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The Stata Journal (2011)
11, Number 4, pp. 605–619

Causal mediation analysis

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Abstract. Estimating the mechanisms that connect explanatory variables with the explained variable, also known as "mediation analysis," is central to a variety of social-science fields, especially psychology, and increasingly to fields like epidemiology. Recent work on the statistical methodology behind mediation analysis points to limitations in earlier methods. We implement in Stata computational approaches based on recent developments in the statistical methodology of mediation analysis. In particular, we provide functions for the correct calculation of causal mediation effects using several different types of parametric models, as well as the calculation of sensitivity analyses for violations to the key identifying assumption required for interpreting mediation results causally.

Keywords: st0243, medeff, medsens, mediation, causal mechanism, direct effects, sensitivity analysis

1 Introduction

The `mediation` package is designed to estimate the role of causal mechanisms that transmit the effect of a treatment variable on an outcome. Causal mechanisms are central to many studies in the social and life sciences, and the statistical analysis of mechanisms is widespread.[1] By positing and empirically testing causal mechanisms, scholars can explain why a relationship exists between two variables. The `medeff` and `medsens` commands contained in the `mediation` package implement the procedures described in Imai, Keele, and Tingley (2010a) and Imai, Keele, and Yamamoto (2010c) for a common set of statistical models.

Earlier approaches to mediation analysis largely relied on a form of structural equation modeling. Unfortunately, these earlier methods were not derived from a formal framework for causal inference and did not permit sensitivity analyses with respect to key identification assumptions. Furthermore, earlier methods were difficult to correctly extend to nonlinear models such as those with binary outcome variables. The tools in the `mediation` package enable users to conduct sensitivity analyses and cover several common statistical models that handle binary dependent variables. Mediation and sensitivity analysis are each implemented with one line of syntax, making the procedure simple for users. In this article, we discuss the foundations of these methods and how to use the `mediation` package. A longer, nontechnical introduction is provided by Imai et al. (2011).

1. For example, a canonical article about the topic by Baron and Kenny (1986) has over 25,741 citations according to Google Scholar (accessed 11 November 2011).

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2 Background

2.1 Notation

The underlying theoretical results that the `mediation` package is based upon formulate the identification of causal mechanisms in the common framework of potential outcomes. First, consider a sample in an experiment with units that are in either the treatment $T_i = 1$ or the control $T_i = 0$ condition. The outcome for observation i in the treatment condition can be denoted as $Y_i(1)$, or more generally $Y_i(T_i)$. In practice, we usually observe each unit in only one condition; thus $Y_i(0)$ is not observed in the above example. This implies that the unit-level treatment effect is unobservable, and researchers typically focus on estimation of the average treatment effect over a population, $E\{Y_i(1) - Y_i(0)\}$.

Mediation analysis moves beyond calculation of average treatment effects and instead seeks to quantify the effect of a treatment that operates through a particular mechanism. Let $M_i(t)$ denote the potential value of a mediator of interest for unit i under the treatment status $T_i = t$. Similarly, let $Y_i(t, m)$ denote the potential outcome if the treatment and mediating variables equal t and m. Here we observe only one of the potential outcomes, and the observed outcome Y_i is $Y_i\{T_i, M_i(T_i)\}$, which depends upon both the treatment status and the level of the mediator under the observed treatment status.

2.2 Quantities of interest

The key quantity of interest is the calculation of how much of the treatment variable is transmitted by the mediating variable. Following Robins and Greenland (1992) and Pearl (2001), we define indirect effects, or causal mediation effects, for each unit i as

$$\delta_i(t) \quad \equiv \quad Y_i\{t, M_i(1)\} - Y_i\{t, M_i(0)\}$$

for each treatment status $t = 0, 1$. This causal quantity is the change in the outcome corresponding to a change in the mediator from the value that would be realized under the control condition, $M_i(0)$, to the value that would be observed under the treatment condition, $M_i(1)$, while holding the treatment status constant at t. For example, if $M_i(1) = M_i(0)$, then the treatment has no effect on the mediator and the causal mediation effect would be zero. Importantly, because the treatment is fixed and only the mediator changes, we isolate the hypothesized mechanism. We can also define the direct effects of the treatment as

$$\zeta_i(t) \quad \equiv \quad Y_i\{1, M_i(t)\} - Y_i\{0, M_i(t)\}$$

for each unit i and each treatment status $t = 0, 1$. This represents all other causal mechanisms linking the treatment to the outcome.

Although we observe $Y_i\{t, M_i(t)\}$ for units with $T_i = t$, we do not observe the counterfactual outcome $Y_i\{t, M_i(1 - t)\}$ in the typical research design with one observation

per unit. This makes identifying causal mechanisms more difficult than identifying treatment effects and requires an additional assumption known as sequential ignorability (SI), discussed below. In practice, just as with treatment effects, we are interested in an average of the mediation effects. This is called the average causal mediation effect (ACME) $\overline{\delta}(t)$ and is defined as $\overline{\delta}(t) \equiv E[Y_i\{t, M_i(1)\} - Y_i\{t, M_i(0)\}]$. Similarly, the average direct effect (ADE) is defined as $\overline{\zeta}(t) \equiv E[Y_i\{1, M_i(t)\} - Y_i\{0, M_i(t)\}]$.

2.3 Identification assumption

The ACME or ADE is not identified in the standard design, where the treatments are randomized or ignorable conditional pretreatment covariates, and the mediator or outcome variables are measured. This is because a potential outcome required for the calculation of indirect and direct effects is never observed. An additional assumption is therefore required: SI. The assumption can be written as follows:

Assumption 2.1 (Sequential ignorability [Imai, Keele, and Yamamoto 2010c])

$$\begin{aligned} \{Y_i(t', m), M_i(t)\} &\perp\!\!\!\perp T_i | X_i = x \\ Y_i(t', m) &\perp\!\!\!\perp M_i(t) | T_i = t, X_i = x \end{aligned}$$

where X_i is a vector of the observed pretreatment confounders, $0 < \Pr(T_i = t \mid X_i = x)$ and $0 < p(M_i = m \mid T_i = t, X_i = x)$ for $t = 0$ and 1, and all x and m in the support of X_i and M_i, respectively.

Assumption 2.1 applies two ignorability assumptions sequentially. In the first step, given the observed pretreatment confounders, the treatment assignment is assumed to be ignorable—statistically independent of potential outcomes and potential mediators. This assumption is common and is also called unconfoundedness, exogeneity, or no omitted variable bias. In experiments, the assumption is expected to hold because treatment is randomized. The second step assumes—given the actual treatment status and pretreatment confounders—the observed mediator is ignorable. While the second step is similar to standard exogeneity assumptions, it is interesting to note that randomizing both the treatment and mediator does not identify the ACME (Imai, Tingley, and Yamamoto forthcoming; Imai et al. 2011).

2.4 Existing methods and practices

The standard approach to mediation analysis can be broken out into either 1) a set of steps where the statistical significance of slope estimates in a regression is evaluated or 2) the multiplication of slope coefficients along the causal path and a test of the significance of the product. An extended discussion of these approaches is contained elsewhere (MacKinnon, Warsi, and Dwyer 1995; Imai, Keele, and Tingley 2010a). The key practical limitations of existing methods are 1) the difficulty in correctly extending to nonlinear models (for example, probit) and 2) the inability to conduct sensitivity analyses to the SI assumption.

Regarding the first limitation, existing suggestions to fit mediation effects with binary models using nonlinear regression models (probit/logit) do not correspond to causal mediation effects (Imai, Keele, and Tingley 2010a). When both the mediator and outcome variable are continuous and fit with a linear regression, the mediation effect under the SI assumption is equivalent to fitting two regressions,

$$M_i = \alpha_2 + \beta_2 T_i + \xi_2^\top X_i + \epsilon_{i2} \quad (1)$$

$$Y_i = \alpha_3 + \beta_3 T_i + \gamma M_i + \xi_3^\top X_i + \epsilon_{i3} \quad (2)$$

and taking the product of the coefficient on the treatment variable in the first model with the coefficient on the mediator model in the second, $\beta_2\gamma$. Sans explicit recognition of the role the SI assumption plays, this is well known (MacKinnon et al. 2007; Baron and Kenny 1986). When outcome variables are binary and mediators are continuous, the product of coefficients (or some transformation of them) does not correspond to the ACME (Imai, Keele, and Tingley 2010a; Pearl 2011) despite advice to the contrary (MacKinnon et al. 2007; Kenny 2008). Similarly, the product of slope coefficients cannot be used when the mediator is binary and a nonlinear model is used (probit/logit) (Li, Schneider, and Bennett 2007; Imai, Keele, and Tingley 2010a). It is broadly accepted that methods developed for linear models do not extend to the use of nonlinear models in the context of mediation analysis (Kohler, Karlson, and Holm 2011).[2]

Regarding the second limitation, the blind application of earlier methods without respect to the nonrandomization of the mediator has led some to advise abandoning the search for mechanisms (Bullock, Green, and Ha 2010). Alternatively, we suggest conducting sensitivity analyses (which we show how to do using our package in section 4) or adopting different experimental designs (Imai, Tingley, and Yamamoto forthcoming). Sensitivity analysis allows the analyst to state how an estimated quantity would change for different degrees of violation of the key identification assumption (Rosenbaum 2002). Because the SI assumption can never be tested directly, sensitivity analysis is a key component of conducting causal mediation analysis.

3 Causal mediation analysis

3.1 The algorithm

The `mediation` package calculates the average mediation and direct effects by simulating predicted values of the mediator or outcome variable, which we do not observe, and then calculating the appropriate quantities of interest (average causal mediation, direct effects, and total effects). We implement the parametric algorithm described in (Imai, Keele, and Tingley 2010a).

2. A related procedure is discussed in Buis (2010). The main differences are that their procedure requires dichotomous outcome variables, represents quantities of interest in terms of log-odds, and does not provide a method for sensitivity analysis to violations of the key identification assumption (see section 4).

Algorithm 3.1 (Parametric inference)

Step 1: Fit models for the observed outcome and mediator variables.

Step 2: Simulate model parameters from their sampling distribution.

Step 3: Repeat the following three steps for each draw of model parameters:

1. Simulate the potential values of the mediator.
2. Simulate the potential outcomes given the simulated values of the mediator.
3. Compute quantities of interest (ACME, ADE, or average total effect).

Step 4: Compute summary statistics, such as point estimates (average) and confidence intervals.

The structure of the algorithm is a function of the theoretical results linking the SI assumption and the mediation effect, and the calculation of uncertainty estimates is based on the quasi-Bayesian Monte Carlo approximation of King, Tomz, and Wittenberg (2000). While this algorithm can be applied to a range of parametric or semiparametric statistical models, mediation currently implements the procedure for the common cases where ordinary least squares (OLS), probit, or logit models are used. Users needing more flexibility, such as the use of quantile regressions, at this point would need to use the R package with the same name, which implements a nonparametric bootstrap version of algorithm 3.1.[3] Sampling weights may also be used.

4 Sensitivity analysis

The preceding section discussed how to analyze data under the sequential ignorability assumption. We cannot test this assumption with the data; hence, a sensitivity analysis should be conducted. The sensitivity analysis investigates how robust the results are to the violation of the SI assumption. The exact form of the sensitivity analysis will depend on the types of parametric models used for the mediator and outcome models. We briefly describe each of the three cases covered by the `mediation` package (continuous mediator and outcome, continuous mediator and binary outcome, and binary mediator and continuous outcome).[4]

4.1 Continuous mediator and outcome variables

When linear models are used for the mediator and outcome variables, sensitivity analysis is based on the linear structural equation models in equations 1 and 2. Here a violation of the SI assumption leads to a correlation between ϵ_{i2} and ϵ_{i3}, which we denote by ρ ($\rho = 0$ under SI). As shown in Imai, Keele, and Yamamoto (2010c), the

3. Imai et al. (2010b) illustrate the use of this software.
4. The following assumes a binary (0/1) treatment variable.

ACME can be expressed as a function of ρ using identifiable parameters. The procedure is essentially an application of the iterative feasible generalized least-squares algorithm of the seemingly unrelated regression (Zellner 1962). Asymptotic variance of the estimated average causal mediation effects can be expressed with the delta method and the confidence intervals can be constructed. Additional mathematical details are provided in Imai, Keele, and Yamamoto (2010c).

4.2 Binary mediator and continuous outcome variables

If the mediator is modeled as a probit regression with an independent and identically distributed error term with standard normal distribution, and a linear normal regression with error variance equal to σ_3^2 for a continuous outcome variable, then a sensitivity analysis is available (Imai, Keele, and Tingley 2010a). Assuming that the two error terms jointly follow a bivariate normal distribution with mean zero and covariance $\rho\sigma_3$, the correlation between the two error terms, ρ, is the sensitivity parameter. Under these assumptions, the causal mediation effects can be written in terms of consistently fitted model parameters and a fixed value of ρ. Uncertainty estimates are computed based on the quasi-Bayesian approach. Mathematical details are provided in Imai, Keele, and Tingley (2010a). As shown below, graphing the ACME as a function of ρ is straightforward.

4.3 Continuous mediator and binary outcome variables

In situations with a binary outcome and continuous mediator, a sensitivity analysis is also available in the mediation package. Here the outcome model is assumed to be a probit regression, which allows us to assume that the error terms are jointly normal with a possibly nonzero correlation ρ. The ACME can once again be written as a function of identifiable parameters, and the confidence intervals are once again approximate with the quasi-Bayesian approach previously discussed. Mathematical details are provided in Imai, Keele, and Tingley (2010a).

4.4 Alternative interpretations based on R^2

Expressing the ACME as a function of ρ is simple. However, interpretation of the magnitude of this correlation coefficient may be difficult. An alternative approach is to express the ACME as a function of R^2's, which will capture how important a confounder is for explaining the mediator or outcome variable. If there is an omitted confounder U_i, then the error term will be a function of this confounder, yielding a decomposition of our error term $\epsilon_{ij} = \lambda_j U_i + \epsilon'_{ij}$ for $j = 2$ or 3 (for the mediator model and outcome model). With this setup, ρ can be expressed as a function of the proportions of previously unexplained variances in the mediator and outcome regressions,[5] or based on the proportions of original variances that are explained by the unobserved confounder in the mediator

5. ${R^*}^2_M \equiv 1 - \mathrm{Var}(\epsilon'_{i2})/\mathrm{Var}(\epsilon_{i2})$ and ${R^*}^2_Y \equiv 1 - \mathrm{Var}(\epsilon'_{i3})/\mathrm{Var}(\epsilon_{i3})$.

and outcome regressions.[6] The relationship between the ACME and R^2 parameters can then be expressed as the product of the R^2 parameters for the mediator and outcome variables. For the case of previously unexplained variances, this is $\rho = \text{sgn}(\lambda_2\lambda_3)R^*_M R^*_Y$, and for original variances, this is $\rho = \text{sgn}(\lambda_2\lambda_3)\widetilde{R}_M\widetilde{R}_Y/\sqrt{(1-R^2_M)(1-R^2_Y)}$. In both cases, ρ is a function of the product of unexplained variance measures. Below we show how the `medsens` command reports the values of ${R^*}^2_M {R^*}^2_Y$ or $\widetilde{R}^2_M\widetilde{R}^2_Y$ such that the ACME is zero. Because these are products, this critical point can occur across a range of values.[7]

When the mediator or outcome variable is binary, we use the pseudo-R^2 of McKelvey and Zavoina (1975). For example, in the binary mediator case, we redefine $\widetilde{R}^2_M = \{1-\text{Var}(\epsilon'_{i2})\}/\{\text{Var}(\widehat{M}^*_i)+1\}$ and $R^2_M = \text{Var}(\widehat{M}^*_i)/\{\text{Var}(\widehat{M}^*_i)+1\}$ in the above formula, where $\widehat{M}^*_i$ represents the predicted value of the latent mediator variable for the probit regression. Thus in all cases considered here, we can interpret ρ using two alternative coefficients of determination. This value can then be used to compare across studies or be evaluated in reference to subject-specific knowledge about the likely magnitude of effect from the confounding variable.

5 The medeff command

5.1 Syntax

`medeff` (*model depvar varlist*) (*model depvar varlist*) [*if*] [*in*] [*weight*] ,
`mediate(`*varname*`)` `treat(`*varname* [# #]`)` [`sims(`#`)` `seed(`#`)` `vce(`*vcetype*`)`
`level(`#`)` `interact(`*varname*`)`]

In the first set of parentheses, the user specifies the model for the mediator variable. In the second set of parentheses, the user specifies the model for the outcome variable. Available model types are OLS regression (`regress`), probit (`probit`), and logit (`logit`). If there will be a restriction on observations, this will apply to both models and is done with the standard `if` or `in` qualifier. `fweights`, `iweights`, and `pweights` are allowed.

6. $\widetilde{R}^2_M \equiv \{\text{Var}(\epsilon_{i2}) - \text{Var}(\epsilon'_{i2})\}/\text{Var}(M_i)$ and $\widetilde{R}^2_Y \equiv \{\text{Var}(\epsilon_{i3}) - \text{Var}(\epsilon'_{i3})\}/\text{Var}(Y_i)$.
7. $\text{sgn}(\lambda_2\lambda_3)$ captures whether the coefficient on the omitted variable is similar or different for the mediator and outcome equations. Hence linking these R^2 measures directly back to the ACME (which can be written as a function of ρ) requires that researchers specify the direction (positive or negative) of confounding for both models. R^2_M and R^2_Y are the coefficients of determination for the mediator and outcome regressions.

5.2 Options

`mediate(`*varname*`)` is required and specifies the mediating variable to use in the analysis.

`treat(`*varname* [# #]`)` is required and specifies the treatment variable to use in the analysis, where the numbers following the treatment name are values to use for the control and treatment conditions, respectively. By default, these are set to 0 and 1.

`sims(#)` specifies the number of simulations to run for the quasi-Bayesian approximation of parameter uncertainty. The default is `sims(1000)`. Higher values will increase the computational time.

`seed(#)` sets the random-number seed for precise replicability (though with sufficient `sims()`, results will be very similar). The default value is the current random-number seed.

`vce(`*vcetype*`)` allows users to specify how the standard errors will be calculated. *vcetype* may be `robust`, `cluster` *clustvar*, `bootstrap`, or `jackknife`.

`level(#)` specifies the confidence level, as a percentage, for confidence intervals. The default is `level(95)` or as set by `set level`.

`interact(`*varname*`)` allows for an interaction between the treatment and mediating variable. Interaction terms must be specified prior to running `medeff` and included in the model for the outcome variable.

5.3 Saved results

`medeff` saves the following in `r()`:

Scalars

`r(delta0)`	point estimate for ACME under the control condition
`r(delta1)`	point estimate for ACME under the treatment condition
`r(delta0hi)`	upper bound of confidence interval for δ_0
`r(delta0lo)`	lower bound of confidence interval for δ_0
`r(delta1hi)`	upper bound of confidence interval for δ_1
`r(delta1lo)`	lower bound of confidence interval for δ_1
`r(tau)`	point estimate for total effect
`r(tauhi)`	upper bound of confidence interval for τ
`r(taulo)`	lower bound of confidence interval for τ
`r(zeta0)`	point estimate for ADE under the control condition
`r(zeta1)`	point estimate for ADE under the treatment condition
`r(zeta0hi)`	upper bound of confidence interval for ζ_0
`r(zeta0lo)`	lower bound of confidence interval for ζ_0
`r(zeta1hi)`	upper bound of confidence interval for ζ_1
`r(zeta1lo)`	lower bound of confidence interval for ζ_1

5.4 Example

To illustrate the use of the `medeff` command, we produce 2,000 observations of simulated data. To do this, we use a system of linear structural equations given in equations 1 and 2, fixing the structural parameters α_2, α_3, β_2, β_3, γ, ξ_2, and ξ_3. All are set to 0.25 for simplicity. Below we implement the case for a continuous mediator and outcome variable using OLS regression for both models, in which case the ACME is equivalent to $\beta_2\gamma$.

```
. *Create simulated data
. set seed 312789
. local n 2000
. set obs `n´
obs was 0, now 2000
. *Population Values
. local alpha_2 .25
. local alpha_3 .25
. local beta_2 .25
. local beta_3 .25
. local gamma .25
. local x_beta .25
. *Draw realizations of error terms and pretreatment covariate x assuming no
> correlation
. matrix m = (0,0,0)
. matrix sd = (1,1,1)
. drawnorm e1 e2 x, n(`n´) means(m) sds(sd)
. *Generate realizations of treatment (T), mediator (M), and outcome (Y)
> variables
. generate T = round(runiform(), 1)
. generate M = `alpha_2´ + `beta_2´*T + `x_beta´*x + e1
. generate Y = `alpha_3´ + `beta_3´*T + `gamma´*M + `x_beta´*x + e2
```

```
. *Conduct mediation analysis
. medeff (regress M T x) (regress Y T M x), treat(T) mediate(M) sims(1000)
Using 0 and 1 as treatment values
```

Source	SS	df	MS		
Model	157.797161	2	78.8985804	Number of obs =	2000
Residual	2081.76275	1997	1.04244504	F(2, 1997) =	75.69
				Prob > F =	0.0000
Total	2239.55991	1999	1.12034013	R-squared =	0.0705
				Adj R-squared =	0.0695
				Root MSE =	1.021

M	Coef.	Std. Err.	t	P>|t|	[95% Conf.	Interval]
T	.2620606	.0456963	5.73	0.000	.1724431	.3516781
x	.2468286	.0231428	10.67	0.000	.201442	.2922152
_cons	.2428821	.0320858	7.57	0.000	.179957	.3058072

Source	SS	df	MS		
Model	376.062154	3	125.354051	Number of obs =	2000
Residual	1998.18054	1996	1.00109245	F(3, 1996) =	125.22
				Prob > F =	0.0000
Total	2374.24269	1999	1.1877152	R-squared =	0.1584
				Adj R-squared =	0.1571
				Root MSE =	1.0005

Y	Coef.	Std. Err.	t	P>|t|	[95% Conf.	Interval]
T	.2632512	.045148	5.83	0.000	.174709	.3517934
M	.2372682	.0219291	10.82	0.000	.1942618	.2802746
x	.2604436	.0233161	11.17	0.000	.2147171	.30617
_cons	.2291991	.0318908	7.19	0.000	.1666563	.2917419

Effect	Mean	[95% Conf.	Interval]
ACME1	.0624004	.0410497	.0888912
ACME0	.0624004	.0410497	.0888912
Direct Effect 1	.2638839	.17911	.351345
Direct Effect 0	.2638839	.17911	.351345
Total Effect	.3262843	.2238124	.4363766

Here Stata reports the results from the two regression models and then the summary estimates of the mediation, direct, and total effects. Estimates for both $\delta(1)$ (`ACME1`) and $\delta(0)$ (`ACME0`) are given.[8] The average effect of the treatment variable on the outcome that operates through the mediator is 0.062. The estimates of the direct effect, $\overline{\zeta}(t)$ (`Direct Effect 1` and `Direct Effect 0`), are equal to 0.26. Finally, `medeff` also reports the average treatment effect, `Total Effect`. As expected, under the sequential ignorability assumption, the estimate of the ACME is nearly identical to the product of coefficients method, even though the `medeff` command uses algorithm 3.1. If an analyst had a binary mediator or outcome variable, then a probit or logit model could be used instead of a regress model. As mentioned above, the product of coefficients in this case

8. In the case with all linear models and no treatment or mediator interaction (an option available to the `medeff` command), these estimates will be identical. With a binary outcome, the estimates can differ because of the nonlinear link functions.

will not correspond to the ACME. In such cases, we recommend using a probit model because this permits sensitivity analyses, which we discuss next.

6 The medsens command

6.1 Syntax

`medsens` (*model depvar varlist*) (*model depvar varlist*) [*if*] [*in*] ,
`mediate(`*varname*`) treat(`*varname*`)` [`sims(#) eps(#) level(#) graph`]

The first part of the `medsens` command follows the format of the `medeff` command in that it gives the required regression models.

6.2 Options

`mediate(`*varname*`)` is required and specifies the mediating variable to use in the analysis.

`treat(`*varname*`)` is required and specifies the treatment variable to use in the analysis.

`sims(#)` specifies the number of simulations to run. The default is `sims(100)`. For final production runs, the number should be set higher (such as 500), but note that this will take longer, especially for models with a binary mediator.

`eps(#)` convergence tolerance parameter for the iterative feasible generalized least squares. Used only when both the mediator model and the outcome model are linear. The default is `eps(.01)`. Typically, users will not change this and if so will only decrease it.

`level(#)` specifies the confidence level, as a percentage, for confidence intervals. The default is `level(95)` or as set by `set level`.

`graph` produces a graph of the results with the confidence intervals. Instead of specifying the `graph` option, you can use the variables generated by the `medsens` command (`_med_`*) with a graph command to produce a graph of your own.

6.3 Saved results

`medsens` saves the following in `r()`:

Scalars

`r(errcr)`	ρ (correlation in error terms) at which ACME = 0
`r(r2s_thresh)`	proportions of residual variance in mediator and outcome explained by hypothesized unobserved confounder
`r(r2t_thresh)`	proportions of total variance in mediator and outcome explained by hypothesized unobserved confounder

6.4 Example

We conduct sensitivity analyses for the previous empirical example. In each case, the `medsens` command is used; it automatically detects which type of sensitivity analysis should be conducted. The value of ρ where the ACME is zero is provided, as well as the sensitivity to both types of R^2 expressions. In addition, information required to graph the ACME as a function ρ is provided.

```
. **Run Sensitivity Analysis
. medsens (regress M T x) (regress Y T M x), treat(T) mediate(M) sims(100)
```

Source	SS	df	MS		
Model	157.797161	2	78.8985804		
Residual	2081.76275	1997	1.04244504		
Total	2239.55991	1999	1.12034013		

Number of obs = 2000
F(2, 1997) = 75.69
Prob > F = 0.0000
R-squared = 0.0705
Adj R-squared = 0.0695
Root MSE = 1.021

M	Coef.	Std. Err.	t	P>|t|	[95% Conf.	Interval]
T	.2620606	.0456963	5.73	0.000	.1724431	.3516781
x	.2468286	.0231428	10.67	0.000	.201442	.2922152
_cons	.2428821	.0320858	7.57	0.000	.179957	.3058072

Source	SS	df	MS		
Model	376.062154	3	125.354051		
Residual	1998.18054	1996	1.00109245		
Total	2374.24269	1999	1.1877152		

Number of obs = 2000
F(3, 1996) = 125.22
Prob > F = 0.0000
R-squared = 0.1584
Adj R-squared = 0.1571
Root MSE = 1.0005

Y	Coef.	Std. Err.	t	P>|t|	[95% Conf.	Interval]
M	.2372682	.0219291	10.82	0.000	.1942618	.2802746
T	.2632512	.045148	5.83	0.000	.174709	.3517934
x	.2604436	.0233161	11.17	0.000	.2147171	.30617
_cons	.2291991	.0318908	7.19	0.000	.1666563	.2917419

Sensitivity results

Rho at which ACME = 0	.2354
R^2_M*R^2_Y* at which ACME = 0:	.0554
R^2_M~R^2_Y~ at which ACME = 0:	.0434

95% Confidence interval

```
. twoway rarea _med_updelta0 _med_lodelta0 _med_rho, bcolor(gs14) || line
> _med_delta0 _med_rho, lcolor(black) ytitle("ACME") title("ACME({&rho})")
> xtitle("Sensitivity parameter: {&rho}") legend(off) scheme(sj)
```

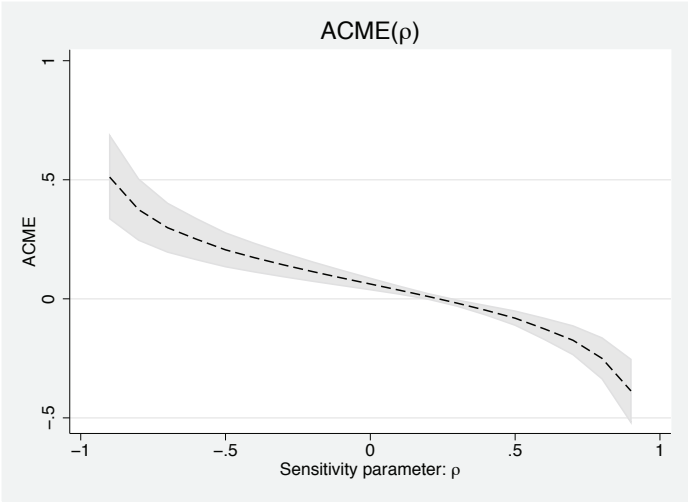

Figure 1. Average causal mediation effect as a function of degree of violation of SI assumption

The results show that for the point estimate of the ACME to be zero, the correlation between ϵ_{i2} and ϵ_{i3} must be approximately 0.24. Alternatively, the product of R^2's measures of sensitivity for the mediator and outcome models, for the residual and total variance, may be examined. For example, an omitted confounder must explain 20% of the remaining variance in the mediator and 27.5% of the remaining variance in the outcome, $0.20 \times 0.275 \approx 0.055$, for the ACME to be zero. Similar calculations can be done for sensitivity with respect to total variation, where the product of R^2's is 0.043.

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