Patent Breadth as an Entry Deterrent:

The Case of Vertically Differentiated Product Innovations

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AAEA Annual Meeting, Long Beach, July 2002

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1. Introduction

Patents provide very important incentives for innovative activity by enabling innovators to appropriate innovation rents through the granting of exclusive rights on their innovations. The limit of these exclusive rights is defined by two elements – patent length and patent breadth. Patent length is the time period during which the innovator has exclusive rights on the innovation and is predetermined by law. Patent breadth defines the technological territory claimed and protected by the patent – the area in the technological space within which competitors cannot offer rival innovations without infringing the patent – and is explicitly chosen by the innovator.

A standard assumption in the economics literature is that an innovator tries to maximize the rents appropriated by his innovation by choosing to claim the maximum patent breadth, thereby deterring the entry of other firms and thus enabling the innovator to earn monopoly rents (Gilbert and Shapiro 1990). Such a strategy, however, fails to recognize that patents are often challenged legally in the Patent Office or in the courts (Cornish 1989, Merges and Nelson 1990). The nature of this challenge is such that the broader is the patent protection, the higher is the probability that the patent will be challenged legally by competitors, that it will overlap another patent and/or that the courts will invalidate it or narrow its scope (Lerner 1994). Given that patent breadth is routinely challenged, the question arises as to whether the innovator is able to choose a patent breadth that deters entry, or whether the innovator is forced to share the market with a new entrant.

The purpose of this paper is to examine the optimal patent breadth strategy that an innovator should employ when faced with the possibility that the patent breadth claimed will be challenged. In this paper, the optimal patent strategy is determined in a sequential game of complete information. The agents in the game are an innovator who seeks patent protection and decides on the patent breadth claimed and a potential entrant who decides on whether to enter the patentee's market and, if entry occurs, where to locate in the vertically differentiated product space. The solution to this game is obtained by backward induction – the problem of the entrant is examined first, followed by the problem of the innovator.

The paper shows that that it is possible under some conditions for an innovator to use patent breadth to deter entry – when this is possible, the optimal patent strategy is to always deter entry. These conditions occur under certain combinations of the entrant's R&D effectiveness and trial cost values (i.e., low R&D effectiveness – which results in high R&D costs – and high trial costs). When these specific conditions do not hold, the optimal strategy for the innovator is to allow a new competitor to enter the market. When allowing entry, the innovator chooses patent breadth so that the benefits of increased product differentiation that result from greater patent breadth are traded off with the increased likelihood of patent challenge that comes with greater patent breadth. One of the conclusions of the paper is that the innovator will only choose the maximum patent breadth when patent infringement is never an optimal strategy for the entrant. This occurs under a very specific set of conditions (i.e., a combination of very high R&D effectiveness and high trial costs values).

The rest of the paper is organized as follows. Section two describes the theoretical development of the strategic patent breadth model; it describes the market conditions, defines patent breadth and models the choice of patent breadth as a sequential game of complete information. Section three provides the analytical solution of the model. Section four concludes the paper.

2. Strategic Patent Breadth Model

The patenting process is modeled as a sequential game of complete information. The agents involved in the game are an incumbent/patentee who applies for a patent and decides on the breadth of patent protection claimed, and a potential entrant who decides where to locate in the product space and who potentially competes with the incumbent in the market. It is assumed that the incumbent has invented a product that meets the patentability requirements and that the regulator (i.e. Patent Office) always grants the patent as claimed; thus the regulator is not explicitly modeled. The latter assumption is made to reflect the case under which the innovator has no help from the Patent Office in determining the breadth of patent protection. The assumption that the Patent Office grants the patent as claimed is a realistic assumption for drastic innovations.

The game consists of three stages. At the first stage of the game the incumbent, having invented a drastic product which will allow him to exert monopoly power in the market and having decided that he wants to patent it, determines the breadth (b) of protection that will be claimed. In the second stage an entrant observes the patentee's product and the breadth of protection granted to it and chooses whether to enter the market or not. If the entrant does not enter, the patentee operates as a monopolist in the third stage of the game. If the entrant decides to enter she does so by choosing the characteristics of her product - i.e., her location in the product space. Once the entrant enters, two cases may prevail in the third stage of the game. If the entrant does not locate within the patentee's claims or if she locates within the patentee's claims and infringement is not found then both the patentee and the entrant market their products and compete in prices. However, if the entrant locates within the patentee's claims and infringement is found then the entrant is not allowed to market her product and the patentee operates as a monopolist at the third stage of the game. It is assumed that both the patentee and the entrant are rational and foresighted. Thus, they both fully anticipate the reaction of their opponent to each of their actions.

In this model, the patentee determines the breadth of protection that will allow the maximum appropriation of innovation rents. The patentee acts strategically taking into consideration the entrant's responses to his choice of patent breadth. He is aware that the probability of the patent being attacked and/or invalidated increases with the breadth of protection and that a broad patent could impede his ability to enforce his rights if the patent is infringed and/or if its validity is directly challenged. In addition, the innovator does not rely on the Patent Office to structure his claims. He is aware of the inefficiencies in the determination of patent breadth in the Patent Office and of the fact that his effort to safeguard his technological territory does not usually conclude with the granting of the patent.

The incumbent's decisions to invent (i.e. his choice of the innovation's specifications) and to patent are not examined – these decisions are treated as exogenous to this game. The only decision that the incumbent has to make involves the breadth of protection that will be claimed – the length of protection is predetermined and is the same for all patents. In addition, it is assumed that the patentee and the entrant produce only one product each and that the entrant does not patent her product since further entry is not anticipated. It is also assumed that the production process is deterministic; once the entrant chooses a location she can produce the chosen product with certainty. Finally, it is assumed that there is no time lag between making and realizing a decision.

2.1 Market conditions and determination of patent breadth

The market in which the patentee and the entrant operate is characterized by the following conditions. The market can only support two products. Every product i (i=1, 2) has an inherent quality represented by the parameter q_i. The quality parameter q_i takes values in the interval $[\underline{q}, \overline{q}]$ where $\underline{q}$ corresponds to the lowest quality that can be allowed in the market while $\overline{q}$ corresponds to the highest quality that is technologically feasible. All consumers agree that higher quality is preferred to lower quality. It is also assumed that every product can be completely described by its quality alone (i.e., only one variety of the product can be produced).

The market consists of differentiated consumers uniformly distributed in the interval [0, 1], each buying one unit of either product 1 or 2 but not both. The consumers differ with respect to some attribute λ uniformly distributed with unity density $f(\lambda)=1$ in the interval $\lambda \in [0, 1]$. The attribute λ determines a consumer's willingness to pay for quality. Consumers differ in their willingness to pay due to differences in income, age, education and/or other characteristics. The utility function for the consumption of product i is given by:

$$U_i = V + \lambda q_i - p_i \qquad (i=1,2) \qquad (1)$$

where p_i is the product price and V is a base level of utility. Product i will be consumed as long as

$U_i \geq 0$ and $U_i > U_j$. It is assumed that V is large enough for $V \geq p_i \; \forall i = 1,2$ to hold true so that the market is always served by at least one product.

It is assumed that the patentee's product is product 1 of quality q_1 and the entrant's product is product 2 of quality q_2. It is also assumed that the entrant enters with a better quality product, such that $q_2 = q_1 + \varepsilon$ where $\varepsilon \in (0, (\bar{q} - q_1)]$. The parameter ε represents the difference between the entrant's and the patentee's quality or equivalently the distance away from the patentee's product that the entrant locates in the quality space. The consumer who is indifferent between products 1 and 2 has a λ value denoted by $\hat{\lambda}$ and given by:

$$U_1 = U_2 \Rightarrow \hat{\lambda} = \frac{(p_2 - p_1)}{(q_2 - q_1)} = \frac{(p_2 - p_1)}{\varepsilon} \quad (2)$$

The marginal consumer $\hat{\lambda}$ determines the market shares of products q_1 and q_2 as depicted in Figure 1.

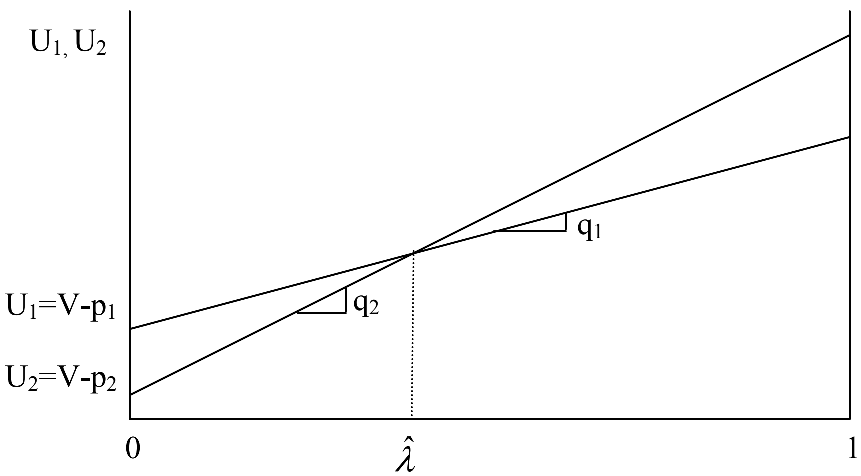

Figure 1. Market Shares of Product 1 of Quality q_1 and Product 2 of Quality q_2

Consumers with a λ value such that $\lambda \in [0, \hat{\lambda}]$ buy the patentee's product (the lower quality product) while consumers with a λ value such that $\lambda \in (\hat{\lambda}, 1]$ buy the entrant's product (the higher quality product). Given that every consumer buys only one unit of the product of his choice, the demand the patentee and the entrant face for their products are given by $y_1 = \hat{\lambda}$ and $y_2 = 1 - \hat{\lambda}$, respectively.

The same production cost structure is assumed for both the patentee and the entrant. The per unit production costs are assumed to be independent of the level of quality and are equal to zero. The production of product 2 of a given quality q_2 requires the incurring of fixed sunk costs (R&D costs), however. These fixed costs, denoted by $F_R(q)$, are an increasing function of quality: $F_R = \beta \frac{q^2}{2}$ where $\beta \geq \frac{4}{9}$. The parameter β represents the effectiveness of the R&D process. Low β values represent high R&D effectiveness and imply low R&D costs while high β values represent low R&D effectiveness and imply high R&D costs.

An important assumption in the model is that reverse engineering is possible and costless, which, in the absence of protection, enables the entrant to reproduce the patented product without incurring the R&D costs. Costless reverse engineering implies that $F_R(q_1) = 0$ for the entrant. Since $q_2=q_1+\varepsilon$ the R&D costs incurred by the entrant for the production of q_2 are a function of ε, namely: $F_R = \beta\frac{\varepsilon^2}{2}$. Given the above, it is increasingly costly for the entrant to locate away from the patentee (i.e., to produce the better quality product) in the quality product space. As will be shown below, the more costly it is for the entrant to produce product 2, the smaller is the degree of differentiation between her product and the patentee's product.

The patentee and the entrant compete in the one-dimensional product space presented in Figure 2 where quality is depicted on the vertical axes. In this product space point A represents product 1 of quality q_1. We denote the breadth of protection claimed and granted to product 1 when it is patented by the variable b. The breadth of patent protection b takes values in the interval $b\in[0, \bar{q}-q_1]$. To normalize the model, we assume that $d = \bar{q} - q_1 = 1$ which implies that $b\in[0, 1]$ and $\varepsilon\in(0, 1]$. When b takes its minimum value of zero the protected area is just a point in the product space, point A. This case represents the minimum breadth of protection granted by the patent, namely zero breadth. With zero breadth of protection, the entrant can locate anywhere in the product space except at point A. Thus, zero breadth protects the patented product only against duplication.

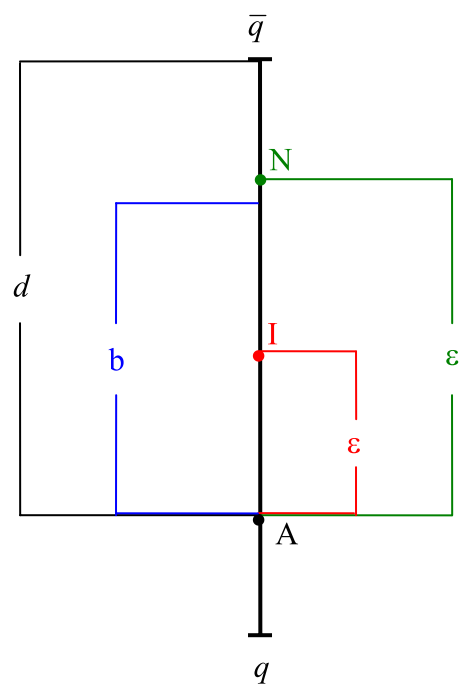

Figure 2. The Product Space and the Breadth of Patent Protection

After observing the quality of the patentee's product (q_1) and the breadth of the patent (b) protection granted to it, the entrant chooses whether or not to enter in the patentee's market. If the entrant decides to enter she has two choices: to locate inside (ε < b – point I in Figure 2) or outside (ε ≥ b – point N in Figure 2) the patentee's claims. The first choice corresponds to a decision to infringe the patent, while the second corresponds to a decision not to infringe the patent. It is assumed that when the entrant locates at a distance ε<b away from q_1, a trial always takes place, either because the patentee files an infringement lawsuit or because the entrant directly challenges the validity of the patent. It is further assumed that the filing of an infringement lawsuit is always met with a counterclaim by the accused

infringer that the patent is invalid.[1] The costs incurred during the infringement trial/validity attack by the patentee and the entrant are denoted by C_P and C_E respectively. These costs are assumed to be independent of the breadth of protection and of the entrant's location. The trial costs will only be incurred if ε<b and they are assumed to be sunk – once made they cannot be recovered by either party.[2]

If the entrant locates within the patentee's claims the patent may not always be found to be valid. Indeed, the greater is the breadth of the patent, the smaller is the probability that the patent will be found to be valid or equivalently that infringement will be found. This assumption follows in part because the broader is the protection claimed, the harder it is to avoid obviousness, to differentiate the innovation from prior art (to show novelty) and to demonstrate that the innovation is enabling. As well, this assumption is in accordance with evidence from the literature that courts tend to uphold narrow patents and invalidate broad ones (Waterson 1990, Cornish 1989, Merges and Nelson 1990). It is also assumed that when the maximum breadth is claimed (b=1), the patent will always be found to be invalid. These assumptions are captured by assuming that the probability that infringement is found, μ, is equal to $\mu = 1 - b$.

The patent system being modeled is assumed to be that of the fencepost type, in which claims define an exact border of protection. Under the fencepost system, infringement will always be found when an entrant locates within the patentee's claims, unless the entrant proves that the patent is invalid (Cornish 1989).[3] In the fencepost system the probability that infringement is found does not depend on how close the entrant has located to the patentee. The implication of assuming a fencepost patent system is that the probability that infringement will be found (given that the entrant has located at $\varepsilon < b$ distance away from q_1) is equal to the probability that the validity of the patent will be upheld. Thus, the fencepost patent system implies that the events that the patent is found to be infringed and that the patent is found to be invalid can be treated as mutually exclusive and exhaustive.

A summary of the formal three stage strategic patent breadth determination game is presented diagrammatically in Figure 3. In Stage one, the patentee chooses patent breadth b. In Stage two, the entrant determines whether to enter in the patentee's market. If the entrant decides not to enter, the patentee makes monopoly profits (Π_m) in Stage three of the game and the entrant makes zero profits (see payoffs at A). If the entrant decides to enter she chooses where to locate in the quality space by choosing the distance ε away from q_1. The choice of ε determines whether a trial occurs. The no trial outcome occurs if the entrant chooses $\varepsilon \geq b$. In this case, at Stage three of the game, the two competitors both produce their respective products and compete in the market by choosing prices. The payoffs for the patentee and the entrant are Π_P^{NI} and Π_E^{NI}, respectively (see payoffs at B). The trial outcome occurs if the entrant chooses $\varepsilon < b$. At trial, there is a probability μ that infringement is found and a probability $1-\mu$ that infringement is not found. If infringement is found during trial, the entrant is not allowed to market her product in Stage three of the game. In this case, at Stage three, the patentee operates as a monopolist while the entrant makes zero profits. If infringement is not found during the trial, then the patentee and the entrant compete by choosing prices. The entrant has no incentive to relocate within the quality space (i.e., the entrant has no incentive to move from point I in Figure 2) as she has already incurred the R&D costs which cannot be recovered. The payoffs for the patentee and the entrant when the entrant chooses $\varepsilon < b$ are $E(\Pi_P^I)$ and $E(\Pi_E^I)$ respectively (see payoffs at C).

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[1] This is a standard defence adopted by accused infringers (Cornish 1989).
[2] With this assumption we exclude the possibility of the court awarding lawyers' fees to either party.
[3] In contrast, a signpost patent system implies that claims provide an indication of protection and the claims are interpreted using the doctrines of equivalents and reverse equivalents.

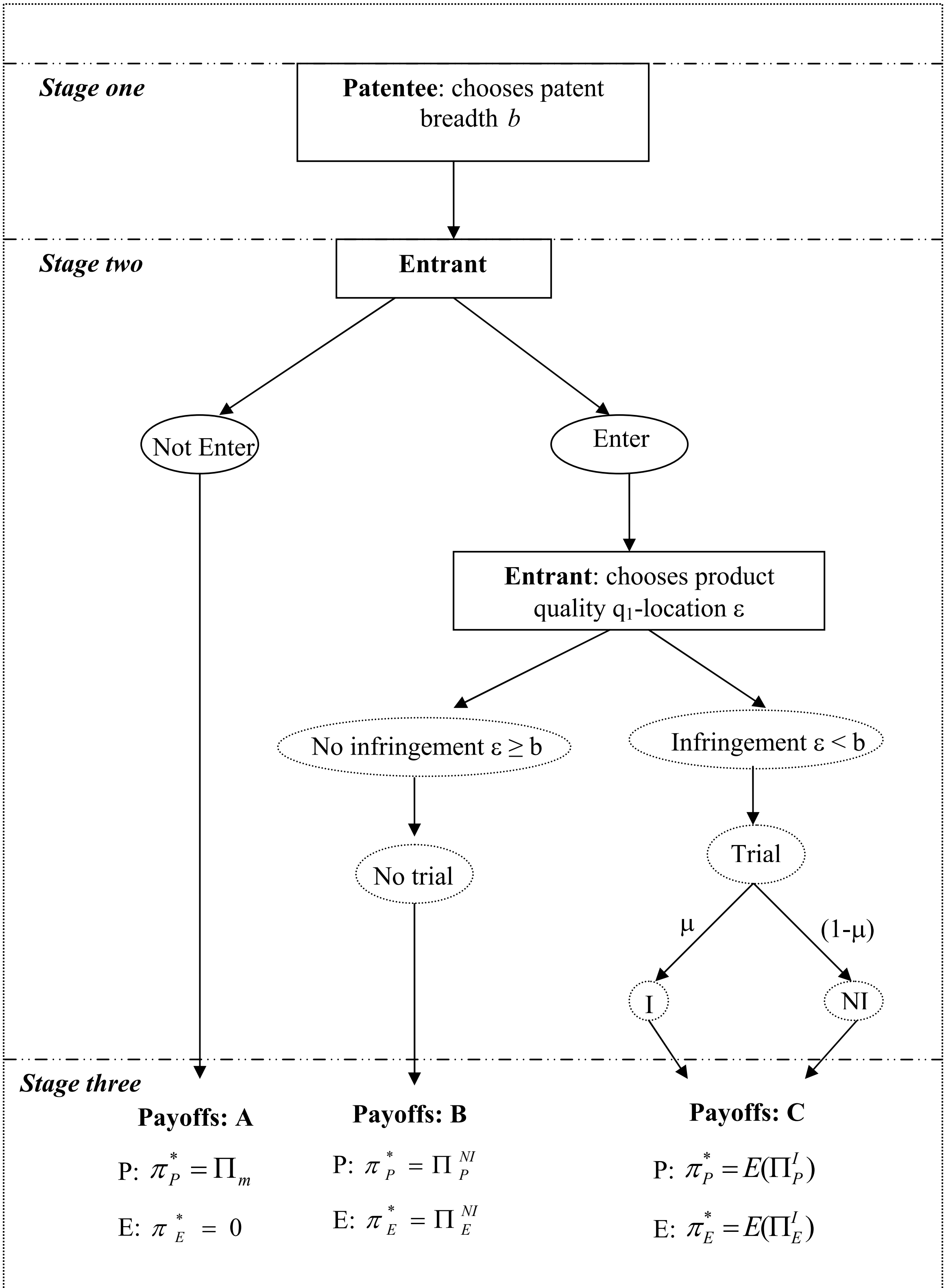

Figure 3. Graphical Representation of the Strategic Patent Breadth Game

3. The Analytical Solution of the Strategic Patent Breadth Game

The sub-game perfect Nash equilibrium of the sequential strategic patenting game is found using backwards induction. The prices that the patentee and the entrant charge for their products when they both operate in the market are determined first. The entrant's quality choice (her location on the quality product space) is derived next and the optimal breadth of patent protection for the patentee is determined last. The use of backwards induction eliminates multiple equilibria that do not represent credible threats and yields the only sub-game perfect Nash equilibrium of the game.

The game is first solved under the assumption of *no patent protection* to determine where it is optimal for the entrant to locate when her choice is not constrained by the breadth of patent protection. This represents the entrant's most preferred location choice which is used as a benchmark for comparison with choices that are available to the entrant when her location choices are constrained by the breadth of patent protection.

3.1 No Patent Protection

- *The Entrant's most preferred location choice*

The entrant's most preferred location is found using backwards induction. Since it is assumed that there is no patent protection there are only two stages in this game. The equilibrium prices of the incumbent's and the entrant's products are determined first. The equilibrium prices determine the Bertrand profits for the two players. The entrant's quality choice, her location on the quality product space is determined last.

The pricing equilibrium. In this stage of the game the quality choice has been made and fixed costs have been sunk. As shown in sub-section 2.1 the demand for the incumbent's and the entrant's product is given by $y_1 = \hat{\lambda}$ and $y_2 = 1 - \hat{\lambda}$, respectively. The incumbent and the entrant choose the prices for their products that maximize their profits, given respectively by:

$$\begin{aligned} &\text{I:} \qquad \max_{p_1} \pi_1^B = p_1 y_1 = p_1 \frac{p_2 - p_1}{\varepsilon} \\ &\text{E:} \qquad \max_{p_2} \pi_2^B = p_2 y_2 = p_2 (1 - \frac{p_2 - p_1}{\varepsilon}) \end{aligned} \tag{3}$$

Optimization of the objective functions in (3) yields the following first order conditions (F.O.C) for a maximum:

$$\begin{aligned} &\frac{\partial \pi_1^B}{\partial p_1} = 0 \Rightarrow p_1^* = \frac{p_2}{2} \\ &\frac{\partial \pi_2^B}{\partial p_2} = 0 \Rightarrow p_2^* = \frac{p_1 + \varepsilon}{2} \end{aligned} \tag{4}$$

Simultaneously solving the equations in (4) yields the equilibrium prices, the quantities supplied and the profits in the final stage of the game, given by:

$$\begin{aligned} &\text{I:} \qquad p_1^* = \frac{\varepsilon}{3},\ y_1^* = \frac{1}{3},\ \pi_1^B = \frac{\varepsilon}{9} \\ &\text{E:} \qquad p_2^* = \frac{2\varepsilon}{3},\ y_2^* = \frac{2}{3},\ \pi_2^B = \frac{4\varepsilon}{9} \end{aligned} \tag{5}$$

Since the entrant has the higher quality product, she charges the higher price. The entrant serves two thirds of the market, while the incumbent serves one third of the market. Profits are increasing in the distance ε between the incumbent's and the entrant's location. The greater is the difference in quality between the two products, the less intense is competition at the final stage of the game and the greater are the profits for both the incumbent and the entrant.[4]

The location choice. The entrant chooses the distance ε away from q_1 that will maximize her profits at this stage of the game. The objective function of the entrant is given by:

$$\text{E:} \qquad \max_{\varepsilon} \Pi_2^0 = \pi_2^B - F_R = \frac{4\varepsilon}{9} - \beta\frac{\varepsilon^2}{2} \tag{6}$$

Optimization of equation (6) yields the following F.O.C. for a maximum:

$$\frac{\partial \Pi_2^0}{\partial \varepsilon} = 0 \Rightarrow \frac{4}{9} - \beta\varepsilon_0 = 0 \Rightarrow \varepsilon_0 = \frac{4}{9\beta} \tag{7}$$

The second order conditions (S.O.C.) for a maximum are satisfied since:

$$\frac{\partial^2 \Pi_2^0}{\partial \varepsilon^2} < 0 \Rightarrow -\beta < 0\,, \qquad \forall\beta \geq \frac{4}{9} \tag{8}$$

The *most preferred* location choice for the entrant is given by ε_0 in equation (7) which holds for $\beta \geq \frac{4}{9}$ since ε∈(0, 1]. The quality of the entrant's product is given by: $q_2 = q_1 + \varepsilon_0 = q_1 + \frac{4}{9\beta}$.

The profits for the incumbent and the entrant under the *no protection* outcome are obtained by substituting equation (7) into their respective profit functions. This substitution yields the following payoffs:

$$\begin{aligned} &\text{I:} \qquad \Pi_1^0 = \frac{\varepsilon_0}{9} = \frac{4}{81\beta} \\ &\text{E:} \qquad \Pi_2^0 = \frac{4\varepsilon_0}{9} - \beta\frac{(\varepsilon_0)^2}{2} = \frac{8}{81\beta} \end{aligned} \tag{9}$$

Proposition 1. *Under no patent protection the less costly it is to produce the better quality product (i.e., the smaller is β), the further away from the incumbent the entrant locates and the greater are profits for both the incumbent and the entrant.*

Proof:

$$\frac{\partial \varepsilon_0}{\partial \beta} = -\frac{4}{9\beta^2} \leq 0 \;\; \forall \beta \geq \frac{4}{9};$$

$$\frac{\partial \Pi_1^0}{\partial \beta} = -\frac{4}{81\beta^2} \leq 0 \, \forall \beta \geq \frac{4}{9};$$

$$\frac{\partial \Pi_2^0}{\partial \beta} = -\frac{8}{81\beta^2} \leq 0 \, \forall \beta \geq \frac{4}{9}. \quad \square$$

When the R&D costs are minimized – this happens when β takes its minimum value $(\beta = \frac{4}{9})$ – the entrant locates at the edge of the market (ε=1) maximizing differentiation between her product and the incumbent's product. The smaller are the R&D costs the greater is the distance ε away from the

[4] This is a well-established result in the product differentiation literature in simultaneous games. When competitors first simultaneously choose their locations in the product space and then compete in prices they choose maximum differentiation to relax competition in the pricing stage that would curtail their profits (Lane 1980, Motta 1993, Shaked and Sutton 1982).

incumbent that the entrant locates and the greater are the profits for both the incumbent and the entrant. Thus, maximum possible product differentiation is desirable by both players.

3.2 Patent Protection b ∈[0,1]

When the incumbent's product is protected by a patent, the entrant's location choices are constrained. Given the assumption of complete information, the patentee knows the entrant's cost structure, her trial costs and can anticipate the entrant's reaction to his choice of patent breadth. Since the entrant's location choice determines the level of the patentee's profits, the patentee chooses the breadth of protection that induces the desired behavior from the entrant.

The patentee knows that there is only one case in which the breadth of the patent does not influence the entrant's location decision. This happens when the entrant's cost structure is such that it is optimal for her to locate at the edge of the market (when $\beta = \frac{4}{9}$ then $\varepsilon_0 = 1$). In this case, irrespective of the breadth of the patent (ε is greater or equal to b for all b∈[0, 1]), the patent is never infringed and profits are maximized for both players. The patentee is free to choose any patent breadth, even the maximum breadth of protection, without triggering the trial outcome and having his patent invalidated.

For any value of $\beta > \frac{4}{9}$, the breadth of the patent may affect the entrant's location decision, which in turn affects the patentee's profits. If the patentee chooses $0 < b \le \varepsilon_0$ it is always optimal for the entrant to enter and to locate at her most preferred location, namely $\varepsilon_0 = \frac{4}{9\beta}$ and no trial will occur. This outcome yields the same payoffs as the *no protection* outcome analyzed above.[5] However, if the patentee chooses $\varepsilon_0 < b \le 1$ the entrant must first decide, depending on the value of patent breadth, whether to enter or not in the patentee's market. If she finds it profitable to enter, she must further decide whether to infringe or not the patentee's product.

A key element in the patentee's decision making is whether there is a value of patent breadth, $\hat{b}$ ∈(ε_0, 1], that can deter market entry. If $\hat{b}$ exists, the patentee can choose this patent breadth and make monopoly profits. This outcome is illustrated in the payoffs at A in Stage Three in Figure 3. It is assumed that the entrant decides not to enter when she is indifferent between entering and not entering the market. Thus, $\hat{b}$ is defined as the patent breadth that makes the expected profits that the entrant realizes when she infringes the patent $(\mathrm{E}(\Pi_E^I))$ and the profits that she realizes when she does not infringe the patent (Π_E^{NI}) less than or equal to zero.

If there is no value of patent breadth that can deter entry in the patentee's market the patentee must find whether there is a value of patent breadth, denoted by $\tilde{b}$ ∈(ε_0, 1], that makes the entrant indifferent between infringing and not infringing the patent. The variable $\tilde{b}$ thus makes the entrant's payoffs at B equal to the payoffs at C in Figure 3. Formally, if $\tilde{b}$ exists it should make Z_E given in equation (10) equal to zero.

$$Z_E = E(\Pi_E^I) - \Pi_E^{NI} \tag{10}$$

If $\hat{b}$ exists the patentee always chooses to deter entry. If $\hat{b}$ does not exist and $\tilde{b}$ exists, the patentee makes a decision of patent breadth by comparing his expected profits when the patent is

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[5] The entrant always finds it optimal to enter since in this case her profits (given by equation (9)) are always positive.

infringed $(\mathrm{E}(\Pi_P^I))$ and his profits when the patent is not infringed (Π_P^{NI}). The difference in the patentee's profits between those two scenarios is denoted by Z_P and is given by:

$$Z_P = E(\Pi_P^I) - \Pi_P^{NI} \tag{11}$$

If $Z_P > 0$ the patentee chooses a patent breadth that induces the entrant to infringe; a patent breadth that makes $Z_E > 0$. If $Z_P \leq 0$ the patentee chooses a patent breadth that results in non-infringement; a patent breadth that makes $Z_E \leq 0$. It is assumed that the entrant chooses not to infringe when she is indifferent between infringing and not infringing the patent.

Since the patentee's profits depend on the entrant's location on the quality product space, the patentee must first solve the entrant's location problem to be able to determine the breadth of protection claimed that maximizes his profits. In other words, the patentee must first determine the values of $\hat{b}$ and $\tilde{b}$, if they exist. Note that both $\hat{b}$ and $\tilde{b}$ are such that, $\hat{b}, \tilde{b} \in (\varepsilon_0, 1]$. As it has been discussed in section 3.2 above, the entrant may find it optimal not to enter or to enter and infringe the patent if and only if $\varepsilon_0 < b \leq 1$; when $b \leq \varepsilon_0$ it is always optimal for the entrant to enter and to locate at her most preferred location ε_0, infringement does not occur and a trial does not take place. To determine the values $\hat{b}$ and $\tilde{b}$ the patentee needs to determine the entrant's expected profits when she infringes the patent and her profits when she does not infringe the patent when $\varepsilon_0 < b \leq 1$. The case where the entrant finds it optimal to infringe the patent is considered first.

3.2.1 The Entrant's location decision when $\varepsilon_0 < b \leq 1$

- *The Entrant's expected profits when she infringes the patent* ($\varepsilon < b$)

When the entrant infringes the patent the trial outcome is triggered. During trial it is determined whether infringement has occurred (or equivalently whether the patent is valid) or whether infringement has not occurred (or equivalently whether the patent is invalid).

<u>The pricing equilibrium</u>. If infringement is found during the trial, the entrant is not allowed to market her product and makes zero profits in the final stage of the game, while the patentee makes monopoly profits:

$$\begin{aligned} &\text{P:} \quad \pi_1^I = \Pi_m \\ &\text{E:} \quad \pi_2^I = 0 \end{aligned} \tag{12}$$

If infringement is not found, the entrant is allowed to remain in the market and to produce the quality of product that she has chosen. In this case, both the entrant and the patentee market their products and compete in prices in the third stage of the game. Their Bertrand profits are determined through the process described in the pricing equilibrium in sub-section 3.1 and are given by:

$$\begin{aligned} &\text{P:} \quad \pi_1^B = \frac{\varepsilon_T}{9} \\ &\text{E:} \quad \pi_2^B = \frac{4\varepsilon_T}{9} \end{aligned} \tag{13}$$

where ε_T is the entrant's optimal location choice under the *trial* outcome.

<u>The location choice</u>. The location of the entrant is determined through the optimization of her expected profits given by:

$$\text{E: } \max_{\varepsilon} \mathrm{E}(\Pi_E^I) = \mu \cdot \pi_2^I + (1-\mu) \cdot \pi_2^B - F_R - C_E = 0 + b \cdot \frac{4\varepsilon}{9} - \beta \frac{\varepsilon^2}{2} - C_E \tag{14}$$

Recall that the probability of the patent being found valid is $\mu = 1 - b$. Optimization of the objective function in equation (12) yields the F.O.C. for a maximum:

$$\frac{\partial E(\Pi_E)^I}{\partial \varepsilon} = 0 \Rightarrow \frac{4}{9}b - \beta\varepsilon_T = 0 \Rightarrow \varepsilon_T = \frac{4}{9\beta}b \tag{15}$$

The S.O.C. for a maximum are satisfied $\forall \beta \geq \frac{4}{9}$:

$$\frac{\partial^2 E(\Pi_E)^I}{\partial \varepsilon^2} < 0 \Rightarrow -\beta < 0 \tag{16}$$

Equation (15) shows that when the entrant infringes the patent she finds it optimal to locate at a distance proportional to the breadth of the patent. Because there is uncertainty with respect to whether the entrant will be able to continue in the market, she 'underlocates'. In order to reduce the R&D costs, which are incurred with certainty, the entrant locates closer to the patentee than she would have done had infringement not been a possibility. Note that when the patentee chooses the maximum patent breadth (b=1) the entrant finds it optimal to locate at her most preferred location, $\varepsilon_T = \varepsilon_0$. This occurs because the entrant knows that she will win at trial with certainty since when b=1 the patent is never found to be valid (i.e., μ=1-b=0).

The entrant's expected profits when she infringes the patent are obtained by substituting the entrant's optimal location under trial from equation (15) into her expected profit function. The substitution yields the following payoffs:

$$\text{E:} \qquad \mathrm{E}(\Pi_E^I) = \frac{8}{81\beta}b^2 - C_E \tag{17}$$

Equation (17) shows that the greater is the breadth of the patent, the greater are the expected profits for the entrant under the trial outcome. This result occurs because the greater is the breadth of the patent, the greater is the probability that infringement will not be found (or equivalently that the patent will be invalidated) and thus, the greater is the probability that the entrant will be able to operate in the market. In addition, the greater are the trial costs that the entrant must incur the smaller are her expected profits when she infringes the patent. Figure 4 depicts the relationship between expected profits under infringement and the breadth of patent protection for different R&D effectiveness and trial cost values.

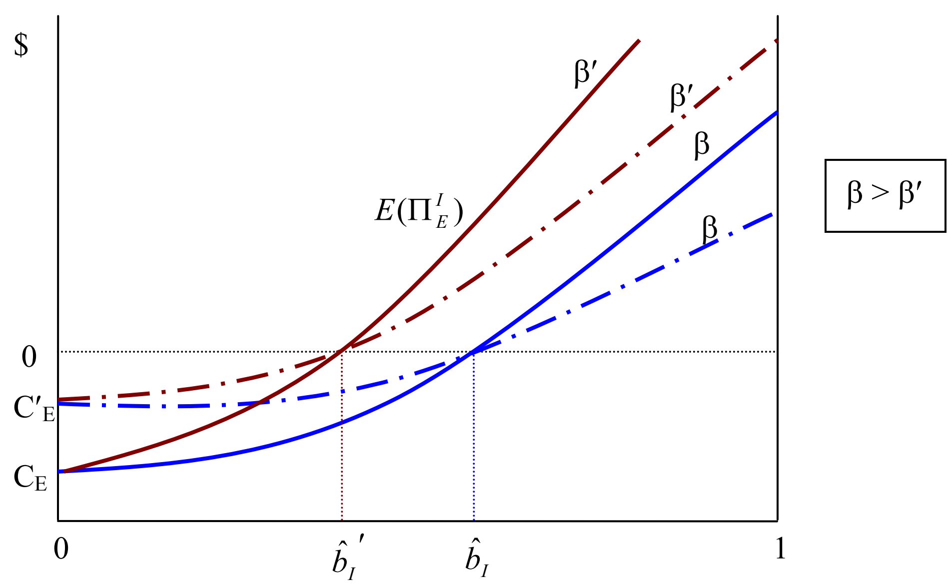


Figure 4. The entrant's Expected Profits Under Infringement

Equation (17) gives one set of conditions under which the entrant can be deterred from entering the market. The entrant will enter into the patentee's market *and* infringe the patent if and only if $\mathrm{E}(\Pi_E^I) > 0$. Thus, the patentee can prevent entry (and subsequently patent infringement) by choosing a patent breadth that makes $\mathrm{E}(\Pi_E^I) \leq 0$, i.e., by choosing a patent breadth that satisfies:

$$b \leq \sqrt{\frac{81\beta C_E}{8}} = \hat{b}_I \tag{18}$$

Thus, $\hat{b}_I$ denotes the breadth of patent protection that makes the entrant indifferent between entering the market *and* infringing the patent on the one hand and not entering on the other hand. Whether $\hat{b}_I \in (\varepsilon_0, 1]$ exists depends on the values of β and C_E.

Equation (18) shows that the greater are the costs of producing the better quality product (i.e., the greater is β) and the entrant's trial costs, the greater is the breadth of the patent that would prevent entry under infringement. These insights are depicted in Figure 4 and the intuition behind it them as follows. The greater are the costs of producing the entrant's product, the closer to the patentee the entrant is forced to locate and the smaller are the profits for the entrant. Similarly, the greater are the trial costs, the less profitable infringement becomes. Under these conditions, infringement is profitable only if patent breadth is relatively large. The greater is patent breadth the larger is the probability that the patent will be invalidated during trial and the greater is the probability that the entrant will be able to operate in the market.

- *The Entrant's profits when she does not infringe the patent ($\varepsilon \geq b$)*

The pricing equilibrium. When the entrant does not infringe the patent both the patentee and the entrant market their products and compete in prices in the final stage of the game. Their profits are determined through the process described in the pricing equilibrium in sub-section 3.1 and are given by:

$$\begin{aligned} &\text{P: } \pi_1^B = \frac{\varepsilon_n}{9} \\ &\text{E: } \pi_2^B = \frac{4\varepsilon_n}{9} \end{aligned} \tag{19}$$

where ε_n is the optimal location choice when the entrant does not infringe the patent.

The location choice. The entrant's optimal location choice under no infringement is determined through the optimization of the profits given by:

$$\begin{aligned} &\text{E: } \max_{\varepsilon} \Pi_E^{NI} = \pi_2^B - F_R = \frac{4}{9}\varepsilon - \frac{\beta}{2}\varepsilon^2 \\ &\qquad s.t. \qquad \varepsilon \geq b \end{aligned} \tag{20}$$

The Lagrangean of the entrant's profit maximization problem is:

$$L = \frac{4}{9}\varepsilon - \frac{\beta}{2}\varepsilon^2 + \lambda(\varepsilon - b)$$

The Kuhn-Tucker conditions for a maximum are:

$$\frac{\partial L}{\partial \varepsilon} \leq 0 \Rightarrow \frac{4}{9} - \beta\varepsilon + \lambda \leq 0, \quad \varepsilon \geq 0, \text{ and } \varepsilon\frac{\partial L}{\partial \varepsilon} = 0$$

$$\frac{\partial L}{\partial \lambda} \geq 0 \Rightarrow \varepsilon - b \geq 0, \quad \lambda \geq 0, \text{ and } \lambda\frac{\partial L}{\partial \lambda} = 0$$

Since $\varepsilon \neq 0 \Rightarrow \dfrac{\partial L}{\partial \varepsilon} = 0 \Rightarrow \dfrac{4}{9} - \beta\varepsilon + \lambda = 0$.

Case 1. If λ=0 then $\varepsilon - b > 0$ and $\frac{\partial L}{\partial \varepsilon} = 0 \Rightarrow \varepsilon_n = \frac{4}{9\beta} = \varepsilon_0$. This solution is rejected since under this case $\varepsilon > b > \varepsilon_0$.

Case 2. If λ>0 then $\varepsilon_n = b$ and $\frac{\partial L}{\partial \varepsilon} = 0 \Rightarrow \lambda = \beta \cdot b - \frac{4}{9}$. The profits for the entrant under this case are given by substituting the solution $\varepsilon_n = b$ into the entrant's profit function. This substitution yields the following profits:

$$\Pi_E^{NI} = \frac{4}{9}b - \frac{\beta}{2}b^2 \tag{21}$$

Equation (21) shows that the greater are the costs of producing the higher quality product (the greater is β) the smaller are the profits for the entrant when she decides not to infringe the patent and locates outside the patentee's claims. Figure 5 depicts the entrant's profits when she does not infringe the patent under different levels of R&D effectiveness.

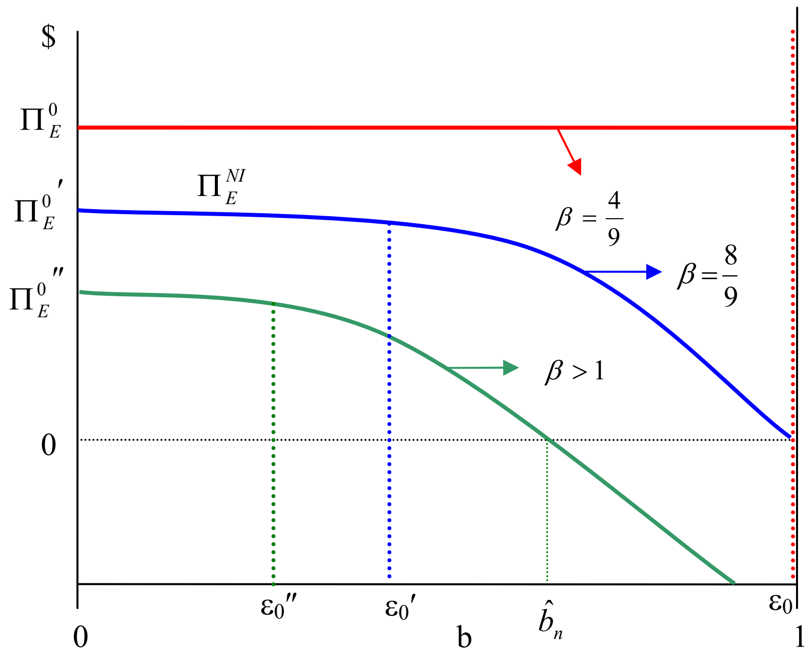


Figure 5. The Entrant's Profits Under No Infringement

Figure 5 shows that the entrant's profits under no infringement are constant for breadth values less than or equal to the entrant's most preferred location ($b \leq \varepsilon_0$) and they are declining for breadth values greater than the entrant's most preferred location ($\varepsilon_0 < b \leq 1$). When patent breadth is less than or equal to the entrant's most preferred location, the entrant always finds it optimal to locate at her most preferred location (ε_0) and makes maximum profits (Π_E^0). However, when patent breadth is greater than the entrant's most preferred location it becomes increasingly costly for the entrant to locate outside the patentee's claims.

The entrant will choose to enter *and not* infringe the patent if and only if $\Pi_E^{NI} > 0$. The patent breadth that makes entry under no infringement non profitable for the entrant ($\Pi_E^{NI} \leq 0$) must satisfy the condition given by:

$$b \geq \frac{8}{9\beta} = \hat{b}_n \tag{22}$$

Thus, $\hat{b}_n$ denotes the breadth of patent protection that makes the entrant indifferent between entering the market *without infringing* the patent on the one hand and not entering on the other hand. Since $\hat{b}_n \in (\varepsilon_0, 1]$ equation (22) implies that $\hat{b}_n$ exists only for β values such that $\beta \geq \frac{8}{9}$.

Equation (22) shows that the greater are the R&D costs (the greater is β) that need to be incurred for the production of the entrant's product, the smaller is the patent breadth that prevents entry under no infringement. This result is depicted in Figure 5 and the intuition behind it is as follows. The entrant must locate outside the patentee's patent breadth for infringement not to occur. If patent breadth is large it may not be profitable for the entrant to locate outside the patentee's claims because it becomes more expensive to produce her product. The greater are the costs of producing the better quality product, the closer to the patentee the entrant is forced to locate and thus the smaller is the patent breadth that makes it unprofitable for the entrant to enter without infringing the patent.

Equations (18) and (22) give the conditions for non-entry under infringement and under no infringement, respectively. The breadth of patent protection that deters entry in the patentee's market, $\hat{b}$, if it exists, must simultaneously satisfy both conditions for non-entry under infringement and under no infringement. Thus, the entry deterrence condition is given by equation (23):

$$\hat{b}_n \leq \hat{b} \leq \hat{b}_I \Rightarrow \frac{8}{9\beta} \leq \hat{b} \leq \sqrt{\frac{81C_E\beta}{8}} \tag{23}$$

Equation (23) shows that patent breadth $\hat{b}$ deters entry if and only if both the entrant's expected profits under infringement and her profits under no infringement are less or equal to zero; for $b = \hat{b}$ $E(\Pi_E^I) \leq 0$ $\wedge$ $\Pi_E^{NI} \leq 0$.

Another important element in the patentee's decision making, besides the existence of the patent breadth $\hat{b}$ that can deter entry, is whether there is a patent breadth $\tilde{b} \in (\varepsilon_0, 1]$ that makes the entrant indifferent between infringing and not infringing the patent. If entry cannot be deterred (i.e., a $\hat{b}$ does not exist), before the entrant enters she must decide whether to infringe or not infringe the patent. As described in section 3.2. patent breadth $\tilde{b}$, if it exists, makes the difference between the entrant's expected profits when she infringes the patent and her profits when she does not infringe the patent (denote by Z_E) equal to zero.

The determination of the entrant's expected profits under infringement and her profits under no infringement allow the patentee to determine the value of Z_E. Substitution of the expressions for the expected profits under infringement and the profits under no infringement given by equations (17) and (21), respectively, into the expression for Z_E, given by equation (10), yields:

$$Z_E = \left(\frac{8}{81\beta} + \frac{\beta}{2}\right)b^2 - \frac{4}{9}b - C_E \tag{24}$$

Equation (24) shows that Z_E is a function of the breadth of the patent (b), the entrant's cost structure (β) and the entrant's trial costs (C_E). The entrant's cost structure and the trial costs are exogenous to the game; these parameters are not affected by the decisions made by the patentee or the entrant. Patent breadth, however, is determined by the patentee. Thus, the breadth of patent protection claimed can determine whether the entrant will find it profitable to infringe or not infringe the patent.

Proposition 2. *When the entrant finds it optimal to enter the market (i.e., when entry cannot be deterred)* *then:*

(a) *The greater is the breadth of patent protection the greater is the entrant's incentive to infringe the patent.*

(b) *The more costly it is to produce the better quality product the greater is the entrant's incentive to infringe the patent.*

(c) *The greater are the entrant's trial costs the smaller is the entrant's incentive to infringe the patent.*

Proofs:

(a) $\frac{\partial Z_E}{\partial b} = (\frac{16}{81\beta} + \beta)b - \frac{4}{9} \geq 0 \;\forall \beta \geq \frac{4}{9} \wedge b \in (\varepsilon_0, 1]$

The greater is the breadth of patent protection the more costly it becomes for the entrant to locate outside the patentee's claims. In addition, the greater is the breadth of patent protection the greater is the probability that the patent will be invalidated and that the entrant will win at trial. Both the above outcomes increase the entrant's incentive to infringe the patent.

(b) $\frac{\partial Z_E}{\partial \beta} = (-\frac{8}{81\beta} + \frac{1}{2})b^2 \geq 0 \;\forall\; \beta \geq \frac{4}{9} \wedge b \in (\varepsilon_0, 1]$

The greater are the costs that need to be incurred for the production of the better quality product the less profitable it becomes for the entrant to locate outside the patentee's claims.

(c) $\frac{\partial Z_E}{\partial C_E} = -1 < 0$. □

The existence of a patent breadth that deters entry, $\hat{b}$, is closely linked to the existence of a patent breadth that makes the entrant indifferent between infringing and not infringing the patent, $\tilde{b}$. Figures 6 and 7 depict different scenarios with respect to the existence of $\hat{b}$ and $\tilde{b}$. Figure 6 depicts two cases under which entry cannot be deterred - a $\hat{b} \in (\varepsilon_0, 1]$ does not exist. Figure 7 depicts three cases under which entry can be deterred - a $\hat{b} \in (\varepsilon_0, 1]$ exists.

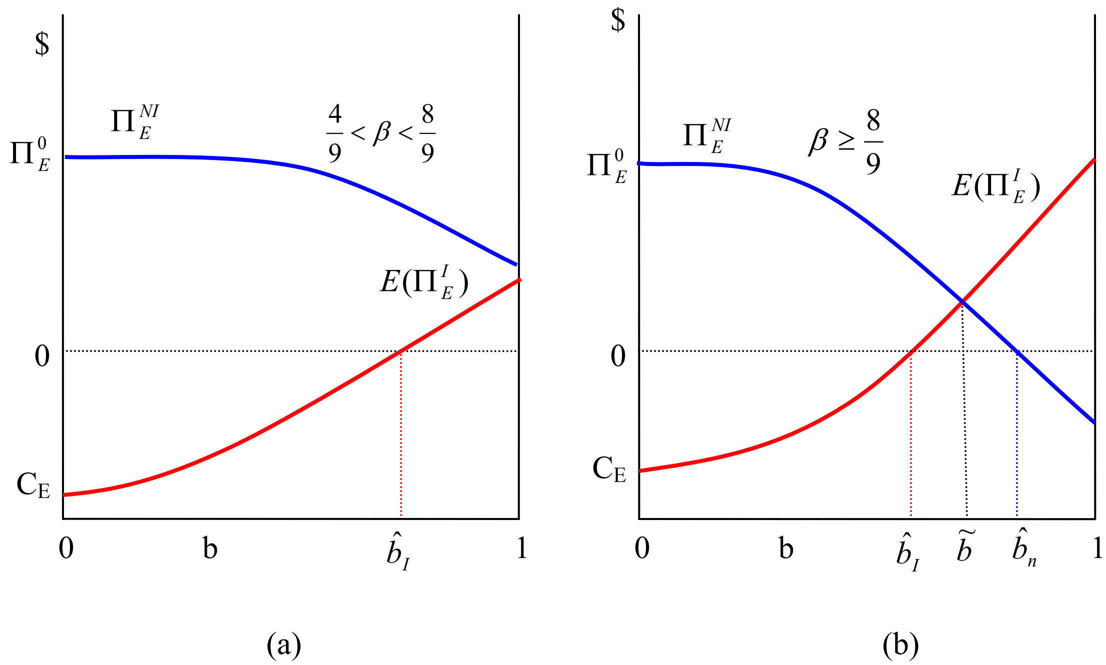


Figure 6. The entrant's profits under infringement and no infringement when entry cannot be deterred – a $\hat{b} \in (\varepsilon_0, 1]$ does not exist

Panel (a) in Figure 6 represents the case where there is no patent breadth that can deter entry in the patentee's market and a $\tilde{b} \in (\varepsilon_0, 1]$ does not exist. In this case non infringement is always an optimal strategy for the entrant as the curve depicting the entrant's profits under no infringement is above the curve depicting the entrant's expected profits under infringement for all patent breadth values (see Proposition 3 for a formal proof). Panel (b) in Figure 6 represents the case where there is a $\tilde{b} \in (\varepsilon_0, 1]$, but $\tilde{b}$ does not satisfy the entry deterrence condition, thus implying that entry cannot be deterred (see Proposition 6 for a formal proof). This result occurs because patent breadth $\tilde{b}$ results in positive profits for the entrant irrespective of whether she infringes the patent or not. Neither $\hat{b}_I$ nor $\hat{b}_n$ can deter entry since none of them satisfies the entry deterrence condition.

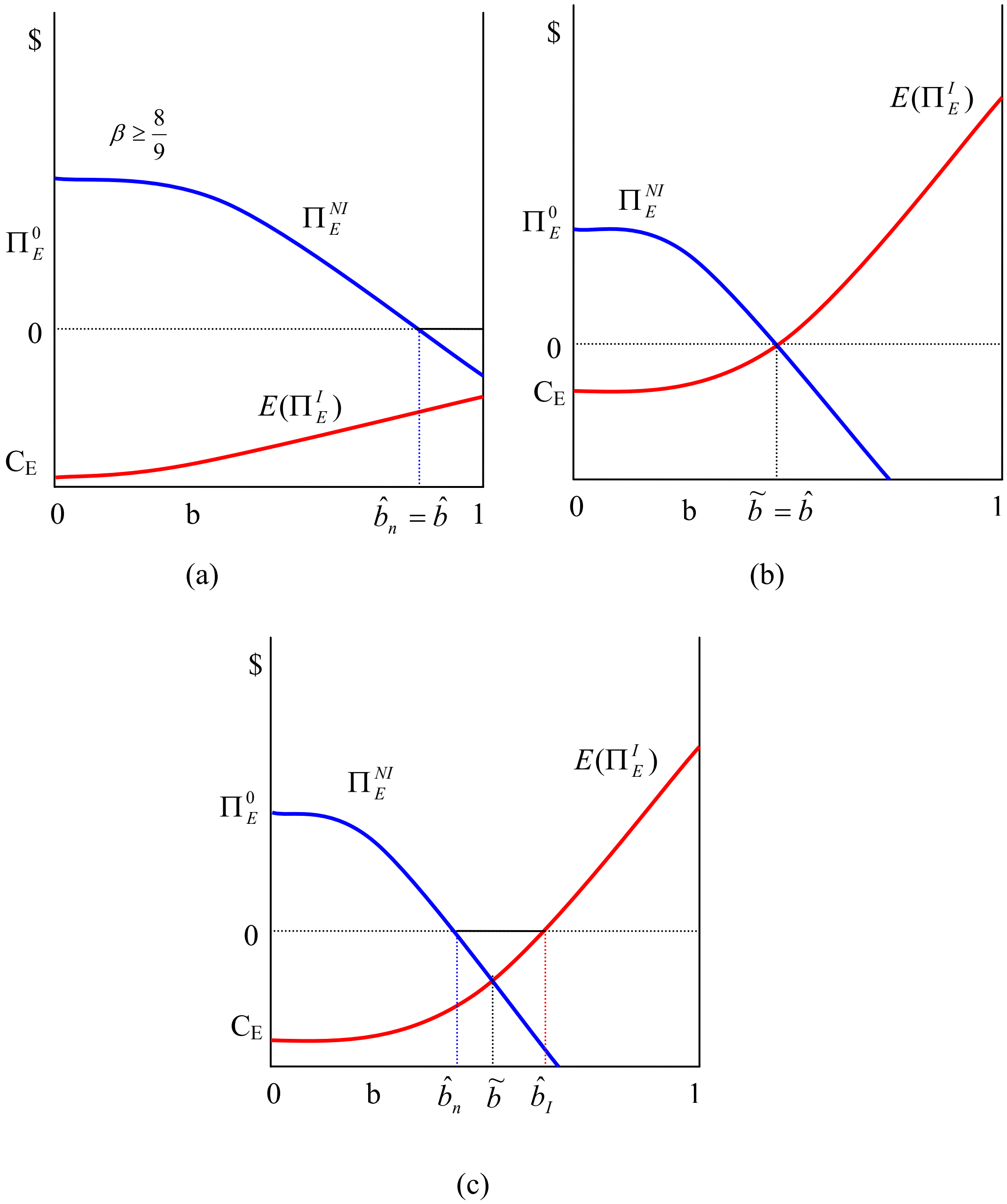


Figure 7. The entrant's profits under infringement and no infringement when entry can be deterred – a $\hat{b} \in (\varepsilon_0, 1]$ exists

Panel (a) in Figure 7 represents the case where entry can be deterred and there is no $\tilde{b} \in (\varepsilon_0, 1]$. Patent breadth $\hat{b}_n$ deters market entry since it satisfies the entry deterrence condition. In fact, any value of patent breadth such that $b \in [\hat{b}_n, 1]$ can deter entry. Panel (b) in Figure 7 represents the case under which $\tilde{b}$ is the only patent breadth that can deter entry. Finally, panel (c) in Figure 7 represents the case where there is a $\tilde{b} \in (\varepsilon_0, 1]$ and $\tilde{b}$ satisfies the entry deterrence condition. In this case, there is a range of patent breadth values that can deter entry in the patentee's market. That is, either $\tilde{b}$, $\hat{b}_I$, $\hat{b}_n$ or any $b \in [\hat{b}_I, \hat{b}_n]$ can deter entry since all the above patent breadth values satisfy the entry deterrence condition.

As it was mentioned above, patent breadth $\tilde{b}$, if it exists, should make $Z_E = 0$. To determine whether a $\tilde{b}$ exists $Z_E = (\frac{8}{81\beta} + \frac{\beta}{2})b^2 - \frac{4}{9}b - C_E = 0$ is solved for b. This solution yields the following two roots: $b_{1,2} = \frac{9(4\beta \pm \sqrt{2}\sqrt{\beta}\sqrt{16C_E + 8\beta + 81C_E\beta^2})}{16 + 81\beta^2}$.

The root $b_2 = \frac{9(4\beta - \sqrt{2}\sqrt{\beta}\sqrt{16C_E + 8\beta + 81C_E\beta^2})}{16 + 81\beta^2} \leq 0 \;\; \forall \;\; \beta \geq \frac{4}{9} \wedge C_E \geq 0$ and it is thus rejected since $\varepsilon_0 < \tilde{b} \leq 1$. The root $b_1 = \frac{9(4\beta + \sqrt{2}\sqrt{\beta}\sqrt{16C_E + 8\beta + 81C_E\beta^2})}{16 + 81\beta^2} \geq 0 \;\; \forall \;\; \beta \geq \frac{4}{9} \wedge C_E \geq 0$ and it is accepted as a possible solution. Thus, if $\tilde{b}$ exists it will be equal to b_1, i.e., $\tilde{b} = b_1 = \frac{9(4\beta + \sqrt{2}\sqrt{\beta}\sqrt{16C_E + 8\beta + 81C_E\beta^2})}{16 + 81\beta^2}$.

The patent breadth $\tilde{b}$ that makes the entrant indifferent between infringing and not infringing the patent is a function of the entrant's cost structure (β) and her trial costs (C_E). Patent breadth $\tilde{b}$ exists only if the values of β and C_E are such that $\varepsilon_0 < \tilde{b} \leq 1$. It is easily verified that the condition $\tilde{b} - \varepsilon_0 > 0$ is satisfied for all β and C_E values. That is, $\tilde{b} - \varepsilon_0 = \frac{9(4\beta + \sqrt{2}\sqrt{\beta}\sqrt{16C_E + 8\beta + 81C_E\beta^2})}{16 + 81\beta^2} - \frac{4}{9\beta} > 0 \;\; \forall \;\; \beta \geq \frac{4}{9} \wedge C_E \geq 0$. The condition $\tilde{b} \leq 1$ is satisfied for certain combinations of β and C_E values. To determine the combinations of β and C_E values which satisfy the condition $\tilde{b} \leq 1$, the pairs of β and C_E values which satisfy the above constraint as an equality ($\tilde{b} = 1$) are determined first. The solution of $\tilde{b} - 1 = 0$ with respect to C_E yields: $C_E = \frac{16 - 72\beta + 81\beta^2}{162\beta}$. The combination of β and C_E values for which $\tilde{b} - 1 = 0$ is represented by the locus $\tilde{b} = 1$ in Figure 8. The area to the right of the locus $\tilde{b} = 1$ represents all combinations of β and C_E for which $\tilde{b}$ exists ($\tilde{b} < 1$); this area includes the dotted and vertically hatched areas in Figure 8. This case is also depicted in panel (b) in Figure 6 and in panel (b) and (c) in Figure 7. The area to the left of the locus $\tilde{b} = 1$ represents all combinations of β and C_E values for which $\tilde{b}$ does not exist ($\tilde{b} > 1$); this area includes the non-shaded and the horizontally hatched areas in Figure 8. This case is also depicted in panel (a) in Figure 6 and panel (a) in Figure 7.

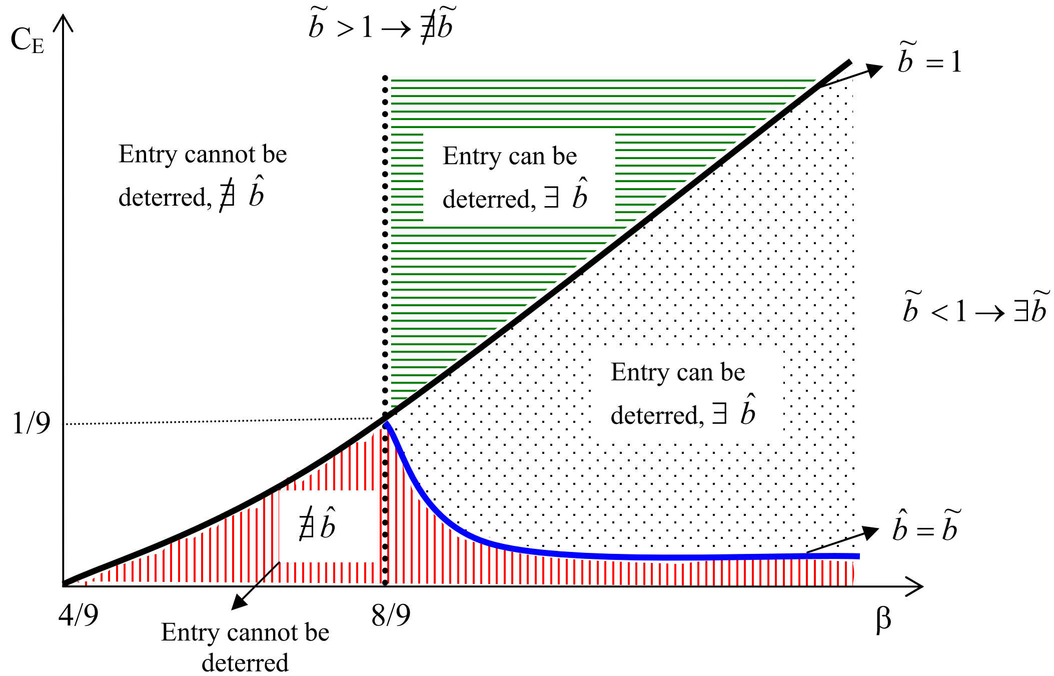


Figure 8. Combinations of C_E and β values for which $\tilde{b}, \hat{b} \in (\varepsilon_0, 1]$ exist

If $\varepsilon_0 < \tilde{b} \leq 1$ exists, it can deter entry if and only if it also satisfies the entry deterrence condition $\frac{8}{9\beta} \leq \tilde{b} \leq \sqrt{\frac{81C_E\beta}{8}}$. The entry deterrence condition is satisfied for $\beta \geq \frac{8}{9}$ and for certain combinations of β and C_E values. To find the combinations of β and C_E values that satisfy the entry deterrence condition, the locus that satisfies the condition as an equality is determined first. The locus $\tilde{b} = \hat{b}$ in Figure 8 refers to the pairs of β and C_E values for which $\frac{8}{9\beta} = \tilde{b} = \sqrt{\frac{81C_E\beta}{8}}$ holds true. Solution of the above condition with respect to C_E yields: $C_E = \frac{512}{6561\beta^3}$. This case is also depicted in panel (b) in Figure 7. All combinations of β and C_E values above the locus $\tilde{b} = \hat{b}$ and below the locus $\tilde{b} = 1$ – the dotted area in Figure 8 – satisfy the entry deterrence condition. This case is also depicted in panel (c) in Figure 7. The combinations of β and C_E values below the locus $\tilde{b} = \hat{b}$ and below the locus $\tilde{b} = 1$ – the vertically hatched area in Figure 8 – do not satisfy the entry deterrence condition. This case is also depicted in panel (b) in Figure 6.

The close relationship between the existence of a patent breadth $\hat{b} \in (\varepsilon_0, 1]$ that can deter entry and a patent breadth $\tilde{b} \in (\varepsilon_0, 1]$ that makes the entrant indifferent between infringing and not infringing the patent is further demonstrated in the propositions that follow.

Proposition 3. *If* $\tilde{b} \in (\varepsilon_0, 1]$ *does not exist it is never optimal for the entrant to infringe the patent.*

Proof:
At the entrant's most preferred location ε_0 non infringement is always more profitable than infringement for the entrant. That is, for $b = \varepsilon_0 = \frac{4}{9\beta}$ $Z_E = -C_E + \frac{128}{6561\beta^3} - \frac{8}{81\beta} < 0 \;\; \forall \;\; \beta \geq \frac{4}{9}$ $\wedge$ $C_E \geq 0$. The above conditions imply that if a $\tilde{b} \in (\varepsilon_0, 1]$ does not exist (i.e., there is no patent breadth that makes $Z_E=0$), then $Z_E<0$ $\forall b \in [0,1]$ which implies that $\Pi_E^{NI} > E(\Pi_E^I)$. This result is depicted in panel (a) in Figure 6 and in panel (a) in Figure 7. □

Proposition 4. *If* $\tilde{b} \in (\varepsilon_0, 1]$ *does not exist, the only patent breadth* $\hat{b} \in (\varepsilon_0, 1]$ *that can deter entry is the patent breadth that satisfies the non-entry condition under no infringement.*
Proof:
From Proposition 3 it is known that for $b=\varepsilon_0$ $Z_E<0$. If $\tilde{b}$ that makes $Z_E=0$ does not exist then $\forall b \in [0,1]$ $Z_E<0 \Rightarrow \Pi_E^{NI} > E(\Pi_E^I)$. If there is a patent breadth $\hat{b}_n$ that satisfies the non-entry condition under no infringement this implies that for $b = \hat{b}_n$ $\Pi_E^{NI} \leq 0$. Given that $\Pi_E^{NI} > E(\Pi_E^I)$, when $b = \hat{b}_n$ the entry deterrence condition is also satisfied. In this case, any $b \in [\hat{b}_n, 1]$ can deter entry. This case is depicted in panel (a) in Figure 7. □

Proposition 5. *If* $\tilde{b} \in (\varepsilon_0, 1]$ *exists:*

(a) The greater are the costs of producing the higher quality product, the smaller is the breadth of the patent that makes the entrant indifferent between infringing and not infringing the patent.
(b) The greater are the trial costs, the greater is the breadth of the patent that makes the entrant indifferent between infringing and not infringing the patent.

Proof:

(a) $$\frac{\partial \tilde{b}}{\partial \beta} = \frac{9\left(4 + \frac{\sqrt{\beta}(8 + 162 C_E \beta)}{\sqrt{2}\sqrt{16C_E + 8\beta + 81 C_E \beta^2}} + \frac{\sqrt{16C_E + 8\beta + 81C_E\beta^2}}{\sqrt{2}\sqrt{\beta}}\right)}{16 + 81\beta^2} -$$
$$\frac{1458\beta(4\beta + \sqrt{2}\sqrt{\beta}\sqrt{16C_E + 8\beta + 81C_E\beta^2}}{(16 + 81\beta^2)^2} \leq 0, \forall \beta \geq \frac{4}{9} and C_E \geq 0$$

The more costly it is to produce the better quality product, the closer the entrant is forced to locate to the patentee and the smaller is the breadth of patent protection that makes it unprofitable for the entrant to locate above the patentee's patent breadth.

(b) $$\frac{\partial \tilde{b}}{\partial C_E} = \frac{9\sqrt{\beta}}{\sqrt{2}\sqrt{16C_E + 8\beta + 81C_E\beta^2}} \geq 0 \;\; \forall \; \beta \geq \frac{4}{9} \;\wedge\; C_E \geq 0$$

The greater are the trial costs the less appealing is infringement to the entrant. The entrant in this case will infringe only if the breadth is so large that her cost structure does not allow her to locate outside the patentee's claims. □

Proposition 6. *If* $\tilde{b} \in (\varepsilon_0, 1]$ *exists and* $\tilde{b}$ *cannot deter entry (i.e.,* $\tilde{b}$ *does not satisfy the entry deterrence condition), then there is no* $\hat{b} \in (\varepsilon_0, 1]$ *that can deter entry.*
Proof:

If $\widetilde{b}$ exists, then for $b = \widetilde{b}$ Z_E=0. Since $\widetilde{b}$ cannot deter entry it follows from equation (10) that at $b = \widetilde{b}$ both $E(\Pi_E^I) > 0 \wedge \Pi_E^{NI} > 0$ must be satisfied. Assume that there is a $\hat{b} > \widetilde{b}$ that can deter entry in the market. Then at $b = \hat{b}$ both $E(\Pi_E^I) < 0 \wedge \Pi_E^{NI} < 0$ should be satisfied. But $\frac{\partial E(\Pi_E^I)}{\partial b} \geq 0$ which, given that at $b = \widetilde{b}$ $E(\Pi_E^I) > 0$, implies that $\forall \hat{b} > \widetilde{b}$ $E(\Pi_E^I) > 0$. Thus, there is no patent breadth $\hat{b} > \widetilde{b}$ that can deter entry. Now assume that there is a $\hat{b} < \widetilde{b}$ that can deter entry in the market. Then at $b = \hat{b}$ both $E(\Pi_E^I) < 0 \wedge \Pi_E^{NI} < 0$ must be satisfied. But Π_E^{NI} is concave in b, $\frac{\partial \Pi_E^{NI}}{\partial b} \geq 0, \frac{\partial^2 \Pi_E^{NI}}{\partial b^2} \leq 0$ which, given that at $b = \widetilde{b}$ $\Pi_E^{NI} > 0$, implies that $\forall \hat{b} < \widetilde{b}$ at $b = \hat{b}$ $\Pi_E^{NI} > 0$. Thus, there is no patent breadth $\hat{b} < \widetilde{b}$ that can deter entry in the market. This case is presented in Figure 6, panel (b). □

Proposition 7. *If* $\widetilde{b} \in (\varepsilon_0, 1]$ *exists and it satisfies the entry deterrence condition as an equality then* $\widetilde{b}$ *is the only breadth of patent protection that can deter entry.*

Proof:

The proof in this proposition is similar to the proof in Proposition 6. Since $\widetilde{b}$ is the breadth of patent protection that makes Z_E=0, if $\widetilde{b}$ makes $E(\Pi_E^I) = 0$ it should also make $\Pi_E^{NI} = 0$ (this follows from equation (10)). Since $\frac{\partial E(\Pi_E^I)}{\partial b} \geq 0$ $\forall$ $\hat{b} < \widetilde{b}$ $E(\Pi_E^I) < 0$ and $\forall$ $\hat{b} > \widetilde{b}$ $E(\Pi_E^I) > 0$. Also, since Π_E^{NI} is concave in b, $\forall$ $\hat{b} < \widetilde{b}$ $\Pi_E^{NI} > 0$ and $\forall$ $\hat{b} > \widetilde{b}$ $\Pi_E^{NI} < 0$. Thus, there is no $\hat{b} \neq \widetilde{b}$ for which $E(\Pi_E^I) < 0$ $\wedge$ $\Pi_E^{NI} < 0$ which implies that there is no $\hat{b} \neq \widetilde{b}$ that satisfies the entry deterrence condition. This case is depicted in panel (b) in Figure 7. □

Proposition 8. *If* $\widetilde{b} \in (\varepsilon_0, 1]$ *exists and it satisfies the entry deterrence condition as a strict inequality then there is a range of patent breadth values in the interval* $[\hat{b}_n, \hat{b}_I]$ *or in the interval* $[\hat{b}_n, 1]$ *that can deter entry.*

Proof:

If $\widetilde{b}$ exists, then for $b = \widetilde{b}$ Z_E=0. If $\widetilde{b}$ can deter entry it follows from equation (10) that at $b = \widetilde{b}$ both $E(\Pi_E^I) < 0 \wedge \Pi_E^{NI} < 0$ must be satisfied. Given that $\Pi_{E\ (b=0)}^{NI} > 0$, Π_E^{NI} is concave in b and at $b = \widetilde{b}$ $\Pi_E^{NI} < 0$, there is a breadth of patent protection $\hat{b}_n \in (0, \widetilde{b})$ such that $\Pi_{E\ (b=\hat{b}_n)}^{NI} = 0$. Similarly given that $\frac{\partial E(\Pi_E^I)}{\partial b} \geq 0$ and at $b = \widetilde{b}$ $E(\Pi_E^I) < 0$ there may exist a $\hat{b}_I \in (\widetilde{b}, 1]$ such that $E(\Pi_E^I)_{(b=\hat{b}_I)} = 0$. This case is represented graphically in Figure 7, panel (c). If $\hat{b}_I \in (\varepsilon_0, 1]$ exists then any $b \in [\hat{b}_n, \hat{b}_I]$ can deter entry. If $\hat{b}_I \in (\varepsilon_0, 1]$ does not exist then any $b \in [\hat{b}_n, 1]$ can deter entry in the market. □

3.2.2 The Patentee's Strategic Patent Breadth Decision

In sub-section 3.2.1 it was shown that the existence of a patent breadth $\hat{b}$ that deters market entry and/or a patent breadth $\widetilde{b}$ that makes the entrant indifferent between infringing and not infringing the patent depends on the entrant's R&D effectiveness (β) (i.e., her R&D cost structure) and her trial costs (C_E). The existence of $\hat{b}$ and $\widetilde{b}$ determines the patentee's optimal patent breadth choice and the profits that can be realized. Different outcomes with respect to the patentee's patent breadth choice and profits emerge under different scenarios regarding the existence of $\hat{b}$ and $\widetilde{b}$. These scenarios and the respective outcomes that emerge are presented in Figure 9.

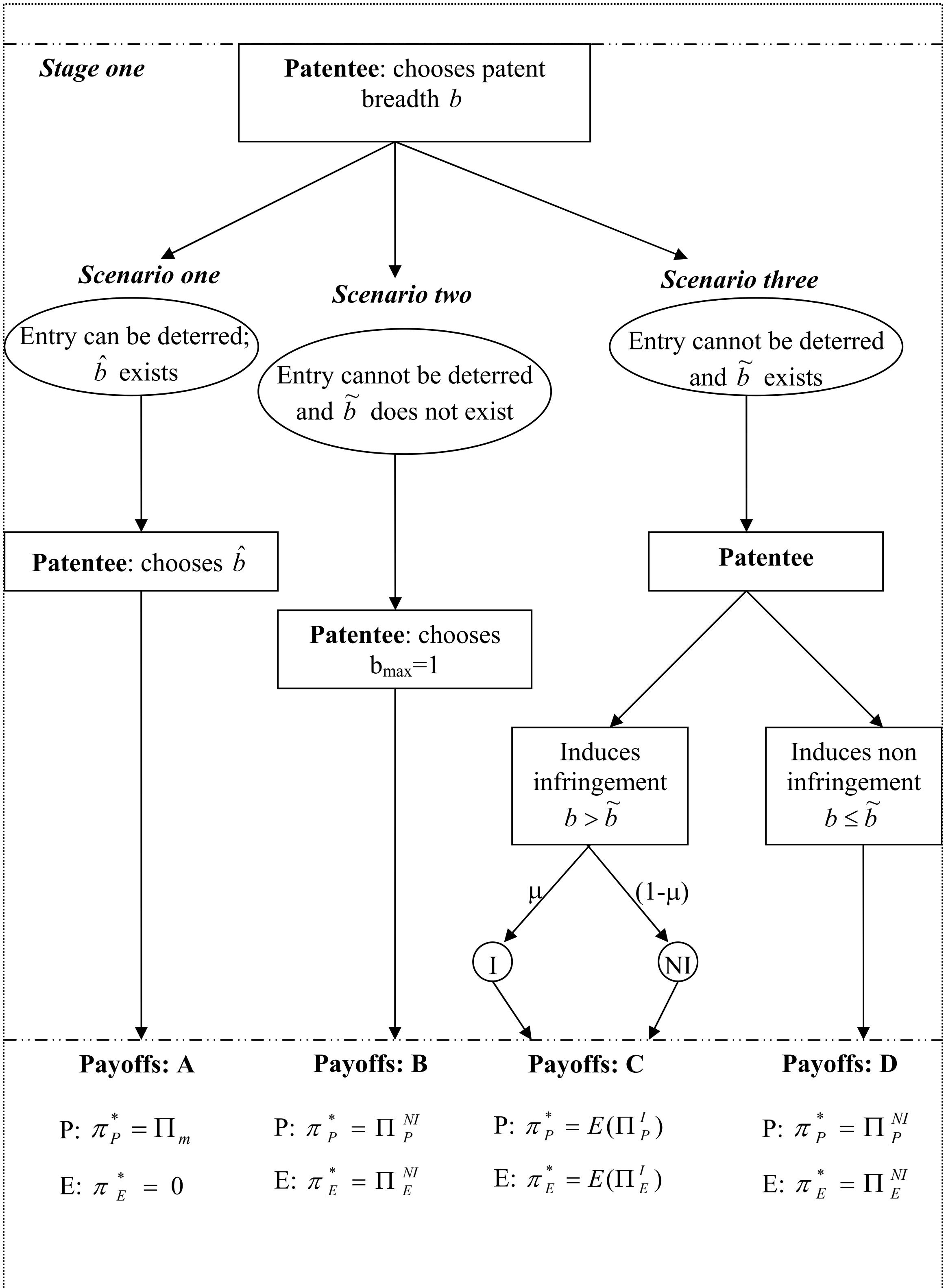


Figure 9. The Patentee's Strategic Patent Breadth Decision

- ***Scenario One*: There is a patent breadth $\hat{b}$ or a range of patent breadth values in the interval $[\hat{b}_n, \hat{b}_I]$ or in the interval $[\hat{b}_n, 1]$ that deter entry.**

Under this scenario, irrespective of whether $\tilde{b}$ exists or not, it is always optimal for the patentee to claim the breadth of patent protection $\hat{b}$ or any breadth values in the interval $[\hat{b}_n, \hat{b}_I]$ or in $[\hat{b}_n, 1]$ that deter entry. By claiming $\hat{b}$ the patentee makes monopoly profits Π_m.

- ***Scenario Two*: There is no patent breadth $\hat{b}$ that can deter entry and there is no patent breadth $\tilde{b}$ that makes the entrant indifferent between infringing and not infringing the patent.**

Under this scenario, as described in Proposition 3, the patent is never infringed. The patentee's profits under no infringement are $\Pi_P^{NI} = \pi_1^B = \frac{\varepsilon_n}{9}$, where $\varepsilon_n = b$ (see sub-section 3.2.1). The patentee chooses the breadth of patent protection that maximizes his objective function given by equation (25):

$$\text{P: } \max_b \Pi_P^{NI} = \frac{b}{9} \tag{25}$$
$$s.t. \quad 0 \le b \le 1$$

Equation (25) shows that the patentee's profits under no infringement are increasing linearly in patent breadth. Given that patent breadth takes values in the interval $0 \le b \le 1$ the patentee's profits under no infringement are maximized for b=1. The patentee's profits when the patent is never infringed are depicted in Figure 10.

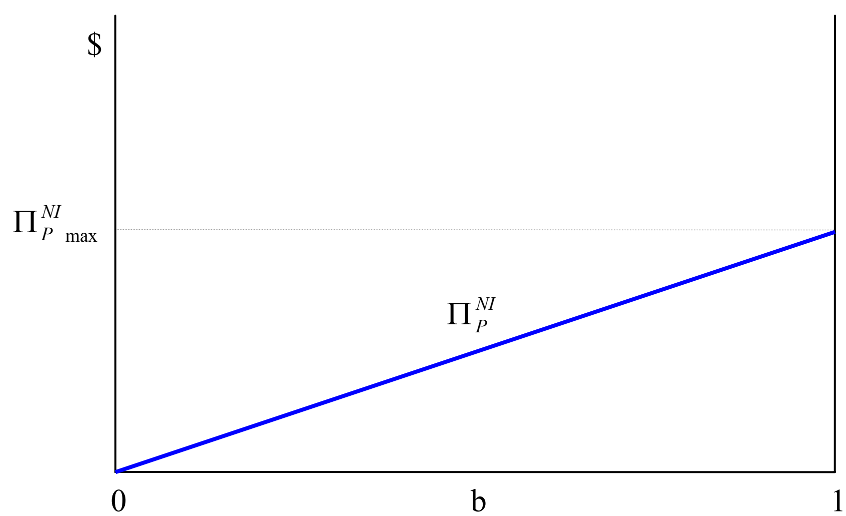


Figure 10. The Patentee's Expected Profits When the Patent is Never Infringed

In Figure 10 the patentee's profits are increasing linearly in patent breadth and are maximized for b=1. The above results show that when it is never optimal for the entrant to infringe the patent (i.e., when there is no $\hat{b}$ or $\tilde{b}$) it is always optimal for the patentee to claim the maximum breadth of patent protection, $b_{\max} = 1$.

The results in scenario two capture the standard assumption made in the patent breadth literature with respect to the patentee's patent breadth decision. The assumption in the patent literature is that the patentee claims the maximum breadth of patent protection (Merges and Nelson 1990, Gilbert and Shapiro

1990, Lanjouw and Schankerman 2001). The above result suggests claiming the maximum patent breadth protection is an optimal strategy for the patentee only if non infringement is an optimal strategy for the entrant.

- ***<u>Scenario Three</u>*: *There is a patent breadth $\tilde{b}$ that makes the entrant indifferent between infringing and not infringing the patent and $\tilde{b}$ cannot deter entry.***

Under this scenario, as it has been shown in Proposition 6, if $\tilde{b}$ cannot deter entry then there is no other breadth of patent protection $\hat{b} \in (\varepsilon_0, 1]$ that can deter entry. In this case, the patentee has to determine whether it is more profitable to induce infringement by claiming a patent breadth $b > \tilde{b}$ or not to induce infringement by claiming a patent breadth $b \le \tilde{b}$. The patentee under this scenario uses the value of $Z_P = E(\Pi_P^I) - \Pi_P^{NI}$ to determine the optimal patent breadth. If $Z_P > 0$ the patentee chooses a $b > \tilde{b}$ that induces the entrant to infringe the patent. If $Z_P \le 0$ the patentee chooses a $b \le \tilde{b}$ that induces non infringement. The optimal patent breadth value is determined through the solution of the patentee's maximization of expected profits under infringement and under no infringement.

- *The Patentee's expected profits when he induces infringement ($b > \tilde{b}$)*

When the patentee claims $b > \tilde{b}$ he knows that the entrant's optimal strategy is to infringe the patent. The patentee makes monopoly profits with probability $\mu = 1 - b$ if his patent is found valid during trial (or equivalently if infringement is found) and duopoly profits with probability $1 - \mu = b$ if his patent is revoked (or equivalently if infringement is not found). The patentee's duopoly profits are given by $\pi_1^B = \frac{\varepsilon_T}{9}$ where $\varepsilon_T = \frac{4}{9\beta} b$ is the entrant's optimal location when she infringes the patent (see sub-section 3.2.1). The patentee also incurs trial costs denoted by C_P which are independent of the breadth of patent protection claimed.

The patentee chooses the breadth of patent protection that maximizes his expected profits under infringement. The patentee's objective function is given by:

$$\text{P: } \max_b E(\Pi_P^I) = \mu \Pi_m + (1-\mu)\pi_1^B - C_P \qquad (26)$$

$$s.t. \quad \tilde{b} + e \le b \le 1 \text{ where } e \to 0$$

The Lagrangean of the patentee's profit maximization problem is given by:

$$L = (1-b)\Pi_m + \frac{4b^2}{81\beta} - C_P + \lambda_1(1-b) + \lambda_2(b - \tilde{b} - e)$$

The Kuhn-Tucker conditions for a maximum are:

$$\frac{\partial L}{\partial b} \le 0 \Rightarrow -\Pi_m + \frac{8b}{81\beta} - \lambda_1 + \lambda_2 \le 0,\ b \ge 0 \text{ and } b\frac{\partial L}{\partial b} = 0$$

$$\frac{\partial L}{\partial \lambda_1} \ge 0 \Rightarrow 1 - b \ge 0,\ \lambda_1 \ge 0 \text{ and } \lambda_1 \frac{\partial L}{\partial \lambda_1} = 0$$

$$\frac{\partial L}{\partial \lambda_2} \ge 0 \Rightarrow b - \tilde{b} - e \ge 0,\ \lambda_2 \ge 0 \text{ and } \lambda_2 \frac{\partial L}{\partial \lambda_2} = 0$$

Since $\tilde{b} + e \le b \le 1 \Rightarrow b \ne 0 \Rightarrow \frac{\partial L}{\partial b} = 0 \Rightarrow -\Pi_m + \frac{8b}{81\beta} - \lambda_1 + \lambda_2 = 0$

Case 1. If $\lambda_1 = \lambda_2 = 0 \Rightarrow \tilde{b} + e < b < 1$ and from $\frac{\partial L}{\partial b} = 0 \Rightarrow b_I = \frac{81\beta\Pi_m}{8}$

The S.O.C. for a maximum are not satisfied, $\frac{\partial^2 L}{\partial b^2} = \frac{8}{81\beta} > 0$ which implies that b_I is a minimum not a maximum and b_I is thus rejected as a solution. The above conditions indicate that there is a corner solution to the expected profit maximization problem. Thus, either b=1 or $b = \tilde{b} + e$ is the breadth of patent protection that maximizes the patentee's expected profits under infringement.

Case 2. If $\lambda_1 > 0$ then b=1 and $\lambda_2 = 0$. In this case, $\lambda_1 = \frac{8}{81\beta} - \Pi_m$ and the patentee's expected profits are:

$$E(\Pi_P^I)_{b=1} = \frac{4}{81\beta} - C_P \tag{27}$$

Case 3. If $\lambda_2 < 0$ then $b = \tilde{b} + e$ and $\lambda_1 = 0$. In this case, $\lambda_2 = \Pi_m - \frac{8(\tilde{b} + e)}{81\beta}$ and the patentee's expected profits are:

$$E(\Pi_P^I)_{b=\tilde{b}+e} = (1 - \tilde{b} - e)\Pi_m + \frac{4}{81\beta}(\tilde{b} + e)^2 - C_P$$

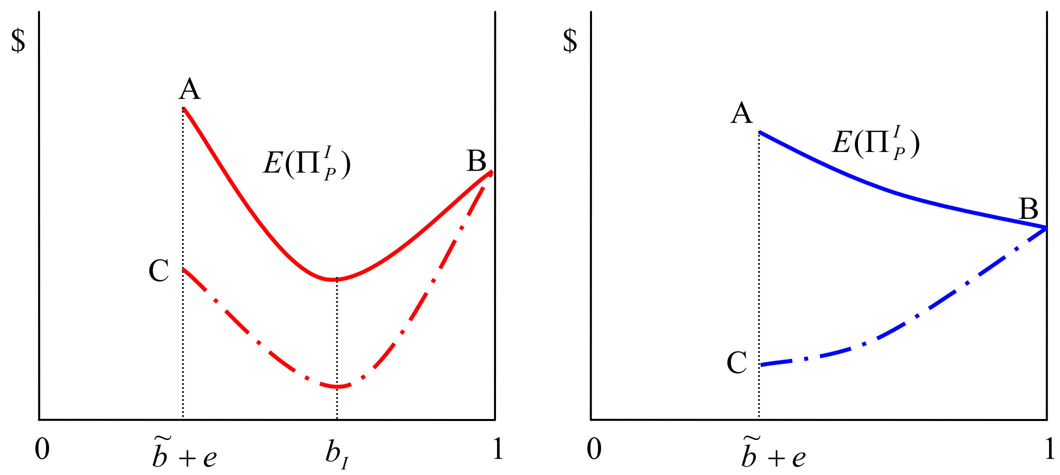
$$\lim_{e \to 0} E(\Pi_P^I)_{b=\tilde{b}+e} = \Pi_m - \tilde{b}\Pi_m + \frac{4}{81\beta}\tilde{b}^2 - C_P \tag{28}$$

where $\tilde{b} = \frac{9(4\beta + \sqrt{2}\sqrt{\beta}\sqrt{16C_E + 8\beta + 81C_E\beta^2})}{16 + 81\beta^2}$

Comparison of the patentee's expected profits when b=1 given by equation (27) to the patentee's expected profits when $b = \tilde{b} + e$ given by equation (28) yields the following results. For monopoly profit values $\Pi_m \geq 0.089$, $E(\Pi_P^I)_{b=1} \leq E(\Pi_P^I)_{b=\tilde{b}+e}$. For monopoly profit values $\Pi_m < 0.089$, $E(\Pi_P^I)_{b=1} > E(\Pi_P^I)_{b=\tilde{b}+e}$. Note that, under scenario three, all values of the entrant's R&D effectiveness (β) and trial costs (C_E) are such that both $C_E \leq \frac{16 - 72\beta + 81\beta^2}{162\beta}$ and $C_E \leq \frac{512}{6561\beta^3}$ are satisfied (i.e., β and C_E values in the vertically hatched area in Figure 8).

The above results show that the smaller are the monopoly profits that the patentee makes when his patent is found valid at trial, the greater is the patentee's incentive to claim the maximum breadth of protection and have his patent revoked. This happens because under infringement the entrant's location is proportional to the breadth of the patent (i.e., $\varepsilon_T = \frac{4}{9\beta}b$) so the greater is patent breadth the further away from the patentee the entrant locates and the greater are the profits at the last stage of the game for both players. In other words, in this case, the effect of the loss of monopoly profits due to the large patent breadth is smaller than the effect of the increased profits brought by the increased level of differentiation between the two products. However, when monopoly profits are large the patentee does not want to risk having his patent revoked by claiming the maximum breadth of patent protection and he claims $b = \tilde{b} + e$ instead. In this case, the effect of the decrease in expected profits due to the decrease in product differentiation is smaller than the effect of the increase in expected profits due to the increased probability that infringement will be found at trial and the patentee will realize monopoly profits.

Given that the level of monopoly profits is unknown, two cases have to be considered. Under the first case $E(\Pi_P^I)_{b=1} > E(\Pi_P^I)_{b=\tilde{b}+e}$, while under the second case $E(\Pi_P^I)_{b=1} \leq E(\Pi_P^I)_{b=\tilde{b}+e}$. The two graphs in Figure 11 depict the above two cases when $b_I \in (\tilde{b}+e\,,\,1)$ (panel (a)) and when $b_I \notin (\tilde{b}+e\,,\,1)$ (panel (b)).



(a): $b_I \in (\tilde{b}+e\,,\,1)$ (b): $b_I \notin (\tilde{b}+e\,,\,1)$

Figure 11. The Patentee's Expected Profits When Infringement Is Induced

As shown in Figure 11 when the patentee's expected profits under infringement are represented by the curve AB the patentee maximizes his profits by choosing the patent breadth $b = \tilde{b}+e$. When the curve CB reflects the patentee's expected profits under infringement then the patentee maximizes his profits by choosing the maximum breadth of patent protection $b = 1$.

- *The Patentee's profits when he induces non infringement ($b \leq \tilde{b}$)*

When the patentee claims $b \leq \tilde{b}$ he knows that the entrant's optimal strategy is to not infringe the patent. The patentee chooses the breadth of patent protection that maximizes his profits under no infringement given by:

$$\text{P: } \max_b \Pi_P^{NI} = \pi_1^B = \frac{\varepsilon_n}{9} = \frac{b}{9} \tag{29}$$

$$s.t. \quad 0 \leq b \leq \tilde{b}$$

Since the patentee's profits under no infringement are increasing linearly in patent breadth the breadth of patent protection that maximizes equation (29) is $b_n = \tilde{b}$. Substituting $b_n = \tilde{b}$ in equation (29) yields the patentee's maximum profits under no infringement:

$$\Pi_P^{NI} = \frac{\tilde{b}}{9} = \frac{4\beta + \sqrt{2}\sqrt{\beta}\sqrt{16C_E + 8\beta + 81C_E\beta^2}}{16+81\beta^2} \tag{30}$$

Having determined the optimal patent breadth and the level of profits under infringement and under no infringement the patentee can determine the value of Z_P. Two cases must be considered depending on whether $b = 1$ or $b = \tilde{b}+e$ is the optimal patent breadth under infringement.

I. $E(\Pi^I_P)_{b=1} > E(\Pi^I_P)_{b=\tilde{b}+e}$

Under this case Z_P is redefined as $Z^1_P = E(\Pi^I_P)_{b=1} - \Pi^{NI}_P$.

Proposition 9. *When $\tilde{b} \in (\varepsilon_0, 1]$ exists and it cannot deter entry, claiming the maximum breadth of patent protection ($b = 1$) is never an optimal strategy for the patentee unless $\tilde{b} = 1$.*

Proof:

$$Z^1_P = E(\Pi^I_P)_{b=1} - \Pi^{NI}_P = \frac{4}{81\beta} - \frac{4\beta + \sqrt{2}\sqrt{\beta}\sqrt{16C_E + 8\beta + 81C_E\beta^2}}{16 + 81\beta^2} - C_P < 0 \; \forall \beta \geq \frac{4}{9} \wedge C_P \geq 0 \wedge C_E \geq 0.$$

Since $Z^1_P < 0$ the optimal strategy for the patentee under scenario three when $E(\Pi^I_P)_{b=1} > E(\Pi^I_P)_{b=\tilde{b}+e}$ is to claim patent breadth $b = \tilde{b}$ which does not induce infringement.

II. $E(\Pi^I_P)_{b=1} \leq E(\Pi^I_P)_{b=\tilde{b}+e}$

Under this case Z_P is redefined as $Z^2_P = \lim_{e \to 0} E(\Pi^I_P)_{b=\tilde{b}+e} - \Pi^{NI}_P$.

Substituting equations (28) and (30) into Z^2_P yields:

$$Z^2_P = \Pi_m - \tilde{b}\Pi_m + \frac{4}{81\beta}\tilde{b}^2 - C_P - \frac{\tilde{b}}{9}$$

where $\tilde{b} = \dfrac{9(4\beta + \sqrt{2}\sqrt{\beta}\sqrt{16C_E + 8\beta + 81C_E\beta^2})}{16 + 81\beta^2}$

The value of Z^2_P cannot be determined without knowledge of the values of the parameters Π_m, β, C_P and C_E. When $\tilde{b} \in (\varepsilon_0, 1]$ exists and it cannot deter entry the optimal breadth of patent protection is either ($b = \tilde{b}$) or ($b = \tilde{b} + e$) depending on the relative values of the parameters Π_m, β, C_P and C_E.

Proposition 10. *When the patentee cannot deter entry (a $\hat{b}$ does not exist) and there exists a patent breadth $\tilde{b}$ that makes the entrant indifferent between infringing and not infringing the patent then:*

(a) *The greater are the patentee's monopoly profits (Π_m) the greater is the patentee's incentive to induce infringement.*

(b) *The greater are the patentee's trial costs (C_P) the smaller is the patentee's incentive to induce infringement.*

(c) *The greater are the entrant's costs of producing the better quality product the greater is the patentee's incentive to induce infringement given that the patentee's monopoly profits are different than zero.*

Proof:

(a) $\dfrac{\partial Z^2_P}{\partial \Pi_m} = 1 - \dfrac{9(4\beta + \sqrt{2}\sqrt{\beta}\sqrt{16C_E + 8\beta + 81C_E\beta^2}}{16 + 81\beta^2} \geq 0$ ∀β, C_E such that,

$C_E \le \dfrac{16-72\beta+81\beta^2}{162\beta} \wedge C_E \le \dfrac{512}{6561\beta^3}$.

Since under this scenario the patentee cannot deter entry, the only case that he can make monopoly profits is if his patent is infringed and he wins at trial. Thus, the greater are the monopoly profits that he anticipates to make the greater is his incentive to claim a patent breadth that will induce infringement.

(b) $\dfrac{\partial Z_P^2}{\partial C_P} = -1 < 0$

(c) $\dfrac{\partial Z_P^2}{\partial \beta} = -A + 162\beta B + 8AB(16+81\beta^2) - \dfrac{1296B}{16+81\beta^2} - \dfrac{4B^2}{\beta}(16+81\beta^2)^2 +$

$\Pi_m(-9B(16+81\beta^2)+1458\beta B) \ge 0$

$\forall\beta$, C_E such that, $C_E \le \dfrac{16-72\beta+81\beta^2}{162\beta} \wedge C_E \le \dfrac{512}{6561\beta^3}$ and $\Pi_m > 0$.

Where $A = \dfrac{4 + \dfrac{\sqrt{\beta}(8+162C_E\beta)}{\sqrt{2}\sqrt{16C_E+8\beta+81C_E\beta^2}} + \dfrac{\sqrt{16C_E+8\beta+81C_E\beta^2}}{\sqrt{2}\sqrt{\beta}}}{16+81\beta^2}$ and

$B = \dfrac{4\beta+\sqrt{2}\sqrt{\beta}\sqrt{16C_E+8\beta+81C_E\beta^2}}{(16+81\beta^2)^2}$.

The intuition behind this result is as follows. The greater are the entrant's R&D costs, the closer the entrant is forced to locate to the patentee. In this case, the patentee has a greater incentive to induce infringement because the closer to the patentee the entrant is forced to locate, the smaller need be the patent breadth that will induce the entrant to infringe and thus, the smaller is the probability that the patent will be invalidated at trial.

The effect that C_E has on the patentee's incentive to infringe the patent is inconclusive, it depends on the values of β and Π_m . □

Figure 12 depicts the patentee's expected profits under infringement and his profits under no infringement as a function of patent breadth.

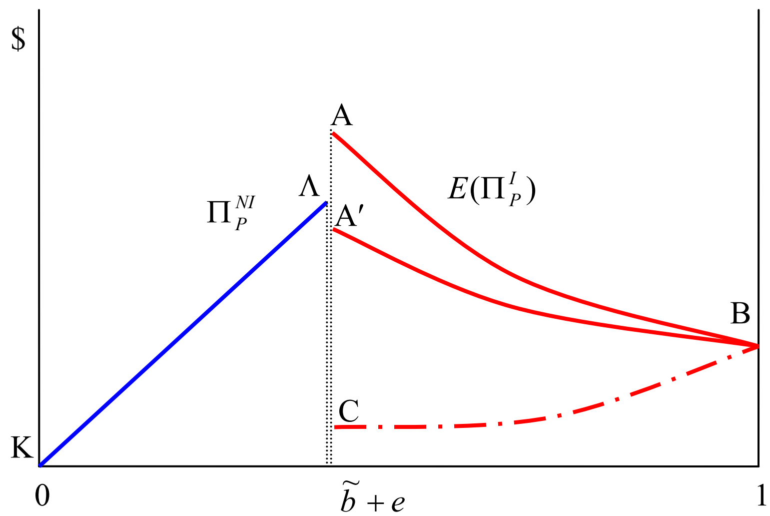


Figure 12. The Patentee's Expected Profits Under Infringement and his Profits Under No Infringement Under Scenario Three

In Figure 12 line KΛ represents the profits that the patentee makes when his patent is not infringed. The curves AB and A′B refer to the expected profits that the patentee makes when his patent is infringed and $E(\Pi_P^I)_{b=1} \leq E(\Pi_P^I)_{b=\tilde{b}+e}$. The curve CB refers to the expected profits that the patentee makes when his patent is infringed and $E(\Pi_P^I)_{b=1} > E(\Pi_P^I)_{b=\tilde{b}+e}$. When the patentee's expected profits under infringement are depicted by the curve A′B or the curve CB, the profits for the patentee are maximized at point Λ where the breadth of patent protection is $\tilde{b}$. When the patentee's expected profits under infringement are depicted by the curve AB profits for the patentee are maximized at point A where the breadth of patent protection is $\tilde{b}+e$.

To summarize the findings of sub-section 3.2.2, the patentee's choice of the optimal patent breadth depends on the entrant's R&D cost structure (β) and her trial costs (C_E). When the combination of β and C_E values is such that both conditions $\beta \geq \frac{8}{9}$ and $C_E \geq \frac{512}{6561\beta^3}$ are satisfied (β and C_E values are in the dotted and horizontally hatched areas in Figure 8), then there exists at least one patent breadth $\hat{b}$ that can deter entry in the market. In this case, the patentee always chooses patent breadth $\hat{b}$ and maximizes his profits (Π_m) operating as a monopolist.

When the combination of β and C_E values is such that both conditions $\frac{4}{9} \leq \beta < \frac{8}{9}$ and $C_E > \frac{16-72\beta+81\beta^2}{162\beta}$ are satisfied (β and C_E values are in the non-shaded area in Figure 8), then there is no patent breadth that can deter entry or that can make the entrant indifferent between infringing and not infringing the patent. The patentee thus finds it optimal to claim the maximum patent breadth (b_{max}=1), since in this case the patent is never infringed and a trial never occurs.

Finally, when the combination of β and C_E values is such that both conditions $C_E \leq \frac{16-72\beta+81\beta^2}{162\beta}$ and $C_E < \frac{512}{6561\beta^3}$ are satisfied (β and C_E values are in the vertically hatched area in Figure 8), then it is optimal for the patentee to choose either patent breadth $\widetilde{b}$ and not induce infringement or $\widetilde{b}+e$ and induce infringement. The choice of the optimal patent breadth in this case depends on the patentee's trial costs (C_P), the monopoly profits that the patentee will make if his patent is found valid at trial (Π_m) and the entrant's R&D cost structure (β) and trial costs (C_E).

4. Concluding Remarks

Economic studies on patents have limited the study of the patenting behavior of the innovator to the analysis of his decision to patent or not to patent the innovation. The innovator's decision to determine the breadth of protection that he will claim, which in turn determines whether the patent will be granted, the breadth of protection granted and the viability of the patent have not been explicitly modeled in the literature. Instead, various studies have assumed that a profit-maximizing innovator will always apply for the broadest protection possible.

In this paper a simple game theoretic model is used to describe the patenting behavior of an innovator who, having invented a drastic product innovation and having decided to seek patent protection, determines the breadth of protection that maximizes the appropriation of the innovation rents from his innovation. To determine the optimal breadth of patent protection claimed, the patentee acts strategically, choosing the breadth of protection that induces the desired behavior by the entrant. The patentee is foresighted and anticipates that he may have to incur costs to enforce his patent rights. The model suggests that the strategic patent breadth, that is, the breadth of patent protection that maximizes the innovators ability to appropriate innovation rents, depends on the entrant's R&D cost structure and her trial costs.

Contrary to what it is traditionally assumed, the results show that it is not always optimal for the patentee to claim the maximum patent breadth possible. In fact, only for certain values of the entrant's R&D effectiveness and trial costs it is optimal for the patentee to claim the maximum breadth of patent protection. The patentee claims maximum patent protection when he cannot deter entry and when the entrant always finds it optimal to not infringe the patent. The maximum breadth of protection is also claimed when the only patent breadth that deters entry $\hat{b}=\widetilde{b}$ is equal to one which occurs for a specific combination of R&D and trial costs (i.e., for $\beta=\frac{8}{9}$ and $C_E=\frac{1}{9}$).

The results hold under the assumption of a fencepost patent system, which implies that the events that the patent is infringed and the patent is invalid can be treated as mutually exclusive and exhaustive. In addition, it has been assumed that the market can only support two products, and that the R&D process is deterministic. Relaxing the above assumptions is the focus of future research.

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