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Estimation of technical efficiency by application of the SFA method for panel data

Abstract. Estimation of the technical efficiency which measures the ability of a company to obtain the maximum output from given inputs or to use the minimum input to achieve given outputs has been considered. Stochastic methods were chosen because of their wide application in research in the whole world. The Translog and Cobb-Douglas stochastic frontiers were fitted in order to estimate the efficiency of milling companies in Poland.

Key words: efficiency, Stochastic Frontier Analysis (SFA), Cobb-Douglas function, Translog function.

Introduction

At the elementary level, the objective of producers can be as simple as seeking to avoid waste, by obtaining maximum outputs from given inputs or by minimizing input use in the production of given outputs. In this case the notion of productive efficiency corresponds to what we call technical efficiency, and the waste avoidance objective of producers becomes the one of attaining a high degree of technical efficiency [Krumbhakar & Lovell 2004]. Generally speaking, the technical efficiency refers to the ability to minimize the input use in production [Krumbhakar & Lovell 2004]. The technical efficiency is a very useful concept to utilize, when firms may be maximizing profits or output subject to profit constraints, as well as when optimizing other goals such as employment. The technical efficiency is a necessary, however not a sufficient condition for profit maximization, and a necessary condition for most of the constrained output maximizations. Therefore, it can be applied within a country to the analysis of firms that have differing objectives [Brada et al. 1997]. The empirical applications of efficiency analysis were conducted in such sectors as accounting, advertising, auditing and law firms, airports, air transport, bank branches, bankruptcy prediction, community and rural health care, dentistry, education, electricity, environment, fishing, forestry, hospitals, hotels, macroeconomics, military activities, rail transport, sports, tax administration, water distribution etc. [Fried et al. 2008].

The measurement of technical efficiency at a business firm level has become a commonplace with the development of frontier production functions. The approach can be deterministic, where all deviations from the frontier are attributed to inefficiency, or stochastic, which is a considerable improvement, since it makes it possible to discriminate between random errors and differences in inefficiency [Wang & Ho 2010]. The main

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methods commonly used to estimate efficiency of a DMU (Decision Making Unit)[2] are the DEA (Data Envelopment Analysis) [Cooper et al. 2007] and the SFA (Stochastic Frontier Approach)[3]. The both methods require all decision making units to have comparable inputs and outputs and both can handle multiple input and multiple output models [Coelli et al. 1998].

The SFA widely uses a stochastic procedure for parametric evaluating the frontier and it is basing on an econometric regression model. The frontier is smooth and appropriately curved. The approach is stochastic, it considers a random variable. The stochastic frontier approach treats deviations from production function as comprising both random error (white noise) and inefficiency [Mortimer & Peacock 2002]. The efficiency score can be measured by applying stochastic frontier techniques to individual annual samples, but in many cases the efficiency differences are notable in a longer time period. For instance in the field of agribusiness, Lakner and Brümmer [2008] apply the stochastic frontier approach to the panel data of German grassland farming; Latruffe, Balcombe, Davidowa and Zawalińska [2002] for Polish farms; Funke and Rahn [2002] for East Germany; Jones, Kleindienst and Rock [1999] for Bulgaria; Kong, Marks and Wan [1999] for China. Nevertheless, there is a lack in the literature of efficiency estimation for food processing companies. In this article, the author has faced this problem and she has carried out a research for a group of Polish and German milling companies. In the milling industry in Poland, concentration processes have been noticed. The small companies fall out from the market which can be caused by a decrease in their efficiency. An affluence of German capital can be observed in Poland which was the second reason for conducting the study. The aim of the paper was to assess and compare the efficiency scores for the companies from both countries.

Measuring efficiency by using the stochastic frontier

The Stochastic Frontier Analysis (SFA) is a method of frontier estimation that assumes a given functional form for the relationship between inputs and an output [Coelli et al. 2005]. The stochastic production function model was proposed independently by Aigner, Lovell and Schmidt [Aigner et al. 1977] as well as by Meeusen and van den Broeck [Meeusen & van der Broeck 1997]. Recently, Kumbhakar, Ghosh and McGuckin [Krumbhakar et al. 1991] and Huang and Liu [1994] proposed stochastic production models that simultaneously estimate the parameters of both the stochastic frontier and the inefficiency functions. Battese and Coelli formulated a stochastic frontier production model similar to that of Huang and Liu and specified it for panel data [Battese & Coelli 1992]. In this paper, the general form of the panel data version by Aigner, Lovell and Schmidt [1977] and the production frontier stated by Coelli, Prasada and Battese [Coelli et al. 1998] is used:

$$\ln y_{it} = f(x_{j.it}, t, \beta) + \varepsilon_{it} \qquad (1)$$

[2] DMUs are the commercial entities that produce tangible goods and services that are sold in the market, enterprises involved in delivering services in the non-market sector, public bodies, the national economic sector etc.

[3] For more information about other panel stochastic frontier models see paper by Wang and Ho [2010].

where $\varepsilon_{it} = v_{it} - u_{it}$ [4],

with $v_{it} \sim N(0, \sigma_v^2)$ and $u_{it} \sim N(\mu, \sigma_u^2)$.

So the equation (1) would be

$$y_{it} = \exp f(x_{j,it}, t, \beta) * \exp(v_{it}) * \exp(-u_{it}) \quad (2)$$

where

f( ) is a suitable functional form (e.g. Cobb-Douglas, Translog),

y_{it} represents the output of the i-th DMU (firm) at time t,

$x_{j,it}$ is the corresponding level of input j of the i-th DMU (firm) at time t, and

β is a vector of unknown parameters to be estimated[5].

The observed deviation of the actual point of production from the frontier $exp(v_{it}-u_{it})$ is a composed error. The v_{it} is a symmetric random error, to account for statistical noise. The symmetric disturbance, v_{it}, is assumed to be due to uncontrollable factors such as weather, making the frontier stochastic. And u_{it} is a nonnegative variable associated with the technical inefficiency of the firm. The statistical noise arises from the inadvertent omission of relevant inputs as well as from measurement errors and approximation errors with the choice of functional form.

Technical efficiency

The technical efficiency of the firm is defined as a ratio of the observed output y_{it} (equation 2) to the maximum[6] feasible output $y_{\max} = \exp f(x_{j,it}; \beta_{it}) * \exp(v_{it})$ in an appropriate environment, defined by a certain level of inputs used by the firm. Thus, the technical efficiency of firm i at time t can be expressed in term of the errors as:

$$TE_i = \frac{y_{it}}{\exp f(x_{j,it}; \beta_{it}) * \exp(v_{it})} \quad (3)$$

so

$$TE_i = \frac{\exp f(x_{j,it}; \beta_{it}) * \exp(v_{it}) * \exp(-u_{it})}{\exp f(x_{j,it}; \beta_{it}) * \exp(v_{it})} \quad (4)$$

$$TE_{it} = E\left[\exp(-u_{it}) \middle| (v_{it} - u_{it})\right] \quad (5)$$

which is the expectation of the exponentiated technical inefficiencies, conditional on the error, ε_{it} (equation 1). Since u_{it} is a nonnegative random variable, these technical efficiencies lie between 0 and unity, where unity indicates that this firm is technically

[4] The value of u_{it} is positive and it decreases the efficiency of an object, therefore we have $-u_{it}$.

[5] The method of maximum likelihood is used for estimation of the unknown parameters, with the stochastic frontier and the inefficiency effects estimated simultaneously.

[6] Maximum feasible output is determined by the firms with inefficiency effect equal to 0 (v_{it}=0).

efficient. Otherwise $TE_i<1$ provides a measure of the shortfall of observed output from maximum feasible output in an environment characterized by $exp(v_{it})$, which allows for variation across producers.

Commonly used method for estimation of a stochastic frontier is a maximum likelihood (ML) method. ML estimations rest on the assumption that the distribution of the errors is actually known. Battese and Coelli (1992) propose a stochastic frontier production function which is assumed to be distributed as truncated normal random variables.

The SFA as a parametric approach requires assuming a specific function form a priori, the frontier is estimated econometrically by some variant of least squares or maximum likelihood approach [Coelli et al. 2005].

Choice of a functional form of the model

When decisions about the function must be made, it is recommended to estimate a number of alternative models and to select a preferred model using the likelihood ratio test [Coelli 1996]. In case of the SFA it is possible to choose one of the following production function models: Cobb-Douglas, CES, Translog, generalised Leontief, normalised quadratic and its variants. The Translog and the Cobb-Douglas production functions are the two most common functional forms which have been used in empirical studies of production, including frontier analyses [Battese & Broca 1997]. However, in many cases a model error is likely to occur because the functional form fitted is usually the Cobb-Douglas, which is highly restrictive. Thus, the adequacy of the Cobb-Douglas should be tested against a flexible functional form, such as the Translog.

A Cobb-Douglas stochastic frontier model takes the form:

$$\ln y_{it} = \beta_0 + \sum_{j=1}^{k} \beta_j \ln x_{j,it} + v_{it} - u_{it} \qquad (6)$$

A Translog stochastic frontier model takes the form:

$$\ln y_{it} = \beta_0 + \sum_{j=1}^{k} \beta_j \ln x_{j,it} + \sum_{j=1}^{k} \sum_{h=1}^{k} \beta_{jh} \ln x_{j,it} \ln x_{h,it} + v_{it} - u_{it} \qquad (7)$$

In the SFA studies, an assumption regarding a specific functional form of stochastic frontier is required a priori. The wrong choice of production function may influence the results. Absolute level of the technical efficiency is quite sensitive to distributional assumptions, rankings are less sensitive.

Application of the SFA model

A stochastic frontier model, of the type originally proposed by Aigner, Lovell and Schmidt [1977], was used. The model allows for decomposing the deviation from production frontier into the statistical noise and inefficiency.

Dataset

The data source contains annual records from the biggest milling companies in Poland and in Germany. The sample includes above 60 companies from both countries. The data include a panel of balance sheets for the period 2004-2007. The production data were all reported as expenditure denominated in PLN in current prices. The production frontiers were fitted for a single output and three inputs. The inputs and the output are identified in Table 1. The input and output variables are described in Table 2.

Table 1. Inputs and outputs used to assess the efficiency scores

Inputs	Outputs
X_1 – costs of production in value terms X_2 – assets in value terms X_3 – mill capacity, tonne	Y – revenue in value terms

Source: own elaboration.

Table 2. Descriptive statistics of the inputs and outputs

Characteristics	Costs of production, PLN thousand	Assets, PLN thousand	Mill capacity, tonne	Revenue, PLN thousand
Mean	189089	50837	292868	188066
Standard error	32664	7730	22755	24472
Standard deviation	488879	115702	340559	366272
Minimum	162	266	10800	554
Maximum	5436338	633596	1402800	2087585

Source: own elaboration.

These inputs and outputs were selected to reflect the cost sources and production possibilities on the input side and the revenue sources on the output side. The dependent variable in such models is often the value added or the profit, but the revenue was preferred because the profit was negative for a certain number of firms, reducing the sample to unacceptable levels.

Specification of the model

It is required to test for the appropriate specification that best represents the data. The stochastic frontier accommodates both Cobb-Douglas and Translog production functions. The functional form of the stochastic frontier was determined by testing the adequacy of the Cobb-Douglas relative to the less restrictive Translog[7]. Thus, the models estimated are defined in equations 6 and 7. The frontier models that are tested are the following:

$$\ln y_{it} = \beta_0 + \sum_{j=1}^{3} \beta_j \ln x_{j,it} + v_{it} - u_{it} \qquad (8)$$

[7] The null hypothesis is that Cobb-Douglas is the appropriate functional form.

and

$$\ln y_{it} = \beta_0 + \sum_{j=1}^{3} \beta_j \ln x_{j,it} + \sum_{j=1}^{3}\sum_{h=1}^{3} \beta_{jh} \ln x_{j,it} \ln x_{h,it} + v_{it} - u_{it} \qquad (9)$$

equation (8) for Cobb-Douglas and equation (9) for Translog respectively. In these equations, ln y is the logarithm of output and the three independent variables (ln x_j) are the logarithms of costs of production, assets, mill capacity in a year of observation. It is important to note that technical efficiencies remain constant over time. The results of testing the functional form of the model were shown in the next part of the paper.

The second test was performed in order to determine whether the inefficiency effects need to be included in the model. The key parameter is $\gamma = \sigma_u^2 / \sigma_v^2$, which lies between zero and unity. If $\gamma = 0$, the technical inefficiency is not present; hence, the null hypothesis is that $\gamma = 0$, indicating that a stochastic frontier model does not need to be estimated and that the mean response function (OLS) is an adequate representation of the data. The closer γ is to unity the more likely it is that the frontier model is appropriate.[8]

Results

The maximum-likelihood estimates of the parameters in the Cobb-Douglas and the Translog stochastic frontier production function models defined by (8) and (9) were obtained using the R-software [A language… 2008]. Hypothesis tests based on the likelihood ratio (LR) test[9] were conducted to select the functional form and to determine the presence of inefficiencies. The likelihood ratio tests (based on log likelihood values for Cobb-Douglas and Translog models) lead to acceptance of the null hypothesis, saying that the Cobb-Douglas is an appropriate functional form (equation 8). Therefore, the empirical results obtained from estimating only the Cobb-Douglas function are reported in this section (Table 3). The summary statistics of obtained technical efficiency scores are presented in Table 4.

The lower part of table 3 reports the results of LR tests of the hypothesis that the technical efficiency effects are not simply random errors. The null hypothesis that the vector γ is equal to zero is decisively rejected, suggesting that inefficiencies are present in the model and that running average production functions is not an appropriate representation of the data. The closer γ is to unity, the more likely it is that the frontier model should be chosen. The value of γ is equal to 0,792 which indicates that 79,2% of the deviation in data is due to the technical inefficiency of enterprises.

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[8] Since γ takes values between 0 and 1, any LR (likelihood ratio) test involving a null hypothesis that includes the restriction that γ has been shown to have a mixed χ^2 distribution, with appropriate critical values [Kodde & Palm 1986].

[9] The likelihood-ratio test statistic, $\lambda = -2\{\log[\text{likelihood}(H_0)] - \log[\text{likelihood}(H_1)]\}$ has approximately χ^2_q distribution with q equal to the number of parameters assumed to be zero in the null hypothesis, where likelihood (H_0) and likelihood (H_1) are the values of the likelihood function under the specification of the null hypothesis and the alternative hypothesis.

Table 3. Final maximum likelihood estimates for the Cobb-Douglas function

Item estimated	Estimate	Std. Error	t value	Pr(>|t|)
Intercept	2.922	0.697	4.191	2.78e-05
LX1	0.489	0.067	7.312	2.64e-13
LX2	0.090	0.035	2.543	0.011
LX3	0.445	0.081	5.496	3.88e-08
σ^2	0.457	0.160	2.847	0.004
γ	0.792	0.092	8.558	2.23e-16
Time	0.007	0.030	0.246	0.805
Log likelihood value	-104,3711			

Source: own calculations based on results from using the R-software [A language… 2008].

By interpreting the results of the inefficiency function one should keep in mind that a negative coefficient reflects reduced firm inefficiency and, hence, increased efficiency. The scores of the technical efficiency are negatively related to all of inputs which indicates that increasing of X_1 (costs of production), X_2 (assets) or X_3 (mill capacity) for producing the same amount of output would lead to a decrease in efficiency, hence an increase of inefficiency. The highest influence on efficiency score was observed in case of the input X_3 i.e. costs of production.

The sum of estimated parameters (exponents, which are elasticity coefficients) for all inputs included in the model informs about the scale effects for the sample. One can observe that the analyzed enterprises operate on the increasing returns to scale (because the sum of all parameters is bigger than 1 [Rembisz 2011]).

The mean efficiency scores for each of four years of analysis are presented in Table 4. In the analyzed period, the efficiency of mills was on the level of 0,65 which indicates a low level of technical efficiency. The milling industry could have produced, on average, the same output by using 35% less of inputs.

Table 4. The mean efficiency scores for period 2004-2007

Characteristics	Year			
	2004	2005	2006	2007
Mean efficiency in a year	0.6528	0.6746	0.6464	0.6382
Average efficiency	0.6530			
Standard deviation	0.0156			

Source: own calculations.

It is to note that the level of the technical efficiency was not very fluctuating over the time period 2004-2007, its average level amounts to 0.653 (standard deviation 0.016). One of the reasons for that could be including the mill's capacity as an input. On the one hand the capacity is an important element of technology and, as Table 3 shows it, this input influences quite strongly the level of efficiency. However, in the analyzed period of time any significant changes in mill size have not been registered.

Conclusions

The traditional econometric belief in the presence of external forces contributing to the random statistical noise is continuously being maintained. Thus, it is desirable for the econometric approach to be relatively more successful than others, so as to provide the basis for a subsequent investigation into determinants of variations in the efficiency. On the other hand, a researcher has to choose the functional form of the frontier and to make an assumption regarding to distribution of variation in inefficiency. A wrong choice may be corrected on the basis of statistical tests (e.g. the likelihood ratio test or, alternatively, the Wald's test).

For estimation of the efficiency scores, the SFA method based on the Cobb-Douglas function was used. The results showed that the scores of the technical efficiency are negatively related to all of inputs which indicates that increasing of X_1 (costs of production), X_2 (assets) or X_3 (mill capacity) for producing the same amount of output would lead to a decrease in efficiency, hence an increase of inefficiency. The milling industry could have produced, on average, the same level of output by using 35% less of inputs. But one can observe that the analyzed sector operate on the increasing returns to scale.

The stochastic frontier approach can be a useful tool for estimating the technical efficiency of firms by including the influence of time. However, the technical efficiency scores obtained from estimation of the stochastic frontier have a little use for policy implications and management purposes if the empirical studies do not investigate the sources of the inefficiency. It is recommended to make an analysis of the sources of technical inefficiency such as, for instance, the degree of competitive pressure, the ownership form, various managerial characteristics, network characteristics and production quality indicators of inputs or outputs.

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