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Abstract. In this article, we discuss generalized method of moments estimation of the covariance structure of longitudinal data on earnings, and we introduce and illustrate a Stata program that facilitates the implementation of the generalized method of moments approach in this context. The program, gmmcovearn, estimates a variety of models that encompass those most commonly used by labor economists. These include models where the permanent component of earnings follows a random growth or random walk process and where the transitory component can follow either an AR(1) or an ARMA(1,1) process. In addition, time-factor loadings and cohort-factor loadings may be incorporated in the transitory and permanent components.

Keywords: st0237, gmmcovearn, permanent inequality, transitory inequality, generalized method of moments, GMM, covariance structure of earnings

1 Introduction

In recent years, an increasing number of articles have examined the covariance structure of longitudinal data on earnings. Prior to work by Hause (1977) and Lillard and Weiss (1978, 1979), much of the literature on earnings differences focused on differences at a point in time, using cross-sectional data. However, with increasing availability of panel data, attention turned to the evolution of individual earnings over time—hence the term covariance structure of earnings.

In this literature, earnings at a point in time reflect both individual-specific timeinvariant characteristics and also transitory but serially correlated error-term variation. Consequently, earnings differences at a point in time may reflect either permanent inequality or transitory inequality, and tackling these different sources of inequality requires different policy measures.

Econometric models of earnings covariance structures are widely used in both macroeconomics and labor economics. Macroeconomists use the estimates as inputs in dynamic stochastic general equilibrium models, where the persistence of shocks has implications for consumption and labor supply over the life cycle. Labor economists are particularly interested in the quantitative importance of the different variance components in explaining residual earnings inequality.

Covariance structure models are related to those used in factor analysis but have been developed in a distinct way by labor economists. Examples of recent research articles that fit earnings covariance structure models include Moffitt and Gottschalk (2002; 2008; 1995), Baker (1997), Baker and Solon (2003), Dickens (2000), Haider (2001), Ramos (2003), Cappellari (2004), Daly and Valletta (2008), Guvenen (2009), Gustavsson (2008), and Doris, O'Neill, and Sweetman (2010a,b).

Estimation of some covariance structure models is possible in Stata using xtmixed. However, xtmixed only allows fitting of basic models and requires strong parametric assumptions. For this reason, research in the area has favored a generalized method of moments (GMM) approach, which is more flexible. The GMM estimator was introduced into the econometrics literature by Hansen (1982); excellent surveys can be found in Hall (2005) and Cameron and Trivedi (2005).

Although Stata's gmm routine, in its moment-evaluator form, could be used to fit each of a number of covariance structure models on a case-by-case basis, this is relatively complicated to do—it involves writing an ado file for each specification—and there are benefits to having a specialist and integrated command that allows researchers to fit many specifications using a single common command syntax. In this article, we introduce gmmcovearn, a user-written program that meets this need.

The rest of the article is set out as follows. In section 2, we briefly review the general principles underlying GMM estimation, and in section 3, we consider this estimator in the context of the covariance structure of longitudinal data on earnings. Our gmmcovearn command is explained in section 4 and is illustrated in section 5 with panel data on earnings for the USA and for Germany. We conclude the article in section 6.

2 A review of GMM estimation

The GMM approach provides a computationally convenient method of performing inference without the need for distributional assumptions. The key to GMM estimation is a set of population moment conditions that are derived from the underlying statistical model. GMM is based on the analogy principle whereby population moment conditions are replaced by their sample analogues. This in turn provides a system of equations that form the basis for the derivation of the GMM estimator. Suppose we have a $k \times 1$ vector m and a $p \times 1$ parameter vector θ such that for a given value θ_0 and data Y,

$$E\left\{m(Y;\theta_0)\right\} = 0$$

The GMM approach replaces the population expectation with the sample moments

$$\overline{m}(\theta) = \frac{1}{N} \sum_{i=1}^{N} m_i(y_i; \theta)$$

and chooses the value of θ that makes $\overline{m}(\theta)$ equal to or close to zero. Formally, the GMM estimator chooses the value of θ so as to minimize the criterion function

$$\overline{m}(\theta)' W_n \overline{m}(\theta)$$

where W_n is a positive-semidefinite weighting matrix that does not depend on θ .

For a just-identified model, any full-rank weighting matrix will lead to the same estimate. For over-identified models, different weighting matrices give rise to different estimators within the GMM class, all of which are consistent under regularity assumptions but differ in terms of efficiency.

The optimal GMM estimator weights by the inverse of the variance matrix of the sample moments. In practice, the optimal weighting matrix has to be estimated. Provided the estimated weighting matrix is consistent, this makes no difference asymptotically. However, estimation of the weighting matrix can lead to biases in finite samples. The problem arises because in small samples, there is a correlation between the moments and the weighting matrix.

Altonji and Segal (1996) and Clark (1996) consider the appropriate weighting matrix in relatively straightforward covariance models, and show that in finite samples the use of the identity matrix is preferable to the optimal weighting matrix. This approach has therefore become common practice when estimating the covariance structure of earnings and is adopted in gmmcovearn and in the remainder of this article. When the identity matrix is used, the GMM objective function resembles that of an equally weighted, nonlinear least-squares model.

The GMM estimator identifies model parameters if the probability limit of the GMM criterion function is uniquely minimized at the true parameter vector, θ_0 . Stock, Wright, and Yogo (2002) provide a very useful summary of the identification issues that arise in GMM estimation, and Doris, O'Neill, and Sweetman (2010b) discuss these issues in the context of the covariance structure of earnings. Weak identification can lead to inconsistent estimates, nonnormality, and size distortions in hypothesis testing even in very large samples. However, if the model is well identified, then under suitable regularity conditions, it can be shown that the limiting distribution of the estimator $\hat{\theta}_{\text{GMM}}$ is as follows:

$$\sqrt{N} \left(\widehat{\theta}_{\text{GMM}} - \theta_0 \right) \stackrel{d}{\longrightarrow} N \left\{ 0, \left(G'_0 G_0 \right)^{-1} \left(G'_0 S_0 G_0 \right) \left(G'_0 G_0 \right)^{-1} \right\}$$

where $G_0 = \text{plim}(1/N) \sum_{i=1}^N \left(\delta m_i / \delta \theta \Big|_{\theta_0} \right)$ and $S_0 = \text{plim}(1/N) \sum_{i=1}^N \left(m_i m'_i \Big|_{\theta_0} \right)$.

In practice, G_0 and S_0 are estimated by evaluating the analytical expressions at the GMM estimate, $\hat{\theta}_{\text{GMM}}$. Hypothesis tests on individual parameters can then be carried out using a Wald test.

For more-general model specification tests, Newey's (1985) test of over-identifying restrictions, based on an adjusted residual sum of squares, can be used. However, as several authors have noted (Baker and Solon 2003; Ramos 2003; Gustavsson 2008), this test almost always rejects the specified model at conventional levels. In addition, Baker (1997) found that such specification tests have inflated size in small samples. For these reasons, this approach is not used as the basis for specification tests in this literature. However, the adjusted or unadjusted residual sum of squares is typically used as a measure of goodness of fit.

3 Estimating the covariance structure of earnings

In the literature on the covariance structure of earnings, the de-meaned logarithm of earnings of worker i at time t, y_{it} , is the sum of a permanent component (for example, due to individual-specific fixed characteristics such as the level of education) and a transitory component (reflecting temporary shocks that affect the individual directly or indirectly via the labor market). Thus in the very simplest specification,

$$y_{it} = \alpha_i + \nu_{it}$$

where α_i is the individual-specific permanent component with variance σ_{α}^2 and ν_{it} are serially uncorrelated transitory shocks with mean zero and variance σ_{ν}^2 . It is assumed that $\operatorname{cov}(\alpha_i, \nu_{it}) = 0$. It follows that the total variance of (log) earnings is $\sigma_{\alpha}^2 + \sigma_{\nu}^2$ and the fraction of total inequality that is permanent is given by $\sigma_{\alpha}^2/(\sigma_{\alpha}^2 + \sigma_{\nu}^2)$. Readers will recognize this as a simple random-effects model, estimable using **xtreg**, **re** or **xtmixed**. Economists have modified this specification in three main ways: 1) they have allowed the relative importance of the permanent and transitory components to change with calendar time and across birth cohorts; 2) they have allowed for persistence in transitory shocks to earnings, using a variety of autoregressive and moving-average specifications; and 3) they have allowed the permanent component of earnings to evolve over time.

When modeling the evolution of the permanent component, two main approaches have been adopted in the literature. In the first approach, individuals have common life-cycle profiles and are subject to shocks that permanently change the individual's place in the earnings distribution (Dickens 2000). This specification is interchangeably referred to as the random walk or unit-root model (Baker 1997; Ramos 2003). In the

second approach, the random growth model, each worker has an individual-specific experience–earnings profile so that earnings growth rates vary across individuals in a systematic way (Haider 2001). In the random walk specification, current earnings are a sufficient statistic for future earnings, while in the random growth model, information in addition to current earnings (for example, initial earnings) may be informative about future earnings. For a detailed discussion and comparison of these two approaches, see Baker (1997) and Guvenen (2009). Additionally, a few articles have fit covariance models that combine the random walk model with the random growth model—see, for example, Ramos (2003) and Moffitt and Gottschalk (2008).

gmmcovearn allows researchers to fit models that incorporate the three key features of earnings dynamics discussed earlier: time and cohort effects in both the permanent and the transitory component; either an AR(1) or an ARMA(1,1) process for the transitory term; and a random growth or random walk model for the permanent component.

The general model we consider assumes that earnings for individual worker i, belonging to birth cohort c and with x years of experience at time t, y_{ict} , are given by

$$y_{ict} = q_c p_t \left(\alpha_i + \beta_i x_{it} + u_{it} \right) + s_c \lambda_t \nu_{it} \tag{1}$$

$$u_{it} = u_{i,(t-1)} + w_{it} \tag{2}$$

where $E(\alpha_i) = E(\beta_i) = E(w_{it}) = E(\nu_{it}) = 0$. α_i and β_i have variances σ_{α}^2 and σ_{β}^2 , respectively, and covariance $\sigma_{\alpha\beta}$. The first two terms inside the parentheses in (1) capture the random-growth component of earnings. Thus each individual may have a different permanent life-cycle growth rate of earnings, and this growth rate may be correlated with initial earnings. The final term inside the parentheses, u_{it} , follows a random walk process, with the variance of w_{it} given by σ_w^2 and $E(u_{i,(t-1)}w_{it}) = 0$.

This process allows for random shocks that have permanent effects. The accumulated variance of the random walk process prior to entry to the labor market cannot be identified in this model and is incorporated into estimation of σ_{α}^2 . This identification problem arises in any model with fixed effects and a unit-root process. p_t and λ_t are factor loadings that allow the permanent and transitory components, respectively, to change over time in a way that is common across individuals; q_c and s_c allow the permanent and transitory cohort. Thus the model allows for time, experience, and cohort effects. These parameters can be separately identified because of nonlinearities in the underlying model.

Serial correlation in the transitory shocks, ν_{it} , is modeled using either an AR(1) or an ARMA(1,1) process, with AR parameter ρ and MA parameter θ . Specifically,

$$\nu_{it} = \rho \nu_{i(t-1)} + \theta \epsilon_{i(t-1)} + \epsilon_{it} \tag{3}$$

where ϵ_{it} is a random variable with variance σ_{ϵ}^2 . The recursive nature of the transitory process necessitates consideration of initial conditions. Panel datasets do not typically

observe individuals from the start of their working lives, and the absence of this information has to be taken into consideration. Because working lives are not infinitely long, it is inappropriate to appeal to a long-run steady-state assumption to resolve the initial-conditions problem (which is the approach taken by **xtmixed** when fitting AR and MA specifications for the transitory component). Instead, we follow the approach suggested by MaCurdy (1982), which has been widely adopted by labor economists, and treat the variance at the start of our sample period, $\sigma_{\nu 1}^2$, as an additional parameter to be estimated.

The GMM estimator matches sample variances and covariances to their population counterparts. In the model specified by equations (1)-(3), the true variance–covariance matrix for cohort c has diagonal elements

$$\sigma_{c1}^{2} = \left\{ q_{c}^{2} p_{1}^{2} \left(\sigma_{\alpha}^{2} + \sigma_{\beta}^{2} X_{c1}^{2} + 2\sigma_{\alpha\beta} X_{c1} + \sigma_{w}^{2} X_{c1} \right) \right\} + \left(s_{c}^{2} \lambda_{1}^{2} \sigma_{\nu_{1}}^{2} \right)$$
(4)

$$\sigma_{ct}^{2} = \left\{ q_{c}^{2} p_{t}^{2} \left(\sigma_{\alpha}^{2} + \sigma_{\beta}^{2} X_{ct}^{2} + 2\sigma_{\alpha\beta} X_{ct} + \sigma_{w}^{2} X_{ct} \right) \right\} + \left\{ s_{c}^{2} \lambda_{t}^{2} \left(\rho^{2t-2} \sigma_{\nu 1}^{2} + K \sum_{w=0}^{t-2} \rho^{2w} \right) \right\}, \ t > 1$$
(5)

and off-diagonal elements

(

$$\operatorname{Cov}\left(y_{ct}, y_{c(t+s)}\right) = q_c^2 p_t p_{t+s} \left\{\sigma_{\alpha}^2 + \sigma_{\beta}^2 X_{ct} X_{c(t+s)} + \sigma_{\alpha\beta} \left(X_{ct} + X_{c(t+s)}\right) + \sigma_w^2 X_{ct}\right\} \\ + s_c^2 \lambda_t \lambda_{t+s} \left(\rho^s \sigma_{\nu 1}^2 + \rho^{s-1} \theta \sigma_{\epsilon}^2\right), \ t = 1 \text{ and } s > 0$$

$$\operatorname{Cov}\left(y_{ct}, y_{c(t+s)}\right) = q_{c}^{2} p_{t} p_{t+s} \left\{\sigma_{\alpha}^{2} + \sigma_{\beta}^{2} X_{ct} X_{c(t+s)} + \sigma_{\alpha\beta} \left(X_{ct} + X_{c(t+s)}\right) + \sigma_{w}^{2} X_{ct}\right\} + s_{c}^{2} \lambda_{t} \lambda_{t+s} \left(\rho^{2t+s-2} \sigma_{\nu1}^{2} + \rho^{s} K \sum_{w=0}^{t-2} \rho^{2w} + \rho^{s-1} \theta \sigma_{\epsilon}^{2}\right), \qquad (6)$$
$$t > 1 \text{ and } s > 0$$

where $K = \sigma_{\epsilon}^2 (1 + \theta^2 + 2\rho\theta)$, X_{ct} is the average experience of individuals in cohort c at time t, and X_{ct}^2 is the average value of experience-squared for cohort c at time t.

The parameter vector to be estimated is given by $\varphi = \left\{ \sigma_{\alpha}^2, \rho, \sigma_{\nu 1}^2, \sigma_{\epsilon}^2, \lambda_2 \dots \lambda_T, p_2 \dots p_T, q_2 \dots q_C, s_2 \dots s_C, \sigma_{\beta}^2, \sigma_{\alpha\beta}, \sigma_w^2, \theta \right\}.$ Identification requires a normalization of the factor loadings and in keeping with the literature, we set p_1, λ_1, q_1 , and s_1 equal to one. We then use this parameter vector to recover the individual components of aggregate inequality. The permanent component at time t is given by the first term in curly braces in (4) or (5), as appropriate, while the second term in curly braces is the transitory component.

The model given in (1)-(3) above encompasses many, but not all, of the models that have been used in the empirical literature. Firstly, the program does not allow for heteroskedasticity in the specified variances, as used, for example, by Baker (1997) and Hoffmann (2009). However, because of identification issues, such specifications have been used only with extremely large administrative datasets, which are typically not publically available. Secondly, earlier work by MaCurdy (1982) and Abowd and Card (1989) found that an MA(2) specification best fits the covariance matrix of earnings differences. While gmmcovearn cannot fit this model directly, our random walk model in levels with an ARMA(1,1) transitory term implies an ARMA(1,2) model in differences (Moffitt and Gottschalk 1995). In practice, the estimated value of the AR parameter is such that the difference between this and the MaCurdy and Abowd–Card specifications is small.

4 The gmmcovearn command

gmmcovearn estimates the parameters of the covariance structure of earnings. The program requires that the panel dataset be in wide format and contain an earnings variable. This variable may be specified in logs or in levels and may refer to actual earnings or to residuals derived from a first-stage regression on a set of observed covariates. If a heterogeneous profiles model is specified (random growth or random walk), the data must also include a labor-market experience variable. For models with cohort effects, the data must contain a cohort indicator variable, and there must be data for every cohort in each year. The program works for both balanced and unbalanced data at the individual level.

The program begins by computing the earnings variances and covariances from the raw data. When the data are unbalanced, each sample moment is constructed using all the available observations for that moment. For each moment expression, average experience (and average-squared experience) is calculated using only the information on individuals who contribute to that moment. The program uses the sample moments; the population expressions given in (4), (5), and (6); and Stata's nl command to recover the parameter estimates. As mentioned earlier, this implies that the weighting matrix used is the identity matrix. The sample moments are provided as part of the program's output. The program adjusts the standard errors of the parameter estimates to take account of the number of observations used in the computation of each moment, following the approach suggested by Haider (2001); this yields valid standard errors whether the data are balanced.

4.1 Syntax for gmmcovearn

The syntax for gmmcovearn is as follows:

```
gmmcovearn earningsvar [if], modeln(#) yearn(#) [expvar(exp) firstyr(#)
cohortn(#) cohortvar(cohvarname) firstcohort(#) cohvarname
stvalue(start_values) newdataname(momdataname) graph(#)]
```

where *earningsvar* is the stub of the name of the earnings variable in the dataset. The numeric suffix attached to the stub identifies the year (or panel wave) of the earnings variable. The program assumes that these suffixes are consecutive integers running from firstyr() through firstyr() + yearn() - 1, where firstyr() is the year associated with the first year observed in the dataset and yearn() is the total number of years observed (see below).

4.2 Options

modeln(1): AR(1), no growth heterogeneity $(\sigma_{\beta}^2 = \sigma_{\alpha\beta} = \sigma_w^2 = \theta = 0)$

modeln(2): ARMA(1,1), no growth heterogeneity $(\sigma_{\beta}^2 = \sigma_{\alpha\beta} = \sigma_w^2 = 0)$

modeln(3): AR(1), random growth $(\sigma_w^2 = \theta = 0)$

modeln(4): ARMA(1,1), random growth ($\sigma_w^2 = 0$)

modeln(5): AR(1), random walk $(\sigma_{\beta}^2 = \sigma_{\alpha\beta} = \theta = 0)$

modeln(6): ARMA(1,1), random walk $(\sigma_{\beta}^2 = \sigma_{\alpha\beta} = 0)$

modeln(7): AR(1), combined random growth and random walk ($\theta = 0$)

modeln(8): ARMA(1,1), combined random growth and random walk

- yearn(#) specifies the total number of years over which earnings are observed. yearn()
 is required.
- expvar(exp) specifies exp as the stub of the name of the experience variable and must be
 used for specifications that allow for growth heterogeneity in the life-cycle earnings
 profile (modeln(3)-modeln(8)). In the dataset, these suffixes must follow the same
 convention as for earningsvar, described above.
- firstyr(#) specifies the numeric suffix attached to the first year of earnings data
 observed in the sample. The default is firstyr(1).
- cohortn(#) specifies the number of cohorts used for the analysis. The default is cohort(1).
- cohortvar(*cohvarname*) identifies *cohvarname* as the variable that distinguishes the different cohorts. The default is cohortvar(cohort).
- firstcohort(#) specifies the value of cohvarname for the first cohort. The default is
 firstcohort(1).
- cohvarname is assumed to be coded in consecutive integers from firstcohort() to firstcohort() + cohortn() 1. For example, in a model with four cohorts, the values of cohvarname could be 1 to 4 (in which case firstcohort() is 1) or 1994 to

1997 (in which case firstcohort() is 1994). Nonconsecutive suffixes, such as 1960, 1970, 1980, and 1990, would have to be recoded before being used.

stvalue(*start_values*) specifies the starting values for the estimation. For T years of data and C cohorts, the values are entered in the following order, separated by commas: sigalpha, rho, sigv1, sige, 12–1T, p2–pT, q2–qC, s2–sC, sigbeta, covalphabeta, sigw, theta, corresponding to parameters σ_{α}^2 , ρ , $\sigma_{\nu 1}^2$, σ_{ϵ}^2 , $\lambda_2 \dots \lambda_T$, $p_2 \dots p_T$, $q_2 \dots q_C$, $s_2 \dots s_C$, σ_{β}^2 , $\sigma_{\alpha\beta}$, σ_w^2 , and θ .

The default value for each of the 1, p, q, and s parameters is 1; for sigalpha and rho, it is 0.5; for sigv1 and sige, it is 0.1; for sigbeta, covalphabeta, and sigw, it is 0; and for theta, it is -0.5. The user should specify starting values only for the parameters actually estimated in the chosen model (see modeln() above).

newdataname(momdataname) allows the user to create the new dataset momdataname consisting of two or more variables, depending on the model used. The first variable, moment, contains the sample moments calculated by the program. The second variable, nobsmoment, contains the number of observations used in calculating the corresponding moment. If a random growth or a random walk model is specified (modeln(3)-modeln(8)), the dataset will also contain the average of experience, aveexp, and the average of squared experience, aveexp2.

For the moment variable, the first T observations are the T sample variances for the first cohort, the next T observations are the variances for the second cohort, and so on for each of the C cohorts. Starting at the $(T \times C + 1)$ th element, the next (T-1) elements are the (T-1) first-order covariances for the first cohort, beginning with the earnings covariance between the first and second year. The next (T-1) elements refer to the first-order covariances for the second cohort, and so on. This pattern is repeated for the higher-order covariances so that the final C observations are the C(T-1)th-order covariances.

graph(#) requests a graphical display of the predicted permanent and transitory components of inequality [calculated using (4) and (5)], along with predicted and actual aggregate inequality. Predicted aggregate inequality is simply the sum of the predicted permanent and transitory components. The default is graph(0). The data underlying these graphs are available in the saved results (see below).

4.3 Saved results

gmmcovearn saves the following in e():

```
Scalars
    e(numoment)
                     number of moment conditions used in fitting the model
Macros
    e(cmd)
                     gmmcovearn
    e(cmdline)
                     command as typed
    e(properties)
                    b V
Matrices
                    coefficient vector: the parameters of the covariance structure model
    e(b)
    e(V)
                    adjusted variance-covariance matrix of the parameters
    e(momentc)
                    sample moments for the earnings variable for cohort c,
                       c = firstcohort(), \dots, firstcohort() + cohortn() - 1 (these are
                       presented in a variance–covariance matrix for each cohort)
                     observations used in calculating each of the sample moments for cohort c,
    e(nobsc)
                       c = firstcohort(), \dots, firstcohort() + cohortn() - 1 (these
                       are presented in the same format as the corresponding variance-covariance
                       matrices)
    e(permc)
                     predicted permanent component of earnings variance in each of the yearn()
                       years for cohort c, c = firstcohort(), \dots, firstcohort() + cohortn() - 1
    e(tempc)
                     predicted transitory component of earnings variance in each of the yearn()
                       years for cohort c, c = firstcohort(), \dots, firstcohort() + cohortn()-1
```

5 Examples

In this section, we use gmmcovearn to analyze the covariance structure of earnings with panel datasets for the USA and Germany. The examples are for illustrative purposes only. The example for the USA uses publically available data and illustrates the use of our estimator when the data are provided in long format. The German example estimates a random growth model of earnings and also shows how cohort effects are accounted for in our estimation procedure. Both examples use unbalanced data.

5.1 National Longitudinal Survey (NLS) data

In this example, we use the NLS panel dataset used in Wooldridge (2002) and available for download within Stata. The dataset provides an unbalanced panel of data on earnings, schooling, and demographic information for 530 individuals from the NLS for the years 1981 to 1987.

```
. use http://www.stata.com/data/jwooldridge/eacsap/nls81_87.dta
```

To begin, we fit a random-effects model with an AR(1) transitory error term, modeln(1), similar to that fit by Lillard and Weiss (1978). A simpler version of this model could be fit using xtmixed; however, xtmixed uses a steady-state assumption to handle initial conditions, which is unlikely to be appropriate, as discussed in section 3. In contrast, gmmcovearn follows the approach suggested by MaCurdy (1982) and treats the variance at the start of our sample period, $\sigma_{\nu_1}^2$, as an additional parameter to be estimated.

To fit this model with gmmcovearn, we first reshape the data into long format:

```
. keep id year lwage
```

```
. reshape wide lwage, i(id) j(year)
  (output omitted)
```

To illustrate the naming conventions used in our program, we next describe the variables in this dataset:

. describe					
Contains data					
obs:	530				
vars:	8				
size:	15,900				
	storage	display	value		
variable name	type	format	label	variable label	
id	int	%9.0g		identifier	
lwage81	float	%9.0g		81 lwage	
lwage82	float	%9.0g		82 lwage	
lwage83	float	%9.0g		83 lwage	
lwage84	float	%9.0g		84 lwage	
lwage85	float	%9.0g		85 lwage	
lwage86	float	%9.0g		86 lwage	
lwage87	float	%9.0g		87 lwage	

Sorted by: id

This dataset contains seven years of earnings data, from 1981 to 1987. The program requires the user to input *earningsvar*, which is the stub of the name of the earnings variable. In this example, *earningsvar* is **lwage**. In these data, the numeric suffix attached to **lwage** refers to the year of observation. The lowest suffix is **firstyr()**, the numeric year attached to the first year of earnings data, which in this case is 81.

A summary of the variables used is provided below:

. summarize					
Variable	Obs	Mean	Std. Dev.	Min	Max
id	530	991.3925	660.7559	4	2229
lwage81	242	9.388543	.5187844	6.71772	11.503
lwage82	261	9.457249	.4130265	7.855649	10.69415
lwage83	312	9.498774	.4851994	6.579612	11.12857
lwage84	349	9.518088	.4505686	7.406881	10.80471
lwage85	359	9.551186	.506268	6.031743	10.79433
lwage86	361	9.643893	.5299772	7.141506	11.53664
lwage87	379	9.704476	.5805771	7.841583	12.80161

This table clearly illustrates the unbalanced nature of the data. For example, there are 242 observations used to calculate the sample variance for 1981 but 379 observations

used to calculate the final-year sample variance. To see the number of observations that will be used to calculate the covariance of earnings between 1981 and 1982, we can issue the count command, as follows:

```
. count if lwage81~=. & lwage82~=.
 193
```

As we will see later, all information on the number of observations used can be recovered from the program's saved results. The gmmcovearn command is issued as follows:

. gmmcovearn (obs = 28)	lwage, yearn(7) modeln(1)	firstyr	(81)		
Iteration 0:	residual SS =	.1643298				
Iteration 1:	residual SS =	.05161				
Iteration 2:	residual SS =	.0017779				
Iteration 3:	residual SS =	.0016158				
Iteration 4:	residual SS =	.001615				
Iteration 5:	residual SS =	.001615				
Iteration 6:	residual SS =	.001615				
Iteration 7:	residual SS =	.001615				
Iteration 8:	residual SS =	.001615				
Iteration 9:	residual SS =	.001615				
Source	SS	df	MS			
				Nu	mber of obs =	28
Model	.796840451		802528		squared =	0.9980
Residual	.001614961	12 .00	013458		j R-squared =	0.9953
					ot MSE =	.0116009
Total	.798455413	28 .028	516265	Re	s. dev. =	-193.8376
moment	Coef.	Std. Err.	t	P> t	[95% Conf.	Interval]
/sigalpha	.0683058	.0088071	7.76	0.000	.0491168	.0874948
/sigalpha /rho	.0683058 .3130349	.0088071	7.76 5.14	0.000	.0491168 .1804137	.0874948
/sigalpha /rho /sigv1	.0683058 .3130349 .201089	.0088071 .0608686 .0144893	7.76 5.14 13.88	0.000 0.000 0.000	.0491168 .1804137 .1695194	.0874948 .4456561 .2326586
/sigalpha /rho /sigv1 /sige	.0683058 .3130349 .201089 .0588356	.0088071 .0608686 .0144893 .0375667	7.76 5.14 13.88 1.57	0.000 0.000 0.000 0.143	.0491168 .1804137 .1695194 0230152	.0874948 .4456561 .2326586 .1406864
/sigalpha /rho /sigv1 /sige /12	.0683058 .3130349 .201089 .0588356 1.209775	.0088071 .0608686 .0144893 .0375667 .3380594	7.76 5.14 13.88 1.57 3.58	0.000 0.000 0.000 0.143 0.004	.0491168 .1804137 .1695194 0230152 .4732069	.0874948 .4456561 .2326586 .1406864 1.946343
/sigalpha /rho /sigv1 /sige /12 /13	.0683058 .3130349 .201089 .0588356 1.209775 1.497133	.0088071 .0608686 .0144893 .0375667 .3380594 .496198	7.76 5.14 13.88 1.57 3.58 3.02	0.000 0.000 0.143 0.004 0.011	.0491168 .1804137 .1695194 0230152 .4732069 .4160105	.0874948 .4456561 .2326586 .1406864 1.946343 2.578256
/sigalpha /rho /sigv1 /sige /12 /13 /14	.0683058 .3130349 .201089 .0588356 1.209775 1.497133 1.142064	.0088071 .0608686 .0144893 .0375667 .3380594 .496198 .3835471	7.76 5.14 13.88 1.57 3.58 3.02 2.98	0.000 0.000 0.143 0.004 0.011 0.012	.0491168 .1804137 .1695194 0230152 .4732069 .4160105 .3063868	.0874948 .4456561 .2326586 .1406864 1.946343 2.578256 1.977742
/sigalpha /rho /sigv1 /sige /l2 /l3 /l4 /l5	.0683058 .3130349 .201089 .0588356 1.209775 1.497133 1.142064 1.317238	.0088071 .0608686 .0144893 .0375667 .3380594 .496198 .3835471 .4160183	7.76 5.14 13.88 1.57 3.58 3.02 2.98 3.17	0.000 0.000 0.143 0.004 0.011 0.012 0.008	.0491168 .1804137 .1695194 -0230152 .4732069 .4160105 .3063868 .4108118	.0874948 .4456561 .2326586 .1406864 1.946343 2.578256 1.977742 2.223664
/sigalpha /rho /sigv1 /sige /12 /13 /14	.0683058 .3130349 .201089 .0588356 1.209775 1.497133 1.142064	.0088071 .0608686 .0144893 .0375667 .3380594 .496198 .3835471	7.76 5.14 13.88 1.57 3.58 3.02 2.98	0.000 0.000 0.143 0.004 0.011 0.012	.0491168 .1804137 .1695194 0230152 .4732069 .4160105 .3063868	.0874948 .4456561 .2326586 .1406864 1.946343 2.578256 1.977742
/sigalpha /rho /sigv1 /sige /12 /13 /14 /15 /16 /17	.0683058 .3130349 .201089 .0588356 1.209775 1.497133 1.142064 1.317238 1.438042	.0088071 .0608686 .0144893 .0375667 .3380594 .496198 .3835471 .4160183 .464473	7.76 5.14 13.88 1.57 3.58 3.02 2.98 3.17 3.10	0.000 0.000 0.143 0.004 0.011 0.012 0.008 0.009	.0491168 .1804137 .1695194 -0230152 .4732069 .4160105 .3063868 .4108118 .4260424	.0874948 .4456561 .2326586 .1406864 1.946343 2.578256 1.977742 2.223664 2.450042
/sigalpha /rho /sigv1 /sige /12 /13 /14 /15 /16	.0683058 .3130349 .201089 .0588356 1.209775 1.497133 1.142064 1.317238 1.438042 1.706241	.0088071 .0608686 .0144893 .0375667 .3380594 .496198 .3835471 .4160183 .464473 .5657103	7.76 5.14 13.88 1.57 3.58 3.02 2.98 3.17 3.10 3.02	0.000 0.000 0.143 0.004 0.011 0.012 0.008 0.009 0.011	.0491168 .1804137 .1695194 -0230152 .4732069 .4160105 .3063868 .4108118 .4260424 .4736643	.0874948 .4456561 .2326586 .1406864 1.946343 2.578256 1.977742 2.223664 2.450042 2.938818
/sigalpha /rho /sigv1 /sige /12 /13 /14 /15 /16 /17 /p2	.0683058 .3130349 .201089 .0588356 1.209775 1.497133 1.142064 1.317238 1.438042 1.706241 .9159306	.0088071 .0608686 .0144893 .0375667 .3380594 .496198 .3835471 .4160183 .464473 .5657103 .0803285	$\begin{array}{c} 7.76\\ 5.14\\ 13.88\\ 1.57\\ 3.58\\ 3.02\\ 2.98\\ 3.17\\ 3.10\\ 3.02\\ 11.40 \end{array}$	0.000 0.000 0.143 0.004 0.011 0.012 0.008 0.009 0.011 0.000	.0491168 .1804137 .1695194 -0230152 .4732069 .4160105 .3063868 .4108118 .4260424 .4736643 .7409098	.0874948 .4456561 .2326586 .1406864 1.946343 2.578256 1.977742 2.223664 2.450042 2.938818 1.090951
/sigalpha /rho /sigv1 /sige /12 /13 /14 /15 /16 /17 /p2 /p3	.0683058 .3130349 .201089 .0588356 1.209775 1.497133 1.142064 1.317238 1.438042 1.706241 .9159306 1.112308	$\begin{array}{c} .0088071\\ .0608686\\ .0144893\\ .0375667\\ .3380594\\ .496198\\ .3835471\\ .4160183\\ .464473\\ .5657103\\ .0803285\\ .0998032 \end{array}$	$\begin{array}{c} 7.76\\ 5.14\\ 13.88\\ 1.57\\ 3.58\\ 3.02\\ 2.98\\ 3.17\\ 3.10\\ 3.02\\ 11.40\\ 11.15\end{array}$	0.000 0.000 0.143 0.004 0.011 0.012 0.008 0.009 0.011 0.000 0.000	.0491168 .1804137 .1695194 0230152 .4732069 .4160105 .3063868 .4108118 .4260424 .4736643 .7409098 .894856	.0874948 .4456561 .2326586 .1406864 1.946343 2.578256 1.977742 2.223664 2.450042 2.938818 1.090951 1.329761
/sigalpha /rho /sigv1 /sige /12 /13 /14 /15 /16 /17 /p2 /p3 /p4	.0683058 .3130349 .201089 .0588356 1.209775 1.497133 1.142064 1.317238 1.438042 1.706241 .9159306 1.112308 1.307378	$\begin{array}{c} .0088071\\ .0608686\\ .0144893\\ .0375667\\ .3380594\\ .496198\\ .3835471\\ .4160183\\ .464473\\ .5657103\\ .0803285\\ .0998032\\ .1214745 \end{array}$	$\begin{array}{c} 7.76\\ 5.14\\ 13.88\\ 1.57\\ 3.58\\ 3.02\\ 2.98\\ 3.17\\ 3.10\\ 3.02\\ 11.40\\ 11.15\\ 10.76\end{array}$	$\begin{array}{c} 0.000\\ 0.000\\ 0.000\\ 0.143\\ 0.004\\ 0.011\\ 0.012\\ 0.008\\ 0.009\\ 0.011\\ 0.000\\ 0.000\\ 0.000\\ 0.000\\ \end{array}$.0491168 .1804137 .1695194 0230152 .4732069 .4160105 .3063868 .4108118 .4260424 .4736643 .7409098 .894856 1.042708	.0874948 .4456561 .2326586 .1406864 1.946343 2.578256 1.977742 2.223664 2.450042 2.938818 1.090951 1.329761 1.572048

Estimated parameters with corrected standard errors below

	Coef.	Std. Err.	z	P> z	[95% Conf.	Interval]
sigalpha	.0683058	.024722	2.76	0.006	.0198516	.1167599
rho	.3130349	.077011	4.06	0.000	.1620961	.4639736
sigv1	.201089	.0402344	5.00	0.000	.122231	.279947
sige	.0588356	.0456725	1.29	0.198	0306809	.1483521
12	1.209775	.4187484	2.89	0.004	.3890433	2.030507
13	1.497133	.6406559	2.34	0.019	.2414706	2.752796
14	1.142064	.4497494	2.54	0.011	.2605716	2.023557
15	1.317238	.5380952	2.45	0.014	.2625906	2.371885
16	1.438042	.5672408	2.54	0.011	.3262706	2.549814
17	1.706241	.6850775	2.49	0.013	.3635139	3.048968
p2	.9159306	.1452523	6.31	0.000	.6312413	1.20062
- p3	1.112308	.2170894	5.12	0.000	.6868211	1.537796
- p4	1.307378	.237724	5.50	0.000	.8414475	1.773308
- p5	1.449588	.2657075	5.46	0.000	.9288111	1.970366
- p6	1.466273	.270121	5.43	0.000	.9368458	1.995701
p7	1.470464	.3122724	4.71	0.000	.8584214	2.082507

The output header includes the standard information on goodness of fit produced by the **nl** command. This includes the residual sum of squares (unadjusted) which, as noted earlier, is sometimes used as a measure of fit in the empirical literature.

The parameters sigalpha, rho, sigv1, sige, 12–17, and p2–p7 correspond to σ_{α}^2 , ρ , $\sigma_{\nu 1}^2$, σ_{ϵ}^2 , $\lambda_2 - \lambda_7$, and $p_2 - p_7$. The result for rho indicates moderate persistence in the transitory shock; the factor loadings 12–17 and p2–p7 indicate rising transitory and permanent variances over time.

The standard errors reported in the top panel of the output are incorrect: they fail to take into account the number of individuals used when calculating the sample moments. Correctly adjusted standard errors are reported in the bottom panel. The corresponding adjusted variance-covariance matrix is saved in e(V).

Hypothesis tests can be carried out using test after running the gmmcovearn command. For example, a test that the permanent factor loadings, p_t , are constant over time can be carried out using a Wald test, as follows:

In this example, we reject constant permanent factor loadings at the 6% significance level.

To illustrate the saved results from gmmcovearn, we use ereturn list.

As noted above, gmmcovearn begins by computing the earnings covariances from the raw data. Because we have only one cohort in this dataset, there is only one sample covariance matrix saved in e(moment1).

. m	atrix list	e(moment1)					
sym	metric e(mo	ment1)[7,7]					
	81	82	83	84	85	86	87
81	.26913726						
82	.14437909	.17059092					
83	.08859929	.11214142	.23541845				
84	.12305372	.09880327	.13358462	.20301206			
85	.09517703	.08657077	.13002885	.15184436	.25630733		
86	.10260867	.09372365	.11757618	.13149989	.1893622	.28087588	
87	.0913199	.09370207	.11586105	.13049657	.16674991	.19970039	.33706979

The number of observations reported in the header of the output table for gmmcovearn refers to the number of moment conditions used in the analysis and not the number of individuals used in the estimation. The number of individuals contributing to each of the sample moments is saved in e(nobs1).

These sample sizes are used in the calculation of the corrected variance-covariance matrix for our parameter estimates, which are saved in e(V).

The components of the variance decomposition are saved in e(perm1), which contains the permanent component of inequality, and e(temp1), which is the transitory component. To recover the permanent components, we simply type

```
. matrix list e(perm1)
e(perm1)[7,1]
perm
cohort1year1 .06830577
cohort1year2 .05730368
cohort1year3 .08450995
cohort1year4 .11675074
cohort1year5 .114353134
cohort1year5 .11465449
cohort1year7 .114769514
```

The results reported above indicate a steady increase in permanent inequality over this time period.

5.2 German earnings data

To illustrate the use of gmmcovearn for a more complicated model that includes cohort effects, we use a data extract from the eight waves of the European Community Household Panel for Germany. The years covered by the survey are 1994–2001. As in the previous example, the data are unbalanced. The earnings variable for each year is the residual from a first-stage regression of earnings on potential experience and potential experience squared. The earnings variables are yi1994-yi2001 and the individual experience variables are potexp1994-potexp2001. In the dataset, the cohort indicator variable is birthcoh, which takes the values 1–4 corresponding to the four cohorts in the data. The number of individuals in each cohort is given below.

. tab birthco	oh		
birthcoh	Freq.	Percent	Cum.
1 2 3 4	1,650 1,109 1,547 2,453	24.41 16.41 22.89 36.29	24.41 40.82 63.71 100.00
Total	6,759	100.00	

The following shows the estimates of a random growth model with cohorts and an AR(1) specification for the transitory error term.

```
. gmmcovearn yi, yearn(8) modeln(3) cohortn(4) expvar(potexp) firstyr(1994)
> cohortvar(birthcoh)
(obs = 144)
Iteration 0: residual SS = .4483834
Iteration 1: residual SS = .0186083
Iteration 2: residual SS = .0086681
Iteration 3: residual SS = .0070441
Iteration 4: residual SS = .0070062
Iteration 5: residual SS = .0070061
Iteration 6: residual SS = .0070061
Iteration 7: residual SS = .0070061
Iteration 8: residual SS = .0070061
```

Source	SS	df		MS			
Model Residual	2.16612446 .007006143	26 118		312479 059374	R-s Adj	nber of obs = squared = j R-squared = ot MSE =	0.9968 0.9961
Total	2.17313061	144	.015	091185			-1021.378
moment	Coef.	Std.	Err.	t	P> t	[95% Conf.	Interval]
/sigalpha	.4386704	.0605	5217	7.25	0.000	.318821	.5585198
/rho	.3417541	.0361	788	9.45	0.000	.2701102	.4133979
/sigv1	.0742456	.0062	2163	11.94	0.000	.0619355	.0865556
/sige	.0301694	.0112	2735	2.68	0.009	.0078448	.0524939
/12	1.498371	.2427	683	6.17	0.000	1.017624	1.979118
/13	1.312379	.2569	202	5.11	0.000	.8036072	1.821151
(output omit	ted)						
/sigbeta	.0003872	.0000	489	7.92	0.000	.0002904	.0004841
/covalphab~a	012158	.001	729	-7.03	0.000	0155819	0087341

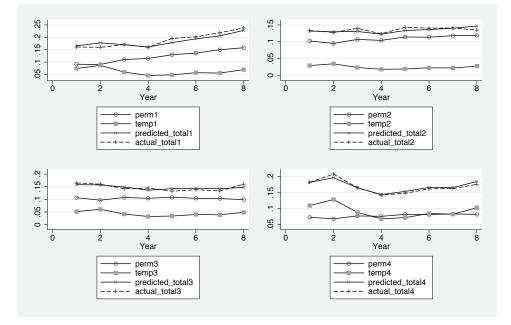
Parameters	with	corrected	standard	errors	below
------------	------	-----------	----------	--------	-------

	Coef.	Std. Err.	z	P> z	[95% Conf.	Interval
sigalpha	.4386704	.1706614	2.57	0.010	.1041801	.7731606
rho	.3417541	.0361377	9.46	0.000	.2709254	.4125827
sigv1	.0742456	.0118533	6.26	0.000	.0510135	.097477
sige	.0301694	.0104324	2.89	0.004	.0097223	.050616
12	1.498371	.2208264	6.79	0.000	1.065559	1.93118
13	1.312379	.2312379	5.68	0.000	.8591613	1.76559
14	1.162531	.1944341	5.98	0.000	.7814474	1.54361
15	1.193413	.1942618	6.14	0.000	.8126667	1.57415
16	1.297056	.2167269	5.98	0.000	.8722795	1.72183
17	1.279366	.2119609	6.04	0.000	.8639299	1.69480
18	1.428739	.2584156	5.53	0.000	.9222535	1.93522
p2	.9842998	.0313717	31.38	0.000	.9228123	1.04578
p3	1.07534	.0433522	24.80	0.000	.9903711	1.16030
- p4	1.08418	.050535	21.45	0.000	.9851328	1.18322
- p5	1.148706	.0694794	16.53	0.000	1.012529	1.28488
 p6	1.168077	.0789378	14.80	0.000	1.013362	1.32279
p7	1.205651	.0887034	13.59	0.000	1.031796	1.37950
- p8	1.219382	.0922591	13.22	0.000	1.038558	1.40020
- q2	.989772	.111584	8.87	0.000	.7710713	1.20847
q3	.7322496	.1311763	5.58	0.000	.4751487	.989350
q4	.4866869	.1032265	4.71	0.000	.2843667	.68900
s2	.6293472	.0628975	10.01	0.000	.5060703	.752624
s3	.8336288	.0684399	12.18	0.000	.6994891	.967768
s4	1.214287	.0917179	13.24	0.000	1.034523	1.39405
sigbeta	.0003872	.0001758	2.20	0.028	.0000426	.000731
covalphabeta	012158	.005614	-2.17	0.030	0231613	001154

The parameters sigalpha to covalphabeta correspond to σ_{α}^2 , ρ , $\sigma_{\nu 1}^2$, σ_{ϵ}^2 , $\lambda_2 \dots \lambda_8$, $p_2 \dots p_8$, $q_2 \dots q_4$, $s_2 \dots s_4$, σ_{β}^2 , and $\sigma_{\alpha\beta}$. The cohort loadings show that the youngest cohort (birthcoh = 4) has the lowest permanent variance and the highest transitory variance. The variance of the random growth parameter, sigbeta, is significant, which supports the heterogeneous growth-profiles model. The covariance between the random

effect and the random growth parameter, covalphabeta, is negative and significant, indicating that those with lower initial earnings have higher earnings growth rates.

Adding graph(1) as an option in the above command returns a graph of the predicted transitory and permanent components of inequality [calculated using (4) and (5)] as well as the predicted and actual aggregate inequality, as shown below.



The data for the predicted permanent and transitory components used in these graphs are saved in e(perm1)-e(perm4) and e(temp1)-e(temp4).

Because researchers in this area sometimes report convergence problems, it is good practice for users to experiment with a range of starting values by using the **stvalue()** option to check the robustness of the reported parameter estimates. In the following example, we use a starting value of ρ of 0.9, which is far away from our previous parameter estimate. In this example, the model quickly converges to point estimates that are practically identical to those reported earlier.

```
gmmcovearn yi, yearn(8) modeln(3) cohortn(4) firstyr(1994) cohortvar(birthcoh)
> expvar(potexp)
(obs = 144)
Iteration 0:
           residual SS =
                        5.183927
Iteration 1:
           residual SS =
                        .6704438
           residual SS =
Iteration 2:
                        .3053995
           residual SS =
Iteration 3:
                        .0147202
Iteration 4: residual SS =
                        .0079693
Iteration 5: residual SS =
                        .0070087
Iteration 6: residual SS =
                        .0070062
```

Iteration 7: Iteration 8: Iteration 9: Iteration 10:	residual SS = residual SS = residual SS = residual SS	.0070061	L			
Source	SS	df	MS			
Model Residual	2.16612446 .007006143 2.17313061	118 .000	3312479 0059374 5091185	R- Ad Ro	mber of obs = squared = j R-squared = ot MSE = s. dev. =	0.9968 0.9961
Total	2.17313061	144 .015	091185	ке	s. dev. =	-1021.378
moment	Coef.	Std. Err.	t	P> t	[95% Conf.	Interval]
/sigalpha	.4386703	.0605215	7.25	0.000	.3188213	.5585194
/rho	.3417535	.0361787	9.45	0.000	.2701098	.4133973
/sigv1	.0742456	.0062163	11.94	0.000	.0619356	.0865557
/sige	.0301693	.0112735	2.68	0.009	.0078446	.052494
/12	1.498373	.2427693	6.17	0.000	1.017624	1.979122
/13	1.312381	.2569215	5.11	0.000	.8036066	1.821156
(output omit	ted)					
/sigbeta	.0003872	.0000489	7.92	0.000	.0002904	.0004841
/covalphab~a	012158	.001729	-7.03	0.000	0155819	0087341

Parameters	with	corrected	standard	errors	below	

	Coef.	Std. Err.	z	P> z	[95% Conf.	Interval]
sigalpha	.4386703	.1706613	2.57	0.010	.1041804	.7731602
rho	.3417535	.0361377	9.46	0.000	.270925	.412582
sigv1	.0742456	.0118533	6.26	0.000	.0510136	.0974777
sige	.0301693	.0104324	2.89	0.004	.0097221	.0506165
12	1.498373	.220828	6.79	0.000	1.065558	1.931188
13	1.312381	.2312394	5.68	0.000	.8591603	1.765602
14	1.162533	.1944351	5.98	0.000	.7814469	1.543618
15	1.193415	.1942636	6.14	0.000	.8126652	1.574165
16	1.297058	.2167281	5.98	0.000	.8722788	1.721837
17	1.279368	.2119622	6.04	0.000	.8639294	1.694806
18	1.428741	.2584167	5.53	0.000	.9222536	1.935228
p2	.9842999	.0313717	31.38	0.000	.9228124	1.045787
р3	1.07534	.0433522	24.80	0.000	.9903712	1.160309
p4	1.08418	.050535	21.45	0.000	.9851329	1.183226
p5	1.148706	.0694793	16.53	0.000	1.012529	1.284883
p6	1.168077	.0789377	14.80	0.000	1.013362	1.322793
p7	1.205651	.0887032	13.59	0.000	1.031796	1.379506
p8	1.219382	.092259	13.22	0.000	1.038558	1.400207
q2	.9897719	.111584	8.87	0.000	.7710713	1.208473
q3	.7322497	.1311767	5.58	0.000	.4751481	.9893513
q4	.4866869	.1032268	4.71	0.000	.2843661	.6890078
s2	.6293474	.0628975	10.01	0.000	.5060705	.7526243
s3	.8336278	.0684398	12.18	0.000	.6994883	.9677673
s4	1.214286	.0917179	13.24	0.000	1.034522	1.39405
sigbeta	.0003872	.0001758	2.20	0.028	.0000426	.0007319
covalphabeta	012158	.005614	-2.17	0.030	0231613	0011547

6 Conclusion

Models of the earnings covariance structure are widely used in both labor economics and macroeconomics. The most common approach to fitting these models entails the use of GMM. However, there is no routine that allows for easy and fast estimation of these models. gmmcovearn is a user-written Stata program that is designed to meet this need.

The program first computes earnings variances and covariances from the raw data. These sample moments, combined with appropriate population expressions are then used with Stata's nl command to recover the parameter estimates. The program uses these parameter estimates to decompose aggregate inequality into its permanent and transitory components and provides a graphical display of this decomposition.

The program has a number of attractive features. First, it is not written for one specific data structure; it allows for balanced and unbalanced data and has flexibility with respect to the number of time periods and the number of cohorts. In addition, a wide range of models can be fit, covering the majority of those used in the empirical literature. Moreover, it calculates standard errors that are correct for the given data structure. The program also allows the user to easily experiment with alternative starting values, which may be important in practice. These features combine to facilitate easy and fast estimation of earnings covariance models.

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