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Choices by Poor Households when the Interest Rate for Deposits Differs from the Interest Rate for Loans

INTRODUCTION

A dynamic model of optimal decisions by a poor household, with an infinite horizon and rational expectations over uncertain future income, can be solved and simulated using the technique of orthogonal polynomial projection. The household faces a credit limit, and the interest rate on savings (deposits) differs from the interest rate for loans (borrowing). The change in the spread between the interest rate for deposits and for loans, and the effects of that spread on household decisions, suggest that attention should be paid to access to formal financial services and to the effects of decreasing the transaction costs associated with them.

The model to be used incorporates five basic features of a poor household and its financial contracts. First, poor households both borrow and save. They borrow from formal or informal lenders, and households save in financial deposits or in real goods. Second, poor households face a credit limit. Third, financial contracts take place through time. Resources are lent in the present for the promise to repay in the future, so saving/borrowing choices in the present affect consumption in the future. Fourth, poor households earn less for saving than they pay for borrowing. Fifth, income for poor households is variable and uncertain (Besley, 1995).

The model also omits at least 10 basic features of the financial contracts used by poor households. First, and most importantly, the possibility, prevention and punishment of default affect financial contracts. Second, households smooth both consumption and income, so production and consumption choices depend on each other (Morduch, 1995). Third, the transaction costs of small loans or deposits swamp the interest earned or paid. We model changes in transaction costs as changes in the spread between the interest rates for deposits and loans. This makes transaction costs vary with the size of the loan or deposit. In reality, most transaction costs are fixed, regardless of the size of the loan or deposit. Fourth, we model financial contracts as credit cards or passbook accounts. Real financial contracts often involve multi-period commitments such as instalment loans or certificates of deposit. Fifth, most loans require collateral. Sixth, both savings and borrowing may be non-zero at once. Seventh, households engage in non-financial saving and borrowing. Eighth, contracts may have non-divisibilities.

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Ninth, households may save not only for precautionary motives but also for investment, speculation and convenience. Tenth, and finally, interest rates and institutions are determined endogenously in general equilibrium.

Because of algebraic complexity, no single analytic model has captured more than a couple of these features (for example, Mendelson and Amihud, 1982; Helpman, 1981). Many models omit credit limits, but without explicit restrictions on the utility function, the optimal decision is then to play a Ponzi game. Few analytic models recognize the fact that borrowing costs more than saving pays.

The results extend those of Deaton (1991; 1992). Simulations suggest that more favourable interest rates increase the mean of consumption and decrease its variance. Thus access to formal financial services and/or lower transaction costs for financial transactions can improve the welfare of poor households. This is necessary but not sufficient to justify interventions in financial markets designed to help households.

The remainder of the paper consists of a presentation of the model, with discussion of optimal decision rules, followed by examination of the long-run distribution of consumption.

THE MODEL

The decision problem of the poor household is formulated as a Bellman equation. Time is indexed by t. If the household lives 40 years and makes financial decisions weekly or monthly, the horizon is effectively infinite. The household has rational expectations over labour income $\tilde{y}_t$. Labour income is an independent identically distributed (iid) random variable realized at the start of each period. The per-period discount rate is δ. The time-separable, time-invariant, per-period utility function $U(\cdot)$ is defined over a single composite consumption good c_t whose price is unity. More consumption increases utility but at a decreasing rate, so the household is risk-averse.

The poor household chooses a level of net saving s_t. Borrowing is negative net saving. With formal financial contracts or with low transaction costs, deposits earn an interest rate of d_f and loans cost an interest rate of l_f. In contrast, the interest rates with informal contracts or with high transaction costs are d_i and l_i. Formal deposits earn more than informal savings, and formal loans cost less than informal loans:

$$r(s_t) = \begin{Bmatrix} d \text{ if } s_t > 0 \\ l \text{ if } s_t \leq 0 \end{Bmatrix}, \text{ where}$$

$$\mathrm{d} = \begin{Bmatrix} d_f \text{ with formal savings or low transactions costs} \\ d_i \text{ with informal savings or high transactions costs} \end{Bmatrix}, \qquad (1)$$

$$l = \begin{Bmatrix} l_f \text{ with formal loans or low transactions costs} \\ l_i \text{ with informal loans or high transactions costs} \end{Bmatrix},$$

$$d_f > d_i, l_f < l_i \text{ and } d_k < l_k, k = i, f.$$

On the savings side, several forces make the rate of return to informal saving low and even negative: households usually lend informally to friends or relatives for low or no interest; stocks of grain or building materials depreciate; inflation erodes cash balances; and relatives seek gifts from liquid households (Binswanger and Rosenzweig, 1986). In contrast, formal deposits hide wealth from light-fingered relatives and provide safer, higher returns.

On the borrowing side, formal loans should be cheaper than informal ones. For example, moneylenders often charge astronomical rates. In addition, the reduced transaction costs implicit in loans from friends or relatives are more than overcome by the opportunity cost of maintaining the social ties required to get informal loans. The revealed preference of borrowers and savers in developed economies for formal financial contracts shows that, at least in deep financial markets, formal contracts offer more than informal contracts.

The household starts each period with wealth w_t, the sum of labour income, net saving from the past period and any interest from net saving in the past period:

$$w_t = y_t + s_{t-1} \cdot [1 + r(s_{t-1})]. \tag{2}$$

The household allocates wealth between consumption and savings:

$$w_t = c_t + s_t. \tag{3}$$

New households have no savings. Borrowing is less than the credit limit k, and saving is less than wealth:

$$k \le s_t \le w_t. \tag{4}$$

The value function $V(w_t)$ is the sum of current and discounted expected future utility, given current wealth and optimal decisions in all future periods. The Bellman equation for the household's maximization problem is:

$$V(w_t) = \max_{k \le s_t \le w_t} U(w_t - s_t) + \left(\frac{1}{1+\delta}\right) \cdot E_t V\{\tilde{y}_{t+1} + s_t \cdot [1 + r(s_t)]\}, \quad \tilde{y}_t \sim \text{iid}, \tag{5}$$

with $r(s_t)$ defined as in (1).

Equation (5) is a functional equation in $V(\cdot)$. Since w_t is continuous, the solution function $V(\cdot)$ must make (5) hold at an infinite number of values of w_t. Savings is the function $f(w_t)$ that maximizes (5). Given assets and savings, (3) gives consumption.

The parameterization of (5) follows Deaton (1992). Utility is CARA(2). This assumption has some empirical support (Hildreth and Knowles, 1982; Kydland and Prescott 1982; Friend and Blume, 1975; Tobin and Dolde, 1971). What matters for the results is not the exact number used but rather the fact that the poor household is risk-averse.

With favourable interest rates, deposits earn 5 per cent and loans cost 25 per cent. When rates are unfavourable, savings earn –5 per cent and loans cost 50 per cent. The credit limit is 10. Again the result does not depend on the exact numbers but rather on the fact of the credit limit and the changes in the spread between the favourable and unfavourable cases. Income is normal with mean 100 and standard deviation 10. The discount rate δ is 10 per cent. These choices match those of Deaton (1992) and Dercon (1992). We cannot defend these choices as empirical facts – they are made to facilitate comparisons between the simple model of Deaton (1992) and the same model with a credit limit and a spread between the interest rate on savings and loans.

Miranda (1994) and Judd (1991) show why numerical solutions of (5) by orthogonal polynomial projection are more accurate, elegant and quick than the grid techniques of Deaton (1991; 1992). The value function is represented by a polynomial with nice approximation properties. Given an initial guess for $V(\cdot)$ at a few well-chosen levels of wealth, we use the first-order conditions of (5) to solve for the level of savings that maximizes $V(\cdot)$, taking the current approximation to $V(\cdot)$ as given when evaluating the right-hand side of (5). We approximate the distribution of the income shock with Gaussian quadrature. This process iterates until $V(\cdot)$ converges.

OPTIMAL DECISIONS

Figure 1 shows optimal savings as a function of wealth. Consumption is wealth less savings. The solid line stands for choices with favourable interest rates, and the dashed line stands for choices with unfavourable interest rates. The 'wiggles' reflect approximation error.

Four insights can be gleaned from Figure 1. First, low levels of wealth lead to borrowing and net saving is negative. In fact, a household may borrow so much that the credit limit binds, as at wealth levels below 75 units for poor households in the favourable case. The cheaper the loan, the higher the level of wealth at which a household will start to borrow. In practice, more poverty means a hungry household waits longer before it will borrow.

Second, households can sometimes consume all their assets and neither save nor borrow. That is, net saving is zero. This flat stretch of the net-savings function comes from the unequal interest rates for saving and borrowing. It disappears when the two rates are the same, as most analytical models assume (for example, Deaton, 1992; Dercon, 1992). This is how the flat stretch comes about. For some levels of wealth, one more unit of consumption in the present is worth more than the discounted expected value of one more unit plus interest in the next period, but less than the discounted expected value of not having to repay an extra unit plus interest in the next period. The range of disintermediation decreases as the spread between the interest rates for loans and deposits decreases. This flat stretch in the net savings function may be part of the answer to the puzzle of why so many poor households have no deposits or loans at all (Hubbard *et al.*, 1994). With a low reward for deposits and a high price for loans, a poor household might maximize utility by living hand-to-mouth.

Third, the household saves at high levels of wealth. Furthermore, the interest elasticity of saving increases as the return to saving increases. Not only does

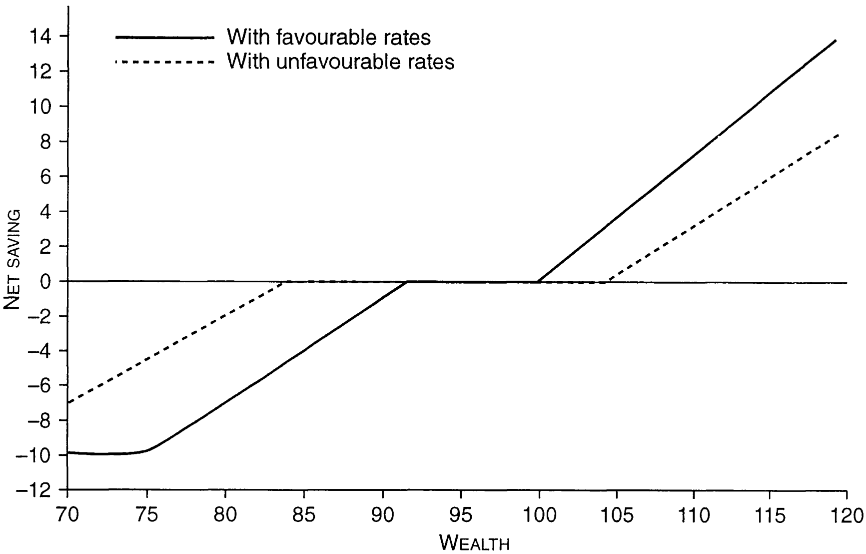

FIGURE 1 *Optimal decisions with different interest rate spreads*

the household begin saving at lower levels of wealth, but the rate at which the household increases savings as wealth increases also increases. This matches the stylized fact that, although rich and poor both save, the rich save a larger percentage of their income than the poor. For this parameterization, increasing the return to savings increases savings more than decreasing the cost of borrowing decreases savings, since cheaper loans reduce the need for a buffer of savings. In results not shown here, deposits decrease as loans get cheaper and the need to self-insure falls, all else constant.

Fourth, poor households will save even with negative returns and borrow even at exorbitant rates, since they want to avoid episodes of low consumption so much. The desire to borrow when consumption is low helps explain the high rates charged by loan sharks and moneylenders (Adams and Fitchett, 1992).

Figure 1 shows decision rules. Given wealth, it depicts the level of net savings that maximizes the sum of current and discounted expected future utility over an infinite horizon. The decision rules alone do not, however, reveal the particular levels of savings and consumption of a poor household using the optimal decision rule through time. Nor do they reveal how interest rates affect the way in which the household can smooth consumption.

THE LONG-RUN DISTRIBUTION OF CONSUMPTION

To approximate the long-run distribution of consumption for both the favourable and unfavourable scenarios, the behaviour of a poor household can be simulated using the decision rules in Figure 1 for 100,000,000 periods (Figure 2). In the unfavourable case (dashed line), the mean of consumption is 99.88 with a standard deviation of 8.34. In the favourable case (solid line), the mean is 100.14 with a standard deviation of 6.02.

More favourable interest rates smooth consumption in two ways. First, cheaper loans help to avoid low consumption. The extreme left tail of the distribution of consumption is thinner with favourable rates than with unfavourable rates. Second, more rewards for saving decreases episodes of high consumption. The extreme right tail of the distribution of consumption with favourable rates is inside the extreme right tail of the distribution with unfavourable rates. Increased savings and the higher interest earnings pad the buffer of the household against poor income draws.

Figure 2 highlights two other insights. First, savings and loans both buffer consumption, but not in the same way, skewing consumption to the left. The credit limit means the poor household can avoid gluts more easily than famines. In addition, loans cost more than savings earn. Second, the distribution of

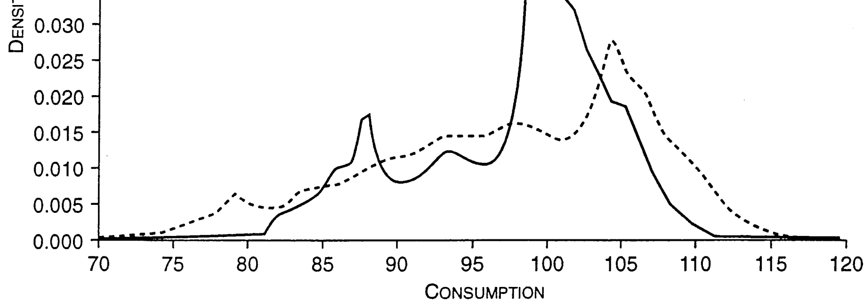

FIGURE 2 *Long-run distribution of consumption with different interest rates*

consumption has three modes. Roughly speaking, this happens because the overall distribution is a mixture of the distributions of current assets conditional on the levels of net savings in the past period. Only the tail modes require explanation, and the modes in the left tail (91 units with favourable rates and 82 units with unfavourable rates) are the most interesting. These peaks happen because, when wealth is near the range where borrowing starts, a wide range of wealth maps into a narrow range of consumption. For example, consumption is almost the same when wealth is just below the point where nothing is saved or borrowed as when wealth is just above that point. The need to repay old debt and interest means that the conditional mean of wealth is lower if the household borrowed in the past period. This increases the likelihood of wealth being in the range where nothing is borrowed or saved or just in the range where something is borrowed. The same argument holds for savings accounts for the modes in the right tail (103 with favourable rates and 109 with unfavourable rates).

CONCLUSION

An attempt has been made to solve and simulate a model of financial choices by a poor household with favourable and unfavourable interest rates. The model accounts for the uncertainty of income, the intertemporal nature of financial contracts and the reality of credit limits and of different interest rates for loans and deposits.

Incorporating the features often missed by analytic models makes a difference. In particular, the spread between the interest rates for deposits and loans means that it is sometimes optimal neither to save nor to borrow. This disintermediation creates extra modes in the long-run distribution of consumption. Simulations suggest that favourable interest rates help the household increase mean consumption and decrease its variability. These results strengthen the idea that formal finance and/or decreased transactions costs can improve the welfare of poor households. They do not, however, justify interventions in financial markets. All that is suggested is that benefits could be positive, though neither the level of benefits or that of costs has been measured.

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