respectively, are analyzed and evaluated from cottonseed crushers' perspective.

The central hypothesis of this study is that even though no active futures market exists for whole cottonseed and cottonseed crush, processors can reduce input and output price risk through cross-hedging. The input risk can be reduced by cross-hedging the whole cottonseed with soybean and the price risk can be reduced through cross-hedging cash cottonseed products with soybean products, commodities having established futures markets. Additionally, it is hypothesized that the relationship between cash cottonseed products prices and the soybean products futures prices are strong enough such that cross-hedging can be efficiently executed. The final hypothesis is that net realized prices from cross-hedging will exhibit risk efficiency superior to cash pricing. Cross-hedging cottonseed hulls and linters are, however, not considered in this study, since time series data are not available for these products.

By definition, cross-hedging is the pricing of a cash commodity position by using futures for different commodities. Simple cross-hedging uses futures of one commodity to offset a cash position, and multiple cross-hedging uses two or more different commodities. However, cross-hedging is more complicated than a direct hedge. Difficulties arise in selecting the appropriate futures contracts as cross-hedging vehicles and determining the size of the futures position to be established. Potential cross-hedging vehicles must be commodities that are likely to demonstrate a strong direct or inverse price relationship to the cash commodity. This analysis is concerned only with simple cross-hedging. Soybean, soybean oil and soybean meal are selected as cross-hedging vehicles for this analysis. Soybean oil and soybean meal are close substitutes for cottonseed oil and cottonseed meal, respectively, and they are thought to be influenced by many of the same supply and demand factors. Soybean is also regarded as a close substitute for cottonseed with respect to protein concentration.

While cross-hedging their inputs and outputs cottonseed crushers' make two very important decisions. First, they decide how much of the commodities they should hedge. Second, they decide when to place and lift the hedge. The more accurate they are in these two risky decisions, the more risk free they are. The goal of this study is to present a set of time-specific cross-hedging strategies for cottonseed and cottonseed products which are optimal under risky decision making but easy to manage. In order to meet the goal, Bayesian decision science is chosen over the standard sampling theory approach as the Bayesian approach is appropriate for decision-making under estimation risk.

Employing Bayesian decision science, expected utility and profit maximizing cross-hedge ratios are estimated for eight different hedging horizons, viz., May 4-week, May 8-week, May 12-week, May 24-week, October 4-week, October 8-week, October 12-week, and October 24-week. It is found that optimal level of profit as well as utility increases the longer the term of cross-hedging when placed by the end of May, and decreases with the length of hedge placed by the end of October. Finally, the optimal Bayesian cross-hedging marketing strategies are applied to cross-hedge cottonseed, meal and oil in the year of 1998-99 for different risk-aversion level. The results of the application support empirical findings with little exceptions.

A Brief Review of Literature

An extensive theoretical description of cross-hedging for a commodity for which no futures exists is provided by Anderson and Danthine (1981). Assuming a non-stochastic production process (no yield risk), Anderson and Danthine considered the problem of hedging in a single futures market but with many possible trading dates. Their cross-hedging model used a mean-variance framework to derive an optimal hedging strategy assuming that the agent had knowledge of the relevant moments of the probability distribution of prices. Kahl (1983) illustrated the derivation of optimal hedging ratios under different assumptions about the cash position. She argued that, when the futures and cash positions were endogenous, the optimal hedging ratio was independent of risk aversion. Comparing the study of Heifner (1972, 1973) to that of Telser (1955, 1956), she showed that the optimal hedging ratio was not dependent on the risk parameter. Following Wilson (1989), the optimal hedge ratios obtained from minimizing the variance of revenue were equivalent to parameters estimated from ordinary least squares regression (OLS) of cash price changes on future price changes. Ames et al. (1992) investigated the possibility of cross-hedging canola with a complex soybean meal and soybean oil futures contract. They found that canola could be effectively hedged using soybean oil and meal futures. A minimum variance method was adopted in their study as a measure of hedging effectiveness. Fackler and McNew (1993) discussed the derivation and estimation of optimal hedge positions for firms that deal in multiple commodities and have multiple relevant futures contracts available for hedging. They pointed out that soybean crush hedging must account for the existing relationships among the cash prices of soybeans, soybean oil and soybean meal. They showed that hedge ratios derived from separate single commodity estimates are also sub-optimal. It was found that multi-product optimal hedge positions provide significant risk reductions relative to simpler approaches.

Following Fackler and McNew, Dahlgran (2000) presented a cross-hedging consulting study performed for a cottonseed crusher. He examined how futures markets should be used to hedge cottonseed crushing. He applied a soybean crushing spread in a cross-hedging context with a portfolio risk minimization objective to develop the desired hedge ratios for a variety of cross-hedging portfolios and for several hedge horizons. Risk minimizing hedge ratios were derived by regressing changes in prices for cottonseed, cottonseed hulls, cottonseed meal, and cottonseed oil against changes in prices of potential hedge vehicles, such as futures contracts for the soybean complex; futures contracts for feed grains; US wheat futures contract; futures contract for cotton, the dollar index, and the Japanese yen; and Canadian futures contracts for flaxseed, rapeseed, oats, and wheat. Stepwise regression was used to select the most effective cross-hedge utilizing a given number of futures contracts. Dahlgran reported that the effectiveness increased the longer the term of the hedge. Based on follow-up discussions, his article reported whether the recommended hedging strategies were adopted, how they were applied, the difficulties in implementing these strategies, and differences between managerial and academic perceptions of hedging strategies. His observations imply that the economics of hedge management might be as important as the underlying risk aversion in determining hedging behavior. But, Dahlgran used a very wide range of futures contracts in his hedge vehicle pool, which was difficult to manage. Moreover, he did not consider the appropriate time of placing and lifting hedge while examining different horizon.

Estimation Risk

Whenever economic analysis involves incorporating estimated parameters into theoretically derived decision rules, the optimal outcome depends on the estimation procedure. This problem is called estimation risk (Bawa, Brown, and Klein). Estimation risk is ever-present in economic problems. Typically, it is ignored and sample parameter estimates are directly substituted for the true but unknown parameters in theoretical decision rules.

Lence and Hayes (1994) showed that the optimal futures position estimated by means of the parameter certainty equivalent (PCE) approach lacks normative value because it is generally sub-optimal when there is uncertainty regarding the actual parameter values. They provided a model based on a Bayesian decision criterion that can be used to obtain an optimal futures position in the realistic situation where the decision maker has sample information and prior beliefs regarding the relevant parameters. They claimed that their model nested both the theoretical model with perfect parameter information and the PCE formula, and yielded the perfect parameter information paradigm when the decision maker is completely confident about his or her prior information relative to the sample information. The PCE formula was nested within their model when the quality of the sample information was infinitely larger than the quality of the prior and the sample size is infinite. Lence and Hayes also presented the results of some simulations regarding the futures position obtained by means of Bayesian criterion, the PCE approach, and the PPI (perfect parameter information, assuming that the priors equal the true parameters) case. The simulations demonstrated the sensitivity of the optimum futures position to the method that was used. They inferred that the differences in the optimal futures position implied a large monetary value to investors using the proposed method.

Theoretical Background

The principal theory of choice underlying risky decision making is the expected utility theory. A complete understanding of the approach can be found in the landmark work of Von Neumann and Morgenstern (1944). The expected utility model follows that an optimal choice under risk is the one which is based on the maximization of expected utility. The expected utility model provides a single-valued index that ranks decision alternatives. The formation of the utility function is based on the individual's attitude towards risk. Three alternative forms of subjective risk attitude are risk neutrality, risk aversion, and risk preference.

A linear utility function implies risk neutrality, a function concave to the origin implies risk aversion, and a convex function implies risk preference.

A risk-averse cottonseed crusher has a utility function $u(p)$, where p is the total profit from crushing, characterized by two important properties: nonsatiation and decreasing marginal utility of returns ($u'(p) > 0$, $u''(p) < 0$). These two properties imply that the utility function is concave. Because of the shape of the utility function, a risk-averse individual prefers a sure amount to taking a risk with the same expected payoff; i.e., $u[E(p)] > E[u(p)]$. The certainty equivalent, CE, is the amount in units of p that gives the same utility as some risk taking decisions. Risk-averse individuals are willing to pay an insurance premium to avoid the uncertainty involved in the risky decisions. The risk premium is the amount of p that will make an individual indifferent between receiving the certain amount, CE, and taking the risk. For risk-averse individuals, CE is always less than the expected return and the risk premium is always positive. Typically, it is assumed that a risk averse decision maker has a negative exponential utility function of the form:

$$U(p) = -e^{-\phi p} \quad (1)$$

$$\phi = -U''(p)U'(p) \quad (2)$$

In the above equations, ϕ denotes the Arrow-Pratt coefficient of absolute risk aversion. Constant Absolute Risk Aversion (CARA) is commonly used to analyze producer's decisions under risk (e.g., Antle and Goodger, 1984; Babcock, Choi, and Feinerman, 1993; Buccola, 1980; Chalfant, Collender, and Subramanian, 1990; Lee and Brorsen, 1994; Lence and Hayes, 1995; and Yassour, Zilberman, and Rausser, 1981). The value of ϕ generally lies between 0.001 and 0.000001, with smaller values implying less risk aversion. In general, the expected utility depends on the entire probability distribution of the outcomes. However, without assuming any particular form of distribution of the prices or of the profit function, expected utility can be evaluated using the above function with the observations at some particular points of the similar time rotations (e.g., week, month or year).

Following the analyses of Kallberg and Ziemba (1979), the risk aversion level can be defined as $\alpha = \phi\omega$, where α is the level of risk aversion, ϕ is the coefficient of absolute risk aversion and ω is initial wealth. If gross returns per dollar of initial wealth are around unity, then moderate risk aversion corresponds to α values between 2 and 4. In the present case of cross-hedging, it was difficult to calculate the actual wealth at risk. However, it was found that the operating capital (capital used to purchase the raw materials and operation of the crushing plant) of a plant capable of crushing 1000 ton of cottonseed is around $200,000. Approximately, 50-75% of this amount was borrowed capital (Southern Cotton Oil, Valdosta, Georgia). So, we set ω equal to $50,000 and $100,000. Defining risk aversion levels as moderate-to-low (α = 1), moderate (α = 2) and moderate-to-high (α = 3), the corresponding coefficients of absolute risk aversions are:

	$\alpha = 1$	$\alpha = 2$	$\alpha = 3$
ω = $ 50,000	$\phi = 0.00002$	$\phi = 0.00004$	$\phi = 0.00006$
ω = $ 100,000	$\phi = 0.00001$	$\phi = 0.00002$	$\phi = 0.00003$

In equations 1 and 2, π is the total profit from cottonseed crushing. The profit function of a cottonseed crusher includes futures prices of soybean and soybean products along with the cash prices of cottonseed and cottonseed products, crushing cost and the corresponding hedge ratios. Consider a cottonseed crusher with a crushing plant capable of crushing 1000 tons (2,000,000 lbs.) of cottonseed cost effectively. According to the NCPA (National Cottonseed Products Association), crushing 1000 tons of cottonseed produces 900,000 lbs of cottonseed meal, 320,000 lbs of cottonseed oil, 540,000 lbs of hulls and 160,000 lbs of linters. Using the cash and futures market prices we can calculate the profit from crushing 1000 tons of cottonseed by applying the following formula.

$$p = -(2{,}000{,}000 \text{ x } P_c) + ß_c \text{ x } 2{,}000{,}000(P_{sl} - P_{sp}) + (900{,}000 \text{ x } P_{cm}) + ß_m \text{ x } 900{,}000(P_{smp} - P_{sml})$$
$$+ (320{,}000 \text{ x } P_{co}) + ß_o \text{ x } 320{,}000(P_{sop} - P_{sol}) + 540{,}000 \text{ x } P_{ch} + 160{,}000 \text{ x } P_{cl} - C_c \text{ x } 100. \quad (3)$$

where,

P_c = Cash price of cottonseed per pound

P_{cm} = Cash price of cottonseed meal per pound

P_{co} = Cash price of cottonseed oil per pound

P_{ch} = Cash price of hulls per pound

P_{cl} = Cash price of cottonseed linters per pound

P_{sp} = Soybean futures price at the time of placing hedge

P_{sl} = Soybean futures price at the time of lifting hedge

P_{smp} = Soybean meal futures price at the time of placing hedge

P_{sml} = Soybean meal futures price at the time of lifting hedge

P_{sop} = Soybean oil futures price at the time of placing hedge

P_{sol} = Soybean oil futures price at the time of lifting hedge

$ß_c$ = Hedge ratio for soybean

$ß_m$ = Hedge ratio for soybean meal

$ß_o$ = Hedge ratio for soybean oil

C_c = crushing cost per ton

In equation (3), profit from hedges are calculated by considering the differences between the futures prices at the time of placing and lifting hedge, according to the futures position taken by the crusher. A risk minimizing crusher establishes a long position by buying soybean futures contracts and offsets it by selling the same number of contracts at the time of buying cash cottonseed. Therefore, the profit from soybean futures transaction is determined by the difference of soybean futures price at the time of lifting hedge from the price at the time of placing hedge. On the other hand, the crusher establishes short positions by selling soybean meal and oil futures contracts. She offsets the short positions by buying the futures contracts at the time of selling cash cottonseed meal and oil. So, the profit from soybean meal and oil futures transactions are the differences of the futures prices at the time of placing hedge from the prices at the time of lifting hedge. It should also be mentioned here that the costs of rollovers are hidden in the hedge profit terms.

The average cash price for hulls ($0.0605 per pound) in Atlanta is obtained from Feedstuffs magazine. The approximate average price of linters is $0.15 per pound (Mr. Jerry Wiseman) and the average crushing cost is approximately $50.00 per ton (Southern Cotton Oil, Valdosta, Georgia). Using these data, the above profit function can be reduced to:

$$p = 2,000,000 \text{ x } [-P_c + ß_c(P_{sl} - P_{sp})] + 900,000 \text{ x } [P_{cm} + ß_m(P_{smp} - P_{sml})] + 320,000 \text{ x } [P_{co} + ß_o(P_{sop} - P_{sol})] + 6,670.00 \quad (4)$$

The utility of the crusher can be obtained by substituting the profit function in the negative exponential utility function. With estimation risk, the method by which the optimal decision can be obtained in a manner consistent with expected utility maximization is Bayesian decision criterion (Klein et al.; DeGroot; Berger). In the special case of a negative exponential utility function as used in this study, the Bayesian decision vector $(ß_c\ ß_m\ ß_o)'$ is the solution that maximizes expected utility. The basic idea here is to select a current strategy using past data that optimizes a Bayesian loss function.

Data and the Bayesian Test for Market Efficiency

The data used in this analysis are constructed from three sources. The cash cottonseed and cottonseed meal price data for three locations--Los Angeles, Memphis and San Francisco--are obtained from *Feedstuffs* magazine. The observations are Wednesday closing prices from July 6, 1994 through September 15, 1999. Cottonseed oil market prices were not available on a local or regional basis. Monthly average prices for cottonseed oil are obtained from *Oil Crops Situation and Outlook Yearbook* published by Economic Research Service of the United States Department of Agriculture. The soybean complex, soybean meal and soybean oil futures prices are obtained from Chicago Board of Trade. The soybean and soybean meal futures prices are also the Wednesday closing prices for the same time period and are always for the contract nearest to maturity. The soybean oil futures prices are monthly averages for the contract nearest to maturity.

Bayesian tests for market efficiency are performed on all cash and futures prices, using the procedures developed in Dorfman (1993). The Bayesian test for market efficiency is essentially a comparison of the probability of a nonstationary root versus the probability of a stationary dominant root, assuming an autoregressive time series model for the data series being tested. After setting the prior distribution on the roots of the time series and specifying a likelihood function, Monte Carlo integration techniques are employed to numerically approximate the posterior probabilities in favor of and against stationarity. In the tests performed here, two prior specifications are used—a beta distribution on each root and an uninformative prior—and two likelihood functions are also investigated—one nonparametric and a standard Gaussian (normal) one. Posterior probabilities are calculated numerically by Bayes' Theorem which states the posterior is proportional to the prior times the likelihood. Posterior odd ratios can then be formed from the posterior probabilities by dividing one posterior probability by the other; an odds ratio greater than one shows posterior support for the hypothesis placed on the top of the odds ratio. The posterior odds ratios were computed for all combinations of prior distributions and related assumptions.

The test results are depicted in table 1. An odds ratio greater than one implies an efficient market, while an odds ratio less than one implies an inefficient market. The test results reject market efficiency (the presence of nonstationary roots) except for the cottonseed price series of Forth Worth and soybean meal futures price series. Of the 48 odds ratios, only four are greater than unity. The test on cottonseed cash prices for Fort Worth strongly supports an efficient market when employing the nonparametric density. The test on the soybean meal futures contracts also favors an efficient market under nonparametric density. But when assuming a normal distribution for price changes, the tests show very little posterior support for unit roots and the corresponding market efficiency. Such market inefficiency implies that the futures prices could be predicted accurately enough to earn a risk adjusted economic profit, opening the way for potentially profitable hedging and/or cross-hedging strategies.

Table 1. Results of Bayesian test for Market Efficiency.

Sample	K_{nb}	K_{nf}	K_{gb}	K_{gf}
Cottonseed (Fort Worth)	1.2751*	1.3706*	0.2912	0.2917
Cottonseed (Los Angeles)	0.0681	0.1194	0.0430	0.0232
Cottonseed (Memphis)	0.0632	0.0526	0.0843	0.0645
Cottonseed (San Francisco)	0.2328	0.1844	0.0688	0.0336
Cottonseed meal (Fort Worth)	0.6960	0.7734	0.1724	0.1634
Cottonseed meal (Los Angeles)	0.3752	0.4051	0.3602	0.3768
Cottonseed meal (Memphis)	0.0018	0.0018	0.2343	0.2375
Cottonseed meal (San Francisco)	0.0298	0.0327	0.2736	0.2715
Cottonseed oil	0.0941	0.0264	0.0994	0.0204
Soybean futures	0.3368	0.3076	0.2348	0.2508
Soybean meal futures	1.3453*	1.0753*	0.3376	0.3423
Soybean oil futures	0.1675	0.0618	0.1576	0.0539

K is the posterior odds ratio in favor of a nonstationary dominant root, the subscripts representing innovation density and prior, respectively. Subscript n stands for the nonparametric density; g for the Gaussian (normal) density; b the beta-prior; and f the flat prior. Odds ratios marked by asterisks support efficient markets.

Optimal Cross-Hedging Strategies

The purpose of this section is not only to calculate the optimal hedge ratios but also to find out the optimal time and duration of cross-hedging activity. In order to estimate the optimal cross-hedge ratios, simulations were performed using eight data sets. In the case of placing a hedge during the end of May, four data sets were constructed with four different durations: four, eight, twelve, and twenty-four weeks. Four similar data sets were constructed in the case of placing hedges during the end of October. May and October were chosen as the times of placing hedges by considering expected and actual cottonseed production. Cotton is typically planted throughout March and early April and harvested in September through November (NCPA). So, by the beginning of May, a cottonseed crusher would have an estimated amount of cottonseed production. To protect herself from fluctuations in cottonseed, meal and oil prices, she would like to place cross-hedges around May-June. As the cotton growing season progresses, yields may be estimated with greater accuracy. Finally, at the end of October, the cottonseed crusher would have the actual amount of cottonseed produced. She would also have the estimated amount of meal, oil, hulls and linters. So, there may be some potential for placing cross-hedges during the end of October.

Each of the eight data sets was constructed with the Memphis cash prices of cottonseed, cottonseed meal and cottonseed oil along with the CBOT (Chicago Board of Trade) futures prices of soybean, soybean meal and soybean oil. Both cash and futures prices at the time of placing and lifting hedges were obtained for ten consecutive years, 1988-89 through 1997-98. Employing Bayesian decision science, simulations were performed with each of the data sets. The prior belief here is that the hedge ratios lie between 0 and 1.2, but prior belief is uniformly distributed within this range. Hedge ratios smaller than zero and greater than one imply speculation, so the upper limit of the hedge ratio being set to 1.2 allows for a slight speculation in order to reduce the net hedging gap.

Simulations were performed in six steps. First, using the observations of cash and futures prices and the hedge ratios to be selected ($ß_c$, $ß_m$, and $ß_o$), a profit function was constructed according to equation 4. For a fixed set of hedge ratios, 10,000 values for all the unknown parameters and random elements of the models presented are drawn from their posterior distributions. The expected value of the profit was calculated by simply taking the mean value of profit over the 10,000 calculated values (one for each drawn set of parameters). Third, the value of the profit was substituted in the negative exponential utility function and the expected utility from profit was calculated. That finishes the simulation for a fixed set of hedge ratios. The above steps were repeated for all possible combinations of hedge ratios from 0 to 1.2, initially with 0.1 increments for each of the parameters. Resulting expected profits and expected utility of profits were saved in a matrix along with the corresponding values of the parameters. The optimal hedge ratios, which gave the maximum expected profit and the maximum expected utility, were then separated from the saved matrix. Finally, the fourth and fifth procedures were repeated with 0.01 increments around the initial estimates of the parameters. Finding the optimal hedge ratios from this second, finer scan provides the Bayesian estimators of the optimal hedge ratios for the whole cottonseed, cottonseed meal and cottonseed oil, respectively.

Empirical Results

Using the procedures described above, the eight different cross-hedging horizons were evaluated for five different values of the coefficient of absolute risk aversion. Simulation results are summarized in tables 2, 3, and 4. The expected profit-maximizing Bayesian cross-hedge ratios, along with the corresponding optimum profits for all the marketing alternatives, are shown in table 2. The expected utility maximizing Bayesian cross-hedge ratios and corresponding optimal utilities under different levels of risk aversion are presented in tables 3 and 4. It is evident that the choice of a cross-hedging strategy based on expected profit maximization is insensitive to risk preference. However, optimal cross-hedge ratios based on expected utility maximization vary with the risk aversion coefficient.

Table 2 shows the Bayesian cross-hedge ratios for the whole cottonseed ($ß_c$), cottonseed meal ($ß_m$) and cottonseed oil ($ß_o$) for the eight alternative hedging horizons. The estimated cross-hedge ratios are either 1.2 or zero. The expected profit maximizing simulation procedure gives the extreme values of the parameters based on the historical patterns in prices. This implies that a cottonseed crusher can make profit on average by cross-hedging when the average price change over those ten years is favorable (positive price change for cottonseed meal and oil and negative price change for the whole cottonseed). The empirical results suggest that cross-hedging cottonseed is always profitable if the hedge is placed by the end of October and by the end of May only for four weeks. Cross-hedging cottonseed oil is always profitable if the hedge is placed by the end of May and never profitable if placed by the end of October. The May 8-week, October 4-week and October 8-week cross-hedges of cottonseed meal do not give any profit on average.

The results also suggest that cross-hedging cottonseed, cottonseed meal and cottonseed oil at the same time is not profitable unless the hedge is placed by the end of May only for four weeks. The corresponding optimum profits for all the alternative cross-hedging strategies are also presented in Table 2. It is clear that the May 24-week cross-hedging of cottonseed meal and oil, ($ß_c$=0.0, $ß_m$=1.2, and $ß_o$=1.2), gives the maximum expected profit among the eight marketing strategies. The May 4-week cross-hedging of cottonseed, meal and oil, ($ß_c$=1.2, $ß_m$=1.2, and $ß_o$=1.2), gives the minimum expected profit among all of the strategies. It is also noticeable that the October 4-week and 8-week cross-hedging of cottonseed only give high volume of profits.

Table 2. Profit Maximizing Cross-Hedge Ratios of Cottonseed, Meal and Oil

Cross-hedging Horizon	Cross-hedge Ratios			Optimum Profit
	$ß_c$	$ß_m$	$ß_o$	($)
May 4-week	1.2	1.2	1.2	3436.32
May 8-week	0.0	0.0	1.2	2475.28
May 12-week	0.0	1.2	1.2	19246.8
May 24-week	0.0	1.2	1.2	34946.4
October 4-week	1.2	0.0	0.0	31358.6
October 8-week	1.2	0.0	0.0	26627.4
October 12-week	1.2	1.2	0.0	24131.8
October 24-week	1.2	1.2	0.0	19543.4

Considering five different risk aversion coefficients, the expected utility maximizing Bayesian cross-hedge ratios and the corresponding optimal expected utility levels for alternative May cross-hedging are presented in tables 3 and 4. From table 3, it is clear that the cross-hedge ratios are identical to the expected profit maximizing ones when the absolute risk aversion coefficient is very low (f = 0.00001). However, the May 8-week cross-hedge ratio for cottonseed meal increases with the coefficient of absolute risk aversion. On the other hand, the May 24-week cross-hedge ratio for cottonseed meal decreases as f rises. An abrupt decrease in the May 4-week cross-hedge ratio for cottonseed oil is also observed when f = 0.00006. On the other hand, an increase in the October 4-week cross-hedge ratios for cottonseed meal (0.0 to 0.8) and oil (0.0 to 0.4) is observed when f = 0.00006. The October 8-week cross-hedge ratio for meal increases gradually with the risk aversion coefficient up to f = 0.00004 but falls abruptly (1.2 to 0.1) when f = 0.00006. A similar abrupt decrease (1.2 to 0.3) is also observed in the October 8-week cross-hedge ratio for the whole cottonseed. The October 12-week and 24-week cross-hedge ratios for cottonseed fall with the increase in the risk aversion coefficient. However, increase in f shows strong support in favor of cross-hedging cottonseed oil (0.0 to 1.2) using the October 8-week, 12-week and 24-week terms.

Table 4 summarizes the resulting expected utilities from the eight alternative May and October cross-hedging strategies under different level of risk aversion. Table 4 shows that expected utility increases with hedge length for the May hedges, with a 24-week cross-hedging strategy giving the highest level of expected utility among the four alternatives under all levels of risk aversion. For October hedges, expected utility decreases the longer the term of hedge and a 4-week cross-hedging strategy gives the highest level of expected utility among the four alternatives under all five risk aversion coefficients. This is the exact opposite case to that of May cross-hedging, due to differences in the prevalent price dynamics in the spring and the fall.

Table 3. Expected Utility Maximizing Cross-Hedge Ratios for Under Different Risk Aversion Coefficients

Risk Aversion Coefficient	May 4-week Cross-hedge Ratios			May 8-week Cross-hedge Ratios			May 12-week Cross-hedge Ratios			May 24-week Cross-hedge Ratios		
	$ß_c$	$ß_m$	$ß_o$	$ß_c$	$ß_m$	$ß_o$	$ß_c$	$ß_m$	$ß_o$	$ß_c$	$ß_m$	$ß_o$
f=0.00001	1.2	1.2	1.2	0.0	0.0	1.2	0.0	1.2	1.2	0.0	1.2	1.2
f=0.00002	1.2	1.2	1.2	0.0	0.1	1.2	0.0	1.2	1.2	0.0	1.2	1.2
f=0.00003	1.2	1.2	1.2	0.0	0.4	1.2	0.0	1.2	1.2	0.0	0.9	1.2
f=0.00004	1.2	1.2	1.2	0.0	0.5	1.2	0.0	1.2	1.2	0.0	0.6	1.2
f=0.00006	1.2	1.2	0.3	0.0	0.8	1.2	0.0	1.2	1.2	0.0	0.4	1.2

Risk Aversion Coefficient	Oct. 4-week Cross-hedge Ratios			Oct. 8-week Cross-hedge Ratios			Oct. 12-week Cross-hedge Ratios			Oct. 24-week Cross-hedge Ratios		
	$ß_c$	$ß_m$	$ß_o$	$ß_c$	$ß_m$	$ß_o$	$ß_c$	$ß_m$	$ß_o$	$ß_c$	$ß_m$	$ß_o$
f=0.00001	1.2	0.0	0.0	1.2	0.0	0.0	1.2	1.2	0.0	1.2	1.2	0.0
f=0.00002	1.2	0.0	0.0	1.2	0.4	1.2	1.2	1.2	0.7	1.2	1.2	1.2
f=0.00003	1.2	0.0	0.0	1.2	1.0	1.2	0.8	1.2	1.2	1.2	1.2	1.2
f=0.00004	1.2	0.0	0.0	1.2	1.2	1.2	0.3	1.2	1.2	1.0	1.2	1.2
f=0.00006	1.2	0.8	0.4	1.2	0.1	1.2	0.0	1.2	1.2	0.8	1.2	1.2

Table 4: Resulting Utility from Alternative Cross-Hedging Strategies Under Different Risk Aversion Coefficients

Risk Aversion Coefficient	May 4-week Cross-hedging	May 8-week Cross-hedging	May 12-week Cross-hedging	May 24-week Cross-hedging
f=0.00001	-0.9843	-0.9886	-0.8372	-0.7247
f=0.00002	-1.0075	-1.0064	-0.7247	-0.5523
f=0.00003	-1.0751	-1.0529	-0.6515	-0.4367
f=0.00004	-1.1976	-1.1321	-0.6099	-0.3507
f=0.00006	-1.6721	-1.4247	-0.6017	-0.2332
Risk Aversion Coefficient	**Oct. 4-week Cross-hedging**	**Oct. 8-week Cross-hedging**	**Oct. 12-week Cross-hedging**	**Oct. 24-week Cross-hedging**
f=0.00001	-0.7484	-0.8037	-0.8313	-0.8727
f=0.00002	-0.5887	-0.7045	-0.7863	-0.8459
f=0.00003	-0.4870	-0.6621	-0.8181	-0.8849
f=0.00004	-0.4223	-0.6695	-0.9009	-0.9890
f=0.00006	-0.3527	-0.7866	-1.2266	-1.3706

These results can be used as a guide for the cottonseed crushers to protect themselves against the input and output price risks. Applying the results of this analysis, an expected profit maximizing crusher can meet her objectives by cross-hedging whole cottonseed using an October 4-week strategy, ($ß_c$=1.2, $ß_m$= 0.0, and $ß_o$= 0.0), and cottonseed meal and oil employing a May 24-week cross-hedging strategy, ($ß_c$=0.0, $ß_m$=1.2, and $ß_o$=1.2). A risk averse crusher who tries to maximize expected utility would reach her goal by choosing the same hedge horizons, but she has to determine the optimal hedge ratios corresponding to her risk aversion coefficient. For example, a moderate-to-high risk averse cottonseed crusher (f = 0.00006) would choose the strategy of cross-hedging cottonseed meal and oil using a May 24-week strategy with hedge ratios of $ß_c$=0.0, $ß_m$= 0.4 and $ß_o$ =1.2, and using an October 4-week strategy with hedge ratios of $ß_c$=1.2, $ß_m$=0.8 and $ß_o$=0.4.

A further empirical example follows.

An Application of the Strategies

The results in the previous section provide the appropriate cross-hedge ratios and the information for the proper cross-hedging strategy to use. While using soybean, soybean meal and soybean oil as the cross-hedging vehicles for the whole cottonseed, cottonseed meal and cottonseed oil respectively, a cottonseed crusher would establish a long position by buying soybean futures contracts and short positions by selling soybean meal and soybean oil futures contracts.

Establishing the appropriate size of the futures position to be taken, the number of contracts of the cross-hedging vehicle required to equate to a given cash position needs to be multiplied by the cross-hedge ratio. Suppose that a cottonseed crusher in Georgia is planning to process 1,000 tons (2,000,000 pounds) of cottonseed from which approximately 900,000 pounds of meal and 320,000 pounds of oil would be produced. In order to protect herself from the fluctuations of prices in the cash markets, she would like to place cross-hedges against cottonseed, cottonseed meal and cottonseed oil using soybean, soybean meal and soybean oil futures respectively. The soybean futures trading unit at Chicago Board of Trade is 5,000 bushels, which is equivalent to 300,000 pounds (60 lb/bu). So, the number of soybean contracts equivalent to 1,000 tons of cottonseed is 6.67 (2,000,000 lb/ 300,000 lb). To cross-hedge 1,000 tons of cottonseed, the crusher has to take long position of $ß_c$ x 6.67 soybean futures contracts. On the other hand, the trading units of soybean meal and soybean oil futures contracts at CBOT are 100 tons (200,000 lb) and 60,000 pounds, respectively. Thus, in order to cross-hedge cottonseed meal and oil, the crusher has to short $ß_m$ x 4.5 soybean meal futures contracts and $ß_o$ x 5.33 soybean oil futures contracts, respectively. Using the Bayesian cross-hedge ratios presented in the previous section, the results of cross-hedging for all of the eight alternative strategies under different levels of risk aversion can be evaluated.

The resulting expected profit and utility from the May and October cross-hedging alternatives for different sets of cross-hedge ratios under the three risk aversion coefficients are presented in Table 5 using data from 1998 to evaluate the alternative strategies. Expected profit and utility from cash pricing, assuming a moderate to low risk averse crusher (f = 0.00001), on the same dates of offsetting cross-hedges are also listed in Table 5 for comparison. From the upper part of Table 5 it is clear that with respect to the resulting expected utility all of the May cross-hedging alternatives are superior to cash pricing. With respect to expected profit, all of the May cross-hedging strategies are superior to cash pricing except for the May 4-week with $ß_c$=1.2, $ß_m$=1.2 and $ß_o$=1.2. It is also evident that the May 12-week cross-hedging is superior to the May 24-week, the May 24-week cross-hedging is superior to the May 8-week and the May 8-week cross-hedging is superior to the May 4-week cross-hedging in general. Particularly, the May 12-week cross-hedging strategy with $ß_c$= 0.0, $ß_m$= 1.2 and $ß_o$= 1.2 is the most preferable among the May marketing alternatives. The May 24-week cross-hedging is inferior to the May 12-week because of an unusually abrupt decrease in cottonseed meal cash price at the end of 1998. Thus, with a little exception, 1998 May cross-hedging results confirm the findings of Bayesian cross-hedging method described in the previous section.

The resulting profit and utility from different October cross-hedging under the three risk aversion coefficient are presented in the lower part of Table 5. Results confirm that utility decreases the longer the term of cross-hedging without any exception. The October 4-week cross-hedging with $ß_c$= 1.2, $ß_m$= 0.0 and $ß_o$= 0.0 is found to be the most effective among all. Table 5 also shows that only the October 4-week cross-hedging with $ß_c$= 1.2, $ß_m$= 0.0 and $ß_o$= 0.0, October 8-week cross-hedging with $ß_c$= 1.2, $ß_m$= 1.0 and $ß_o$=1.2 or with $ß_c$= 0.3, $ß_m$= 0.1 and $ß_o$=1.2, and October 12-week cross-hedging with $ß_c$= 0.8, $ß_m$= 1.2 and $ß_o$= 1.2 or $ß_c$= 0.0, $ß_m$= 1.2 and $ß_o$= 1.2 are superior to cash pricing. All other October cross-hedging strategies are inferior to cash pricing.

Thus, this study reveals that soybean, soybean meal and soybean oil futures can be successfully used as cross-hedging vehicles for cottonseed, cottonseed meal and cottonseed oil, respectively. The empirical results imply that a cottonseed crusher has to be careful about choosing the proper time of placing hedges, along with the appropriate size of the contracts and optimal lengths of the hedges. The Bayesian cross-hedging rules suggest that hedges for cottonseed meal and oil should be placed by the end of May for longer terms, i.e., for 12 to 24 weeks, and the hedge for cottonseed should be placed by the end of October for shorter terms, e.g., 4 weeks. Above all, May 12 and 24 weeks cross-hedging of cottonseed meal and oil and October 4 weeks cross-hedging of the whole cottonseed are the most effective marketing strategies.

Table 5: Resulting profit and utility from cash pricing and May cross-hedging

Risk Aversion Coefficient	May 4-week Cross-hedging		May 8-week Cross-hedging		May 8-week Cross-hedging		May 8-week Cross-hedging	
	Profit	Utility	Profit	Utility	Profit	Utility	Profit	Utility
Cash Pricing	-6206.0	-1.0640	9400.0	-0.9103	17692.0	-0.8378	14402.0	-0.8659
f= 0.00001	-1084.4	-1.0109	13496.8	-0.8727	32245.6	-0.7244	22744.4	-0.7966
f= 0.00003	-1084.4	-1.0331	18428.8	-0.5753	32245.6	-0.3801	22204.0	-0.5137
f= 0.00006	2351.6	-0.8684	17780.8	-0.3441	32245.6	-0.1445	20656.4	-0.2896

Risk Aversion Coefficient	October 4-week Cross-hedging		October 8-week Cross-hedging		October 12-week Cross-hedging		October 24-week Cross-hedging	
	Profit	Utility	Profit	Utility	Profit	Utility	Profit	Utility
Cash Pricing	-14146	-1.1520	-30150	-1.3519	-44676	-1.5632	-33612	-1.3995
f= 0.00001	-6706	-1.0694	-33750	-1.4041	-49608	-1.6423	-55704	-1.7455
f= 0.00003	-6706	-1.2228	-26450	-2.2111	-40344	-3.3546	-55704	-5.3181
f= 0.00006	-7714	-1.5886	-24560	-4.3649	-34104	-7.7387	-47224	-17.0038

Total profit and expected utility from cash pricing are calculated assuming a moderate-to-low risk averse crusher (f= 0.00001) who does not use the futures market.

Conclusions

The investigation, using Bayesian decision science, reveals that soybean, soybean meal and soybean oil futures can be successfully used as cross-hedging vehicles for cottonseed, cottonseed meal and cottonseed oil, respectively. The empirical results imply that a cottonseed crusher has to be careful about choosing the proper time of placing hedges, along with the appropriate size of the contracts and optimal lengths of the hedges. The Bayesian cross-hedging rules suggest that hedges for cottonseed meal and oil should be placed by the end of May for longer terms, i.e., for 12 to 24 weeks, and the hedge for cottonseed should place by the end of October for shorter terms, e.g., 4 weeks. Particularly, net realized prices from four weeks cross-hedging of cottonseed, twelve or twenty four weeks cross-hedging of cottonseed meal and four, eight, twelve or twenty four weeks cross-hedging of cottonseed oil placed by the end of May and October exhibit risk efficiency superior to cash pricing. Above all, May twelve and twenty four weeks cross-hedging of cottonseed meal and oil and October four weeks cross-hedging of the whole cottonseed are the most effective marketing strategies. However, these inferences are drawn from the application of the Bayesian cross-hedging results only to data from 1998. Further study is necessary to evaluate the Bayesian cross-hedging rules with historical data for better outcomes. Whether the cash prices of cottonseed and cottonseed products follow any pre-harvest or post-harvest trend and how the weather condition affects the production and price of these commodities could also be examined further. Extended research to explore how the demand and supply responses of these products change with other environmental and economic phenomena will be helpful.

In his consulting study of cross-hedging cottonseed crushing, Dahlgran (2000) estimated the risk minimizing hedge ratios by regressing changes in spot prices for cottonseed and its products against changes in price of selected futures contracts. He used stepwise regression to select the most effective cross-hedge, utilizing a given number of futures contracts and hypothetical weekly, monthly, quarterly and semiannual hedge horizons. He found that the hedging effectiveness increased the longer the term of the hedge. Dahlgran reported that the major difficulties involved in the application of cross-hedging cottonseed crush were the range of futures contracts to include in the hedge vehicle pool, the hedge horizons, the ultimate portfolio size and the hedge management costs. In contrast to Dahlgran's analysis, the present study of cross-hedging cottonseed and its products uses only soybean complex futures contracts which is not as difficult and costly to manage as the huge hedge vehicle pool used by Dahlgran. This research also shows that the superiority of one hedging horizon over the other depends not only upon the appropriate size of the futures contracts but also on the time of placing hedge. Thus, this approach to cross-hedging of cottonseed and its products eliminates the difficulties reported in Dahlgran's study.

Finally, this study provides alternative marketing strategies for cottonseed products that improve profitability of cottonseed crushing. The empirical findings suggest that using the Bayesian cross-hedging strategies, an expected utility-maximizing cottonseed crusher can easily minimize her risk by simple cross-hedging of cottonseed, meal and oil with soybean complex futures. The employment of Bayesian decision science in cross-hedging is new in the study of producer price risk management using futures. It may be useful to apply this method to cross-hedge other commodities for which there is no active futures market.

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