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The determinants of technology adoption by UK farmers using Bayesian model averaging: the cases of organic production and computer usage

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We review and implement a reversible jump approach to Bayesian model averaging for the Probit model with uncertain regressors. Two applications are investigated. The first is the adoption of organic systems in UK farming, and the second is the influence of farm and farmer characteristics on the use of a computer on the farm. While there is a correspondence between the conclusions we would obtain with and without model averaging results, we find important differences, particularly in smaller samples. Concerning the adoption of an organic system, we find that attitudes to the sustainability of the current system along with the ability of organic farms alone to satisfy society's needs for food are influential. Additionally, the source of management information used by the farmer has a significant impact. Regarding the adoption of computers, we confirm the findings of previous work that the level of education affects uptake and that age is a factor determining adoption. We also find that dairy and organic farms are more likely to use a computer. The physical size of the farm is positively associated with the probability of computer use while net farm income has a limited impact.

Key words: Bayesian model averaging, reversible jump algorithm, technology adoption.

1. Introduction

The aim is to analyse adoption decisions made by farmers. In particular, we examine the factors influencing the decision to adopt organic farming practices and to use a computer in the management of the farm. Our analysis of the adoption of organic farming re-examines a data set originally collected by Burton *et al.* (1999), and the analysis of the adoption of computers uses data collected as part of the UK government's Farm Business Survey. The UK Department of the Environment Food and Agriculture reports that 'the market for organic products is generally buoyant and expanding. Retail sales of organic produce are now worth approximately £1.2 billion per year with considerable opportunity for import substitution through increasing home production'. Understanding the characteristics of farmers and their environments, which mitigate for and against the adoption of this technology, is important in exploiting these opportunities. In the adoption of computers, agricultural producers have lagged behind other businesses in computer

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ownership and use. Despite the rapid adoption of computer technology by British farmers in recent years, there has been little in the way of formal econometric analysis of why farmers purchase computers, what they use them for and whether computers are making a positive impact on farm profitability.

In both cases we apply a Probit model and adopt an empirical approach to the selection of the explanatory variables. Recognising the well-documented pitfalls of adopting automated model selection strategies of the 'stepwise' kind (Miller 1984), we employ an approach that has not been used in this type of analysis to date. An appealing alternative to estimating one very large model or searching for a better performing sub-model is to take an average over many models. While classical statistics struggles to give any formal basis for averaging over models,¹ a Bayesian approach provides a theoretical underpinning, along with a clear methodology for implementing model averaging. Final estimates can be obtained by taking a weighted average of estimates over models, where a model that is highly supported by the data will be given a higher weight than one which is less supported by the data. Importantly, this approach can also deliver a measure of the probability that a given variable enters the model.

The construction of the weights used for Bayesian model averaging (BMA) is performed using the Bayesian 'marginal likelihood' (ML). Unlike the likelihood function, the ML does not depend on the parameter values because these have been integrated out of the expression.² Thus, it is a function of the data, the prior and the model, but not the parameters. Where two models are thought equally likely, *a priori*, the 'posterior odds' for two models is equal to the ratio of their ML (also known as the Bayes factor or ratio). Therefore, the ML can be used to give a weight to a given model with BMA.

Bayesian model averaging can be difficult to implement because the ML is often hard to compute. Under these circumstances, BMA is practicable only over a small number of models. For the standard linear regression model, the ML can be expressed analytically and computed quickly. Even so, where the set of models is defined by all the combinations of regressors that can enter the regression, the model space can be infeasibly large.³ Therefore, it can still be impractical to estimate every model and assign a weight to each model. Bayesian computation can solve this problem by employing an algorithm where only a relatively small subset of the models require estimation. The ML can provide a basis for choosing models as part of the algorithm that 'jumps' between one model and another. This class of algorithms are an extension of the Monte Carlo Markov Chain (MCMC) algorithms that are employed to estimate many Bayesian models.

¹ Martin *et al.* (2004) develop a hybrid approach to model averaging, but this could not be strictly labelled Bayesian or classical.

² The likelihood can be viewed as the probability of the data conditionally on the parameters. The ML can be viewed as the expected likelihood, given the model, model priors and data.

³ A model with k variables has 2^k sub-models.

This paper uses and explains a Bayesian reversible jump (RJ) procedure for Probit model selection in which the probability of regressors entering the model is estimated along with the parameters themselves in the Probit equation. The estimates obtained from this procedure include the probability that a variable enters the model along with model-averaged estimates (and standard deviations) of the Probit parameters. The RJ method is an approach to model averaging, which can be applied to the selection of models where the number of potential models is very large. The general RJ approach to estimation of models was developed by Green (1995), and a general approach to the estimation of limited dependent variable data was outlined by Holmes and Held (2006). The RJ approach was applied to a linear time series model by Balcombe and Rapsomanikis (2010), but so far, there have been no applications of the RJ procedure for a Probit model within the Agricultural Economics literature. Applications of the RJ Probit within the Economics literature are few. An exception is Leon-Gonzalez and Scarpa's approach (2008), which applied this algorithm in a contingent valuation setting.

It is not our aim to compare classical with Bayesian methods, but to compare results with and without model averaging. In introducing the RJ approach, we are mindful that agricultural economists will be less concerned with theoretical arguments for BMA, but more concerned with its practicality, and how it may change the inferences obtained from a given data set. BMA is certainly practical. The models in this paper take less than half an hour to estimate on a modern computer, even though the data sets would be regarded as relatively large. Moreover, there is no doubt that BMA can sometimes have a substantive influence on inferences drawn from a given set of data. For example, Balcombe and Rapsomanikis (2010) show that in the context of a time series model, the use of BMA lead to quite different conclusions. With noninformative priors and large sample sizes, Bayesian and classical approaches generally lead to comparable point estimates and confidence intervals (see Mittelhammer *et al.* 2000, pp. 661–666). Therefore, although we use Bayesian methods throughout this paper, we would expect a comparison between classical and BMA results to be similar to our comparison of conventional Bayesian and BMA results.

The paper proceeds by discussing the estimation of the Probit using the RJ method in Section Section 2. Section 3 introduces the data and presents and discusses the empirical results. Section 4 concludes. Mathematical details are presented in an Appendix.

2. Model and estimation

A common Bayesian approach to estimation is to simulate the posterior distribution for the parameters of a model using MCMC algorithms (e.g. Chib and Greenberg 1994). The RJ approach is an extension of the MCMC algorithms. The difference is in that, when using the RJ approach, the model is also drawn from its posterior distribution, not just the parameters.

Using the notation $f(x|y)$ to denote the conditional distribution of x given y , MCMC algorithms operate by drawing from $f(x|y)$ and then using the draw of x to draw y from $f(y|x)$. Subject to certain conditions, this leads (provided the sequence is repeated many times) to draws from $f(x, y)$. Within the standard MCMC approach, the quantities y and x can represent parameters or latent data. With the RJ approach, they can also represent models. In this section, we describe the model and estimation procedure in more detail.

2.1. The model

The model employed within this paper is of a standard binomial Probit form:

$$\begin{aligned} y &= xb + e & (1) \\ y &= (y_1, \dots, y_T)'; e = (e_1, \dots, e_T)'; \text{ and} \\ x &= (x_1, \dots, x_T)', \end{aligned}$$

where $x_i = (1, x_{i,1}, x_{i,2}, \dots, x_{i,k})'$. It is assumed that $e_i \sim^{iid} N(0, 1)$. The restriction on the variance of e_i is the usual identifying assumption for the Probit model. The data y_i is not observed for the Probit model. Instead, we observe the indicator variable $d = (d_1, \dots, d_n)'$ where $d_1 = 1$ where $y_i > 0$ and $d_i = 0$ otherwise. The Bayesian approach to estimation requires a prior distribution for b . Where this is specified as $f(b) = N(\beta_0, M_0^{-1})$, then

$$f(b|y) = N(\beta_2, M_2^{-1}) \text{ and } y_i = TN^+(xb, 1)\mathbf{1}(d_i = 1) + TN^-(xb, 1)\mathbf{1}(d_i = 0) \quad (2)$$

where $M_1 = x'x$; $M_2 = M_0 + M_1$; $\beta_1 = M_1^{-1} x'y$; $\beta_2 = M_2^{-1} (M_0\beta_0 + M_1\beta_1)$, and $TN^+(TN^-)$ denotes a positively (negatively) truncated distributed normal distribution and $\mathbf{1}(\cdot)$ denotes an indicator function.

2.2. Estimation

Where the regressors are known, estimation can proceed through simulation by drawing b from $f(b|y)$, then y , then $f(y|b, d)$ and so on, recording the draws of b so as to simulate the marginal posterior distribution. The RJ algorithm that we employ only involves a further step by augmenting the sequence by drawing from $f(m|y, d)$, where m denotes the model (the choice of regressors). The last step is achieved by proposing a new model m^* in a ‘symmetric fashion’,⁴ then accepting this new model (rather than the old model m) with probability

⁴ This means that the probability of proposing a move from m to m^* is equal to the probability of proposing a move from m^* to m .

$$p = \min \left(\frac{\frac{|M_2(m^*)|^{-\frac{1}{2}}}{|M_0(m^*)|^{-\frac{1}{2}}} \times e^{-\frac{J_{m^*}}{2}}}{\frac{|M_2(m)|^{-\frac{1}{2}}}{|M_0(m)|^{-\frac{1}{2}}} \times e^{-\frac{J_m}{2}}}, 1 \right) \quad (3)$$

where

$$J_m = (y - x_m \beta_{0,m})' (y - x_m \beta_{0,m}) - (y - x_m \beta_{0,m})' x_m M(m)_2^{-1} x_m' (y - x_m \beta_{0,m}) \quad (4)$$

and x_m are the regressors for model m , and $\beta_{0,m}$ $M(m)_0$ are the priors for the parameters under model m , and $M(m)_2 = (M_{0,m} + x_m' x_m)$. Derivations of Equations (3) and (4) are left for an appendix (see The marginal likelihood). The validity of the ‘model step’ above follows from the fact that the conditional distribution of the model $f(m|y, d)$ simplifies to $f(m|y)$ because any admissible set of latent (y) is sufficient to deduce the observed data (d). Therefore, the model step within the RJ algorithm for the Probit model is almost identical to the model step within the normal linear model, except that the variance is set to one.

The Priors adopted in this survey are the ‘G-Priors’ (Zellner 1986) with $\beta_{0,m}' = 0$ for all models. Using this construction, the priors are $M_{0,m} = \frac{x_m' x_m}{n}$. For the rationale behind the use of these priors, readers are referred to the discussion and further references within Fernandez *et al.* (2001). Within our analysis, the priors over all models are uniform (each model is, *a priori*, equally likely as another). In principle, informative priors could be placed over the model space if some variables were thought more likely to determine adoption than others. However, we prefer to use noninformative priors over the model space.

3. Empirical section

Our analysis within this section will examine two different data sets: organic technology adoption and computer adoption in agriculture. As discussed previously, our analysis throughout will be Bayesian. Classical point estimates and confidence intervals for the data sets in this paper are similar to the results we present below without using BMA. In our conclusions, we contrast results that would be obtained with a classical-type hypothesis test for an exclusion restriction with the results that would be obtained using the probability of inclusion computed with the BMA model. Although it is not entirely consistent with the Bayesian literature, we denote significance at the 5 per cent level if the coefficient in the standard model has a 95 per cent Bayesian confidence interval (also known a high density region) that excludes zero. Bayesian significance has a slightly different interpretation to that of classical significance.⁵ However, as already noted, the Bayesian confidence intervals

⁵ e.g. see Mittelhammer *et al.* (2000, chapter 24).

presented herein are similar to those obtained by a classical analysis; thus, the exclusion of zero from the Bayesian confidence interval for the standard model would also indicate significance in the classical sense. Therefore, we continue to label the parameter 'significant' if its confidence interval excludes zero.

3.1. Adoption of organic farming

The data on the adoption of an organic farming system are composed of 237 horticultural producers from the UK, of which 151 were conventional producers and 86 had adopted organic practices. Those adopting organic practices comprise two groups: 71 are registered with the UK Register of Organic Food Standards and 15 declare themselves to be organic although they are not formally registered as such. The survey was conducted in 1996. The sources and summary of these data are described in the study Burton *et al.* (1999). The data with descriptions can also be found in the Economic and Social Research Council data archive.

Burton *et al.* (1999) run a Logit regression of the decision to adopt organic practices on a set of explanatory variables. These are listed in Table 1. As our aim is to compare the model-averaged results with those obtained from a model similar to that used by Burton *et al.* (1999), we employ exactly the same variables used in their analysis. We report the conventional Bayesian

Table 1 Definition of variables for model of adoption of organic farming

| | |
|---------|---|
| hysize | The size of the farm household (no.) |
| fem | Gender of the farmer (= 1 for female, = 0 for male) |
| age | The age of the farmer at the date of the survey (years) |
| hefe | If the farmer has had further or higher education = 1, = 0 otherwise |
| yagric | If income from agriculture is the main source of income for the household = 1, = 0 otherwise |
| toha | The size of the farm (ha) |
| enviss | If the farmer is concerned about local, national or global environmental issues = 1, = 0 otherwise |
| conin | If the farmer believes that 'current practice in conventional farming will sustain farm productivity indefinitely' = 1, = 0 otherwise |
| orgff | If former believes that organic farming alone can 'satisfy society's needs for food and fibre' = 1, = 0 otherwise |
| fsv | If the farmer believes that the trend to larger farm sizes is detrimental to the environment = 1, = 0 otherwise |
| maxcon | If the farmer tries to maximise the proportion of own consumption which is supplied by his/her farm = 1, = 0 otherwise |
| memenv | If farmer is a member of a countryside or environmental organisation = 1, = 0 otherwise |
| mempga | If farmer is member of a producer's group = 1, = 0 otherwise |
| infpss | If main information source is the press = 1, = 0 otherwise |
| infbuy | If main information source is buyers/merchants = 1, = 0 otherwise |
| inffmrs | If main information source is other farmers = 1, = 0 otherwise |
| infadas | If main information source is the Agricultural Development and Advisory Service = 1, = 0 otherwise |

Probit results for their model in Table 1 below, on the left-hand side. We also report 95 per cent highest posterior density intervals for the estimates. Where these intervals exclude zero, implying that associated variables are significant at the 5 per cent level, a star is given in superscript. In terms of significance, these are broadly the same as those reported by Burton *et al.* (1999).

The BMA results are reported on the right-hand side of Table 2. The estimates and the standard deviations are based on the posterior distributions for both the standard and BMA results. The columns headed *LB* and *UB* are the upper and lower bounds of 95 per cent Bayesian confidence intervals. The figures in the column *prob* give the probabilities that the relevant variables are included in the BMA model. In the columns headed *meffects*, the table reports the mean of the marginal effects distribution computed by evaluating a marginal effect at each draw from the posterior using mean values of the explanatory variables. Apart from the intercept, we can see that four variables: *conin*, *orgff*, *infpss* and *infbuy* (*conin* = 1 if farmer believes that current practices will sustain farm productivity, *orgff* = 1 if farmer believes that farming alone can satisfy societies needs for food and fibre, *infpss* (*infbuy*) = 1 if main source of information is press (merchants)) are included with probability 0.99 or above, closely followed by *infadas*, which is included with probability of 0.971. Notably, all these are also significant at the 5 per cent level in the conventional model. More generally, across most of the variables, there is a correspondence between the probability of being included in the model and the significance of the associated coefficient. Generally, the more significant a variable is (the further away the interval is from zero), the more likely that the variable is included in the model. This said, we would revise the importance of variables in the light of the probabilities in the eighth column. First, it would be inaccurate to conclude that a variable that is significant at the 5 per cent level should be in the model with 95 per cent probability. For example, *inffmrs* (*inffmrs* = 1 if main source of information is other farmers) is significant at the 5 per cent level, but is included in the model with a probability of 78 per cent. Perhaps more notable is the variable *age* that is close to significance at the 5 per cent level, yet it only has a probability of inclusion of 45 per cent. By contrast, the variable *maxcon* (*maxcon* = 1 if maximiser of consumption of own production) was insignificant at the 5 per cent level, but it was included with a probability of 61 per cent. Other variables of note include the variable for memberships of an environmental organisation (*memenv*), which is marginally insignificant (the lower bound of the conventional confidence interval is -0.001). The BMA probability of inclusion for this variable is 60 per cent, however. While the conventional results imply the somewhat surprising result that this variable should be omitted therefore, the BMA results are much more equivocal in this regard. Finally, we note that the model-averaged results differ substantively from the standard results. For variables that are not included with a probability close to 1, the estimates are substantially lower in absolute value, reflecting the high probability that they are zero.

Table 2 Factors influencing the uptake of organic farming

| Estimate | Standard | | | | Model averaged | | | | | | |
|----------|----------|-------|--------|---------|----------------|--------|-------|--------|--------|---------|--------|
| | SD | LB | UB | Effects | Estimate | SD | Prob | LB | UB | Effects | |
| int | -1.338* | 0.246 | -1.847 | -0.886 | -1.156 | 0.229 | 1.000 | -1.619 | -0.721 | | |
| hhsize | 0.272* | 0.095 | 0.089 | 0.462 | 0.026 | 0.202 | 0.114 | 0.870 | 0 | 0.381 | 0.030 |
| fern | 1.465* | 0.554 | 0.428 | 2.582 | 0.125 | 0.971 | 0.657 | 0.800 | 0 | 2.036 | 0.127 |
| age | -0.028 | 0.015 | -0.057 | 0.001 | -0.001 | -0.011 | 0.015 | 0.448 | -0.042 | 0 | -0.000 |
| hefe | -0.166 | 0.333 | -0.803 | 0.488 | -0.006 | -0.010 | 0.124 | 0.139 | -0.342 | 0.255 | -0.000 |
| yagric | -0.045 | 0.379 | -0.821 | 0.663 | -0.001 | -0.034 | 0.163 | 0.152 | -0.510 | 0.097 | -0.003 |
| totha | -0.002 | 0.001 | -0.005 | 0.000 | -0.000 | -0.002 | 0.002 | 0.694 | -0.005 | 0 | -0.000 |
| enviss | 0.992 | 0.513 | -0.030 | 1.972 | 0.095 | 0.593 | 0.632 | 0.583 | 0 | 1.792 | 0.104 |
| conin | -2.404* | 0.505 | -3.417 | -1.460 | -0.183 | -2.221 | 0.459 | 1.000 | -3.111 | -1.335 | -0.254 |
| orgff | 1.205* | 0.335 | 0.549 | 1.866 | 0.106 | 1.199 | 0.322 | 0.996 | 0.569 | 1.819 | 0.154 |
| fsv | 0.136 | 0.356 | -0.530 | 0.859 | 0.010 | 0.043 | 0.172 | 0.173 | -0.103 | 0.650 | 0.004 |
| maxcon | 0.771 | 0.455 | -0.098 | 1.665 | 0.078 | 0.522 | 0.533 | 0.606 | 0 | 1.505 | 0.084 |
| memenv | 0.677 | 0.340 | -0.001 | 1.349 | 0.060 | 0.422 | 0.434 | 0.598 | 0 | 1.222 | 0.053 |
| mempga | -0.937* | 0.373 | -1.695 | -0.225 | -0.073 | -0.711 | 0.460 | 0.813 | -1.439 | 0 | -0.073 |
| infss | -1.361* | 0.352 | -2.055 | -0.684 | -0.110 | -1.217 | 0.332 | 0.998 | -1.896 | -0.601 | -0.145 |
| infbuy | -1.298* | 0.352 | -1.993 | -0.608 | -0.100 | -1.163 | 0.343 | 0.993 | -1.834 | -0.500 | -0.131 |
| inffmrs | 0.833* | 0.334 | 0.203 | 1.501 | 0.068 | 0.424 | 0.777 | 0 | 1.285 | 0.073 | |
| infadas | -1.112* | 0.365 | -1.798 | -0.367 | -0.086 | -1.057 | 0.387 | 0.971 | -1.676 | 0 | -0.120 |

LB, lower bounds; UB, upper bounds; * indicates that the 95% interval does not include zero.

The coefficients of the model-averaged results are generally smaller (in absolute value) than for the standard results, but in nearly all cases retain their original signs. Those variables that enter the models a relatively small proportion of the time have correspondingly smaller values in absolute terms. In this sense, the BMA results represent the middle ground between a model selection strategy in which only the most general model is estimated and one where insignificant variables are eliminated from the model. The strategy of excluding insignificant variables from the model is an extreme one and arguably does not truly reflect the nature of our uncertainty about the role of the variable. Thus, the BMA results represent a more balanced approach between two polar approaches that are commonly employed in the literature.

In terms of the size of the marginal effects, the variables having the biggest impact on the probability of adopting organic farming reflect the beliefs of the farmer, their sources of information and their gender. The pair of variables that relate to attitudes to farming practices (*conin* and *orgff*) imply that the farmers who are confident that organic farming is capable of meeting the entire needs of society for food and fibre are more likely to adopt farming, while those who believe that conventional practices are sustainable are less likely to do so. Having the latter attitude reduces the probability of adoption by 25 percentage points, while the former increases it 15 percentage points. Farmers who rely on the press as a source of information are less likely to adopt organic farming by 14 percentage points as are those who rely on buyers and merchants (13 percentage points) and those who rely on the ADAS.⁶ While its impact is smaller, reliance on other farmers for information increases the probability of adoption by 7 percentage points. This finding is in conformity with other studies and suggests that the information given by conventional routes may be more appropriate to conventional farming and less well suited to the needs of organic farming. The importance of farmer provided advice to organic farmers, emphasises the reliance of these farmers on informal networks for the provision of information. As in other studies, we find that female farmers are more likely to adopt organic methods by 12 percentage points. Finally, it is evident that the results do not give strong support to the view widely reported elsewhere, that the age of the farmer is important in determining the uptake of an organic system as an increase in age of one year only increases the probability of adoption by 0.03 percentage points and there is only a 42 per cent probability of this variable appearing in the model.

3.2. Computer adoption

The literature suggests various factors that may affect the diffusion of farm-based computer technology in England and Wales. The likelihood of computer adoption within a farm business depends on the characteristics of the

⁶ ADAS is the privatised Agricultural Development and Advisory Service.

farmer and his/her business. The age and education of the farmer have been found to be significant determinants in the adoption process (Lazarus and Smith 1988; Putler and Zilberman 1988; Batte *et al.* 1990; Woodburn *et al.* 1994; Hoag *et al.* 1999; Lewis 1998; Ascough *et al.* 2002). Older farmers have been found not to use as many sources of information as their younger colleagues and are more dependent on their experience in farming. Moreover, older and more experienced farm decision-makers tend to maintain less-complicated record types, which may reduce their demand for computer-based management innovation. As an exception, however (Jarvis 1990; Baker 1992), it was found that managers' age and education are insignificant in determining computer adoption among Texan rice producers and New Mexico nonfarm agri-businesses, respectively. In addition, Woodburn *et al.* (1994), Ortmann *et al.* (1994) and Ascough *et al.* (2002) found that farmers' self-rating of financial, computer and management skills to be significant factors in the adoption process.

Results from a number of studies (Lazarus and Smith 1988; Putler and Zilberman 1988; Batte *et al.* 1990; Jarvis 1990; Baker 1992; Woodburn *et al.* 1994; Lewis 1998) indicate that gross farm income or farm size is a significant factor in computer adoption. In the UK, Warren (2000) found a clear positive relationship between increasing use of computer technology and increasing farm size, as well as a tendency for cattle and sheep farms to have lower levels of adoption than other farm types. Woodburn *et al.* (1994) also found that the probability of computer adoption is lower on farms with beef enterprises in Natal, South Africa, while Batte *et al.* (1990) found adoption rates among Ohio commercial farms to be highest for mixed livestock and dairy producers. The reasoning for these conflicting results may lie in the degree of livestock production intensity in the different regions and the availability of appropriate livestock production decision analysis and record-keeping software. Further significant positive factors in the decision to adopt computer technology include ownership of farm sales-related businesses (Putler and Zilberman 1988; Baker 1992), the presence of off-farm employment and higher proportions of rented land (Woodburn *et al.* 1994), and reduced levels of enterprise diversification (Putler and Zilberman 1988) and off-farm investments (Ortmann *et al.* 1994).

3.2.1. Data on computer adoption

The Department for Environment, Food and Rural Affairs (Defra) (2001) survey of computer use in England found that 35 per cent of holdings had computer access. Moreover, 25 per cent of holdings owned a computer but do not use it for farm business. In the 2002/03 FBS survey period, 74 per cent of 2366 farmers had access to a computer, and 75 per cent of these farmers used computers for farm business purposes. Of those farm business computer users, 62 per cent made at least some use of the computer for office management functions, 53 per cent for farm management accounts, 43 per cent for livestock enterprise management, 36 per cent for statutory records, 32 per

cent for tax accounts, 28 per cent for arable enterprise management and 16 per cent for the farm's payroll. In this paper, we use data from Defra's Farm Business Survey data for 2366 farms in England and Wales over the 2002/2003 financial year. There are 453, 748 and 563 farms in the north, east and west of England, respectively, and 602 in Wales. There are 1259 full owner-occupied farms and 359 full tenanted farms, and 748 have a mixed tenure status. The sample includes 874 small farms, 865 medium-sized farms and 627 large farms.

The average age of the farmers in the sample is 54 years. Of the total sample of farmers, 824 (35 per cent) have a 'school only' highest education level, while 1192 (50 per cent) have General Certificate of Education (GCE) 'O' or 'A' levels or the equivalent and 263 (11 per cent) have a degree or postgraduate qualification. Of the four regions surveyed, the east of England has the lowest proportion of 'school only' educated farmers and the highest proportion of farmers with GCE and university qualifications, while the reverse is true for Wales. The average age of farmers with a 'school only' education is 58 years, while those with a GCE/college education and university graduates average 51 and 52 years, respectively.

Table 2 gives a summary of the use of computers on the farms in the sample. Of the total sample of farms, 432 (25 per cent) did not have access to a computer, 314 (18 per cent) used a computer for personal/family purposes only and 972 (57 per cent) used a computer for farm or related business use. Farmers are extending their use of computers for farm or related business use, specifically various farm, financial and record management purposes.

3.2.2. Results for computer adoption

The dependent variable in our analysis is whether the farmer owns and uses a computer on the farm. Table 3 details the definitions of the explanatory variables for which the variable name is insufficient to convey their precise interpretation. Table 4 reports the results where the columns have the same structure as Table 2. As with organic data, there is a close correspondence between the significance of the variable and the probability that it will enter the model. In nine of the 12 significant variables, the probability that the variable enters the model exceeds 0.95. In a number of cases, the significant variables were deemed to be in the model with probability near 1. However, *North*, *Dairy* and *Degree* have probabilities of entering the model of 34 per cent, 71 per cent and 75 per cent, respectively, even though they are significant at the 5 per cent level. Thus, as in the case of the organic data, the BMA results differ substantively from the standard one in some important respects.

Of the variables appearing in the model with >80 per cent probability, the number of workers has by far the biggest impact with an increase in the number of paid workers by one from the average of 1.57, increasing the probability of adoption by 47 percentage points. The presence of organic enterprises on the farm increases the probability of a computer being used by 26 percentage points. This is perhaps an indication of the progressive nature of such

Table 3 Definitions of selected variables used in computer model

| Farm type | |
|---------------------|--|
| Cereals | Farms on which cereals, oilseeds, peas and beans were harvested; dry and set-aside lands account for over two-thirds of their total SGM |
| General crop | Farms with over two-thirds of their total SGM in arable crops (including field-scale vegetables) or a mixture of arable and horticultural crops; holdings where arable crops account for more than one-third of total SGM and no other grouping accounts for more than one-third |
| Horticultural | Farms where fruit (including vineyards), hardy nursery stock, glasshouse flowers and vegetables, market garden-scale vegetables and outdoor bulbs and flowers account for more than two-thirds of total SGM |
| Specialist pig | Farms on which pigs account for over two-thirds of their total SGM |
| Dairy | Farms where the dairy enterprise, including followers, accounts for over two-thirds of their total SGM |
| Cattle and sheep | Farms with more than two-thirds of their total SGM in cattle and sheep except holdings classified as dairy. A farm is classified as in the LFA if 50% or more of its total area is in the EC Less Favoured Area |
| LFA | |
| Cattle and sheep | Farms with more than two-thirds of their total SGM in cattle and sheep except holdings classified as dairy. A farm is classified as 'lowland' if < 50% of its total area is in the EC Less Favoured Area |
| Lowland | |
| Farm size | |
| Small | 1–2 person years of standard labour requirement |
| Medium | 2–3 person years of standard labour requirement |
| Large | > 3 person years of standard labour requirement |
| Education | |
| School only | Highest level of education attained is school. |
| GCE college | Highest level of education attained is college. |
| Degree | Highest level of education attained is at least first degree. |
| Other | e.g. Apprenticeship |
| 'Ownership' | |
| Sole | Farm owned only by the farmer |
| Partner | Farm is owned as a partnership |
| Other | e.g. Share farming or company |
| Other attributes | |
| Net farm income | Net profit after adding back Interest (net of any interest received) and Ownership Charges, minus Unpaid Manual Labour Costs and the emoluments of the principal director(s) |
| Organic enterprises | Are organic enterprises present on the farm? |
| Off-farm work | Does the farm have employment away from the farm? (yes = 1) |
| Off-farm income | Are there sources of off farm income? (yes = 1) |
| Social payment | Is the farmer in receipt of a social payment? (yes = 1) |

farmers. Region is also important with farms in the east and west of England, both more likely by 19 percentage points to adopt a computer relative to the reference region of Wales. While the probability of including *degree* in the model is relatively low at 75 per cent, when it is included its impact is large, and thus in the model-averaged results, its marginal impact is to increase the probability of adoption by 11 percentage points. This illustrates

Table 4 Factors influencing the on-farm use of a computer

| | Standard | | | | | | Model averaged | | | | |
|--|----------|-------|--------|--------|---------|----------|----------------|-------|--------|--------|---------|
| | Estimate | SD | LB | UB | Effects | Estimate | SD | Prob | LB | UB | Effects |
| Intercept | 0.271 | 0.032 | 0.208 | 0.334 | | 0.263 | 0.032 | 1.000 | 0.200 | 0.327 | |
| Area (Base = south) | 0.208 | 0.090 | 0.029 | 0.383 | 0.076 | 0.070 | 0.342 | 0 | 0.301 | 0.027 | |
| North | 0.569 | 0.099 | 0.378 | 0.766 | 0.209 | 0.511 | 0.092 | 1.000 | 0.336 | 0.694 | 0.194 |
| East | 0.548 | 0.090 | 0.366 | 0.718 | 0.202 | 0.499 | 0.088 | 1.000 | 0.329 | 0.676 | 0.189 |
| Main farm activity (Base = other) | -0.049 | 0.126 | -0.302 | 0.193 | -0.018 | 0.003 | 0.026 | 0.045 | 0 | 0 | 0.001 |
| Cereals | -0.199 | 0.143 | -0.477 | 0.078 | -0.073 | -0.008 | 0.043 | 0.057 | -0.058 | 0 | -0.003 |
| General crop | -0.153 | 0.162 | -0.472 | 0.163 | -0.056 | -0.002 | 0.028 | 0.038 | 0 | 0 | -0.001 |
| Horticultural | -0.050 | 0.169 | -0.375 | 0.292 | -0.018 | 0.002 | 0.030 | 0.035 | 0.000 | 0 | 0 |
| Specialist pig and poultry | -0.277 | 0.118 | -0.512 | -0.046 | -0.102 | -0.160 | 0.121 | 0.709 | -0.343 | 0 | -0.061 |
| Dairy | -0.096 | 0.122 | -0.330 | 0.145 | -0.035 | -0.007 | 0.035 | 0.065 | -0.076 | 0 | -0.003 |
| Cattle and sheep less Favoured Area | 0.084 | 0.131 | -0.166 | 0.348 | 0.031 | 0.025 | 0.073 | 0.137 | 0 | 0.231 | 0.010 |
| Cattle and sheep Lowland | 0.265 | 0.084 | 0.104 | 0.432 | 0.097 | 0.290 | 0.096 | 0.968 | 0 | 0.436 | -0.116 |
| Farm size (Base = medium) | -0.304 | 0.072 | -0.440 | -0.161 | -0.112 | -0.304 | 0.077 | 0.996 | -0.450 | -0.152 | -0.110 |
| Small | 0.265 | 0.084 | 0.104 | 0.432 | 0.097 | 0.290 | 0.096 | 0.968 | 0 | 0.436 | 0.110 |
| Large | 0.019 | 0.062 | -0.100 | 0.144 | 0.007 | 0.000 | 0.010 | 0.030 | 0 | 0 | 0 |
| Tenancy status (Base = mixed, owned and tenanted) | 0.022 | 0.088 | -0.156 | 0.190 | 0.008 | 0.001 | 0.014 | 0.029 | 0 | 0 | 0 |
| Fully owned | | | | | | | | | | | |
| Fully tenanted | | | | | | | | | | | |
| Education of farmer (Base = other e.g. apprenticeship) | -0.206 | 0.151 | -0.505 | 0.090 | -0.076 | -0.246 | 0.150 | 0.776 | -0.440 | 0 | -0.094 |
| School only | 0.099 | 0.147 | -0.184 | 0.386 | 0.036 | 0.056 | 0.131 | 0.283 | -0.036 | 0.404 | 0.021 |
| GCE college | 0.390 | 0.176 | 0.055 | 0.740 | 0.143 | 0.294 | 0.224 | 0.746 | 0 | 0.681 | 0.112 |
| Degree | -0.305 | 0.176 | -0.650 | 0.043 | -0.112 | -0.094 | 0.103 | 0.539 | -0.265 | 0 | -0.036 |
| Ownership (Base = other e.g. share farming or company) | -0.135 | 0.177 | -0.478 | 0.217 | -0.050 | 0.019 | 0.068 | 0.182 | 0 | 0.202 | 0.007 |
| Sole | | | | | | | | | | | |
| Partner | | | | | | | | | | | |

Table 4 (Continued)

| | Standard | | | | Model averaged | | | | | | |
|---------------------------|----------|-------|--------|--------|----------------|----------|-------|-------|--------|--------|---------|
| | Estimate | SD | LB | UB | Effects | Estimate | SD | Prob | LB | UB | Effects |
| Other farm(er) attributes | | | | | | | | | | | |
| No. of unpaid workers | 0.051 | 0.077 | -0.105 | 0.200 | 0.019 | 0.001 | 0.014 | 0.028 | 0 | 0 | 0 |
| No. of paid workers | 1.316 | 0.226 | 0.882 | 1.782 | 0.484 | 1.329 | 0.199 | 1.000 | 0.944 | 1.729 | 0.506 |
| Net farm income | 0.017 | 0.009 | -0.001 | 0.035 | 0.006 | 0.002 | 0.006 | 0.125 | 0 | 0.019 | 0.001 |
| Organic enterprises | 0.716 | 0.177 | 0.378 | 1.068 | 0.263 | 0.694 | 0.182 | 0.997 | 0.357 | 1.059 | 0.264 |
| Age of farmer | -0.188 | 0.030 | -0.248 | -0.132 | -0.069 | -0.191 | 0.028 | 1.000 | -0.246 | -0.137 | -0.073 |
| Off-farm work (yes = 1) | 0.263 | 0.086 | 0.097 | 0.432 | 0.097 | 0.262 | 0.081 | 0.975 | 0.109 | 0.422 | 0.010 |
| Off-farm income (yes = 1) | -0.030 | 0.070 | -0.171 | 0.106 | -0.011 | -0.002 | 0.015 | 0.038 | 0 | 0 | -0.001 |
| Social payment (yes = 1) | 0.094 | 0.142 | -0.189 | 0.361 | 0.035 | 0.007 | 0.042 | 0.048 | 0 | 0 | 0.003 |

LB, lower bounds; UB, upper bounds.

the attraction of using model-averaged results. If we adopted a classical approach and concluded on the basis of a hypothesis test that *degree* belongs to the model, we would report a marginal effect of 14 percentage points, even though we still have a degree of uncertainty about whether the variable should be in the model. Instead, the lower BMA value revises the likely impact of the variable to account for our uncertainty.

With regard to farm type, there is evidence that dairy farms are more likely to use a computer than other farming types. This classification variable enters the model with a probability of 71 per cent although the difference in the probability of adoption at 6 percentage points is relatively small. Other farm classifications all enter the model with probabilities < 15 per cent. A number of findings differ from the preceding literature. In many respects, our results confirm findings reported elsewhere. For example, *age* is found to enter the model with a probability of 100 per cent, and each additional year reduces the probability of adoption by 7 percentage points. We also found a positive relationship between farm size and computer use with both size variables entering the model with probabilities > 95 per cent. The relationship is symmetric around the reference medium-sized farm, such that the probability of adoption is increased (reduced) by around 10 percentage points for large (small) farms. However, net farm income (which is positively related to farm size) is found not to be a good predictor of computer use; it appears in the model with a probability of only 13 per cent.

4. Conclusions

This paper outlined the BMA approach to Probit regressions with uncertain regressors and then explores its use in a comparison of two Probit regressions with and without using BMA. We found the BMA method to be fast and added another useful layer of information when interpreting the results. While we found a high correspondence between the results across estimation with and without BMA, there were also some substantive differences. Overall, if a variable was significant at the 95 per cent level, this could not be used as a reliable indicator of whether that variable should be in the model.

With regard to the results on organic adoption, while broadly in accord with Burton *et al.* (1999), some differences were obtained by using BMA. Most notably, we found that using BMA produced considerably weaker evidence that age and membership of an environmental organisation were good predictors of the use of organic technology, once other covariates were taken into account.

With regard to the influence of farm and farmer characteristics on the uptake and use of computers, we found that the number of workers on the farm has had the largest impact. We also found that organic farmers were much more likely to be users of computer technology as are dairy farmers. We found that education to degree level had a large impact on the probability of adoption, but we are less certain about the inclusion of this variable in the model. As a

result, the BMA results are adjusted downwards in comparison with conventional results. With regard to size, we find that the physical size of the business is positively associated with adoption. The age of the farmer is found to be a strong predictor of adoption, with older farmers less likely to adopt.

The use of BMA in this article has been limited to the Probit model with linear effects. However, there are other contexts in which it may have utility. One further application may be in the selection of regressors when using 'flexible functional forms', which are popular in the Agricultural Economics literature. Where the number of explanatory variables is large, flexible functional forms can suffer badly from the 'curse of dimensionality'.

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Appendix

Preliminary definitions

Take the model as defined in the paper. Let V be the prior variance and M_0 be the prior precision for the parameters b that have a prior normal $b \sim N(\beta_0, M_0^{-1})$. Also, let β and M be conformable vectors and matrices indexed by j (defined further below) and define: $Q_j = (b - \beta_j)'M_j(b - \beta_j)$ and $P_j = \beta_j'M_j\beta_j$. Using this notation, the prior distribution for β is: $\pi(b) = 2\pi^{-\frac{p}{2}}|M_0|^{\frac{1}{2}}\exp(-\frac{Q_0}{2})$. The likelihood is $f(y|b) = (2\pi)^{-\frac{p}{2}}\exp\left(-\frac{S(b)}{2}\right)$ where $S(b) = \sum(y_t - x_t'b)^2$. Further define $M_1 = x'x$ and $M_2 = (M_0 + M_i)$. Let $\beta_1 = M_1^{-1}x'y$ and $\beta_2 = M_2^{-1}(M_0\beta_0 + M_1\beta_1)$. Further define $S(\beta_j) = \sum(y_t - x_t'\beta_j)^2$. Three results that are of use in what follows are:

- (See Proof 1)

$$S(b) = S(\beta_1) + Q_1 \quad (5)$$

- (See Proof 2)

$$Q_0 + Q_1 - Q_2 = P_0 + P_1 - P_2 \quad (6)$$

- and (see Proof 3)

$$S(\beta_1) + P_0 + P_1 - P_2 = \underbrace{(y - x\beta_0)'(y - x\beta_0)}_A - \underbrace{(y - x\beta_0)'xM_2^{-1}x'(y - x\beta_0)}_B \quad (7)$$

Deriving the posterior and ML

The posterior

Combining the prior with the likelihood, we define the function $p(\beta|y)$ as follows:

$$p(\beta|y) = \left(\frac{1}{\sqrt{2\pi}} \right)^{T+p} |M_0|^{\frac{1}{2}} \exp \left(- \frac{S(b) + Q_0}{2} \right). \quad (8)$$

Using Equation (5)

$$p(\beta|y) = (2\pi)^{-\frac{T+p}{2}} |M_0^{-1}|^{-\frac{1}{2}} \exp \left(- \frac{S(\beta_1) + Q_1 + Q_0}{2} \right) \quad (9)$$

using Equation (6)

$$p(\beta|y) = (2\pi)^{-\frac{T+p}{2}} |M_0^{-1}|^{-\frac{1}{2}} \exp \left(- \frac{S(\beta_1) + P_0 + P_1 - P_2}{2} \right) \times \exp \left(- \frac{Q_2}{2} \right) \quad (10)$$

therefore $S(\beta_1)$, P_0 , P_1 and P_2 are functions of the data and priors only. It is evident that this joint density is:

$$f(\beta|y) \propto \exp \left(- \frac{(b - \beta_2)' M_2 (b - \beta_2)'}{2} \right) \quad (11)$$

Therefore, $\beta/y \sim N(\beta_2, M_2^{-1})$.

The marginal likelihood

The marginal likelihood is $ML = \int_{\beta} (p(\beta|y)) d\beta$, therefore:

$$\begin{aligned} ML &= (2\pi)^{-\frac{T+p}{2}} |M_0^{-1}|^{-\frac{1}{2}} \exp \left(- \frac{S(\beta_1) + P_0 + P_1 - P_2}{2} \right) \int_{\beta} \times \exp \left(- \frac{Q_2}{2} \right) d\beta \\ &= (2\pi)^{-\frac{T+p}{2}} |M_0^{-1}|^{-\frac{1}{2}} \exp \left(- \frac{S(\beta_1) + P_0 + P_1 - P_2}{2} \right) \times |M_2^{-1}|^{\frac{1}{2}} (2\pi)^{\frac{p}{2}} \end{aligned} \quad (12)$$

Using Equation (7):

$$J = S(\beta_1) + P_0 + P_1 - P_2 = \underbrace{(y - x\beta_0)'(y - x\beta_0)}_A - \underbrace{(y - x\beta_0)'xM_2^{-1}x'(y - x\beta_0)}_B \quad (13)$$

Thus, the ML observes the following proportionality:

$$ML \propto (2\pi)^{-\frac{T}{2}} \frac{|M_2|^{-\frac{1}{2}}}{|M_0|^{-\frac{1}{2}}} \times e^{-\frac{J}{2}}. \quad (14)$$

It is this equation that provides the basis of the Metropolis Hastings acceptance probability in the paper (Eqn 3).

Proofs

Proof 1:

$$\begin{aligned}
 S(b) &= S(\beta_1) + Q_1 = (y - xb)'(y - xb) \\
 &= (y - x\beta_1 - x(b - \beta_1))'(y - x\beta_1 - x(b - \beta_1)) \\
 &= \underbrace{(y - x\beta_1)'(y - x\beta_1)}_{=S(\beta_1)} + \underbrace{(b - \beta_1)'x'x(b - \beta_1)}_{=Q_1} + 2(y - x\beta_1)'x(b - \beta_1) \\
 &\quad = 0
 \end{aligned} \tag{15}$$

Proof 2: First note that for $j = 1, 2, 3$, $Q_j = b'M_jb + P_j - 2b'M_j\beta_j$. Using these conditions

$$Q_0 + Q_1 - Q_2 = P_0 + P_1 - P_2 + K$$

where

$$K = b'M_0b - 2b'M_0\beta_0 + b'M_1b_1 - 2b'M_1\beta_1 - b'M_2b + 2b'M_2\beta_2.$$

We can show that K is zero because $b'M_2b = b'M_0b + b'M_1b$ and $b'M_2\beta_2 = b'M_0\beta_0 + b'M_1\beta_1$:

$$\begin{aligned}
 K &= b'M_0b - 2b'M_0\beta_0 + b'M_1b_1 - 2b'M_1\beta_1 - b'M_0b - b'M_1b \\
 &\quad + 2(b'M_0\beta_0 + b'M_1\beta_1) = 0
 \end{aligned} \tag{16}$$

Proof 3: We need to show that:

$$\begin{aligned}
 J &= S_1 + P_0 + P_1 - P_2 = (y - x\beta_0)'(I - xM_2^{-1}x')(y - x\beta_0) \\
 &= \underbrace{(y - x\beta_0)'(y - x\beta_0)}_A - \underbrace{(y - x\beta_0)'xM_2^{-1}x'(y - x\beta_0)}_B
 \end{aligned} \tag{17}$$

Result 3.1: $S(\beta_1) = y'y - P_1$

- Proof of 3.1

$$\begin{aligned}
 S(\beta_1) &= (y - x\beta_1)'(y - x\beta_1) = y'y - P_1 \\
 &= y'y - P_1 - 2\beta_1'x'y
 \end{aligned} \tag{18}$$

using $\beta_1'x'y = \beta_1'M_1\beta_1 \Rightarrow S(\beta_1) = y'y - P_1$

Result 3.2: $P_2 - P_0 = \beta_0'M_1\beta_0 + (y - x\beta_0)'xM_2^{-1}x'(y - x\beta_0) + 2\beta_0'M_2M_2^{-1}x'(y - x\beta_0)$

• Proof of 3.2

$$\begin{aligned}
 \underbrace{\beta'_2 M_2 \beta_2}_{P_2} &= (\beta_0 + M_2^{-1} x'(y - x\beta_0))' M_2 (\beta_0 + M_2^{-1} x'(y - x\beta_0)) \\
 &= \beta'_0 M_2 \beta_0 + (y - x\beta_0)' x M_2^{-1} x'(y - x\beta_0) + 2\beta'_0 M_2 M_2^{-1} x'(y - x\beta_0) \\
 &= \underbrace{\beta'_0 M_0 \beta_0}_{P_0} + \beta_0 M_1 \beta_0 + (y - x\beta_0)' x M_2^{-1} x'(y - x\beta_0) \\
 &\quad + 2\beta'_0 M_2 M_2^{-1} x'(y - x\beta_0). \tag{19}
 \end{aligned}$$

Therefore, using Result 3.1

$$J = S_1 + P_0 + P_1 - P_2 = y'y + P_0 - P_2 \tag{20}$$

and Result 3.2

$$\begin{aligned}
 J &= y'y - \underbrace{[\beta'_0 M_1 \beta_0 + (y - x\beta_0)' x M_2^{-1} x'(y - x\beta_0) + 2\beta'_0 M_2 M_2^{-1} x'(y - x\beta_0)]}_{P_2 - P_0} \\
 &= y'y - \underbrace{\beta'_0 M_1 \beta_0 - 2\beta'_0 x'(y - x\beta_0)}_{A = (y - x\beta_0)'(y - x\beta_0)} - (y - x\beta_0)' x M_2^{-1} x'(y - x\beta_0). \tag{21}
 \end{aligned}$$