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The Mixed-complementarity Approach to Specifying Agricultural Supply in Computable General Equilibrium Models

INTRODUCTION

In computable general equilibrium (CGE) models, it is typically assumed that agricultural resources are smoothly substitutable in neoclassical production or cost functions, with flexible wages, rents and prices generating market equilibrium in a setting with full resource employment.[1] Although this specification is often adequate, it is also often inadequate, especially when the analysis focuses on resource allocation and production technology issues. With more disaggregation, which is becoming common in CGE models with an agricultural focus, the use of smooth, twice-differentiable, production or cost functions to specify agricultural technology is increasingly unrealistic. The purpose of this paper is to show how CGE models formulated as non-linear mixed-complementarity (MC) problems can incorporate alternative, more realistic, specifications of agricultural technology and supply, drawing on the extensive literature on mathematical programming models applied to agriculture.[2]

First, we present a stylized standard neoclassical CGE model, which is then extended to a CGE–MC format to include Leontief (activity analysis) technology, endogenous determination of the market regime for agricultural factors (unemployment or full employment) and inequality constraints on agricultural factor use. In an analysis of reduced agricultural water supplies in Egypt, it is then shown how such a model can generate realistic results concerning water use and productivity that cannot be captured in a standard CGE model. The main conclusion is that, in analyses focused on agricultural supply issues, CGE–MC models that selectively incorporate features from the mathematical programming literature offer a powerful alternative to standard models. The underlying producer optimization problems for the different situations are presented in an Appendix.

THE STANDARD CGE APPROACH TO TECHNOLOGY AND FACTORS

Table 1 presents a stylized neoclassical CGE model which, like most of those in the literature, is formulated as a system of simultaneous equations, all of

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TABLE 1 *A stylized CGE model*

	Equation
1	$q_s^s = NC(q_{fs}^f) \quad s \in S$
2	$q_{s's}^{int} = \alpha_{s's}^s q_s^s \quad s' \in S, s \in S$
3	$p_s^{va} = p_s^s - \sum_{s'} p_{s'}^s \alpha_{s's}^s \quad s \in S$
4	$w_{fs}^s = \frac{\partial q_s^s}{\partial q_{fs}^f} p_s^{va} \quad f \in F, s \in S$
5	$w_{fs}^s = \overline{w}_{fs}^{dist} w_f \quad f \in F, s \in S$
6	$q_s^h = NC\left(\sum_s p_s^{va} q_s^s, p_s^s\right) \quad s \in S$
7	$q_s^s = q_s^h + \sum_{s'} q_{ss'}^{int} \quad s \in S$
8	$\overline{q}_f^f = \sum_s q_{fs}^f \quad f \in F$
9	$\overline{p} = \prod_s \Omega_s p_s^s$

Notation

Sets

$s, s' \in S$	sectors (commodities)
$f, f' \in F$	factors

Variables

p_s^s	price for sector *s*
p_s^{va}	value-added price for sector *s*
q_s^h	quantity of household demand for output of sector *s*
q_s^s	quantity of output for sector *s*
q_{fs}^f	quantity of demand for factor *f* from sector *s*
$q_{s's}^{int}$	quantity of intermediate demand for commodity *s*′ from sector *s*
w_f	wage of factor *f*
w_{fs}^s	wage of factor *f* in sector *s*

Note: The letters in the column #Eq. refer to the number of elements in the corresponding sets. The domains of some equations (and related variables) are smaller than indicated if each sector does not use all factors or intermediate input commodities. The producer problem is presented in optimization form in the Appendix.

Description	#Eq.	Var.
Sectoral production	S	q_s^s
Intermediate input demand	$S \cdot S$	$q_{s's}^{int}$
Value-added price	S	p_s^{va}
Factor demand	$F \cdot S$	q_{fs}^f
Sectoral factor prices	$F \cdot S$	w_{fs}^s
Household demand	S	q_s^h
Commodity market	S	p_s^s
Factor market	F	w_f
Cost-of-living index	1	—

Parameters

$\alpha_{s's}^s$	quantity of intermediate input s' per unit of output in sector s
Ω_s	household expenditure share for sector s
$\overline{p}$	cost of living index
$\overline{q}_f^f$	supply of factor f
w_{fs}^{fdist}	relative wage distortion for factor f in sector s

Functions

NC	neoclassical function

which are strict equalities. The model is highly simplified – government, foreign trade and savings-investment are omitted – to focus on producer technology and resources.

Producers in each sector maximize profits given their technology, specified by a nested neoclassical value-added function (with factor inputs as arguments) and fixed (Leontief) intermediate input coefficients (equations 1–4). (The underlying producer optimization problems for this and following models are presented in the Appendix.) The treatment of agriculture is the same as for other sectors. Exogenous relative gaps between sectoral factor rents (wages) are permitted (equation 5). Households receive all factor incomes and spend it on the basis of neoclassical demand functions, derived from utility maximization subject to an income constraint (equation 6). The markets for factors and commodities are in equilibrium 7–8) with flexible wages and prices as equilibrating variables. Production techniques are assumed to be sufficiently flexible to ensure that fixed aggregate factor supplies are always fully employed at positive prices. Equation 9 fixes a measure of the aggregate price level, the cost-of-living index, defining the *numéraire*. Given that the real side of the model is homogeneous of degree zero in prices, the model can only determine relative prices. In Table 1, the number of equations exceeds the number of variables by one — with the exception of the last equation, the last column of Table 1 pairs each equation with a variable of identical dimension. However, given Walras' law, one of the equations is functionally dependent. The model has an equal number of variables and independent equations, and a unique solution can almost invariably be found.

A model with this structure (or variations on the theme: for example, with neoclassical substitutability for intermediate inputs) has proved itself to be a dependable workhorse. It is well-behaved, can be implemented with a small data set, and is almost invariably solvable, generating a solution with strictly positive prices. In some contexts, however, it has serious drawbacks – in particular, if the analysis is focused on agricultural technology and resource questions. Neoclassical production functions exaggerate the smoothness of real-world input substitutability and preclude tests of the attractiveness of discontinuous technical alternatives, for example introducing new crop varieties. When viewed from a disaggregated perspective, land and water resources are often unemployed, with zero prices.

In many contexts, these shortcomings can be overcome, or mitigated, if the agricultural supply module of the CGE model incorporates features that are standard fare in agricultural mathematical programming models, such as Leontief technology and inequality constraints for resources and other production aspects. Pathbreaking work in this area is due to Keyzer, who developed a tailor-made algorithm for solving general equilibrium models with complementarity relationships used to capture regime shifts in foreign trade and storage policies (Fischer *et al.*, 1988; Keyzer *et al.*, 1992). Up to this point, such mixed complementarity (MC) CGE models have rarely been used to model the agricultural supply side. Recent advances in computational technology make it possible to solve CGE–MC models at reasonable cost. In the next section, we give a simple example of such a model, with a treatment of agricultural supply that draws on the agricultural mathematical programming literature.

AN AGRICULTURAL CGE–MC MODEL

An MC model consists of a set of simultaneous (linear or non-linear) equations that are a mix of strict equalities and inequalities, with each inequality linked to a bounded variable in a complementary-slackness condition (Rutherford, 1995). Such models are familiar to economists because the Kuhn–Tucker optimality conditions define a mixed-complementarity problem (which is necessary and sufficient for a global optimum for nearly all well-behaved economic linear and non-linear optimization models, including agricultural sector mathematical programming models). Indeed, all programming models can be written as MC problems. From the perspective of this paper, a CGE–MC model can incorporate features found in agricultural mathematical programming models, with inequalities, which cannot readily be captured in strict equality simultaneous equation systems. For example, it is easy to incorporate resource unemployment (with associated zero wages), crop rotations, self-sufficiency production targets, stocking targets and credit rationing.

Table 2 shows a simple CGE–MC model, which is an augmented version of the stylized model in Table 1.[3] Equations with the same number as in Table 1 are unchanged except for slight notational and domain adjustments. New equations are numbered with single or double asterisks. As opposed to the model of Table 1, each sector may generate more than one commodity, with the quantities determined by fixed yield coefficients (equation 1′). This extension is particularly useful when crop–livestock interactions matter.

The model distinguishes between sectors (or activities, the set S) and commodities (produced by sectors, the set C). Sector returns per unit activity are given by the sum of commodity prices times yield coefficients (equation 3′). The model also makes a distinction between (agricultural) sub-factors (the set $FSUB$, here land and water) and factors (the set F), one or more of which are aggregates of the sub-factors (here one of the factors is a land/water aggregate). Sub-factor demand is a Leontief function of the level of the aggregate land/water factor (equation 4′); that is, land and water are used in fixed proportions in the production of a given crop. For each sub-factor, there is an upper limit on the supply share that may be allocated to any single sector (equation 4″). In any applied model, the domain of this equation and associated variables should be constrained to relevant sub-factor–sector combinations. The price of the aggregate land/water factor is a linear function of the prices of the sub-factors and a penalty variable (equation 5′). The penalty variable (or scarcity price) takes on a positive value when needed to ensure that the sub-factor constraint is not violated. More specifically, it enters the complementary slackness condition linked to the sub-factor constraint (equation 4″): if the constraint is (not) binding, the penalty is positive (zero). The market equilibrium conditions of the sub-factors (equation 8′) are inequalities linked to the corresponding prices in complementary slackness conditions: if the price is positive, the resource is fully employed; if it is zero, unemployment is permitted. (Cf. the note at the bottom of Table 2.) Accounting for one dependent equation, the model has an equal number of variables and independent equations.

TABLE 2 *A stylized CGE–MC model*

	Equation
1	$q_s^s = NC(q_{fs}^f) \quad s \in S$
1′	$q_c^c \sum_s \gamma_{cs} q_s^s \quad c \in C$
2	$q_{cs}^{int} = \alpha_{cs}^s q_s^s \quad c \in C, s \in S$
3	$p_s^{va} = p_s^s - \sum_c p_c^c \alpha_{cs}^s \quad s \in S$
3′	$p_s^s = \sum_c \gamma_{cs} p_c^c \quad s \in S$
4	$w_{fs}^s = \frac{\partial q_s^s}{\partial q_{fs}^f} p_s^{va} \quad f \in F, s \in S$
4′	$q_{fs}^{fsub} = \alpha_{fs}^{fsub} q_{f's}^f \quad f \in FSUB; s \in S; f' = land/water$
4″	$\Psi_{fs}^{\max \bar{q}_f fsub} \geq q_{fs}^{fsub} \quad f \in FSUB, s \in S \quad [w_{fs}^{\max} \geq 0]$
5	$w_{fs}^s = \overline{w}_{fs}^{dist} w_f \quad f \in FF, s \in S$
5′	$w_{f's} = \sum_{f \in FSUB} \alpha_{fs}^{fsub} \left(w_f^{sub} + w_{fs}^{\max} \right) \quad s \in S, f' = land/water$
6	$q_c^h = NC\left(\sum_s p_s^{va} q_s^s, p_c^c \right) \quad c \in C$
7	$q_c^c = q_c^h + \sum_s q_{cs}^{int} \quad c \in C$
8	$\bar{q}_f^f = \sum_s q_{fs}^s \quad f \in FF$
8′	$\bar{q}_f^{fsub} \geq \sum_s q_{fs}^{fsub} \quad f \in FSUB \quad [w_f^{sub} \geq 0]$
9	$\bar{p} = \prod_s \Omega_s p_s^s$
1*	$\sum_f w_{fs}^s \alpha_{fs}^f \geq p_s^{va} \quad s \in S \quad [q_s^s \geq 0]$
4*	$q_{fs}^f = \alpha_{fs}^f q_s^s \quad f \in F, s \in S$

New notation

Sets

$c \in C$	commodities
$f, f' \in FF (\subset F)$	factors without sub-factors (all except land/water)
$f \in FSUB$	sub-factors (land, water; sub-factors to land/water aggregate)

Variables

p_c^c	price for commodity c
q_c^c	quantity (production level) for commodity c
q_{fs}^{fsub}	quantity of demand for sub-factor f in sector s
q_{cs}^{int}	quantity of intermediate demand for commodity c from sector s
w_f^{fsub}	wage of sub-factor f
$w_{fs}^{\max}$	penalty on sub-factor f in sector s

Note: Equations with same number as in Table 1 are unchanged except for domain changes. Equations 1* and 4* replace 1 and 4 for a model with Leontief technology also for all factors. Variables entering the associated complementary slackness condition are provided in brackets after the inequalities; for example, the following complementary slackness condition is linked to equation 8′ and the lower bound on

Description	#Eq.	Var.
Sectoral production	S	q_s^s
Commodity production	C	q_c^c
Intermediate demand	$C \cdot S$	q_{cs}^{int}
Value-added price	S	p_s^{va}
Sector price	S	p_s^s
Factor demand	$F \cdot S$	q_{fs}^f
Sub-factor demand	$FSUB \cdot S$	q_{fs}^{fsub}
Sub-factor constraint	$FSUB \cdot S$	w_{fs}^{Max}
Sectoral factor price	$FF \cdot S$	w_{fs}^s
Sectoral sub-factor price	S	$w_{lw,s}^s$
Household demand	C	q_s^h
Commodity market	C	p_c^c
Factor market	FF	w_f
Sub-factor market	$FSUB$	$w_f{}^{sub}$
Cost-of-living index	1	—
Leontief first-order condition for profit-max. (replacing 1)	S	q_s^s
Leontief factor demand (replacing 4)	$F \cdot S$	q_{fs}^f

Parameters

α_{fs}^f	quantity of factor f per activity unit in sector s
α_{fs}^{fsub}	quantity of sub-factor f per unit of factor f in sector s
α_{cs}^s	quantity of intermediate input c per unit of output in sector s
Ω_c	consumption expenditure share for commodity c
γ_{cs}	yield of commodity c per activity unit in sector s
Ψ_{fa}^{Max}	maximum share of the supply of factor f used in sector s
$\bar{q}_f^{fsub}$	supply of sub-factor f

the sub-factor price:

$w_f^{sub}\left(\bar{q}_f^{fsub} - \sum_s q_{fs}^{fsub}\right) = 0, f \in FSUB$. The two producer problems are presented in optimization form in the Appendix.

Alternatively, Leontief technology may be extended to all factors by substituting equations 1* and 4* for equations 1 and 4. The new profit-maximization condition, with the associated complementary slackness condition, states that marginal value-added product is less than or equal to the marginal factor cost and that, if the sector activity is positive, marginal value-added product and marginal cost are equal. This condition is written as an inequality to allow the specification of several activities for each 'crop' (combination of commodity outputs), some of which may not be operated. If the model is limited to one activity per crop, the range of input substitutability would typically be understated. While it is feasible to permit multiple outputs for sectors in a standard CGE model, allowing factor unemployment, constraints on factor use and the use of Leontief technology, all involving inequality constraints, requires an MC formulation.

AN APPLICATION TO EGYPT

In order to demonstrate the significance of the MC approach to CGE modelling, we here briefly present results from experiments using a dynamic (recursive) CGE–MC model of Egypt with a detailed treatment of agriculture.[4] The model is solved for 1990 (the base year), 1993 and 1995, and every five years thereafter until 2020. Apart from being dynamic, this model differs from the stylized model in Table 2 in that it portrays an open economy with a more complete set of domestic institutions (including government and enterprise sectors), as well as investment and savings.

The agricultural supply side of the model quite closely follows the basic version of Table 2 (that is, the one with activity analysis technology limited to sub-factors). One difference is that the land sub-factor is disaggregated by season (summer and winter). Hence crops may be classified according to whether they use water in summer, winter, or in both seasons (for perennials). Upper limits on sub-factor use are only imposed for cotton use of summer land: following Egypt's standard crop rotation, cotton is not permitted to occupy more than one-third of the land not covered by perennial crops. An additional equality constraint (with an associated penalty variable) makes sure that the areas for cotton and a short winter clover crop (typically preceding cotton) are equal. Outside agriculture, an MC formulation is used for labour to permit endogenous choice of market regime (unemployment or full employment). The model is solved in GAMS, using PATHS or MILES, two solvers for MC problems.[5]

One set of experiments explored the impact of a gradual reduction of agricultural water supplies, reflecting some combination of reduced supplies from the Nile or the transfer of increasing volumes to non-agricultural sectors. In the experiments, agricultural water supplies were reduced in steps of 10 per cent, with declines ranging from zero to 60 per cent, taking place gradually between 1990 and 2020. At the aggregate level, the impact is quite manageable. As the cut in water supplies changes from zero to 60 per cent, annual growth in real GDP at factor cost for 1990–2020 falls from 5.2 to 4.8 per cent. However, the impact on the agricultural sector is more severe: its annual growth rate falls

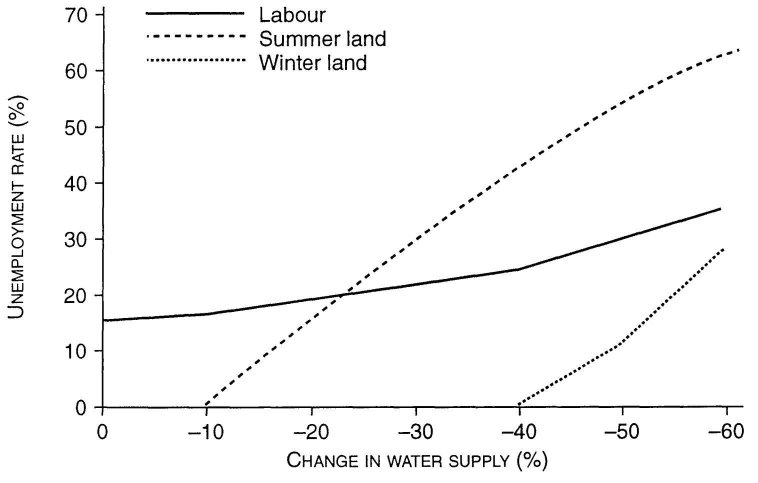

FIGURE 1 *Factor unemployment rates with reduced water supplies, tiger scenarios*

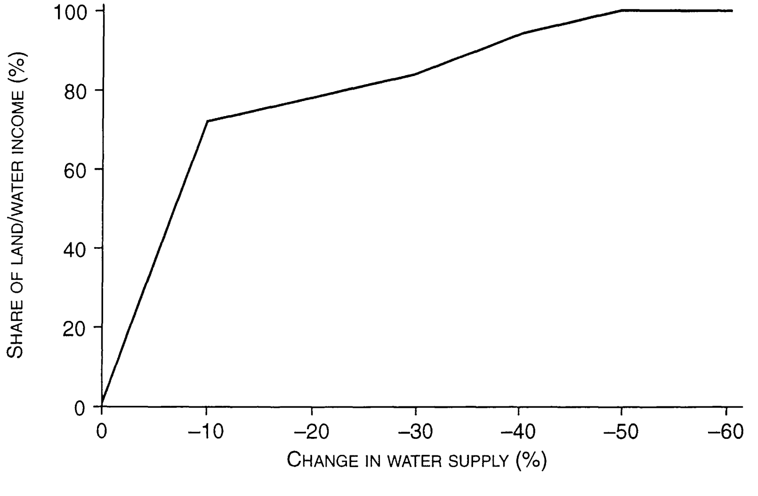

FIGURE 2 *Water share in total land/water income, tiger scenarios, 2020*

from 3.5 to 2.0 per cent. At the micro level, the mix between labour, capital and, for crop activities, a land/water aggregate is driven by profit maximization subject to a CES function. Given this flexibility, the marginal return to the land/water aggregate is always positive. It is allocated to the land/water sub-factors (water, winter land and summer land) some, but not all, of which may be slack.

Figure 1 shows that, with no cut in water supplies, both land types are fully employed in 2020 while the labour unemployment rate is 15 per cent. When the water supply cut has reached 10 per cent, summer land is taken out of production. Part of the winter land becomes idle when the cut exceeds 40 per cent. For labour, unemployment increases gradually from 15 per cent for no water cut to 34 per cent when the water cut reaches 60 per cent. Accordingly, Figure 2 shows that, as water becomes scarce and excess supplies emerge for both land types, the water share in total land/water income gradually moves from zero to 100 per cent: that is, while initially water has excess supply and a zero rent, it eventually becomes binding while both types of land become partly unemployed, with zero rent. In this model, endogenous determination of the factor market regime (unemployment or full employment) is highly significant. In the background, inequality constraints on the cropping pattern ensured that the production structure remained agronomically feasible.

CONCLUDING REMARKS

In analysis focused on agricultural supply issues, CGE–MC models, which selectively incorporate features from the mathematical programming literature, offer a powerful alternative to standard approaches. The strength of the CGE–MC formulation is that it can capture critical aspects of the institutional and technological structure of agricultural production. Moreover, this is one of the rare occasions when the lunch is free – there is no sacrifice of other features, including the treatment of foreign trade and policy tools, that have made CGE models attractive.

NOTES

[1]Early CGE models specified sectoral production functions and derived factor demand functions. Many models now start with cost or profit functions. Chambers (1988) discusses the use of cost functions in agriculture. Computationally, the approaches are essentially identical.

[2]See Agrawal and Heady (1972) and Hazell and Norton (1986).

[3]The model in Table 2 draws on formulations in Robinson and Gehlhar (1996) and Löfgren *et al.* (1996).

[4]For additional details, including discussion of the 'tiger' and 'turtle' scenarios (the former is mentioned in the figures), see Löfgren *et al.* (1996).

[5]For GAMS, see Brooke *et al.* (1988). Rutherford (1995) provides more information on PATH and MILES.

REFERENCES

Agrawal, R.C. and Heady, E.O. (1972), *Operations Research Methods for Agricultural Decisions*, Ames, Iowa: Iowa State University Press.
Brooke, A., Kendrick, D. and Meeraus, A. (1988), *GAMS: A User's Guide*, Redwood, California: The Scientific Press.
Chambers, R.G. (1988), *Applied Production Analysis*, Cambridge, Mass: Cambridge University Press.
Fischer, G., Frohberg, K., Keyzer, M.A. and Parikh, K.S. (1988), *Linked National Models: A Tool for International Food Policy Analysis*, International Institute for Applied Systems Analysis, Boston: Kluwer Academic Publishers.
Hazell, P.B.R. and Norton, R.D. (1986), *Mathematical Programming for Economic Analysis in Agriculture*, New York: Macmillan.
Keyzer, M., van Veen, W. and Tims, W. (1992), 'The SOW Applied General Equilibrium Model', in E. Thorbecke (ed.), *Adjustment and Equity in Indonesia*, Paris: OECD Development Centre.
Löfgren, H., Robinson, S. and Nygaard, D. (1996), *Tiger or Turtle? Exploring Alternative Futures for Egypt to 2020*, Discussion Paper 11, Trade and Macroeconomics Division, August, Washington, DC: International Food Policy Research Institute.
Robinson, S. and Gehlhar, C.G. (1996), Impacts of Macroeconomic and Trade Policies on a Market-Oriented Agriculture', in L.B. Fletcher (ed.), *Egypt's Agriculture in a Reform Era*, Ames, Iowa: Iowa State University Press.
Rutherford, T. (1995), 'Extensions of GAMS for Complementarity Problems Arising in Applied Economic Analysis', *Journal of Economic Dynamics and Control*, **19**, 1299–1324.

APPENDIX

In the CGE models in the main body of the paper, the equations relevant to producer behaviour are written in the form of first-order conditions. We will here present the underlying producer optimization problems using the same notation as in Tables 1 and 2. In the model of Table 1, the producer in sector S (agricultural or non-agricultural) is represented by equations 1–4. Producer technology is specified as a nested neoclassical value-added function and fixed (Leontief) intermediate input coefficients. In condensed form, the optimization problem for the producers in sector S is to select q^f_{fs} for $f \in F$ so as to maximize

$$\pi_s = p^s_s NC(q^f_{fs}) - \sum_{s' \in S} p^s \alpha^s_{s's} NC(q^f_{fs}) - \sum_f w^s_{fs} q_{fs} \tag{A1}$$

where π_s is profit in sector S. In the process of embedding producer behaviour in the full CGE model, new equations defining $q^s_s, q^{int}_{s's}$, and p^{va}_s are added (equations 1–3 in Table 1) while the first-order condition (derivative of (A1) with respect to q^f_{fs} set to zero) is rearranged and simplified (equation 4).

In Table 2, two alternative CGE–MC model versions are presented. For the first, behaviour and technology for sector S is represented by equations 1, 2, 3, 3′, 4, 4′, 4″, 5′. The new elements in producer technology are (1) that one of the arguments in the value-added function is a land/water aggregate, made up of land and water in fixed proportions; and (2) a constraint on sectoral factor use that may reflect agronomic considerations or policy. The condensed version of the underlying profit-maximization problem for S is to select q^f_{fs} for $f \in F$ so as to maximize

$$\pi_s = \sum_{c \in C} p^c_c \gamma_{cs} NS(q^f_{fs}) - \sum_{c \in C} p^c_c \alpha^s_{cs} NC(q^f_{fs}) - \sum_{f \in FF} w^s_{fs} q_{fs} - \sum_{f \in FSUB} \sum_{f'=lw} w^{sub}_f \alpha^{fsub}_{fs} q^f_{f's} \tag{A2}$$

subject to

$$\sum_{f'=lw} \alpha^{fsub}_{fs} q^f_{f's} \leq \Psi^{max}_{fs} \bar{q}^{fsub}_f \quad f \in FSUB$$

where

$$lw = \text{land/water}$$

In Table 2, the first-order conditions (derivatives of the Lagrangean with respect to q^f_{fs} and $w^{\max}_{fs}$, the constraint function multiplier, both set to zero) are manipulated and simplified to yield equations 4 and 4″, drawing on definitions of $q^s_s, q^{int}_{cs}, p^{va}_s, p^s_s, q^{fsub}_{fs}$ and w^s_{fs} (the latter for f = land/water aggregate), represented by equations 1, 2, 3, 3′, 4′ and 5′.

In the second model version in Table 2, with Leontief technology for all inputs (factors and intermediates), equations 1* and 4* replace 1 and 4. The optimization problem for sector S producers is to select q^s_s so as to maximize

$$\pi_s = \sum_{c \in C} p^c_c \gamma_{cs} q^s_s - \sum_{c \in C} p^c_c \alpha^s_{cs} q^s_s - \sum_{f \in FF} w^s_{fs} \alpha^f_{fs} q_{fs} - \sum_{f \in FSUB} \sum_{f'=lw} w^{sub}_f \alpha^{fsub}_{fs} \alpha^f_{f's} q^s_s$$

subject to equation (A2). The full CGE–MC representation of the producer problem is found by adding the same definitions as for the preceding problem, with the exception that an equation is needed for q^f_{fs} (4*) instead of q^s_s. After manipulation, the first-order conditions (derivatives of the Lagrangean with respect to q^s_s and $w^{\max}_{fs}$ set to zero) can be restated as 1* and 4″.