# MANAGING PHOSPHOROUS SOIL DYNAMICS OVER SPACE AND TIME

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## Abstract

Understanding the relationship between soil fertility dynamics and crop response is conceptually appealing. Even more appealing is comprehension of the spatial and temporal heterogeneity of these connections over a production surface and across seasons. Knowledge of these interactions is complicated because nutrient carryover dynamics and crop response to inputs are determined simultaneously on the one-hand, and sequentially on the other. A second problem enters when crops are rotated, for example, in the corn-soybean system commonly practiced in the Corn Belt. This paper examines the nutrient carryover-crop response nexus using data from a corn-soybean, variable-rate nitrogen (N) and phosphorous (P) experiment conducted over five years. Site-specific corn response to N and P and soybean response to P are simultaneously estimated with a P carryover equation. These estimates are used in a dynamic programming model to map site-specific optimal N and P fertilizer policies, soil P evolution, and profitability. The net present value of managing N and P site-specifically is compared to a strategy where these inputs are managed uniformly following extension guidelines. The results suggest that when P-carryover is managed, site-specific returns to the variable-rate strategies are higher than returns to a conventional, uniform strategy.

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## 1. Introduction

Phosphorous in surface runoff and tile outflow from cropland is an important environmental problem in some areas. Among other problems, phosphorous contributes to eutrophication in lakes and ponds. Phosphorous is also a relatively expensive fertilizer. The general goal of this study was to develop a method for site-specific management of phosphorous soil fertility that took into account both temporal and spatial variability. The initial focus was on optimizing the net present value of phosphorous fertilizer with the hope of reducing phosphorous application above economic rates. This study could inform farmers, fertilizer dealers, crop production economists and those interested in better management of water and soil resources.

Managing phosphorous soil fertility is complicated by the fact that phosphorous carries over from year to year and the rate of carryover varies widely within fields. Some phosphorous fertilizer recommendations assume that soil phosphorous should be built up over time until an optimum level is reached, and then fertilizer application is reduced to maintain optimal levels (Black, 1993 for some examples). Depending on soil chemistry, the rate at which soil phosphorous accumulates varies widely within fields. In some areas fertilizer phosphorous converts almost directly as equivalent soil phosphorous. In other areas it is almost impossible to build soil phosphorous, even with large applications. This means that with uniform-rate applications, phosphorous is overapplied in some parts of fields, increasing the chance that excess phosphorous finds its way into waterways, and underapplied in other parts of fields, reducing yields and profits. Knowledge about crop response functions to inputs over the production surface and over time and information about site-specific nutrient carryover dynamics enables producers to optimally manage inputs using variable rate technologies (VRT).

One of the first analyses of the impact of nutrient carryover dynamics on VRT profitability was Schnitkey et al.'s (1996) study investigating P and K management. The results of their simulation found that precision application of P and K was profitable over a thirty-year horizon. What is unclear is how generalizable these results are to other production fields in large part because the source of the response and nutrient carryover parameters are not well-documented. Lowenberg-DeBoer and Reetz (2002) borrowed Schnitkey et al's (1996) carryover parameters to compare a VRT program that used rapid soil P and K build-up to a fertilizer program that followed a gradual nutrient build-up strategy using regional fertilizer rates recommended by research extension. They found that the economically preferred strategy was one where fertilizer applications build soil P and K to carrying capacity levels in the first year followed by maintenance applications to sustain optimal soil nutrient levels thereafter. Popp et al. (2002) simulated P carryover dynamics over a ten-year production period for a rice-soybean rotation, but their focus was on VRT, the dynamics particular to different soil types, and profitability. They found that VRT profitability over a ten year horizon depended on soil clay content.

The fundamental model shared by studies that have investigated how site-specific management is affected by nutrient carryover dynamics was originally developed by Kennedy et al. (1973) and later refined by Kennedy (1986). In these seminal studies optimal fertilizer application was framed as a discrete time, optimal nutrient replacement problem conditional upon residual effects of fertilizer. The producer's intertemporal optimization problem was therefore complicated by the impact of fertilizer carryover from applied fertilizer and residual soil nutrients from previous applications on the time preference for money.

There are few studies investigating the effects of nutrient carryover dynamics on VRT as a fertilizer management strategy perhaps because, until recently, multi-year data sets from controlled VRT experiments with both yield and soil test data were unavailable. Another reason may be that the parameters regulating nutrient carryover dynamics are challenging to estimate. First, there is the issue of functional form choice available to describe carryover dynamics (Black, 1993, pp. 519-563). Kennedy (1986) demonstrated that linear carryover equations are especially useful for determining the impact of residual fertilizer on input management decisions because the producer's intertemporal optimization problem with respect to fertilizer use is reduced to a Markov decision making problem. This simplification lends itself to an appealing economic interpretation with respect to sequential, intertemporal optimality conditions. Secondly, estimation of carryover dynamic coefficients is data-intensive and econometrically challenging, requiring systems of equations with the usual problems associated with endogeneity (see Jomini, 1990 as an example). This estimation challenge is further complicated by the spatial nature of VRT studies, and the fact that agronomic data is oftentimes heteroskedastic or spatially autocorrelated (Anselin et al., 2004; Hurley et al., 2003). Additional modeling complications arise when crops are seasonally rotated (for example, soybeans following corn).

This paper examines the nutrient carryover-crop response nexus using data from a corn-soybean, variable rate nitrogen (N) and phosphorous (P) experiment conducted over five years in Southern Minnesota. Site-specific corn response to N and P and soybean response to P is simultaneously estimated with a P carryover equation using three-stage least squares. The estimates are used in a dynamic programming model to map site-specific optimal N and P fertilizer policies, soil P evolution, and profitability.

The study is a departure from previous VRT studies in that production data from a multi-period, controlled variable rate application (VRA) nitrogen and phosphorous (VRA-NP) experiment are used to simultaneously estimate site-specific response functions for corn and soybean with a P carryover equation. Subsequently, the estimated parameters are used in a dynamic programming environment comparing a uniform management strategy (UNI) to three VRA management strategies, and the effects P carryover dynamics have on the intertemporal management of N and P. Although the carryover equation used in this study is different than the functional forms used in the Kennedy studies, the same optimality condition derived by Kennedy (1986), and most recently by Thomas (2003) is obtained after differentiation of Bellman's equation and application of the envelope theorem. The results found in this study have implications for the spatial management of P and N in particular, and development of dynamic fertilizer models in general. The results are intended as a methodological example of how crop response to variable rate inputs and nutrient carryover dynamics can be simultaneously modeled.

The profitability analysis is ex post because it assumes that the producer knows, in hindsight, yield response and optimal N and P applications (Bullock and Bullock, 2000). Ex post analyses are useful as starting points in economic assessments of solutions to management problems. While ideally such evaluations should be based on ex ante decision-making, ex post estimates provide insight into possible outcomes of VRA-UNI comparisons. If ex post results are not profitable, then ex ante results are unlikely to be profitable.

Alternatively, if the ex post approach does show profitable results, there still remains the question of how well it would do in an ex ante decision making context.

## 2. Methodology

### 2.1. Model Development

Consider a producer who maximizes net present value (NPV) from a corn-soybean rotation system over future periods. Assume that crop response to inputs and soil fertility is heterogeneous across the production field and that plant response to inputs covaries with soil fertility. The next assumption is that the physical and chemical processes that regulate nutrient carryover (for example, chelate formation, cation exchange, and denitrification) vary continuously over the production surface. Third, assume that input factor prices and technology are constant. Fourth, the producer can choose to manage *z* areas site-specifically if yield response to inputs and fertility of these sites is known. Or, the producer can choose to apply a uniform fertilizer rate following extension recommendations at the beginning of production cycle *m*. Lastly, assume that the producer controls the level of total available phosphorous (TAP) by choosing applied fertilizer P to maximize the NPV of the *m*th corn-soybean rotation; $TAP_m = \lambda F_m + P_m$ with $\lambda \in [0,1]$ (see Kennedy et al. (1973), Kennedy (1986), Jomini (1990), Tré (2000) and Thomas (2003) for similar specifications). Total available phosphorous at the beginning of the *m*th rotation is a function of applied fertilizer P at the beginning of rotation *m* ($F_m$) and phosphorous available in the soil solution ($P_m$) before fertilizer application. The parameter $\lambda$ measures the substitution between applied P fertilizer and soil phosphorous.

Profit in any given corn-soybean rotation *m* in site *z* is:

$$\Pi_{m,z} = \delta p_c f_{z,m}\left(N_{m,z}, TAP_{m,z}\right) + \delta^2 p_s g_{z,m}\left(P_{c,m}\right) - \mathbf{r}'_m \mathbf{X}_{m,z} - FIXED\ COSTS_m \qquad (1)$$

where $p_c$ and $p_s$ are corn ($c$) and soybean ($s$) prices, $f_{z,m}(\cdot)$ and $g_{z,m}(\cdot)$ are concave corn and soybean SSCRF's corresponding with site $z$; $\mathbf{X}_{m,z}$ is an input vector applied at the beginning of rotation $m$ in site $z$ before corn is planted (here, $\mathbf{X}_{m,z} = \left[N_{m,z}, F_{m,z}\right]'$); $\mathbf{r}_m$ is a corresponding vector of input prices at the beginning of rotation $m$ with $r_P$ the price of P fertilizer and $r_N$ the price of N; $P_{c,m}$ is fertilizer P carryover following corn production in rotation $m$; and $\delta = \left(1+\rho\right)^{-t}$ is a discount factor with the opportunity cost of capital $\rho$. Fixed costs include, but are not limited to, soil test costs, mapping fees, and fertilizer application costs. Corn is the first crop planted by the producer at the beginning of the $m$th of $M$ sequences in this rotation system (Figure 1). It is also assumed that no N or P is applied between corn harvest and soybean planting, and that N is not a decision variable with respect to soybean production.

A phosphorous carryover function describes the evolution of phosphorous within and between corn-soybean rotations. For development of the model consider the most general form of a state transition function: new stock = weathering + old stock + recharge – draw down. Or in the present case $P_{m+1} = c_0 + \alpha(P_m + \lambda F_m) - \gamma Yield_m$. The carryover function is identical to the function described by Kennedy (1986, page 173), but includes an intercept term representing unavailable P tied up in the soil solution ($c_0$), and the marginal contributions of carryover P ($\alpha$), applied P ($\alpha\lambda$), and plant P use ($\gamma$) in the current state to P carryover in the next state. When the state transition function is linear the decision making sequence faced by the producer reduces to a Markov decision process. The dynamic problem is memoryless because the distribution of $F_{m+1}$, conditional on history of the process through production cycles is only determined by $F_m$. Therefore, given $F_m$, $F_{m+1}$ is independent of the prior realizations of the process (Miranda and Fackler, 2002). Put another way, the only history that matters is current history in this problem. All relevant information with respect to input management is embodied in the knowledge regarding the current production cycle.

Corn can use soil phosphorous from the previous corn-soybean rotation in the current *m*th rotation ($P_m$) (Figure 1). Part of this carryover stock is used in corn production (nutrient draw-down), some of the applied fertilizer P is converted into the soil P stock, while the remaining stock is either re-incorporated as P soil stock for soybeans in the next season or tied up in the soil solution. This conversion path of soil P following corn production ($P_{c,m}$) is summarized as $P_{c,m} = P_{c,m}\left(TAP_{m,z}, f_{z,m}\left(N_m, TAP_{m,z}\right), s_{c,m,z}\right)$, where $s_{c,m,z}$ are other soil characteristics (for example, pH, potassium, or organic matter content). Recalling the general linear carryover model, $\partial P_{c,m}/\partial P_m = \alpha \in [0,1]$, $\partial P_{c,m}/\partial f_{z,m} = \gamma_c < 0$, and $\partial P_{c,m}/F_m = \alpha\lambda \in [0,1]$. The phosphorous available following corn production is either used in soybean plant growth, tied up in the soil solution, lost to P run-off attached to sediment, erosion, or carried over into the next rotation as available P for corn. This dynamic is summarized with the function $P_{m+1} = P_{m+1}\left(P_{c,m}, g_{z,m}\left(P_{c,m}\right), s_{c,m,z}\right)$ where $\partial P_{m+1}/\partial P_{c,m} = \alpha \in [0,1]$ and $\partial P_{m+1}/\partial g_{z,m} = \gamma_s < 0$. The partial derivatives $\partial P_{m+1}/\partial P_{c,m}$ and $\partial P_{c,m}/\partial P_m$ are equal because the soil physiochemical processes regulating the P-stock carryover dynamic are assumed to be the same between crop productions. Because P is not applied before the soybeans $\partial P_{c,m}/F_m$ does not appear in this carryover function. It is also assumed that applied N does not contribute to P carryover.

Assume nitrogen is applied uniformly or site-specifically. If the producer chooses SSM quasi-fixed costs are subtracted from the profit objective, including soil-sampling costs, management zone identification costs (for example, maps), and variable rate application costs (Bullock et al., 2002). The producer choosing the UNI management strategy incurs uniform application costs which are lower than VRA costs (Aghib and Lowenberg-DeBoer, 1999). Any soil test costs used to adjust application to extension recommended input rates are deducted from the objective of the producer choosing the UNI strategy.

The producer maximizes the expected value of the stream of future discounted profits from *m* to the terminal period *M* because production and carryover dynamics are stochastic:

$$\max_{F_{m,z}} E_m \left( \sum_{m=1}^{M} \sum_{z=1}^{Z} \delta^{m-1} \omega_z \Pi_{m,z} \right) \quad \text{s.t. } V(P_M) = 0 \text{ (boundary condition)} \tag{2}$$

The proportion of the field covered by management zone *z* is $\omega_z$ with $\sum_z \omega_z = 1$. For the moment the site-specific subscripts are suppressed for exposition. Let $V(P_m)$ be the maximum value of equation 2 in rotation *m*. The NPV can also be expressed using Bellman's equation according to the maximum principle of dynamic programming (Léonard and Van Long, 1992, page 176):

$$V_m(P_m) = \max_{F_m} E_m \left\{ \Pi_m + \delta^2 V_{m+1}(P_{m+1}) \right\} \tag{3}$$

The available soil phosphorous $\left(P_m\right)$ at the beginning of rotation $m$ is the state variable and $F_m$ (applied fertilizer) is the control variable. The producer's problem is broken down into a sequence of choices made with respect to applied fertilizer P, and the recurrence relation $\left(V_{m+1}\left(P_{m+1}\right)\right)$ represents the impact of these decisions.

Kennedy (1986) and Dillon and Anderson (1990) show that dynamic programming problems characterizing soil fertility management are solved recognizing that the decisions made between two periods will hold for any $m$ → $m$+1 sequence when the state transition function is linear. To develop these ideas, consider a producer's intertemporal optimization problem as one involving only fertilizer and carryover phosphorous. For the general case describing P management from rotation $m$ to $m$ + 1,

$$V_m\left(P_m\right)=\max_{F_m}\left\{\delta p_c f_m\left(TAP_m\right)-r_P F_m+\delta^2 p_s g_m\left(P_{c,m}\right)+\delta^2 V_{m+1}\left(P_{m+1}\right)\right\} \tag{4}$$

with the first order condition (FOC) with respect to fertilizer P

$$\frac{\partial V_m}{\partial F_m}=\delta p_c\frac{\partial f_m}{\partial TAP_m}\lambda-r_P+\delta^2 p_s\frac{\partial g_m}{\partial P_{c,m}}\left(\alpha\lambda+\gamma_c\frac{\partial f_m}{\partial TAP_m}\lambda\right)+\delta^2\frac{\mathrm{d}V_{m+1}}{\mathrm{d}P_{m+1}}\frac{\mathrm{d}P_{m+1}}{\mathrm{d}TAP_m}\lambda=0 \tag{5}$$

Multiplying equation 5 by $\lambda^{-1}$ yields the discounted marginal value products (MVP) of total available P for corn and soybean production in rotation $m$:

$$MVP_{corn,m}+MVP_{soy,m}+\delta^2\frac{\mathrm{d}V_{m+1}}{\mathrm{d}P_{m+1}}\frac{\mathrm{d}P_{m+1}}{\mathrm{d}TAP_m}=MFC\lambda^{-1} \tag{6}$$

with the marginal factor cost (MFC) being the fertilizer P input price ($r_P$), MFC$\lambda^{-1}$ the marginal cost of soil P, $MVP_{corn,m} = \delta p_c \frac{\partial f_m}{\partial TAP_m}$, and $\delta^2 p_s \frac{\partial g_m}{\partial P_{c,m}}\left(\alpha + \gamma_c \frac{\partial f_m}{\partial TAP_m}\right) = MVP_{soy,m}$.

The envelope theorem is applied to find $\mathrm{d}V_{m+1}/\mathrm{d}P_{m+1}$ using equation 4:

$$\frac{\partial V_m}{\partial P_m} = \delta p_c \frac{\partial f_m}{\partial TAP_m} + \delta^2 p_s \frac{\partial g_m}{\partial P_{c,m}}\left(\alpha + \gamma_c \frac{\partial f_m}{\partial TAP_m}\right) + \delta^2 \frac{\mathrm{d}V_{m+1}}{\mathrm{d}P_{m+1}} \frac{\mathrm{d}P_{m+1}}{\mathrm{d}TAP_m} = 0 \tag{7}$$

Assuming input prices are constant and updating $\partial V_m/\partial P_m$ by one period, the result is that $\mathrm{d}V_{m+1}/\mathrm{d}P_{m+1} = MFC\lambda^{-1}$, or when marginal value of carryover soil stock P is equal to the cost of soil P. This identity yields the following optimality condition; $MVP_m = MFC\left(1-\delta^2\alpha\right)\lambda^{-1}$, where $MPV_m = MVP_{corn,m} + MVP_{soy,m}$ from equation 6. Kennedy (1986, page 174) and Thomas (2003) derived similar results. This result extends their findings by including nutrient draw down effects and by considering crop rotation. The cost of P application increases when $\lambda$ approaches zero because it is the substitutability of soil P for applied P. Conversely, MFC decreases as $\lambda \rightarrow 1$. The benefit received by managing P carryover is the expression just to the left of the MFC. Therefore fertilizer P costs are reduced by optimally managing P soil stock. The optimality condition holds for any site $z$; $MVP_{m,z} = MFC\left(1-\delta^2\alpha\right)\lambda^{-1}$.

## 2.2. Econometric Model to Estimate P Carryover

The fertilizer carryover models of Jomini (1990), Tré (2000), and Thomas (2003) are modified to estimate $\lambda$, $\alpha$, $\gamma_c$, $\gamma_s$, $\partial f / \partial TAP$, and $\partial g / \partial P_{c,m}$ :

$$P_{j,m,z} = c_0 + \alpha TAP_{m,z} + \gamma_j y_{j,m,z} +$$

$$\sum_p^S \left( \varphi_p s_{p,m,z} + \psi_p TAP_m s_{p,m,z} + \xi_p s_{p,m,z} s_{q,m,z} \right) + u_{j,m,z},\ p \neq q \tag{8}$$

$$\begin{aligned} y_{j,m,z} = {} & \beta_{0,j,m,z} + \sum_z \delta_{z,0} d_{z,0} + \left( \beta_{N,j,m,z} + \sum_z \delta_{z,N} d_{z,N} \right) N_{m,z} + \left( \beta_{N^2,j,m,z} + \sum_z \delta_{z,N^2} d_{z,N^2} \right) N^2_{j,m,z} + \\ & \left( \beta_{P,j,m,z} + \sum_z \delta_{z,P} d_{z,P} \right) TAP_{j,m,z} + \left( \beta_{P^2,j,m,z} + \sum_z \delta_{z,P^2} d_{z,P^2} \right) TAP^2_{j,m,z} \\ & + \left( \beta_{NxP,j,m,z} + \sum_z \delta_{z,NxP} d_{z,NxP} \right) N_{m,z} TAP_{j,m,z} + \varepsilon_{j,m,z} \end{aligned} \tag{9}$$

$$TAP_{j,m,z} = \begin{cases} \lambda F_{m,z} + P_{s,m} & \text{for corn} \\ P_{c,m} & \text{for soybeans} \end{cases} \tag{10}$$

where $y_{j,m,z}$ is yield from crop $j$ ($j$ = corn or soybean) in cycle $m$; $P_{j,m}$ is the phosphorus stock following crop $j$ in cycle $m$ (kg ha$^{-1}$); $F_m$ is applied P in cycle $m$ (kg ha$^{-1}$); $s_{p,z,m}$ are $p$ additional site-specific soil characteristics ($p$ = percent organic matter, K, and pH); and $u_{j,m,z}$ and $\varepsilon_{j,m,z}$ are disturbance terms. Note that when $j$ = corn and $k$ = soybean, $m = m - 1$ for $P_{j,m}$. Conversely for $P_{j,m}$, when $j$ = soybean and $k$ = corn, $m = m$.

Equation 8 estimates the whole field phosphorous carryover dynamic. Equation 9 estimates the site-specific yield response to total phosphorous and nitrogen for corn and total phosphorous for soybean. The yield response parameters $\beta_i$ are the weights that determine how much total available P and applied N are used in plant production. The coefficients of the site-specific indicators ($d_{m,z}$) are constrained as $\sum_z \delta_{z,i} = 0$. Equation 10 is the total phosphorous available for plant production in period $m$ for crop $j$.

Hoeft and Peck (2000) estimate that about 25% of the applied fertilizer P is expected to contribute to available P stocks. The expected value of $\alpha\lambda$ should be near this value. That is, approximately 4.48 kg $ha^{-1}$ of elemental P is needed to increase soil test P levels 1.12 kg $ha^{-1}$. Because P is not applied before the soybean cycle $\alpha\lambda$ does not appear in the P carryover equation following soybeans.

Crop removal estimates for corn are about 0.007 kg of P $kg^{-1}$ of corn, and 0.013 kg of P $kg^{-1}$ of soybeans (Hoeft and Peck, 2000). The expected values of $\gamma$ for corn and soybean should be close to these numbers, although values lower than these have been used (Schnitkey et al., 1996).

The $\lambda$ parameter measures the rate of substitution of fertilizer P for soil P where $E[\lambda] \in [0,1]$. The contribution of applied P to plant growth is appreciable when $\lambda \approx 1$. This parameter has no direct effect on soybean response because P is not applied in the soybean year. Likewise $\beta_3$, $\beta_4$, and $\beta_5$ do not appear in the soybean response equation because the focus of the field trial was not on N for soybeans.

A schematic of the carryover-production system and associated parameters is presented in Figure 2. The system was estimated using three stage least squares. Parameter significance was tested using White's heteroskedastic-robust standard errors.

### 2.3. Corn and Soybean Crop Response Data

The research site (Windom, Minnesota, $43^{\circ}90'\mathrm{N}, 85^{\circ}05'\mathrm{W}$ ) was established in the fall of 1996. The 12.2 ha production field has been in a corn-soybean rotation for the last 20 years with no manure applied during that time frame. The site was extensively grid sampled in the fall of 1996 and annually thereafter. Soil tests were taken in the control strips, only. Soil organic matter contents ranged from less than 2% to 10%. Phosphorous (P) soil tests ranged from very low (<5 parts per million, ppm) to very high (>15 ppm), and soil pH ranged from 6.0-8.0.

The experimental design was three replications of 13 treatments set out in a split plot arrangement of a randomized complete block design where P was the main plot. Nitrogen rates were randomized in the P treatments. Treatments were applied at constant rates in strips across the entire field. The P treatments were randomized within each replication. All treatments were repeated three times (a total of 39 strips). Nitrogen treatments ranged from 0, 67, 112, 157, and 202 kg $\mathrm{ha}^{-1}$, while P treatments ranged from 0, 56, and 112 kg $P_2O_5$ $\mathrm{ha}^{-1}$. Urea was applied in fall 1996, 1998, and spring 2001 (before planting). In the fall of 1998 and 2000 P fertilizer treatments were applied to the same areas within the field that had received the treatments in 1996. The experiment was designed to accommodate extra N treatment strips that would ensure zero N rate treatments were not placed in the same area as previous zero N rate treatments.

The 2001 growing season provided the fifth year of research information from this site, and the third year for corn production. Grain yield determination was made every 15 m through each treatment strip using a Model 8 Massey Ferguson plot combine (AGCO Corp., Duluth, GA) equipped with a ground distance monitor and a computerized HarvestMaster weigh cell (HarvestMaster, Logon, UT). Every 15 m the combine was stopped and the harvest grain was georeferenced and weighed. P-Olsen and P-Bray tests were doubled to proxy available soil P as kg ha$^{-1}$. When pH was less than 7.4 P-Bray tests were used to proxy soil P content.

Expected corn and soybean response functions were estimated using the corn response functions from the 1997, 1999, and 2001 production years, and the soybean response functions for the 1998 and 2000 years. These expected functions are the weighted average of the estimated response functions. The yearly weights ($w_t$) were calculated as $w_t = \prod_{l,t} \phi(ppt_{l,t}) \big/ \sum_t \prod_{l,t} \phi(ppt_{l,t})$; $l$ = April, May,…, September; $t$ = 1997, 1999, and 2001 for the corn years, and 1998, 2000 for the soybean years; *ppt* is the total precipitation observed in Cottonwood County, MN, in month $l$, year $t$; and $\phi(\cdot)$ is the normal probability density function (pdf). This weighting plan is based on the general rule of probability multiplication, $P\left( A_1 \bigcap_{i \in [1,\ldots,N-1]} A_i \right) = \prod_{i \in [1,\ldots,N]} P(A_i)$ assuming that rainfall in month $l$ is independent of rainfall in month $l$-1. Expected response coefficients are estimated as the weighted average of the response coefficients, $\sum_t w_t \beta_{i,t}$. These weights capture some of the stochastic effects attributable to variable rainfall and other weather-related events in each of the production years. Cottonwood County MN precipitation data from 1991 to 2003 was used to estimate these weights (Minnesota State Climatology Office-Department of Natural Resources and Water, 2004).

## 2.4. Management Zone Determination

There are hundreds of classifications whereby management zones could be constructed. For example, management zone classifications could be organic matter, soil type, elevation, soil depth, or pH. In this study, an unsupervised *K*-means cluster analysis available in PROC FASTCLUS (SAS, 2000) was used to isolate clusters identifying portions of the field that exhibit similar yield response patterns. The following steps were used to identify clusters for management zone identification. First, the study site was subdivided into 69 sub-blocks following the yield response estimation methodology of Mamo et al. (2003). For the corn years each sub-block had 13 yield points. For the soybean years sub-blocks included 9 observations because the focus of the soybean study was not on N. This is the highest resolution with respect to identifying yield response heterogeneity of the site. Second, yield response was estimated for each sub-block using a quadratic function. Third, because the objective of this study is soil P fertility management, intercept terms corresponding with each block and each production year were used in the clustering algorithm to delineate fertility management zones. The intercept terms predict what corn or soybean growth would be without N or P fertilizer. Therefore, they represent the fertility of a given response block, conditional upon plant response to N and P. Last, the clusters were then used in Equation 8 to proxy fertility management and plant production zones.

## 2.5. Prices

The average market prices (1997 to 2001, NASS, 2002) for corn and soybean observed in Cottonwood County, Minnesota, were used as output prices (\$0.079 $kg^{-1}$ and \$0.175 $kg^{-1}$, corn and soybean, respectively). Input prices were \$0.077 and \$0.118 $kg^{-1}$ for N and P, respectively. Information costs for SSM included costs for map making and management zone development ($2.96 per map), and soil sampling, including lab fees and collection ($13.59 ha$^{-1}$). These are the costs reported in the 1997 Akridge and Whipker dealership survey. Variable rate application costs ($13.21 ha$^{-1}$) were the average of the VRT costs reported in Akridge and Whipker (1997), Akridge and Whipker (1999), and Whipker and Akridge (2001). A uniform application cost of $9.88 ha$^{-1}$ is assumed for the UNI budget analysis (Aghib and Lowenberg-DeBoer, 1999). These costs are doubled because P and N are usually applied at different times. Because the UNI strategy uses soil test information to follow extension recommendations for P, a WF soil test fee of $0.82 ha$^{-1}$ is charged under the uniform strategy. Mapping and soil test fees are charged once (in the 1997 year) because these products are assumed to have a useful life of approximately four years (Swinton and Lowenberg-DeBoer, 1998). Uniform and VRT fees are only charged before the corn production years. The NPV of the VRT strategy is compared to the NPV of returns from the UNI N and P fertilization strategy.

## 3. Cluster Analysis Results and Management Zone Characteristics

According to the cubic clustering criterion (Sarle, 1983) the optimal number of clusters was three, indicating that this was the optimal number of management zones using the intercept terms as fertility proxies ($R^2$ = 0.65). Corn and soybean yields were lowest in cluster grouping (CG) 2, and highest in CG 1 (except in 2001). CG 2 had the highest average %OM (4.51%), followed by CG 1 (4.42%) and CG 3 (4.14%). All pH test averages were similar across clusters (~7.1). CG 3 recorded the highest soil P levels (7.4 ppm), followed by CG 1 (7.2 ppm P), then CG 2 (7.0 ppm P). Potassium soil test levels were lowest in CG 2.

The yield rankings suggest that the CG 1 clusters are high fertility areas, followed by CG 3, then CG 2. Proportions of the field represented by CG 1, CG 2, and CG 3 were 26%, 25%, and 49% (Table 1). From here on, cluster groupings CG 1, CG 2, and CG 3 correspond with management zones $z_1$, $z_2$, and $z_3$, respectively.

## 4. Carryover and Crop Response Regression Results

### 4.1. Carryover Parameters

The P carryover models (equation 8) explained between 69% and 81% of the variation in the soil stock P data (Table 2). The marginal contribution of soil stock P in period $m \rightarrow m+1$ was significant ($\alpha$ = 0.82, T = 10.69), indicating that 82% of soil P carries over into future stock. The $\lambda$ parameter indicates that 25% of fertilizer P is used in corn production in any given rotation. Jomini (1990) estimated the analogue phosphate fertilizer parameter of 87% for millet in Niger, while Tré (2000) estimated a fertilizer potassium parameter of 52% for plantains in Nigeria. Phosphorous uptake efficiency by plants is compromised in soils with relatively high pH readings, and this lower value is not too surprising.

The soil P drawn-down parameters for corn ($\gamma_c$ = -0.00009) and soybean ($\gamma_s$ = -0.001) were lower than the expected values (-0.007 and -0.013 for corn and soybean, respectively). However, the P draw-down coefficient for soybean estimated here is similar in magnitude to those used by Schnitkey et al. (1996) and Lowenberg-DeBoer and Reetz (2002) (soybean = -0.0007). For corn phosphate drawdown Schnitkey et al. and Lowenberg-DeBoer and Reetz used a coefficient of -0.0013. The marginal contribution of applied fertilizer P to P carryover

($\alpha\lambda$ = 21%) was significant (T = 10.84) but not different at the 5% level from the expected value (25%) reported in the agronomic literature (likelihood ratio test = 3.47, P = 0.06).

### 4.2. Yield Response Parameters

Corn yield response equations explained 20%, 42%, and 8% of the variation in the corn yield data in 1997, 1999, and 2001 (Table 3). The low adjusted $R^2$ value in the 2001 year is not surprising because there were many zero yields recorded in this year caused by the extremely wet planting season. Soybean response models explained 39% and 42% of the soybean yield data in 1998 and 2000, respectively. Linear and quadratic terms did not carry the expected signs in all years for corn. However, expected signs were obtained taking the weighted average of the annual crop responses.

## 5. Economics Analysis

### 5.1. Optimization Methods

The first order conditions of equation 1 with respect to N and P were simultaneously solved for a single period. Optimal values for applied P and N fertilizer were used to update yields and the P carryover equation. These new values were used as starting points in the next iteration for five rotations.

### 5.2. Management Scenarios

Four management scenarios were compared in the dynamic analysis. Soil P starting values for management zones 1, 2, and 3 were 16.5, 15.6, and 19.0 kg ha$^{-1}$, respectively, as determined from the P soil test value taken before the experiment (fall 1996) for all scenarios. For potassium and pH levels, the site-specific averages of potassium and pH soil test levels over the course of the experiment were used (Table 1) for all scenarios. All scenarios span five corn-soybean rotations (a ten-year time horizon). In the baseline scenario (UNI) the producer applies N and P uniformly at the extension recommendation rates of 90 kg ha$^{-1}$ N (Randall and Schmitt, 2002) and 56 kg ha $^{-1}$ P (Rehm and Schmitt, 1993) before planting corn every rotation. In scenario 1 (VRA-NP), the producer applies N and P site-specifically. It is assumed that corn and soybean yield response to inputs is known, and economically optimal rates are applied at the beginning of each rotation. In scenario 2 (VRA-P), the producer applies N uniformly at the extension rate. Phosphorous is applied site-specifically at an economically optimal rate. In the last scenario (VRA-N), the producer applies P uniformly at the extension rate. Nitrogen is applied site-specifically at an economically optimal rate.

As a sensitivity analysis the baseline and VRA-NP NPVs and input levels were compared when the discount rate was, ceteris paribus, (i) increased to 12.5%, (ii) the N cost kg$^{-1}$ was increased to \$0.10, (iii) the cost of fertilizer P was increased 10 \$0.15 kg$^{-1}$, (iv) and the corn and soybean prices decreased to marketing year low values (\$0.063 and \$0.153 kg$^{-1}$, respectively).

## 6. Economic Results and Discussion

### 6.1. Comparison of Baseline and SSM Strategies

The pattern observed for P dynamics was a rapid build-up scenario when P was a decision variable, which was anticipated given the linear carryover specification (Table 4, 5). The rapid build-up scenario resulted in a constant NPV and corn and soybean yield levels in all rotations following the first rotation. When P was not a decision variable but N was a gradual build-up scenario for P resulted because an N carryover mechanism was not included in the optimization. The P build-up process takes longer in the baseline and VRA-N strategies because of the gradual build-up philosophy embodied in the extension recommendations. The optimal time paths for N were curvilinear, which was expected because there are no N dynamics linked to input decision-making. This resulted in sub-optimal management strategies in a given site. When P was a decision variable in the producer's optimization problem, returns to variable rate management were always higher than the baseline NPV.

Optimization of site 3 was problematic in that the expected corn yields and optimal N and P input rates well exceeded experimental levels (>30000 kg ha$^{-1}$ for corn, 300 kg ha$^{-1}$ for P and 200 kg ha$^{-1}$ for N). Expected soybean yields were only 244 kg ha$^{-1}$ indicating that soybeans production was occurring in production stage three. These results occurred because the expected corn response to P in this site was very flat, with biologically optimal yield peaking around 300 kg ha$^{-1}$ P. The quadratic P coefficient for this site was 95% and 98% smaller than those observed in sites 1 and 2. Some of the year-specific responses were not concave leading to expected response curves not being well-behaved (Table 2). Additionally the N by P interaction term in this site was relatively larger than the other sites, causing expected corn yields and applied N and P levels to overshoot experimental levels. The system of equations was re-optimized with constraints placed on N and P levels for site 3. The upper N and P bounds chosen were determined using the corn 1999 year due to the idea that a grower probably would not exceed the good year application on average.

In general the results indicate that there is potential for higher yields in some portions of the site with better P management. Site-specific, optimal time paths for yields, N and P fertilizer, and soil P variables were similar in zone 1 under the VRA-NP and VRA-P scenarios. The expected NPV under the VRA-P strategy in site 1 was higher than the VRA-NP scenario because of the extra cost of variable rate application. This pattern was not observed in zone 2 because, unlike the N rates in zone 1 the optimal N rate in the VRA-NP scenario is considerably lower than the N rate applied following extension recommendations. Because the optimization was constrained in site 3 the VRA-NP and VRA-P results are sub-optimal as indicated by the lack of an immediate plateau of yields, NPV, soil P, and inputs following the first rotation. By the third (scenario 1) and fourth (scenario 2) rotations soil P plateaus and the change between NPV becomes negligible in the VRA-NP and VRA-P strategies.

Optimal TAP levels are constant over all rotations, and can be calculated using $\lambda$ , the optimal amount of fertilizer applied at the beginning of a rotation, and the P soil stock carried over from the previous rotation (Tables 4, 5). In the scenarios where P was not a decision variable display a curvilinear path that gradually increases over time, approaching optimal TAP levels. Sub-optimal TAP observed in the VRA-N and UNI strategies approach optimal TAP levels after three to four rotations because of the slow build-up philosophy embodied in extension recommendations.

The VRA-NP strategy produced the largest expected NPV over the ten years (\$3845 ha$^{-1}$), followed by the VRA-P and VRA-N strategies (\$3666 and \$3393 ha$^{-1}$, respectively), and then the baseline strategy (\$3330 ha$^{-1}$). As a sensitivity test the system was evaluated assuming the producer had information about corn and soybean response at the whole-field

(WF) level. Whole-field responses were evaluated at optimal N and TAP levels, resulting in a NPV of \$3511 ha$^{-1}$ with TAP starting at 83 kg ha$^{-1}$ then sustained at 63 kg ha$^{-1}$ each rotation. Applied N rates were 93 kg ha$^{-1}$. In general these results are consistent with Lowenberg-DeBoer and Reetz's (2002) study and with the theoretical expectations of the sequential Markov chain problem. The best management strategy is to apply just enough fertilizer P at the beginning of a production sequence to reach the long run economically optimal P soil test, and then apply enough P to maintain this equilibrium in subsequent production periods.

## 6.2. Input/Output Price, Opportunity Cost, and Planning Horizon Sensitivity Analysis

The VRA-NP and baseline management strategies decreased by \$1126 ha$^{-1}$ and \$935 ha$^{-1}$, respectively, when the opportunity cost of capital was increased to 12.5%. Under the VRA-NP strategy with the higher opportunity cost P and N fertilizer application only decreased by about 1 kg ha$^{-1}$. When the cost of N fertilizer was increased to \$0.10 kg$^{-1}$ returns to VRA-NP decreased by \$7 ha$^{-1}$ and the NPV of the baseline strategy increased by \$62 ha$^{-1}$. This increase was not enough to change the ranking between the VRA-NP and baseline strategies. Optimal N rates under the higher N price only decreased by about 1 kg ha$^{-1}$ in the VRA-NP strategy. Likewise optimal P rates and soil P carrying capacity levels only decreased by about 1 kg ha$^{-1}$ when the P fertilizer price was increased. In this scenario the VRA-NP returns decreased by \$22 and ha$^{-1}$, while the baseline NPV increased by \$57 ha$^{-1}$. The gains in baseline strategy were not enough to change the ranking between these strategies. At the lower corn and soybean prices NPVs for VRA-NP and the baseline management strategies decreased by \$468 ha$^{-1}$ and \$342 ha$^{-1}$, respectively, but P and N levels only decreased by about 1 kg ha$^{-1}$ in the VRA-NP strategy.

To test whether P management strategies differed for owner operators and renters, a single-period corn production was optimized. The idea behind this scenario was that farmers who rent land on the typical one year lease have less of a stake in building up then maintaining P soil levels because they do not own the land. In this scenario the renter does not account for the value of carryover in deciding on an optimal fertilizer management strategy. In site 3 P applications were identical for the single and multi-period models because of the response convexity. Nitrogen application in this site was 2% less compared to the multi-period N results. In sites 1 and 2 P application levels decreased by 7% and 61%, respectively. N application decreased by 1% and increased by 11% in sites 1 and 2, respectively. In the U.S., renters with one year leases often farm the same rented land for many years. Through a series of P applications that optimize single year returns, renters can build soil P levels, but usually that build up is slower than the soil fertility increase for an owner operator.

## 7. Spatial Variability of P Build-up and Carryover and Management Implications

Inclusion of the interaction terms in the carryover equation allows $\alpha$ ($\partial P_{t+1}/\partial P_t$) and $\alpha\lambda$ ($\partial P_{t+1}/\partial F_t$) to be evaluated over the production surface with respect to spatial variability of pH, organic matter, and potassium. To understand the effects of these variables on P carryover, steady-state soil P levels without cropping is examined. P steady-state (PSS) is estimated as $c_0/(1 - \alpha)$. These parameters were most sensitive to changes in organic matter and potassium, but not pH (Figure 4). Soil P build-up capacity decreases in organic-rich areas of the field, but increases in parts of the field with higher potassium levels. Soil pH does not appear to affect soil P build-up capacity, but higher pH levels decrease PSS levels. Over the relevant range of potassium levels, PSS levels increase. In parts of the field with high organic matter levels, PSS plateaus. These findings have implications for variable rate P management and P build-up strategies. In some areas of the field the best strategy is to ‘feed the crop’ because P build-up is not feasible because of soil P interactions with other soil characteristics. In other parts of the field a rapid build-up strategy is a feasible management alternative.

## 8. Summary and Conclusions

Estimated coefficients were used in a dynamic programming model to determine optimal N and P fertilizer policies over five corn-soybean rotations, or a ten-year horizon. A uniform management strategy was compared to three VRA strategies. The NPV of a variable rate NP management strategy was highest in all scenarios. When P alone was managed site-specifically NPV returns were always higher than the baseline strategy. When N was managed site-specifically returns were higher than the baseline scenario, but lower than the P optimal management strategies. This result occurs because there is no mechanism linking soil N dynamics to N application decisions.

The NPV results are ex post because it is assumed the producer knows beforehand yield response functions. In other words the producer assumed in this analysis has much more information than most farmers currently have. In all cases where P was optimally managed the best strategy was to apply remedial P applications to build soil P levels up to carrying capacity, and subsequently apply maintenance levels of P fertilizer to maintain the carrying capacity. For sites 1 and 2 both this research and the extension recommendations tend toward the same long term P soil test levels; the extension rates just take longer to get there.

To our knowledge this is the first attempt to simultaneously estimate SSCRFs with a nutrient carryover equation. But there remains much work to be done with respect to estimating nutrient carryover dynamics and then applying this information in economically meaningful ways. Previous studies have relied upon simulation to estimate carryover parameters, or these parameters were cobbled together from various sources in the agronomic literature. Other studies have estimated carryover equations, but not simultaneously with crop response equations.

The results presented here are not without problems. First, there were problems with respect to response function concavity in some production years. In order to approximate reasonable input levels and yields upper bounds needed to be imposed during optimization. Second, the study ignores sub-soil plant-nutrient dynamics. The soil test information used in this analysis applied to the 0-6 inch horizon. This is a non-trivial point given that a significant portion of plant growth occurs in and below this horizon. Soil test information taken below the 6 inch horizon would add a third dynamic dimension to the analysis.

Some studies suggest that the optimal policy in any scenario would be to build up P stock as quickly as possible. These findings are in agreement with those studies. However, this strategy is conditional upon the physical processes particular to a given field, that is, the carryover parameters that regulate P dynamics. Put another way, one size may not fit all. It should not be expected that nutrient pathways observed in one experiment are generalizable across all fields. Black (1993, page 563) mentions that management strategies that call for intermittent, heavy applications to reach target soil nutrient levels are good strategies only when residual amounts remain available in the soil. Of course, the results observed here are driven by the soil variables not included in the carryover model and the actual data used in estimation. Alternative definitions of the carryover model could include soil test variables.

Finally, it is well-known that agronomic data are usually spatially correlated. No attempts were made in this analysis to test for the presence of spatial dependence, or correct for it if it were present. This exercise is left for further studies.

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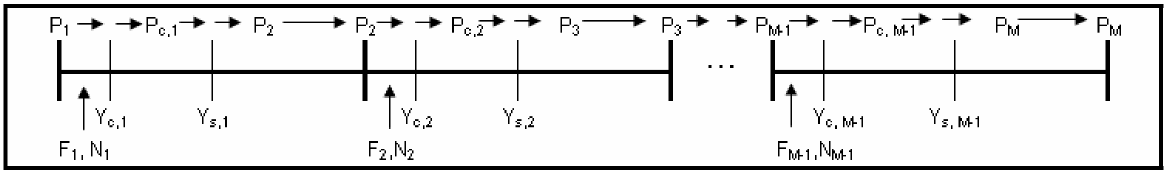

Figure 1. Schematic diagram of production process through M corn-soybean rotations.

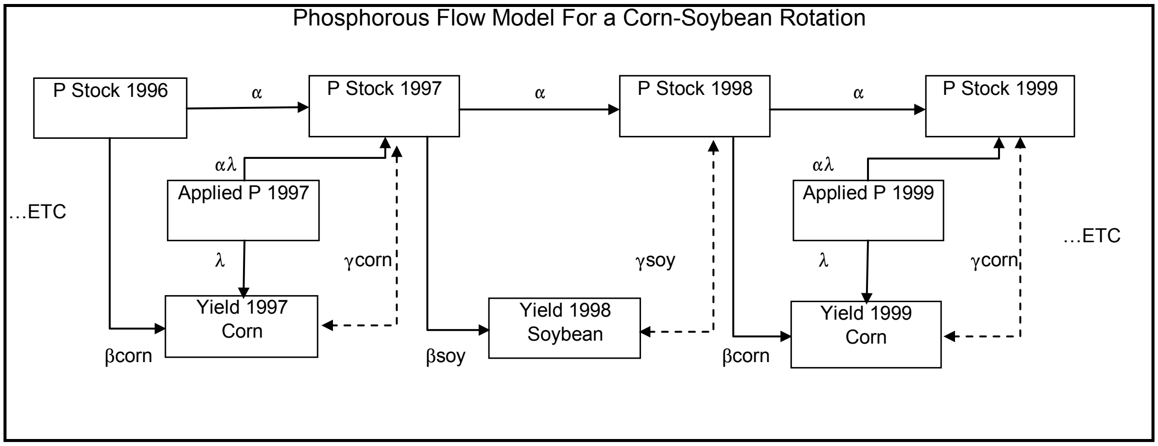

Figure 2. Phosphorous carryover model. Hashed lines corresponding with $\gamma$ indicate that yield in from crop $j$ has a drawdown effect on the P stock in the state preceding the next crop.

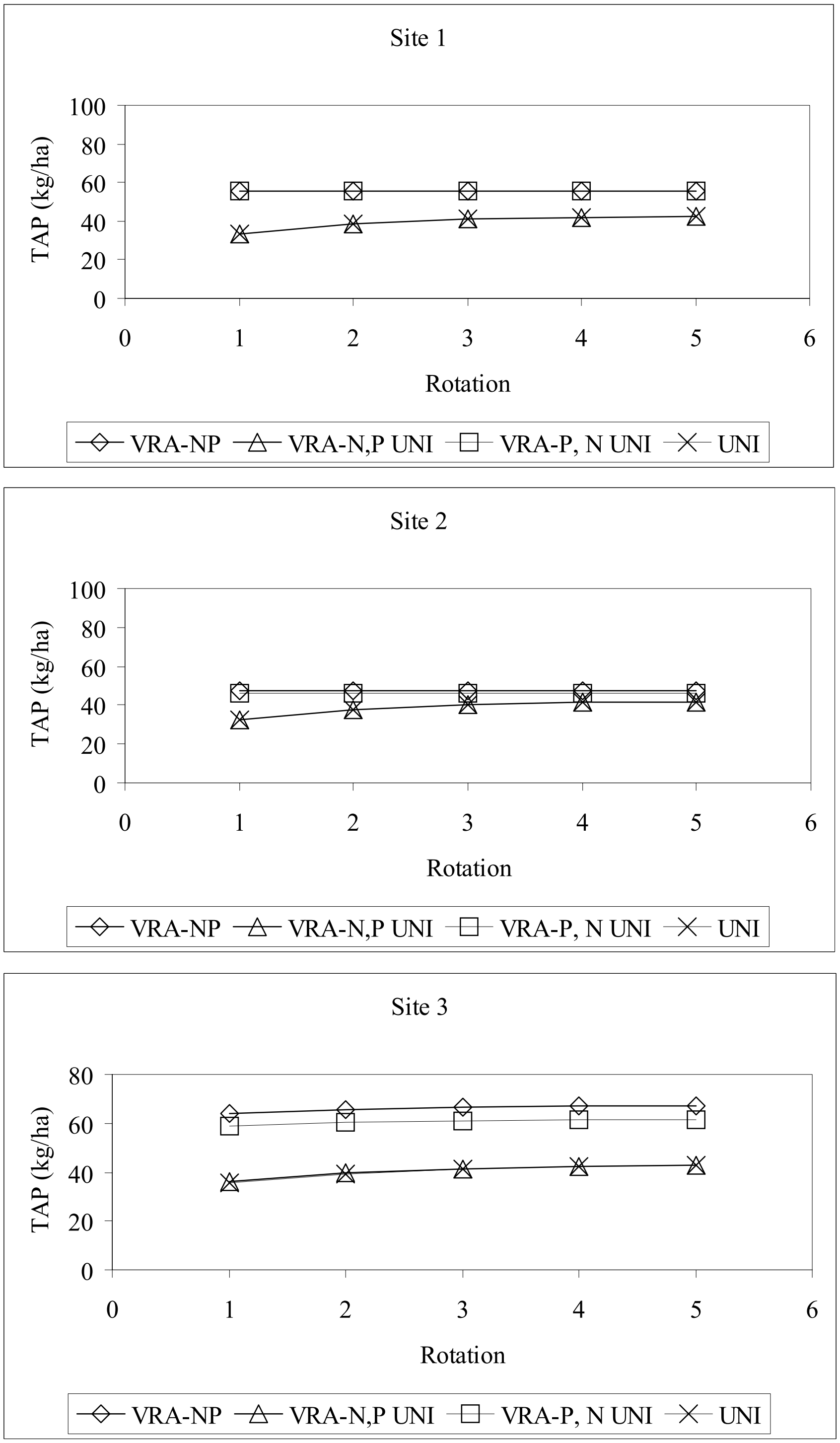

Figure 3. Total available phosphorous (kg ha$^{-1}$) under each management scenario.

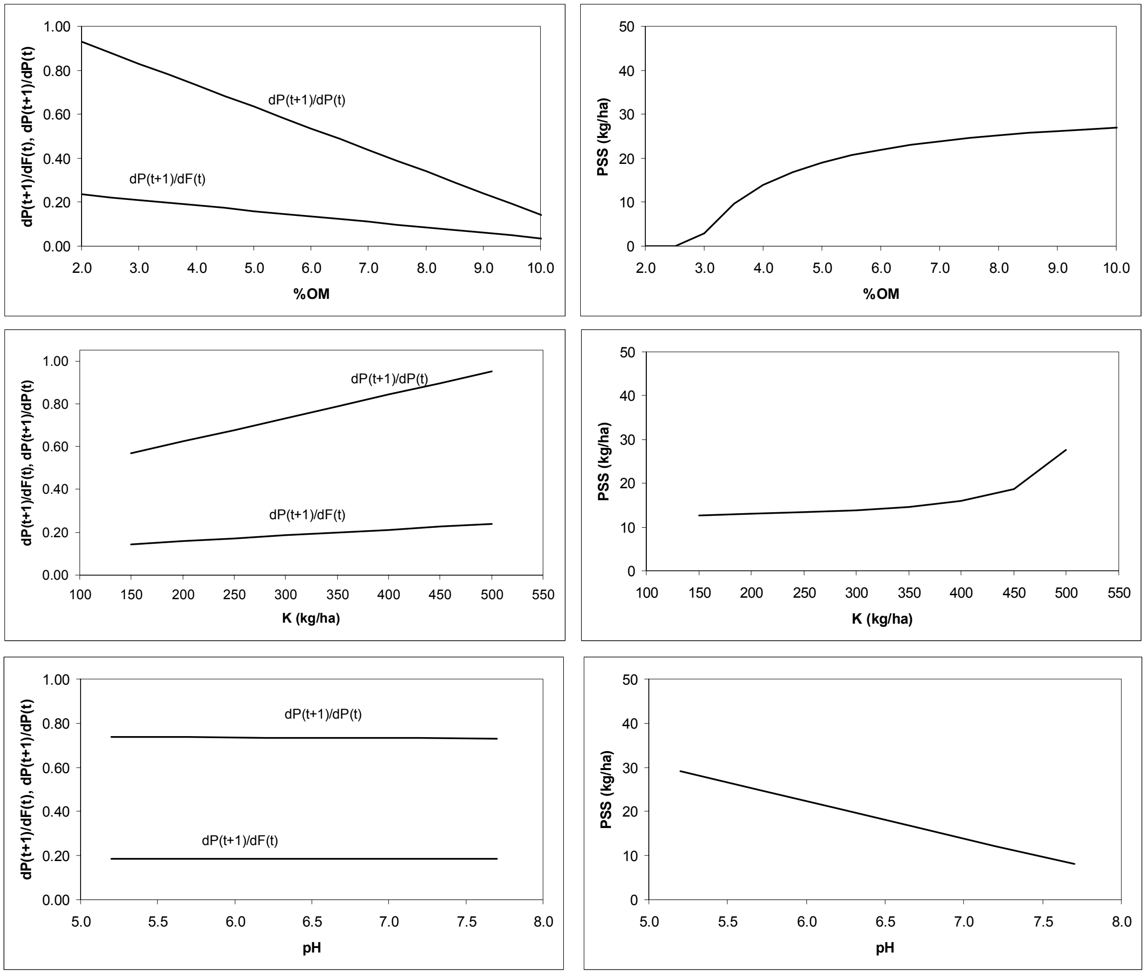

Figure 4. Change in P carryover parameters and steady-state soil P levels (without cropping) with respect to changes in soil characteristics.

Table 1. Descriptive statistics (averages) for soil characteristics and yearly corn and soybean yields in sites.

| *Variable* | *Cluster 1*‡ | *Cluster 2* | *Cluster 3* |
|---|---|---|---|
| K (ppm) | 135 | 132 | 134 |
| P(ppm) | 7.2 | 7.0 | 7.4 |
| pH | 7.07 | 7.09 | 7.07 |
| %OM | 4.42 | 4.51 | 4.14 |
| | | | |
| Yield 1997 (Corn) | 7774† | 7139 | 7403 |
| Yield 1998 (Soybean) | 2977 | 2889 | 2924 |
| Yield 1999 (Corn) | 8813 | 8695 | 8747 |
| Yield 2000 (Soybean) | 2461 | 2372 | 2399 |
| Yield 2001 (Corn) | 6089 | 6735 | 6410 |

† kg ha$^{-1}$; ‡ Proportions of the field represented by site (cluster) $z$: $z_1$ = 26%; $z_2$ = 25%; $z_3$ = 49%.

SOURCE: Authors' estimates.

Table 2. Carryover equation estimates with asymptotic t-values and fit statistics.

| Parameter | Estimate | T-value |
|---|---|---|
| $c_0$ | -132.042 | -11.21 |
| $\alpha$ | 0.818 | 10.69 |
| $\lambda$ | 0.253 | 19.92 |
| $\gamma$ (corn) | -9.000E-05 | -1.42 |
| $\gamma$ (soybean) | -0.001 | -3.80 |
| | | |
| $\varphi_{\%\mathrm{OM}}$ | 11.666 | 3.98 |
| $\varphi_{\mathrm{pH}}$ | 19.940 | 12.66 |
| $\varphi_{\mathrm{K}}$ | 0.304 | 9.57 |
| | | |
| $\psi_{\mathrm{pH}\ \mathrm{x}\ TAP}$ | -1.676 | -4.52 |
| $\psi_{\%\mathrm{OM}\ \mathrm{x}\ TAP}$ | 0.011 | 5.19 |
| $\psi_{\mathrm{K}\ \mathrm{x}\ TAP}$ | -0.051 | -11.68 |
| | | |
| $\xi_{\%\mathrm{OM}\ \mathrm{x}\ \mathrm{pH}}$ | -0.003 | -0.92 |
| $\xi_{\%\mathrm{OM}\ \mathrm{x}\ \mathrm{K}}$ | -0.098 | -7.08 |
| $\xi_{\mathrm{K}\ \mathrm{x}\ \mathrm{pH}}$ | 0.001 | 4.52 |
| | | |
| Carryover year | Root Mean Squared Error | Adjusted $R^2$ |
| 1997 | 6.11 | 0.81 |
| 1998 | 6.00 | 0.76 |
| 1999 | 6.51 | 0.71 |
| 2000 | 6.12 | 0.69 |
| 2001 | 5.48 | 0.81 |

SOURCE: Authors’ estimates.

Table 3. Site-specific yield response estimates for restricted model.

| Site | $\beta_0$ | $\beta_N$ | $\beta_{NN}$ | $\beta_P$ | $\beta_{PP}$ | $\beta_{NxP}$ |
|---|---|---|---|---|---|---|
| Corn 1997 | | | | | | |
| 1 | 5985.49 | 29.47 | -0.28 | 69.29 | -0.77 | 0.37 |
| 2 | 6646.00 | 23.61 | -0.28 | -8.75 | -0.50 | -0.23 |
| 3 | 5644.30 | 41.04 | -0.30 | 27.46 | 0.15 | 1.12 |
| Soybean 1998 | | | | | | |
| 1 | 1944.54 | | | 75.58 | -0.64 | |
| 2 | 1664.67 | | | 100.57 | -1.07 | |
| 3 | 1941.87 | | | 76.07 | -0.69 | |
| Corn 1999 | | | | | | |
| 1 | 4218.02 | 37.94 | -0.20 | 254.49 | -2.75 | -0.13 |
| 2 | 4882.91 | 44.01 | -0.20 | 183.11 | -1.53 | -0.01 |
| 3 | 5461.60 | 49.60 | -0.25 | 122.94 | -0.63 | -0.43 |
| Soybean 2000 | | | | | | |
| 1 | 321.40 | | | 194.32 | -3.10 | |
| 2 | 668.39 | | | 152.41 | -2.14 | |
| 3 | 1138.81 | | | 92.14 | -0.71 | |
| Corn 2001 | | | | | | |
| 1 | 2284.29 | 29.87 | -0.15 | 178.16 | -0.89 | -0.15 |
| 2 | 3718.42 | 43.67 | -0.18 | 94.31 | 0.03 | 0.04 |
| 3 | 4602.18 | 39.89 | -0.18 | 18.90 | 0.42 | -0.44 |
| | | | | | | |
| Year | Crop | | | | RMSE* | Adj. $R^2$ |
| 1997 | Corn | | | | 1509 | 0.20 |
| 1998 | Soybean | | | | 541 | 0.39 |
| 1999 | Corn | | | | 1488 | 0.42 |
| 2000 | Soybean | | | | 582 | 0.42 |
| 2001 | Corn | | | | 3478 | 0.08 |
| | | | | | | |
| Site | Weighted Coefficients‡ | | | | | |
| 1 | 5069.55 | 31.85 | -0.24 | 132.95 | -1.33 | 0.17 |
| 2 | 5820.87 | 31.55 | -0.25 | 56.08 | -0.72 | -0.14 |
| 3 | 5473.23 | 43.26 | -0.27 | 52.79 | -0.03 | 0.51 |
| | | | | | | |
| 1 | 1630.08 | | | 98.58 | -1.11 | |
| 2 | 1471.65 | | | 110.61 | -1.28 | |
| 3 | 1786.29 | | | 79.18 | -0.69 | |

*Root mean squared error; ‡ Weights are: 1997 = 0.61, 1999 = 0.28, and 2001 = 0.12 for corn years; and 1998 = 0.81 and 2000 = 0.19 for soybean years.

SOURCE: Authors' estimates.

Table 4. Comparison of VRA-NP and uniform (UNI) input management strategies.

| | *VRA-NP* | | | | | | | *UNI* | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | Zone 1 | | | | | | | Zone 1 | | | | | | |
| Period | NPV# | $Y_{soy}$† | $Y_{corn}$† | $P_{corn}$‡ | $P_{soy}$ | N¶ | F¶ | NPV | $Y_{soy}$ | $Y_{corn}$ | $P_{corn}$ | $P_{soy}$ | N | F |
| 0 | . | . | . | . | 16 | . | . | . | . | . | . | 16 | . | . |
| 1 | 1178 | 3810 | 10146 | 42 | 32 | 84 | 154 | 1117 | 3488 | 9465 | 27 | 22 | 90 | 67 |
| 2 | 1202 | 3810 | 10146 | 42 | 32 | 84 | 93 | 1153 | 3608 | 9737 | 31 | 24 | 90 | 67 |
| 3 | 1202 | 3810 | 10146 | 42 | 32 | 84 | 93 | 1167 | 3654 | 9839 | 32 | 25 | 90 | 67 |
| 4 | 1202 | 3810 | 10146 | 42 | 32 | 84 | 93 | 1173 | 3674 | 9882 | 33 | 26 | 90 | 67 |
| 5 | 1202 | 3810 | 10146 | 42 | 32 | 84 | 93 | 1176 | 3683 | 9901 | 33 | 26 | 90 | 67 |
| Period | Zone 2 | | | | | | | Zone 2 | | | | | | |
| 0 | . | . | . | . | 16 | . | . | . | . | . | . | 16 | . | . |
| 1 | 1051 | 3804 | 7482 | 37 | 28 | 48 | 126 | 997 | 3505 | 7302 | 27 | 21 | 90 | 67 |
| 2 | 1056 | 3804 | 7482 | 37 | 28 | 48 | 79 | 1015 | 3643 | 7262 | 30 | 23 | 90 | 67 |
| 3 | 1056 | 3804 | 7482 | 37 | 28 | 48 | 79 | 1021 | 3693 | 7232 | 32 | 24 | 90 | 67 |
| 4 | 1056 | 3804 | 7482 | 37 | 28 | 48 | 79 | 1023 | 3713 | 7216 | 32 | 25 | 90 | 67 |
| 5 | 1056 | 3804 | 7482 | 37 | 28 | 48 | 79 | 1024 | 3722 | 7208 | 33 | 25 | 90 | 67 |
| Period | Zone 3 | | | | | | | Zone 3 | | | | | | |
| 0 | . | . | . | . | 19 | . | . | . | . | . | . | 19 | . | . |
| 1 | 1502 | 4005 | 14063 | 49 | 37 | 139 | 179 | 1217 | 3493 | 10670 | 29 | 23 | 90 | 67 |
| 2 | 1525 | 4019 | 14272 | 50 | 38 | 140 | 115 | 1256 | 3593 | 11031 | 31 | 24 | 90 | 67 |
| 3 | 1533 | 4025 | 14378 | 51 | 38 | 141 | 115 | 1275 | 3639 | 11209 | 33 | 25 | 90 | 67 |
| 4 | 1537 | 4028 | 14432 | 51 | 38 | 142 | 115 | 1284 | 3661 | 11298 | 33 | 26 | 90 | 67 |
| 5 | 1539 | 4029 | 14459 | 51 | 38 | 142 | 115 | 1289 | 3672 | 11342 | 34 | 26 | 90 | 67 |

†Corn and soybean yield, kg ha$^{-1}$; ‡ P carryover following corn and soybean cycles (kg ha$^{-1}$); ¶ applied N and P fertilizer (kg ha$^{-1}$); # Net present value (\$ ha$^{-1}$).

SOURCE: Authors' estimates.

Table 5. Comparison of VRA-N and VRA-N input management strategies.

| | *VRA-N, P UNI* | | | | | | | *VRA-P, N UNI* | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | Zone 1 | | | | | | | Zone 1 | | | | | | |
| Period | NPV# | $Y_{soy}$† | $Y_{corn}$† | $P_{corn}$‡ | $P_{soy}$ | N¶ | F¶ | NPV | $Y_{soy}$ | $Y_{corn}$ | $P_{corn}$ | $P_{soy}$ | N | F |
| 0 | . | . | . | . | 16 | . | . | . | . | . | . | 16 | . | . |
| 1 | 1101 | 3489 | 9501 | 27 | 22 | 76 | 67 | 1181 | 3811 | 10143 | 43 | 32 | 90 | 155 |
| 2 | 1163 | 3610 | 9763 | 31 | 24 | 78 | 67 | 1214 | 3811 | 10143 | 43 | 32 | 90 | 93 |
| 3 | 1176 | 3656 | 9862 | 32 | 25 | 78 | 67 | 1214 | 3811 | 10143 | 43 | 32 | 90 | 93 |
| 4 | 1182 | 3676 | 9903 | 33 | 26 | 79 | 67 | 1214 | 3811 | 10143 | 43 | 32 | 90 | 93 |
| 5 | 1185 | 3685 | 9921 | 34 | 26 | 79 | 67 | 1214 | 3811 | 10143 | 43 | 32 | 90 | 93 |
| Period | Zone 2 | | | | | | | Zone 2 | | | | | | |
| 0 | . | . | . | . | 16 | . | . | . | . | . | . | 16 | . | . |
| 1 | 981 | 3506 | 7615 | 27 | 21 | 52 | 67 | 982 | 3787 | 7125 | 36 | 27 | 90 | 120 |
| 2 | 1028 | 3643 | 7602 | 30 | 23 | 51 | 67 | 1014 | 3787 | 7125 | 36 | 27 | 90 | 75 |
| 3 | 1034 | 3693 | 7584 | 32 | 24 | 50 | 67 | 1014 | 3787 | 7125 | 36 | 27 | 90 | 75 |
| 4 | 1036 | 3713 | 7573 | 33 | 25 | 50 | 67 | 1014 | 3787 | 7125 | 36 | 27 | 90 | 75 |
| 5 | 1037 | 3722 | 7568 | 33 | 25 | 49 | 67 | 1014 | 3787 | 7125 | 36 | 27 | 90 | 75 |
| Period | Zone 3 | | | | | | | Zone 3 | | | | | | |
| 0 | . | . | . | . | 19 | . | . | . | . | . | . | 19 | . | . |
| 1 | 1189 | 3499 | 10853 | 29 | 23 | 112 | 67 | 1383 | 3955 | 12889 | 45 | 34 | 90 | 158 |
| 2 | 1256 | 3597 | 11255 | 32 | 24 | 116 | 67 | 1427 | 3971 | 13022 | 46 | 35 | 90 | 104 |
| 3 | 1276 | 3641 | 11456 | 33 | 25 | 117 | 67 | 1433 | 3979 | 13089 | 47 | 35 | 90 | 104 |
| 4 | 1286 | 3663 | 11556 | 33 | 26 | 118 | 67 | 1436 | 3982 | 13123 | 47 | 35 | 90 | 104 |
| 5 | 1291 | 3673 | 11606 | 34 | 26 | 119 | 67 | 1437 | 3984 | 13140 | 47 | 36 | 90 | 104 |

†Corn and soybean yield, kg $ha^{-1}$; ‡ P carryover following corn and soybean cycles (kg $ha^{-1}$); ¶ applied N and P fertilizer (kg $ha^{-1}$); # Net present value ($ $ha^{-1}$).

SOURCE: Authors' estimates.