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Evaluating environmental policies under uncertainty through application of robust nonlinear programming

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Environmental policy evaluation is characterised by a paucity of information. The novel technique of robust mathematical programming is introduced as a means to proactively account for this uncertainty in policy analysis. The procedure allows identification of expected bounds on the range of abatement costs associated with environmental policy. It also has the advantage of not limiting conclusions to realisations of specific point estimates or probability distributions. Empirical insights are provided in an application to a New Zealand inland lake threatened by nitrate pollution from dairy farming. Overall, this novel framework is demonstrated to have several key advantages, including explicit treatment of severe uncertainty, capacity to bound the range of expected abatement costs accruing to a given policy instrument, and the opportunity to identify robust plans that are immune to parametric variation.

Key words: nonpoint pollution, policy evaluation, water quality.

1. Introduction

Pollution of the world's aquatic environments is now primarily attributable to nonpoint pollution (United Nations Environment Program 2008) as point sources are generally more easily identified and regulated. Eutrophication of lakes and rivers following nutrient pollution is widespread, with more than three-quarters of fresh water bodies in the United States of America exceeding safe thresholds for total nitrogen and phosphorus, imposing a cost of around 2.2 billion U.S. dollars annually (Dodds *et al*. 2009). Efficient regulation of nonpoint pollution is often problematic given the ambiguity that characterises the formulation of policy instruments. This uncertainty stems from the diffuse nature of pollution, high number of polluters, nonmarket characteristics of most regulatory benefits, presence of complex production relationships (e.g. factor substitution), and the response of economic agents to market and production uncertainty.

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There is a substantial literature exploring the implications of risk for nonpoint pollution policy (Kampas and White 2004). However, the definition of specific probability distributions for model parameters is difficult when modelling complex systems, as (i) adequate information may be unavailable to guide their estimation, (ii) additional data can be costly to obtain, (iii) information gathering can be complicated by measurement error, and (iv) future values (e.g. for market prices) are often difficult to estimate (Doole and Kingwell 2010). Indeed, the evaluation of nonpoint pollution policies is characterised by uncertainty that can raise doubts about the validity of using standard expected-value analysis considering risk (Shaw and Woodward 2008).

Mathematical programming (MP) is widely used for policy analysis given its capacity to optimise; its ability to provide a consistent, coherent, and flexible framework for describing systems; and its ability to efficiently solve large, complex problems. Inclusion of risk aversion in policy models has long been recognised as an important means to inject greater realism and dampen the elastic behaviour of linear optimisation frameworks (Hazell and Norton 1986). Indeed, applications of MP incorporating this feature, especially using MOTAD (Minimisation of the Total Absolute Deviation) and target MOTAD formulations, have been numerous over the last 30 years (e.g. Adesina and Ouattara 2000). Economists have also widely applied the methods of stochastic programming (Rae 1971; Kingwell *et al.* 1993), chance-constrained programming (Zhu *et al.* 1994), and structured sensitivity analysis (Pannell 1997) to gain insight into the impacts of stochastic features on decision problems when distributional information is available. Nevertheless, the treatment of pure uncertainty—where the distribution of key parameters incorporated in the decision model is unknown—has not been considered in empirical policy models given a lack of a suitable methodological framework.

Hansen and Sargent (2001) introduced robust control as an elegant means to consider ambiguity in conceptual economic models. This approach was subsequently adopted in the analysis of natural resource problems, with applications to water management (Roseta-Palma and Xepapadeas 2004) and species preservation (Woodward and Shaw 2008). Robust control is a powerful technique for small and weakly nonlinear models, particularly those typically used by economic theorists. However, this method does not naturally extend to large empirical models of the type generally used for environmental policy analysis. Thus, ‘‘development of appropriate frameworks for decision making in light of [ambiguity] is an important challenge to economists today’’ (Woodward and Shaw 2008, p. 603).

In the light of this challenge, this study presents the first empirical application of the robust nonlinear programming (RNP) framework introduced by Doole and Kingwell (2010), which proactively deals with ambiguity. RNP uses closed intervals to describe the variation of uncertain coefficients. Each outcome is assumed to occur with the same probability, as there is insufficient reason to believe one result is more likely than another in a state of uncertainty (Jaynes 2003). RNP offers several benefits for environmental policy evaluation, including the representation of uncertainty aversion, solution using standard MP algorithms, and capacity to represent complex systems in models incorporating thousands of equations. The utility of the procedure is assessed in the context of a case study concerning the mitigation of nitrate pollution of a New Zealand inland lake. Economic analysis of this problem is pertinent because factor substitution and manipulation of the productive characteristics of livestock can potentially offset abatement costs associated with regulation.

The paper is structured as follows. Section 2 describes the modelling framework. Section 3 describes the model used to evaluate various policy options for the case study. Section 4 presents an empirical application of this model. Section 5 concludes the paper.

2. Robust nonlinear programming

This section presents a concise description of RNP. More information is available in the study by Doole and Kingwell (2010). A benefit of this formulation is that any number of coefficients may be defined as uncertain in an optimisation problem.

Suppose the decision-maker knows that the values of uncertain parameters occur in a closed interval, $C = [c^{L}, c^{U}]$, where c^{L} and c^{U} are respectively the lower and upper bounds of the interval, without further information on the probability distribution.

A generic nonlinear programming (NLP) problem can be defined: $\max_x J = \pi(x)$, subject to $g(x) \leq 0$ and $x \geq 0$, where $\pi(x)$ is a profit function and $g(x)$ denotes the constraint functions. Assume the functions $g(x)$ and $\pi(x)$ are uncertain and hence described by closed intervals. The midpoint of such intervals is $C^{M} = (c^{L} + c^{U})/2$, while their range is $C^{R} = (c^{U} - c^{L})/2$. This definition of the range is thus half of its conventional magnitude, in line with its typical use in interval mathematics (e.g. Alefeld and Herzberger 1983).

Doole and Kingwell (2010) outline that the solution may be identified through (RNP): $\max_x J = (\pi^{M'} - \Omega^{L}\pi^{R'})x + (\pi^{M'} + \Omega^{U}\pi^{R'})x$, subject to $(g^{M}(x) + \Lambda g^{R}(x)) \leq 0$ and $x \geq 0$, where Ω and Λ are exogenous trade-off parameters ($\Omega = [0, 1]$ and $\Lambda = [0, 1]$) and the {L, U} superscripts represent variations of the trade-off parameters defined for the objective function. The objective function is linear additive in the case of parametric uncertainty. However, the retention of the both lower and upper bounds in the objective function in RNP ensures the identification of a nondominated solution to the maximisation problem (Wu 2008) and provides the range of the objective function for any given optimal solution.

Trade-off parameters specify the proportion of the variation in the uncertain parameter (i.e. the difference between the midpoint and the range) that is considered in the determination of the robust plan. Note that the addition of the range to the midpoint in the less-than constraint in RNP is consistent with maintaining feasibility if parameters vary within their closed interval. This is performed to the degree determined by the trade-off parameter. For example, the plan is robust to all expected outcomes if the trade-off parameter is set to unity. Trade-off parameters are a simple measure of uncertainty aversion as they represent the degree of conservatism that a decision-maker wishes to consider in formulating the optimal plan.

A pedagogical example of a deterministic MP problem is: $\max_x J = 3x_1 - 2.25x_2$, subject to $1x_1 - 0.6x_2 \leq 10$, $x_1 \geq 0$ and $x_2 \leq 0$. Assume that all coefficients are subject to bounded uncertainty, with the interval-valued programming problem stated as: $\max_x J = [2,4]x_1 + [-3.5,-1]x_2$, subject to $[0.9, 1.1]x_1 + [-0.7, -0.5]x_2 \leq 10$, $x_1 \geq 0$ and $x_2 \leq 0$.

Transcription to the RNP formulation requires calculation of the midpoint and range for each interval. The first interval [2,4] in the objective function has $C^{M} = 3$ and $C^{R} = 1$. The second interval [-3.5,-1] in the objective function has $C^{M} = -2.25$ and $C^{R} = 1.25$. The first interval [0.9,1.1] in the constraint has $C^{M} = 1$ and $C^{R} = 0.1$. The second interval [−0.7,−0.5] in the constraint has $C^{M} = -0.6$ and $C^{R} = 0.1$. This yields the RNP formulation as: $\max_x J = (3 - \Omega_1 \cdot 1)x_1 + (3 + \Omega_1 \cdot 1)x_1 + (-2.25 - \Omega_2 \cdot 1.25)x_2 + (-2.25 + \Omega_2 \cdot 1.25)x_2$, subject to $(1 + \Lambda_1 \cdot 0.1)x_1 - (0.6 + \Lambda_2 \cdot 0.1)x_2 \leq 10$, $x_1 \geq 0$ and $x_2 \leq 0$. Note how the negative and positive differences from the midpoint are retained in the objective function to recover the bounds of this function in the optimal solution.

Inherent conservativeness is justified in a state of uncertainty (Woodward and Shaw 2008). This framework simultaneously considers the lower and upper bounds between which all outcomes of the objective function are expected to lie during the optimisation, for given values of Ω and Λ. The worst-case (minimax) outcome is the lower bound of the objective function when all trade-off parameters are set to unity and the constraint functions are positive.[1] The best-case (maximax) outcome or other situations where Λ are negative (i.e. constraint parameters are estimated to fall between the midpoint and their most optimistic bound) are not considered as this optimistic approach is seldom justified in a state of uncertainty (Hansen and Sargent 2001). The possibility of irreversible environmental degradation further motivates the adoption of a conservative view (Pindyck 2007). The formulation of constraint functions in RNP ensures that the robust plan remains feasible in the light of any variability when $\Lambda = 1$. However, the worst-case approach can be relaxed through changing the value of trade-off parameters away from unity. For example, the model in which uncertain variables are defined as their medians is recovered when $\Lambda = \Omega = 0$. (This is called the midpoint model throughout the rest of the paper for simplicity.) In a geometric sense, these trade-off parameters determine the placement of

[1] Whether ranges are added to or subtracted from the midpoint for the constraint functions depends on their sign and the direction of the constraint. See Doole and Kingwell (2010) for more information and examples. Also, see Appendix S1 for a discussion in the context of the case study application.

the constraints that delineate the feasible region between their lower and upper bounds.

Trade-off parameters should be estimated to provide a robust plan that meets the requirements of decision-makers. Indeed, results in Sections 4.4 and 4.5 show that the use of a trade-off parameter of unity for all uncertain coefficients in the empirical model incurs a substantial cost. This highlights the need to estimate trade-off parameters using the best information possible. Various means may be used to estimate trade-off parameter values:

1. Search procedures can be applied to identify the values of the trade-off parameter which calibrate model solutions to reported behaviour. For example, the degree of uncertainty considered by farmers with respect to pasture production is identified in this paper through the use of a global search metaheuristic (Section 3.3).
2. Model solutions may be generated for a range of values of the trade-off parameter(s). The optimal solution may then be chosen by a decision-maker (Candler and Boeljhe 1977; Weersink *et al*. 2002).
3. Worst-case analysis is performed where trade-off parameters are set to unity. This approach is justified when a high level of conservatism is warranted (Hansen and Sargent 2001; Woodward and Shaw 2008).
4. Trade-off parameters may be subjectively elicited through surveys or focus groups. These should focus on the identification of what proportion of outcomes that decision-makers want to ensure against when robust solutions are implemented. (For example, for what proportion of outcomes should we ensure that the optimal plan does not violate the target for the level of nitrate leaching?) These proportions directly yield the values of the trade-off parameters.

These methods are based, in part, on those strategies used to estimate risk aversion parameters listed by McCarl and Spreen (2007). The numerical application presented in this paper utilises the first three of these methods.

The developed method is a practical and useful extension of classical methods of considering parametric variation in economic models. Chance-constrained programming involves the definition of probability distributions for constraint coefficients and allows the infeasibility of these restrictions to be tolerated with some probability (Prekopa 1995). RNP is conceptually similar in that the model involves specification of the proportion of total variation in the uncertain parameter that is insured against in the identification of an optimal solution. However, RNP has a number of distinct benefits. First, the definition of uncertainty using a bounded set with RNP offsets the need to estimate a more detailed, but potentially inaccurate, probability distribution. Second, the solution of models involving chance constraints is typically difficult, as it is rare for problems to be both convex and possess probabilities that can be efficiently computed (Nemirovski and Shapiro 2006). In contrast, the solution of RNP models is straightforward.

A common alternative approach to modelling risk in decision models is to define conservative estimates of coefficients (Rae 1971; McCarl *et al.* 1977; McCarl and Spreen 2007). RNP formalises this approach, while also incorporating the degree of parametric uncertainty that a decision-maker wishes to be insured against in the identification of robust plans. Moreover, RNP differs from both chance-constrained programming and the definition of conservative estimates through simultaneous consideration of both the lower and upper bounds of the objective function during solution through employment of the concept of nondominated solutions from multiobjective programming (Doole and Kingwell 2010). Minimisation of total absolute deviations (MOTAD) modelling has been broadly applied in agricultural economics as a linear approximation of expected value-variance modelling (McCarl and Spreen 2007). MOTAD in its typical form involves identification of an optimal solution that restricts the deviation of objective function values from their expected value (McCarl and Onal 1989). RNP is highly disparate from the MOTAD formulation, as it considers bounded uncertainty rather than the definition of more detailed probability distributions, incorporates the definition of uncertain coefficients in the constraints, and identifies a nondominated solution using both lower and upper bounds of the objective function.

3. Application

3.1. Nitrate pollution of New Zealand freshwater resources

The New Zealand dairy industry is the country's dominant agricultural industry, with dairy products valued at \$7.5 billion comprising 21 per cent of total merchandise exports in the year ending June 2007 (Statistics New Zealand, 2007). The high prices received for dairy products over the last decade have promoted significant intensification of what historically was a low-input, pasture-based system. However, intensification has led to greater nitrate leaching and subsequent nutrient pollution of freshwater bodies (see Monaghan *et al.* (2007) and references therein).

Lakes Karapiro and Arapuni are hydroelectric dams on the Waikato River, New Zealand's longest watercourse. (These lakes are referred to collectively as 'the lake' from here onwards.) As well as electricity generation, these lakes are important for recreation and tourism. Algal blooms have been observed in the lakes in recent years, as nitrate discharges from dairy farms in the surrounding catchment have decreased water quality (Environment Waikato, 2008). Dairy farming currently covers 42 938 ha of the catchment for these two lakes, nearly three-quarters of agricultural land in this area. Accordingly, there is a need for the regional environmental agency to establish appropriate regulatory tools to manage nutrient pollution. This analysis contributes to this goal through the use of RNP to identify the potential costs of emission standards. This study is also of international relevance given the increasing global awareness of the environmental impacts of dairy production, especially in China and India.

The parameters subject to uncertainty in this model are selected based on data availability, critical importance to the problem, discussions with experienced modellers of New Zealand dairy farming systems, previous modelling work, goals of the application, and *a priori* computation of sensitivity indices (Pannell 1997) to highlight the correlation between model output and perturbations of different coefficients. RNP is applied to proactively deal with uncertainty surrounding nitrate leaching, variable costs, and pasture production (see Section 3.3).

3.2. Model description

This section presents an overview of the RNP model used in the case study. It extends the individual farm model used by Doole (2010) to the catchment scale and to represent uncertainty. A brief description is provided here, with more information presented in Appendix S1.

The equilibrium model describes a management year consisting of 26 fortnightly periods. Cows consume grazed pasture and supplementary feeds: concentrates, grass silage and maize silage. Farm area in each period is grazed, harvested for grass silage, or rested for future use within a rotational grazing system. Grazing or silage production can only occur between pasture biomass thresholds that ensure the maintenance of seasonal feed quality and maximise opportunities for subsequent regrowth. Moreover, silage can only be produced at certain times of the year when pasture supply is excess to livestock requirements. Nitrogen fertiliser application increases pasture biomass in subsequent periods.

The supply of metabolisable energy (ME) for allocation between livestock classes is the sum of all feed sources available in a given period. Feed pools are allocated between livestock classes, each of which requires a given level of ME in each period. There are 216 different sets of cow attributes that may be possessed by individual cows. Each has different temporal energy demands given disparity in calving date, cull status, lactation length and productivity. Feed intake constraints ensure that cows do not consume unrealistic amounts of energy.

Total nitrate load from a given farm is defined as a function of nitrogen fertiliser application, livestock intensity and maize silage consumption. Stocking rate is the primary driver of nitrate leaching in New Zealand dairy farming systems as grazed pastures typically provide more nitrogen than cows require and this is expelled in urine (Monaghan *et al.* 2007). Nitrogen fertiliser plays an indirect role, increasing pasture production and hence stocking rate. In contrast, the low N content of maize silage decreases the N excreted by cows.

The objective function defines the maximisation of farm profit. Total revenue is earned from the sale of milk, culled cows and excess calves. Total cost is the sum of variable costs incurred for each cow, fixed costs incurred per hectare of farm area, cost of silage production, cost of maize silage, cost of concentrates and cost of nitrogen fertiliser.

3.3. Parameter values

Parameter values are taken from scientific publications, expert opinion and survey data. Key sources are summarised by Doole (2010).

There is no experimental information regarding pasture production and its variability in the study region. Also, there is no available information regarding the distribution of variable costs or nitrate leaching. For these reasons, the uncertain coefficients treated in this paper describe pasture production, nitrate leaching and variable costs. Of course, with sufficient investment of resources, detailed distributions could be estimated for all three variables. The paper is about how modellers might cope in the absence of such resources. Its value is reinforced in that the definition of more specific distributions can limit the relevance of model output.

The OVERSEER model (Monaghan *et al.* 2007) is used to estimate leachate burdens for multiple combinations of nitrogen fertiliser, stocking rate and maize silage for different soil types in the study region. OVERSEER output is regressed to form a metamodel for each soil type using SHAZAM econometric software (Whistler *et al.* 2004). Pasture production for 1986–2006 is determined using meteorological data from NZCD (2008) and a variant of the model described by Moir *et al.* (2000). This process provides much guidance to the temporal distribution of pasture production. However, bounded sets are still used to describe the uncertainty inherent in estimating pasture production given intrinsic measurement and modelling errors and a lack of suitable validation data. Variable costs are estimated for a series of years using positive mathematical programming (PMP) (Henseler *et al.* 2009). These are characterised as ambiguous parameters given the use of PMP.

The lower and upper bounds for each coefficient are determined using a structured system of generation and review. Estimates for each parameter are first generated using the empirical processes outlined in the previous paragraph. The final estimates are then broadened to account for errors inherent in the estimation process:

1. The lower and upper bounds were decreased and increased by 10 per cent, respectively, for nitrate leaching. This low rate reflected the extensive validation of the OVERSEER model (Monaghan *etal.* 2007).
2. The lower and upper bounds were decreased and increased by 25 per cent, respectively, for pasture production. This high rate is motivated by the lack of data for the pasture model that complicated validation, measurement error associated with the climate inputs, and error because of the inherent inability of the pasture growth model to describe reality, especially as it is not process-based.
3. The lower and upper bounds were decreased and increased by 25 per cent, respectively, for the parameters of the variable cost function. This high rate is motivated by the uncertainty that characterises the estimation of cost functions using PMP (Henseler *etal.* 2009).

The suitability of the estimated lower and upper bounds for all model coefficients were then discussed with experienced agronomists and modellers and updated where appropriate. The rates at which bounds were inflated reflect subjective opinion. However, they provide the best available information regarding how much parametric variation should be considered. The empirical estimation of data was important to inform this process and guide discussion with agronomists and modellers. It is important to consider that the use of very disparate bounds can complicate the application of RNP as they can (i) reduce the information obtained from the resultant bounds within which the values of the objective function are expected to lie and (ii) lead to infeasibility in the constraint set.

Trade-off parameters for the pollution meta-model and quadratic cost function are set to unity. This indicates that maximum levels of nitrate leaching and variable costs associated with agricultural production are considered in the formulation of the optimal plan. Conservative estimates of these elements may often be appropriate for four reasons:

1. There is broad uncertainty surrounding both variable costs and nitrate leaching, and lower values of uncertainty aversion would disregard this ambiguity in the optimisation.
2. Conservativeness is justified in a state of uncertainty (Woodward and Shaw 2008).
3. The focus of the study is estimating the potential range of abatement costs.
4. A worst-case approach could be taken with respect to environmental degradation (i.e. nitrate leaching) given the possibility of irreversible environmental degradation (Pindyck 2007).

However, a decision-maker may prefer a lower level of conservatism in some circumstances. Thus, Section 4.4 presents the implications of variation in the trade-off parameters for leaching load and quadratic variable costs.

The trade-off parameter for pasture production requires careful estimation, as high values lead to unrealistic management plans incorporating high levels of supplementary feed or infeasible models. It is therefore estimated using a combinatorial search algorithm. The trade-off parameter for pasture production is treated as an unknown, and a simulated-annealing procedure (Doole and Pannell 2008) is coded in GAMS Distribution 22.8 (Brooke *et al.* 2008)[2] to identify that value which minimises the absolute difference between observed and optimal levels of herd size and milk production. This identifies a value consistent with historical management and also reduces some of the burden placed on the quadratic cost function as a calibration instrument. The estimated trade-off parameter for pasture production is 0.8.

[2] A combinatorial search algorithm is required since the presence of numerous logical conditions in the complex constraint set of the model precludes the identification of this parameter using MP in GAMS.

This signifies that the average producer constructs their management plan such that it would be expected to remain feasible in 8 years of each decade. A value of zero indicates that the range of uncertain coefficients is not considered at all in the optimisation, while values between zero and 0.8 indicate consideration of less ambiguity than the base case. The impacts of varying this parameter are explored in Section 4.4 given that a different value may be appropriate in some situations.

3.4. Solution of model with robust nonlinear programming

The RNP problem contains 4349 variables and 6407 constraints. The corresponding GAMS program is available from the authors on request.

The base solution contains output for the standard parameter values used in the model. Environment Waikato currently uses emission controls elsewhere to improve water quality in a lake. So, the primary focus of the study is exploring the abatement cost of emission standards defined between 0 and 50 per cent of current levels under different circumstances. The model is used to investigate a number of scenarios:

1. Abatement costs are determined for the base case.
2. The impact of decreasing/increasing the milk price by $500/t is explored.
3. The implications of defining trade-off parameters for pasture growth of $\Lambda = \{0, 0.5, 0.8, 1\}$ are investigated.
4. Monte Carlo simulation is used to explore the implications of subsequent variability in annual pasture growth for the profitability of cow herd compositions formulated through RNP. Hedging against this source of uncertainty is costly because supplementary feeding is required. For each level of the trade-off parameter listed for the third scenario, this involves the following:

   a. Optimising the model.
   b. Fixing cow herds at their optimal levels.
   c. Re-optimising the remaining decision variables for 100 scenarios in which pasture growth in each period is represented as a uniform random variable defined between the lower and upper bounds identified using the process described in Section 3.3.

4. Results and discussion

4.1. Base solution

The base solution closely describes production behaviour in the study region. The optimal stocking rate is 2.69 cows/ha, 2 per cent higher than the reported 2006/07 stocking rate. Milk production in the optimal solution is 334 kg/cow, 5 per cent higher than mean New Zealand production over 2003–2008 (Livestock Improvement Corporation, 2008). Production results in nitrate leaching of 33.1 kg N/ha/year, a typical load observed in New Zealand dairy systems (Monaghan *et al.* 2007). Hence, the model provides a sufficient description of reality to allow useful insight into the value of alternative environmental policies.

4.2. Restriction of nitrate emissions

The optimal stocking rate and the level of nitrogen fertilisation decrease linearly with the stringency of the emission standards (Table 1). Moreover, although low-protein feeds can decrease leaching load, their overall impact is insufficient to warrant significant factor substitution to increase environmental mitigation. Model output also demonstrates that milk production does not improve substantially under simulated scenarios. Milk production varies, but is never more than 3 per cent of its base value (Table 1). Furthermore, lactation length is never adjusted by more than 2 per cent of its standard magnitude (data not reported). Thus, the best response of producers to emission standards is to unequivocally decrease production intensity without manipulating per cow production, at least given the agronomic, economic and technical reality described by the model.

The interval-valued function delineating the trade-off between optimal profit and the stringency of emission standards is shown in Figure 1. The bounded profit function represents all expected realisations of profit for a given set of trade-off parameter values. In contrast to a stochastic MP model, no probabilistic statement can be made regarding the realisation of a given value, apart from membership in the set of expected outcomes. This arises directly from the description of input data using bounded uncertainty sets, rather than standard probability measures.

The trade-off between environmental improvement and producer profit is not large, especially for decreases in nitrate leaching below 30 per cent. For example, a 25 per cent decrease in nitrate leaching lowers optimal profit from [$1211, $1478] to [$1117, $1382] or by 7.8–12.4 per cent (Figure 1). However,

Table 1 Key model output for proportional reductions in nitrate leaching load

N leaching reduction (%)	Profit range ($/ha)	Stocking rate (cows/ha)	N fertiliser (kg N/ha/year)	Maize silage (kg/ha)	Milk solids production (kg/cow)
0	[1211, 1578]	2.69	120	132	334
5	[1210, 1557]	2.62	106	156	333
10	[1195, 1523]	2.54	92	160	332
15	[1178, 1484]	2.45	80	155	330
20	[1154, 1442]	2.38	65	149	327
25	[1117, 1382]	2.28	54	144	325
30	[1066, 1301]	2.15	47	110	325
35	[1008, 1213]	2.01	40	67	325
40	[936, 1110]	1.85	37	17	327
45	[833, 975]	1.68	36	0	330
50	[711, 827]	1.51	34	0	333

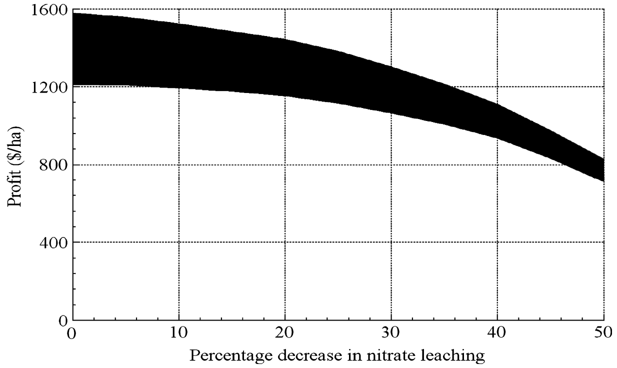

Figure 1 Interval-valued profit function derived for given reductions in nitrate emissions for the standard parameter values.

the profit function decreases as greater percentage reductions in nitrogen are simulated, with abatement costs increasing markedly at emission standards approaching 50 per cent of unregulated levels. Importantly, model output shows that nitrate regulation will incur a cost, even in the long-run when the hypothetical producer has had sufficient time to adjust their farming system in response to regulatory policy.

4.3. Impact of different milk prices

A standard 2008/09 milk payment of $5000/t milk solids (MS) is used in the base model. This could be bounded, but a single value is used because the inclusion of a conservative range of $500/t MS leads to substantial ambiguity surrounding the abatement-cost curve (Figure 2b), compared with the use of a point estimate (Figure 2a). The abatement-cost curves for $4500/t MS and $5500/t MS scenarios have a similar breadth and curvature to that computed in the standard solution (Figure 2a). This arises because the optimal management plan derived for the base case is very robust to output price uncertainty. For example, stocking rate and milk production change by < 1 per cent, relative to the base case, for each scenario. Furthermore, use of nitrogen fertiliser, a key productive input, varies by only 7–8 per cent.

Profit decreases by 46–53 per cent, 41–48 per cent and 39–45 per cent as emission standards are increased from 0 to 50 per cent for output prices of $4500/t MS, $5000/t MS and $5500/t MS, respectively. Thus, although the breadth and curvature of the abatement-cost curves are similar, higher output prices intuitively dampen the cost of environmental regulation, *ceteris paribus*.

4.4. Manipulation of trade-off parameters

A trade-off parameter of unity for pasture growth specifies a robust solution that remains feasible in the light of all uncertainty regarding biomass produc-

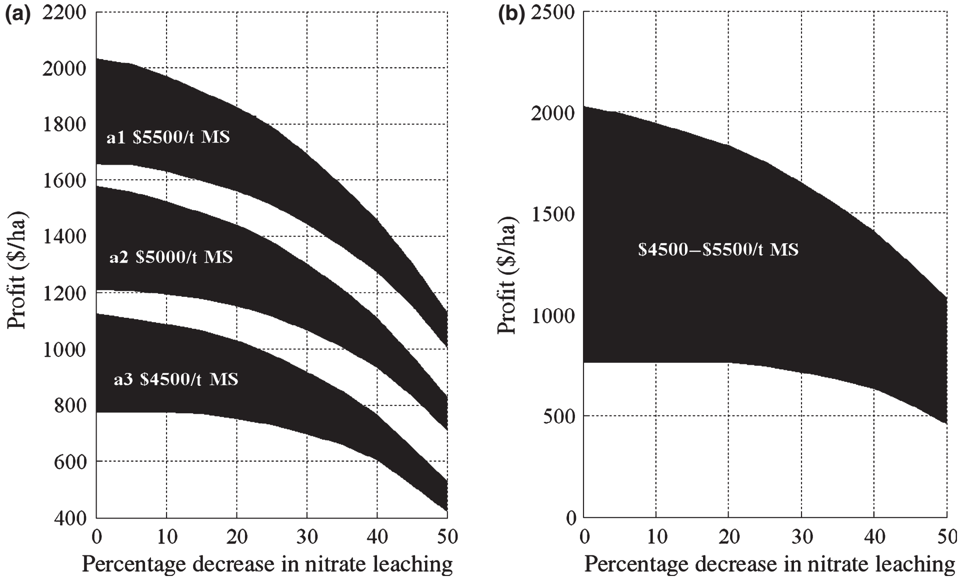

Figure 2 Interval-valued profit functions derived for given reductions in nitrate emissions with (a) discrete prices of milk solids (MS) and (b) a range of prices. Note scenario a2 is the standard case presented in Figure 1.

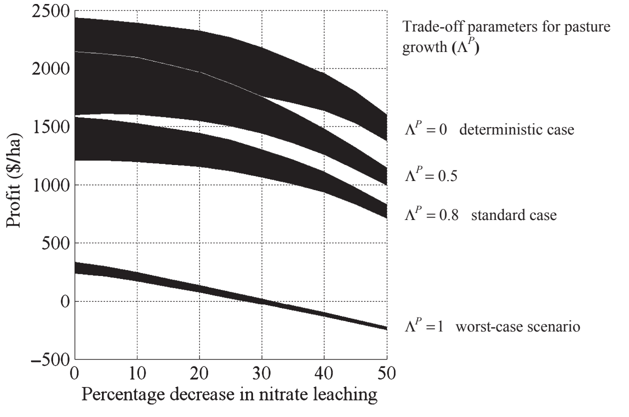

Figure 3 Interval-valued profit functions given trade-off parameters for pasture growth.

tion. This greatly reduces profit relative to the base case (Figure 3), highlighting the inherent conservatism of the worst-case formulation. Moreover, the computed range is narrow as variable cost declines as total herd size almost halves, compared with the standard solution. Profit increases, relative to the base case, as the trade-off parameter is reduced to 0.5 and 0 (Figure 3), as these latter scenarios incorporate more optimistic specifications of pasture growth compared with the worst-case scenario. Nonetheless, the range of the profit function increases in response to inflation of the stocking rate from 2.69 cows/ha in the base case to 3.29 cows/ha when the trade-off parameter is 0.5 and 3.61 cows/ha when the trade-off parameter is 0. Model output is very sensitive to the magnitude of the trade-off parameter defined for pasture growth. This highlights the importance of careful consideration of the level to which decision-makers give weight to parametric uncertainty.

Trade-off parameters for nitrate leaching and quadratic costs are set to unity, but lower values could be appropriate in different scenarios. Figure 4 presents bounded profit functions for alternative levels of N reduction. The breadth of the profit functions narrows as quadratic costs are considered more certain across all N reductions. The profit function is single-valued where the trade-off parameter for quadratic costs is zero, as quadratic costs are the only term considered uncertain included in the objective function. Expected profit intuitively increases as the regulator becomes tolerant of greater variability in nitrate leaching around the catchment goal. Decreases in profit associated with increases in the trade-off parameter are greater as goals for mitigation become more stringent (compare Figure 4a,c). This reflects the convex cost of abatement evident in the profit functions computed for the region (e.g. Figures 1 and 2).

4.5. Stability of income associated with robust plans

Figure 5 highlights a trade-off between the level and stability of income. The {mean, standard deviation} of profit is {1937, 422}, {2020, 195}, {1819, 155}

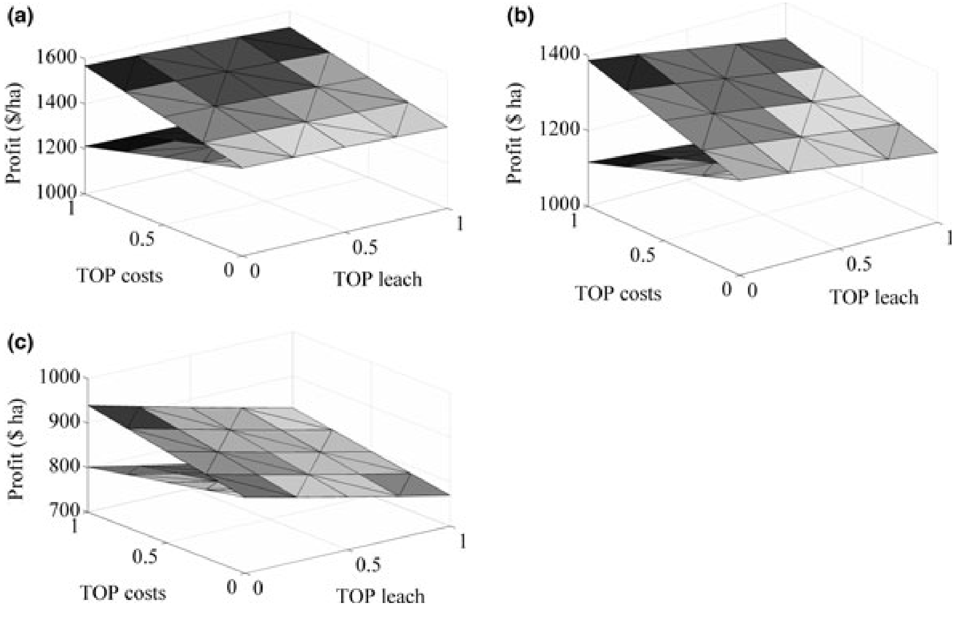

Figure 4 Interval-valued profit functions for different levels of the trade-off parameters for nitrate leaching (TOP leach) and quadratic costs (TOP costs) for reductions in N emissions of (a) 10 per cent, (b) 30 per cent and (c) 50 per cent.

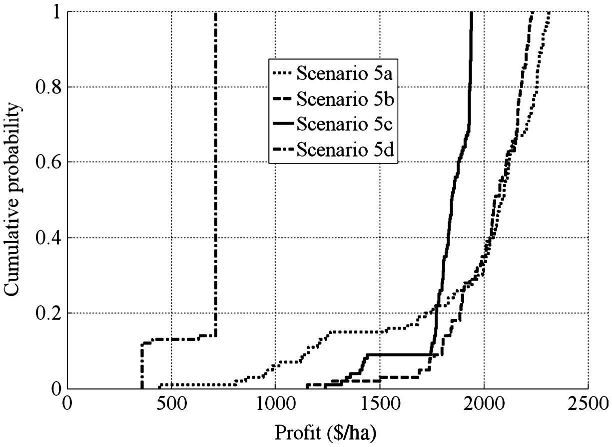

Figure 5 Cumulative distributions of profit given stochastic pasture growth when cow herds are fixed at levels from the optimal plans determined for trade-off parameters for pasture growth (Λ^P) of (a) 0 (consistent with the midpoint model), (b) 0.5, (c) 0.8 (standard case) and (d) 1 (consistent with the worst-case model).

and {714.6, 0.5} for trade-off parameters (Λ^P) of 0 (midpoint solution), 0.5, 0.8 and 1 (fully robust solution), respectively. These yield coefficients of variation (the ratio of the standard deviation and the mean) of 0.22, 0.1, 0.09 and 0.0007 for trade-off parameters of 0, 0.5, 0.8 and 1, respectively.

Mean profit for the midpoint scenario is only 6 per cent higher than that for the base case ($\Lambda^P = 0.8$), mainly because of the high variance of the former (Figure 5, compare Scenarios 5a and 5c). The stocking rate for the midpoint scenario is 25 per cent higher than that for the base case. Thus, profit varies substantially (Figure 5, Scenario 5a) as high levels of supplementary feeding are required when pasture growth is lower than expected. Furthermore, the midpoint of the distribution obtained for $\Lambda^P = 0.8$ is greater than that computed for the midpoint case, and its range is 2.7 times smaller. The cost of the robust formulation used in the standard model is therefore negligible, primarily reflecting its greater stability in the light of subsequent uncertainty. In contrast, the mean and midpoint of the base case is lower than that generated for a trade-off parameter of $\Lambda^P = 0.5$. Nevertheless, the base value is retained, as it provides a better description of observed production given the use of formal calibration of this parameter, as described in Section 3.3.

The fully robust solution, immunised against all uncertainty, is characterised by a minimal variance, but a mean well below that of the midpoint scenario (Figure 5, compare Scenarios 5a and 5d). Moreover, the output generated for this production plan is first-degree stochastically dominated by that of the base and $\Lambda^P = 0.5$ solutions. This illustrates the severe conservatism, and hence limited utility, of the worst-case formulation in this application.

5. Conclusions

Policy evaluation conducted using economic optimisation models suffers from parametric uncertainty given the large size of models, cost of information acquisition, measurement error, and a weak correlation between historical and future states. Economists are slowly beginning to consider such ambiguity in the analysis of natural resource issues (e.g. Roseta-Palma and Xepapadeas 2004). However, the development of appropriate decision frameworks remains an important challenge (Woodward and Shaw 2008), particularly given the limited capacity of robust control to model large, complex systems.

This paper is the first empirical application of the robust nonlinear programming framework of Doole and Kingwell (2010), which allows the explicit treatment of bounded uncertainty in empirical policy models. It has multiple benefits, including (i) removal of the assumption that decision-makers base their plans on certain knowledge, (ii) provision of a precautionary approach to natural resource management, (iii) capacity to bound the range of expected abatement costs accruing to a given policy instrument, (iv) chance to identify robust plans that are immune to parametric variation within the specified bounds, (v) straightforward solution in a MP context and (vi) endogenous stability that can provide more realistic simulation behaviour.

Nonetheless, there is a direct relationship between the conservativeness of the optimal solution and the magnitude of the trade-off parameter(s) that describe the maximum specification error that decision-makers are willing to tolerate. This highlights the need to carefully estimate these parameters through calibration or qualitative methods. Moreover, robust optimisation does not naturally incorporate correlations between random variables and distributional information. Nonetheless, these can be incorporated in a robust optimisation model using stochastic programming if sufficient information is available.

This method is applied to an illustrative example involving regulation of nitrate pollution of two New Zealand lakes. Model output displays that low-protein supplementary feed is not a profitable mitigation practice, even when uncertainty in feed supply is accounted for. Moreover, this analysis highlights that improving per cow milk production through genetics or lactation length is of little or no value in offsetting abatement cost. This reflects the inability of more productive cows to derive sufficient nutrition from a pasture-based diet. Consequently, a cautious approach to policy formulation is recommended.

A number of extensions of this analysis are worthy of further research. First, using robust MP to calibrate individual farms in a catchment context could provide insights into the value of spatially differentiating environmental policy. Second, the trade-off parameter has implicit linkages with the concept of uncertainty aversion and Choquet expected utility theory (Epstein

1999). Formalising these relationships with a focus on estimating the trade-off parameter for empirical work is conceptually interesting and may be practically important.

References

Adesina, A.A. and Ouattara, A.D. (2000). Risk and agricultural systems in northern Cote d'Ivoire, *Agricultural Systems* 66, 17–32.

Alefeld, G. and Herzberger, J. (1983). *Introduction to Interval Computations*. Academic Press, New York.

Brooke, A., Kendrick, D., Meeraus, A. and Raman, R. (2008). *GAMS—A User's Guide*. GAMS Development Corporation, Washington, DC.

Candler, W. and Boeljhe, M. (1977). Use of linear programming in capital budgeting with multiple goals, *American Journal of Agricultural Economics* 53, 325–330.

Dodds, W.K., Bouska, W.W., Eitzmann, J.L., Pilger, T.J., Pitts, K.L., Riley, A.J., Schloesser, J.T. and Thornbrugh, D.J. (2009). Eutrophication of U.S. freshwaters: analysis of potential economic damages, *Environmental Science and Technology* 43, 12–19.

Doole, G.J. (2010). Indirect instruments for nonpoint pollution control with multiple, dissimilar agents', *Journal of Agricultural Economics* 61, 680–696.

Doole, G.J. and Kingwell, R.S. (2010). Robust mathematical programming for natural resource modelling under parametric uncertainty', *Natural Resource Modelling* 23, 285–302.

Doole, G.J. and Pannell, D.J. (2008). Optimisation of a large, constrained simulation model using compressed annealing', *Journal of Agricultural Economics* 59, 188–206.

Environment Waikato (2008). *The Condition of Rural Water and Soil in the Waikato Region: Risks and Opportunities*. Environment Waikato, Hamilton.

Epstein, L.G. (1999). A definition of uncertainty aversion, *Review of Economic Studies* 66, 579–608.

Hansen, L.P. and Sargent, T.J. (2001). Robust control and model uncertainty, *American Economic Review* 91, 60–66.

Hazell, P.B.R. and Norton, R.D. (1986). *Mathematical Programming for Economic Analysis in Agriculture*. Macmillan, New York.

Henseler, M., Wirsig, A., Herrman, S., Krimly, T. and Dabbert, S. (2009). Modelling the impact of global climate change on regional agricultural land use through an activity-based non-linear programming approach, *Agricultural Systems* 100, 31–42.

Jaynes, E.T. (2003). *Probability Theory: The Logic of Science*. Cambridge University Press, Cambridge.

Kampas, A. and White, B. (2004). Administrative costs and instrument choice for stochastic non-point source pollutants, *Environmental and Resource Economics* 27, 109–133.

Kingwell, R.S., Pannell, D.J. and Robinson, S.D. (1993). Tactical responses to seasonal conditions in whole-farm planning in Western Australia, *Agricultural Economics* 8, 211–226.

Livestock Improvement Corporation (2008). *2007/08 Dairy Statistics*. Livestock Improvement Corporation, Hamilton.

McCarl, B.A. and Onal, H. (1989). Linear approximation using MOTAD and separable programming: should it be done?, *American Journal of Agricultural Economics* 71, 158–166.

McCarl, B.A. and Spreen, T.H. (2007). Applied mathematical programming using algebraic systems, [Online]. Available from URL: http://agecon2.tamu.edu/people/faculty/mccarl-bruce/mccspr/newb.pdf [accessed 12 October 2009].

McCarl, B.A., Candler, W.V., Doster, D.H. and Robbins, P.R. (1977). Experiences with farmer oriented linear programming for crop planning, *Canadian Journal of Agricultural Economics* 25, 17–30.

Moir, J.L., Scotter, D.R., Hedley, M.J. and Mackay, A.D. (2000). A climate-driven, soil fertility dependent, pasture production model, *New Zealand Journal of Agricultural Research* 43, 491–500.

Monaghan, R.M., Hedley, M.J., Di, H.J., McDowell, R.W., Cameron, K.C. and Ledgard, S.F. (2007). Nutrient management in New Zealand pastures—recent developments and future issues, *New Zealand Journal of Agricultural Research* 50, 181–201.

Nemirovski, A. and Shapiro, A. (2006). Convex approximations of chance constrained programs, *Siam Journal of Optimization* 17, 969–996.

New Zealand Climate Database (NZCD) (2008). *The national climate database*, NIWA, Hamilton. [Online]. Available from URL: http://cliflo.niwa.co.nz [accessed 15 September 2008].

Pannell, D.J. (1997). Sensitivity analysis of normative economic models: theoretical framework and practical strategies, *Agricultural Economics* 16, 139–152.

Pindyck, R.S. (2007). Uncertainty in environmental economics, *Review of Environmental Economics and Policy* 1, 45–65.

Prekopa, A. (1995). *Stochastic Programming*. Kluwer, Dordrecht.

Rae, A.N. (1971). An empirical application and evaluation of discrete stochastic programming in farm management, *American Journal of Agricultural Economics* 53, 625–638.

Roseta-Palma, C. and Xepapadeas, A. (2004). Robust control in water management, *Journal of Risk and Uncertainty* 29, 21–34.

Shaw, W.D. and Woodward, R.T. (2008). Why environmental and resource economists should care about non-expected utility models, *Resource and Energy Economics* 30, 66–89.

Statistics New Zealand (2007). *New Zealand External Trade Statistics*. Statistics New Zealand, Wellington. [Online]. URL: http://www.stats.govt.nz/products-and-services/global-new-zealand/2007/external-trade-statistics-June-07.htm [accessed: 15 December 2008].

United Nations Environment Program (2008). *Water Quality for Ecosystems and Human Health*. United Nations Global Environmental Monitoring System, Ontario.

Weersink, A., Jeffrey, S. and Pannell, D.J. (2002). Farm-level modelling for bigger issues, *Review of Agricultural Economics* 24, 123–140.

Whistler, D., White, K.J., Wong, S.D. and Bates, D. (2004). *SHAZAM Version 10 User's Manual*. Northwest Econometrics, Gibsons.

Woodward, R.T. and Shaw, W.D. (2008). Allocating resources in an uncertain world: water management and endangered species, *American Journal of Agricultural Economics* 90, 593–605.

Wu, H. (2008). On interval-valued nonlinear programming problems, *Journal of Mathematical Analysis and Applications* 338, 299–316.

Zhu, M., Taylor, D.B., Sarin, S.C. and Kramer, R.A. (1994). Chance constrained programming models for risk-based economic and policy analysis of soil conservation, *Agricultural and Resource Economics Review* 23, 58–65.

Supporting Information

Additional Supporting Information may be found in the online version of this article:

Appendix S1. Evaluating environmental policies under uncertainty through application of robust nonlinear programming.

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