FEASIBLE ESTIMATION OF FIRM-SPECIFIC ALLOCATIVE INEFFICIENCY THROUGH BAYESIAN NUMERICAL METHODS *

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Abstract

The estimation of allocative and technical inefficiency has grown to an enormous body of literature, both theoretical and empirical. Ideally, one would estimate time-varying firm and input-specific parameters describing allocative inefficiency in order to minimize aggregation bias. However, this has never been previously accomplished. Typically, only industry-wide allocative efficiency parameters have been empirically identified. Our proposed solution is to employ Gibbs sampling to approximate posterior distributions from a Bayesian limited information model, embedding GMM moment conditions imposed via an instrumental variables step to obtain plant-specific parameters estimates that vary flexibly over time. For a panel of Chilean hydroelectric power plants, posterior distributions of these estimates display substantial differences in location and precision. By contrast, the standard GMM approach which produces industry-wide, time-varying allocative inefficiency parameters, not only fails to reveal the inter-plant differences by construction, but does not even produce posterior medians that approximate a weighted average of the plant-specific posterior medians.

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1. Introduction

The literature on efficiency measurement has been concerned with measuring economic efficiency in terms of technical and allocative efficiency. Technical efficiency measures the actual input usage relative to the minimum input usage for a given set of outputs or the actual outputs relative to the maximum potential outputs for a given set of inputs. The technical efficiency of a firm is measured relative to the most efficient firm, which defines the production, distance, or cost frontier. Allocative efficiency measures how well firms manage the ratios of inputs in order to minimize the cost of producing a given output level. From the first-order conditions for cost minimization, firms must equate the ratio of marginal products to the ratio of input prices.

Allocative and technical efficiency have been measured using either an error components approach or a so-called parametric approach. With this approach, one estimates parameters that measure allocative efficiency by scaling either input quantities or input prices, yielding shadow input quantities and shadow input prices, respectively. With the error components approach, maximum likelihood techniques are typically used to estimate the parameters that define the distribution of a two-component error term. Firm-specific measures of allocative and technical inefficiency are then obtained from these estimated components. For a survey of this approach see Greene (1997). A major drawback to this technique is that one must assume the correct functional form of the composed error and that the error terms must be uncorrelated with the regressors. However, results may be very sensitive to the assumed functional form and correcting for endogeneity, if it is present, is highly problematic. If the distribution of the composed error is misspecified or if endogeneity is present and not dealt with successfully, the estimated parameters will be inconsistent.

The parametric approach avoids the need to specify the distribution of the errors and the assumption of exogeneity of the regressors. This approach easily allows testing for endogeneity and correcting for its presence using instruments. The simplest parametric approach would be to estimate time-invariant, input-specific allocative efficiency parameters. One can still estimate the effect on input over or under-utilization at the firm level by computing ratios of fitted input quantities. However, the assumption that all firms share common time-invariant allocative efficiency parameters for each input is *a priori* implausible. Estimation of time-varying, firm-specific (or plant-specific if data is available at this level) allocative efficiency parameters is generally preferred in order to reduce aggregation bias.

However, estimation of a full set of firm-specific, time-varying parameters has never been previously accomplished. Rarely, in fact, have researchers been able to estimate a full set of firm-specific, time-invariant allocative efficiency parameters. Even with panel data and models that are highly non-linear in the parameters, which should assist identification, the data do not typically contain enough independent variation to identify the full set of parameters measuring allocative efficiency along with the structural parameters that define stochastic cost, revenue, profit, distance, or production frontiers (the standard dual frontier paradigms). Estimating a shadow cost function with its associated share equations for a cross-section of data, Atkinson and Halvorsen (1998, 1984, and 1980) were able to empirically identify only industry-wide, time-invariant allocative efficiency parameters. The use of panel data allowed identification of additional efficiency parameters for Atkinson and Halabí (2005) and Atkinson and Primont (2002), who estimated a shadow distance system plus associated price equations. In both cases, time-invariant and time-varying allocative efficiency parameters were empirically identifiable, but only at the industry-wide level. Fitting a system comprised of a shadow cost frontier and associated share equations using panel data, Atkinson and Cornwell (1994) were able to estimate time-invariant, firm-specific parameters measuring allocative inefficiency. However, this required eliminating all time-varying allocative efficiency parameters. Successful identification of firm-specific allocative efficiency parameters in this case was most likely due to the use of a panel data set with T large relative to N. Estimating a shadow distance frontier and a set of associated price equations using panel data, Atkinson, Färe, and Primont (2003) were able to compute an even more general specification. They obtained estimates of firm-specific, time-invariant allocative efficiency parameters plus industry-wide, time-varying allocative efficiency parameters. Thus, using non-Bayesian methods, no parametric study has empirically identified a full set of firm-specific, time-constant and time-varying efficiency parameters.

In this paper, we present a solution to this problem by substituting a Bayesian Markov Chain Monte Carlo (MCMC) parametric approach for the standard one. Our MCMC approach follows and extends Atkinson and Dorfman (2005) by employing a limited information likelihood function that minimizes the assumptions required for estimation, making it essentially equivalent to Bayesian Generalized Method of Moments (GMM) with instruments. For a panel of twelve Chilean hydroelectric power plants, we jointly estimate an input distance function and the first-order conditions from the dual shadow-cost minimization model. We obtain the posterior densities for plant-specific, time-constant and time-varying estimates of allocative efficiency, while correcting for any endogeneity of the regressors. We are able to identify and precisely estimate this rich parameterization, when the classical approach has failed to do so, because we iteratively draw from a series of conditional posterior densities for the parameters using MCMC. Time-varying and plant-specific measures of technical inefficiency are computed residually, which provide estimates of productivity change (PC), which can be decomposed into technical change (TC) and efficiency change (EC).

Our empirical findings are that energy is under-utilized in one half of the plants (due to limited water availability) and over-utilized by the rest. Labor is over-utilized for most plants. Little movement toward allocative efficiency is observed over time for any plant. Technical efficiency is low for many plants and productivity change and efficiency change vary widely across plants. For example, PC ranges from 3% to 16% across plants for the final two years of the sample. Finally, we reestimated our model using industry-wide allocative inefficiency parameters in place of plant-specific ones. The aggregate allocative inefficiency parameters mask important differences among the plant-specific parametric estimates with regard to central tendency as well as precision.

2. Firm-Specific Allocative and Technical Inefficiency

To model firm-specific allocative inefficiency we follow Atkinson and Primont (2002) and employ an input distance function in a cost-minimizing framework to derive a set of estimating equations. We generalize their set-up slightly to add the firm-specific inefficiency measures. To begin, an input distance function is defined as

$$D(\mathbf{y}_t, \mathbf{x}_t) = \sup_{\lambda}\{\lambda : (\mathbf{x}_t/\lambda) \in L(\mathbf{y}_t)\}, \tag{2.1}$$

where $\mathbf{y}_t$ is an $M \times 1$ vector of outputs, $\mathbf{x}_t$ is an $N \times 1$ vector of inputs, $L(\mathbf{y}_t)$ is the input requirement set, and $\lambda \geq 1$. Its inverse is the measure of technical inefficiency. Then, we assume the typical firm solves the following cost minimization problem:

$$C(\mathbf{y}_t, \mathbf{p}_t) = \min_{\mathbf{x}_t}\left\{\mathbf{p}_t\mathbf{x}_t \ : \ D(\mathbf{y}_t, \mathbf{x}_t) \geq 1\right\}, \tag{2.2}$$

where $\mathbf{p}_t$ is a $1 \times N$ vector of input prices. Due to constraints on the optimization process, in most cases, the observed value of $\mathbf{x}_t$ will fail to solve (2.2). Denote the "shadow" input quantities that *do* solve (2.2) by $\mathbf{x}_t^* = [k_{1t}x_{1t}, \ldots, k_{Nt}x_{Nt}]$, where the k_{nt} are measures of input-specific departures of shadow input quantities from actual input quantities and are parameters to be estimated. Subject to a normalization for one input, these measures of allocative inefficiency will be estimated for each plant, but plant-identifying subscripts are suppressed here for simplicity.

The first-order conditions corresponding to (2.2) are

$$p_{nt} = \mu \frac{\partial D(\mathbf{y}_t, \mathbf{x}_t^*)}{\partial x_{nt}}, \quad n = 1, \ldots, N, \tag{2.3}$$

where μ is the Lagrangian multiplier and the derivative is evaluated at the shadow values of the inputs.

Two properties of distance functions can be used to transform equation (2.3) into an estimable equation. First, the distance function is linearly homogeneous in the inputs which implies that $\sum_{n=1}^{N} \frac{\partial D(\mathbf{y}_t, \mathbf{x}_t^*)}{\partial x_{nt}} x_n^* = D(\mathbf{y}_t, \mathbf{x}_t^*)$ by Euler's theorem. Second, $D(\mathbf{y}_t, \mathbf{x}_t^*) = 1$ by definition. Thus, if we multiply both sides of (2.3) by optimal input levels $\mathbf{x}_t^*$, sum over all N inputs, and apply these two properties, we obtain

$$p_n = (\mathbf{p}_t \mathbf{x}_t^*) \frac{\partial D(\mathbf{y}_t, \mathbf{x}_t^*)}{\partial x_{nt}}, \quad n = 1, \ldots, N. \tag{2.4}$$

The above set of equations along with the distance function comprise the set of equations we employ to estimate firm-specific technical and allocative efficiency.

2.1. Model Specification

To make our econometric model specific to panel data, the shadow input distance function for firm or plant f and time t can be written as

$$1 = D(\mathbf{y}_{ft}, \mathbf{x}^*_{ft}, t) g(\epsilon_{ft}), \tag{2.5}$$

where ϵ_{ft} is a random error. We adopt the translog functional form for (2.5):

$$\begin{aligned}
0 &= \ln D(\mathbf{y}_{ft}, \mathbf{x}^*_{ft}) + \ln g(\epsilon_{ft}) \\
&= \gamma_o + \sum_m \gamma_m \ln y_{mft} + .5 \sum_m \sum_w \gamma_{mw} \ln(y_{mft}) \ln(y_{wft}) \\
&+ \sum_m \sum_n \gamma_{mn} \ln y_{mft} \ln x^*_{nft} + \sum_n \gamma_n \ln x^*_{nft} \\
&+ .5 \sum_n \sum_l \gamma_{nl} \ln x^*_{nft} \ln x^*_{lft} + \sum_m \gamma_{mt} \ln y_{mft} t \\
&+ \sum_n \gamma_{nt} \ln x^*_{nft} t + \gamma_{t1} t + .5\, \gamma_{t2} t^2 + \ln g(\epsilon_{ft}),
\end{aligned} \tag{2.6}$$

where

$$g(\epsilon_{ft}) = \exp(v_{ft} - u_{ft}), \tag{2.7}$$

v_{ft} is a two-sided disturbance, and $u_{ft} \geq 0$ captures technical inefficiency.

As proposed by Cornwell, Schmidt, and Sickles (1990), we specify the u_{ft} in terms of firm-specific linear and quadratic trends:

$$u_{ft} = \beta_0 + \beta_{f0} D_f + \beta_{f1}\, D_f\, t + \beta_{f2}\, D_f\, t^2, \tag{2.8}$$

where the D_f are firm dummies and the $\beta_{fq},\ q = 0, 1, 2,$ are parameters to be estimated.

In this paper, since we utilize panel data on a group of plants, we introduce plant-specific allocative efficiency parameters which are both time-invariant and time-varying as

$$k_{nft} = \exp(\kappa_{nf} + \kappa_{nf1} t + \kappa_{nf2} t^2), \tag{2.9}$$

which allows for allocative efficiency to change non-monotonically over time. Previous researchers have employed more restrictive specifications for allocative efficiency. Atkinson and Halvorsen (1984) use a cross-section of firms and restrict k_{nft} to

$$k_n = \exp(\kappa_n), \tag{2.10}$$

which allows estimation of only input-specific, time-invariant allocative efficiency parameters. Atkinson and Cornwell (1994) use panel data on firms and restrict k_{nft} to

$$k_{nf} = \exp(\kappa_{nf}), \tag{2.11}$$

which allows for firm-specific allocative efficiency parameters which are time invariant. Atkinson, Färe, and Primont (2003) utilize panel data on firms and restrict k_{nft} to

$$k_{nft} = \exp(\kappa_{nf} + \kappa_{n1}t + \kappa_{n2}t^2), \tag{2.12}$$

which allows for firm-specific, time-invariant parameters but only industry-wide time-varying parameters (κ_{n1} and κ_{n2}) that are shared across firms.

Identification requires a restriction on both the β_{fq} and the κ_{nft}. First, for one firm we set $\beta_{fq} = 0\ \forall q$. Second, due to linear homogeneity of the distance function in input quantities, we must normalize κ_{nft} for some input n for each firm f. This implies that we can only measure the over or under-utilization of one input relative to another; thus, we set $\kappa_{nft} = 1$ for a numeraire input $\forall\, t, f$. The specific choice of the numeraire does not affect any model parameters other than the absolute values of the κ_{nft}; however, their relative values remain unchanged, regardless of the choice of the numeraire.

Our parameterization of the allocative efficiency measures as coefficients which scale input quantities avoids what Bauer (1990) called the Greene problem. Greene utilized maximum likelihood to estimate an error components model and assumed that allocative and technical efficiency measures were independent, which is highly implausible. Bauer (1990) summarizes considerable research devoted to solving the Greene problem using the error components models estimated using maximum likelihood. We avoid both the misspecification problems inherent with the maximum likelihood approach and solve the Greene problem by measuring allocative efficiency using parameters which scale input quantities. This approach does not impose independence between the parameters of the distance function (which include parameters measuring allocative efficiency) and the one-sided residual, $\hat{u}_{ft}$, from which we derive our measure of technical efficiency.

We substitute the restrictions in (2.7), (2.8), and (2.12), along with those that impose symmetry and linear homogeneity (see Atkinson and Primont (2002) for details) into the stochastic translog shadow distance system (2.6). Taking derivatives of (2.6), we can specify (2.4) in terms of the distance function parameters as

$$\begin{aligned}
p_n = (\mathbf{p}_t \mathbf{x}_t^*) \Bigg\{ & \left[\gamma_n + \sum_m \gamma_{mn} \ln y_{mft} + \sum_l \gamma_{nl} \ln x_{lft}^* + \gamma_{nt} t \right] \times \\
& \exp \Bigg[\gamma_o + \sum_m \gamma_m \ln y_{mft} + .5 \sum_m \sum_w \gamma_{mw} \ln(y_{mft}) \ln(y_{wft}) \\
& + \sum_m \sum_n \gamma_{mn} \ln y_{mft} \ln x_{nft}^* + \sum_n \gamma_n \ln x_{nft}^* \\
& + .5 \sum_n \sum_l \gamma_{nl} \ln x_{nft}^* \ln x_{lft}^* + \sum_m \gamma_{mt} \ln y_{mft} t \\
& + \sum_n \gamma_{nt} \ln x_{nft}^* t + \gamma_{t1} t + .5\, \gamma_{t2} t^2 \Bigg] \times \\
& \Bigg[\gamma_o + \sum_m \gamma_m \ln y_{mft} + .5 \sum_m \sum_w \gamma_{mw} \ln(y_{mft}) \ln(y_{wft}) \\
& + \sum_m \sum_n \gamma_{mn} \ln y_{mft} \ln x_{nft}^* + \sum_n \gamma_n \ln x_{nft}^* \\
& + .5 \sum_n \sum_l \gamma_{nl} \ln x_{nft}^* \ln x_{lft}^* + \sum_m \gamma_{mt} \ln y_{mft} t \\
& + \sum_n \gamma_{nt} \ln x_{nft}^* t + \gamma_{t1} t + .5\, \gamma_{t2} t^2 \Bigg]^{-1} \times \left[\frac{1}{x_{nft}^*} \right] \Bigg\}, \quad n = 1, \ldots, N. \qquad (2.13)
\end{aligned}$$

We then append a random error term to the shadow distance function and n derived price equations in (2.13) to obtain a system of $(n+1)$ nonlinear equations with multiple cross-equation restrictions which we refer to as the shadow distance system. Note that the high degree of non-linearity in the unknown parameters in equations (2.13) should aid identification of the allocative efficiency parameters.

2.2. Measurement of Allocative Inefficiency

By taking ratios of equations in (2.4), we obtain the conditions for cost minimization in terms of shadow quantities and actual prices. For firm f at time t, we can directly estimate relative over- and under-utilization of any pair of inputs, x_{nft} and x_{lft}, in comparison to the cost-minimizing ratio, $(k_{nft}x_{nft})/(k_{lft}x_{lft})$, by computing $\hat{k}_{nft}/\hat{k}_{lft}$. We argue that frequently researchers and policy makers are more interested in shadow quantities than shadow prices, which give a fundamental advantage to the shadow distance system over the shadow cost system. Examples include the effects of quotas or restrictive work rules on input usage, inefficient input usage due to rate of return regulation, and the impact of government subsidies or tariffs in agriculture on the input usage.

2.3. Measurement of Technical Efficiency

Following the estimation of (2.6), we compute levels of TE, EC, TC, and PC. Non-negativity of the u_{ft} is not imposed in estimation. Instead by adding and subtracting $\hat{u}_t = \min_f(\hat{u}_{ft})$ from the fitted model, we define the frontier intercept. Let $\ln \hat{D}(\mathbf{y}_t, \mathbf{x}_t, t)$ represent the estimated translog portion of (2.6) (i.e., all terms except $h(\epsilon_{ft})$). Then, adding and subtracting $\hat{u}_t$ yields

$$0 = \ln \hat{D}(\mathbf{y}_t, \mathbf{x}_t, t) + \hat{v}_{ft} - \hat{u}_{ft} + \hat{u}_t - \hat{u}_t = \ln \hat{D}^*(\mathbf{y}_t, \mathbf{x}_t, t) + \hat{v}_{ft} - \hat{u}^*_{ft}, \tag{2.14}$$

where $\ln \hat{D}^*(\mathbf{y}_t, \mathbf{x}_t, t) = \ln \hat{D}(\mathbf{y}_t, \mathbf{x}_t, t) - \hat{u}_t$ is the estimated frontier distance function in period t and $\hat{u}^*_{ft} = \hat{u}_{ft} - \hat{u}_t \geq 0$.

Using (2.14), we estimate firm f's level of technical efficiency in period t, TE_{ft}, as

$$\mathrm{TE}_{ft} = \exp(-\hat{u}^*_{ft}), \tag{2.15}$$

where our normalization of $\hat{u}^*_{ft}$ guarantees that $0 \leq \mathrm{TE}_{ft} \leq 1$. Given the estimates of TE_{ft} obtained from (2.15), we then calculate EC_{ft}, the rate at which a firm is approaching the isoquant, as the change in technical efficiency:

$$\mathrm{EC}_{ft} = \Delta\mathrm{TE}_{ft} = \mathrm{TE}_{ft} - \mathrm{TE}_{f,t-1}. \tag{2.16}$$

We measure TC, the movement inward of isoquants, as a discrete approximation which involves computing the difference between the estimated frontier distance function in periods t and $t-1$ holding output and input quantities constant:

$$\begin{aligned}\mathrm{TC}_{ft} &= \ln \hat{D}^*(\mathbf{y}_t, \mathbf{x}^*_t, t) - \ln \hat{D}^*(\mathbf{y}_t, \mathbf{x}^*_t, t-1) \\ &= \sum_m \hat{\gamma}_{mt} \ln y_{mft}(d_t - d_{t-1}) + \sum_n \hat{\gamma}_{nt} \ln x_{nft}(d_t - d_{t-1}) \\ &+ \hat{\gamma}_t - \hat{\gamma}_{t-1} + (\hat{u}_{t-1} - \hat{u}_t). \end{aligned} \tag{2.17}$$

Thus, the change in the frontier intercept, $\hat{u}_t$, affects TC as well as EC. Finally, given EC and TC, we construct estimates of PC as

$$\mathrm{PC}_{ft} = \mathrm{TC}_{ft} + \mathrm{EC}_{ft}. \tag{2.18}$$

3. Estimation

3.1. An Error Components Approach

One possible estimation procedure would be to completely specify the full likelihood and estimate an error components model in a Bayesian framework. Zellner, Bauwens, and van Dijk (1988) point out the difficulties in accurate specification of the full likelihood. In addition, one must assume that the composed error term is uncorrelated with the explanatory variables. In terms of our distance system, a typical composed error specification is that

$$0 = \ln D(\mathbf{y}_{ft}, k_{ft}\mathbf{x}_{ft}, t) + \ln g(\epsilon_{ft}) = \ln D(\mathbf{y}_{ft}, k_{ft}\mathbf{x}_{ft}, t) + v_{ft} - u_{ft}, \tag{3.1}$$

and

$$p_n = (\mathbf{p}_t\mathbf{x}_t^*)\frac{\partial D(\mathbf{y}_t, \mathbf{x}_t^*)}{\partial x_{nt}} + \eta_{ft}, \quad n = 1, \ldots, N. \tag{3.2}$$

Various assumptions about the independence of the error terms and their joint distribution in these two equations have been made and each may be incorrect, resulting in model misspecification. See Greene (1997). In addition, one must assume that the error terms are uncorrelated with the regressors. Estimated parameters will be inconsistent if regressors are endogenous. Dealing with endogeneity within this context is problematic.

3.2. An Alternative Allowing Endogeneity

To derive estimators for the shadow input distance function system, we use a limited information Bayesian system estimator to avoid the difficulties in accurately specifying the full likelihood. In our estimation approach, we follow and generalize Kim (2002) and

Zellner and Tobias (2001).[1] Kim proves that maximizing entropy subject to a restriction on a Generalized Method of Moments (GMM) criterion function yields an optimal limited-information likelihood function (LILF). We advance Kim's approach by treating the covariance of the errors as well as unknown parameters of our distance system as random variables and constructing a joint LILF. Interested readers can see Atkinson and Dorfman (2005) for more details.

The form of the LILF and the resulting posterior distribution depends on the moment conditions that serve as the basis for the criterion function. Following the development in Kim (2002), we start with a standard first moment condition for the parameter vector γ,

$$E[h(\gamma|\Omega, D)] = 0, \tag{3.3}$$

where γ is a vector of regression model parameters, Ω is the covariance matrix of the regression model's stochastic error terms, and D represents the data, including instruments. Our specification of $h(\gamma)$ is such that this restriction sets the conditional expectation of the error terms equal to zero. A second moment condition on gamma is

$$E[h(\gamma)h(\gamma)'] = S. \tag{3.4}$$

This second restriction sets the conditional error variance to the consistent estimator as in standard GMM estimation.

We now extend Kim (2002) to include the random covariance matrix parameters in our LILF by adding two moment conditions on Ω,

$$E[\mathrm{tr}(\Xi\Omega^{-1})] = \xi, \text{ and } E[\ln|\Omega|] = \tau, \tag{3.5}$$

[1] Zellner and Highfield (1988) and Zellner (1998) develop early Bayesian Method of Moments (BMOM) estimators that clearly presage Kim's approach. Other applications of BMOM can be found in Green and Strawderman (1996) and LaFrance (1999).

where Ξ is the sum of squared residuals matrix, while ξ and τ are scalar constants. These moment conditions impose enough regularity on the otherwise unrestricted distribution of Ω to result in a conditional limited information posterior (LIP) distribution for Ω in the form of the standard inverted Wishart familiar to Bayesian statisticians; see Zellner and Tobias (2001) for details of the univariate case which they pioneered.

Any selection from the set of admissible LILFs is obviously somewhat *ad hoc*, but we defend our selection process as being in the spirit of GMM estimators. Thus, we choose the least informative (most diffuse) LILF from the admissible set in order to impose the minimum amount of assumptions on the estimation procedure.

From the set of admissible functions $\mathcal{F}$ that satisfies the above moment conditions, the least informative LILF, f, is found by solving the optimization problem

$$\text{argmax}_{f \in \mathcal{F}} \; - \int f(\gamma, \Omega|D) ln f(\gamma, \Omega, D) d\gamma d\Omega. \tag{3.6}$$

The solution is

$$\hat{f}(\gamma, \Omega|D) = c_o |\Omega|^{-c_1} \exp\Big[-c_2 h(\gamma)' S^{-1} h(\gamma) - c_3 \text{tr}(\Xi\Omega^{-1})\Big]. \tag{3.7}$$

Inspection of the above LILF shows $\hat{f}$ to be the product of a distribution from the exponential family for γ and an inverted Wishart with respect to Ω, where c_o, c_1, c_2, and c_3 are constants, $S = E[h(\gamma)h(\gamma)']$ and Ξ is the sum of squared errors matrix. If one used the LILF in (3.7) as a likelihood function and found the values of γ and Ω which maximize it, the result is the standard GMM estimator. The proof of this follows easily from that in Kim (2002, eq. 3.8) for the case with a known Ω.

The prior distribution used in our application is a product of independent priors on the structural parameters of the distance function, the prior on the covariance matrix of the vector of errors, and a set of indicator functions that restrict prior support to the region where the theoretical restrictions from economic theory are satisfied. This prior distribution can be written as

$$p(\gamma, \Omega) \propto \mathrm{MVN}(g_o, H_o)|\Omega|^{-(m+1)/2}\mathrm{I}(\gamma, \mathcal{R}), \tag{3.8}$$

where MVN is the multivariate normal distribution, g_o is the vector of prior means on the parameters in γ, H_o is the prior variance-covariance matrix on the same parameters, $I(\gamma, \mathcal{R})$ represents the indicator function that equals one when the restrictions are satisfied and zero otherwise, and m is the number of equations in our system.

The vector g_o is set to zero. The matrix H_o is a diagonal matrix with diagonal elements set to 1000 for the basic structural parameters of the distance function, 0.25 for the plant-specific allocative parameters, and 0.01 for the plant-specific parameters that interact with time.

The indicator function part of the prior restricts positive prior (and posterior) support to the region, $\mathcal{R}$, that satisfies monotonicity for all inputs and for the output. Ideally, monotonicity would be satisfied at 100% of our data points. However, we allow for potential measurement errors by requiring that monotonicity for inputs and the output be satisfied for close to, but still less than, 100% of our observations.

Having derived the limited information likelihood function and defined the prior density $p(\gamma, \Omega)$, we apply Bayes Theorem using the LILF in place of a standard likelihood function, and derive a limited-information posterior distribution

$$f(\gamma, \Omega|D) = p(\gamma, \Omega)\hat{f}(\gamma, \Omega|D)\mathrm{c}^{-1}, \tag{3.9}$$

where c is the normalizing constant. Only values or expressions for the two constants, ξ and τ, in the moment conditions for Ω from equation (3.5) are needed to derive the precise limited-information posterior. Careful choice of these two constants leads to a limited-information posterior with "standard" parameters,

$$p(\gamma,\Omega|D) = c_o|\Omega|^{-(n-k+m+1)/2}\exp\left[-\frac{1}{2}(\gamma-\gamma_p)'\Psi_p^{-1}(\gamma-\gamma_p)-\frac{1}{2}\mathrm{tr}(\Xi\Omega^{-1})\right]\mathrm{I}(\gamma,\mathcal{R}), \quad (3.10)$$

where

$$\gamma_p = \Psi_p(H_o^{-1}g_o + \Psi_m^{-1}\gamma_m), \quad (3.11)$$

$$\Psi_p = (H_o^{-1} + \Psi_m^{-1})^{-1}, \quad (3.12)$$

and Ψ_m is the standard GMM covariance matrix of γ_m, which is the standard GMM estimator of γ given the set of identifying restrictions specified.[2] The limited-information posterior distribution in (3.10) is a truncated version of the standard multivariate normal-inverted Wishart distribution common in Bayesian econometrics, although with a nonstandard mode (due to the replacement of the usual ML estimator with a GMM estimator in ((3.11)) and ((3.12))).

The posterior distribution defined above in (3.10) does not allow for analytical calculation of posterior means and medians of the parameters in γ or functions of those parameters (such as the k_{nft} that we are most interested in). Therefore, numerical methods must be employed to approximate the posterior distribution and estimate the posterior means and medians that are of interest. For this purpose, we used an MCMC approach, specifically Gibbs sampling with an accept-reject step for the imposition of the monotonicity condition. For more details on Gibbs sampling and MCMC methods, see Chib (1995) or Tierney (1994).

[2] The constants ξ and τ could be estimated but that would greatly complicate the estimation algorithm by adding a numerical optimization step requiring a quasi-Newton or equivalent search algorithm for each loop through the Gibbs sampler that will be employed to approximate the posterior. In return, one would get a more accurate marginal posterior distribution for Ω. We choose to simply set the two constants to convenient values instead given the interest is on a subset of the parameters in γ.

The Gibbs sampler essentially consists of repeated draws from conditional distributions of subsets of parameters that are easier to generate random draws from than the full posterior. In this application, the conditional distributions are a truncated MVN for γ conditional on Ω and an inverted Wishart for Ω conditional on γ. The conditional posterior for γ is further broken down into subsets with draws for the $(\kappa_{nf}, \kappa_{nf1}, \kappa_{nf2})$ parameters accomplished conditional on the covariance matrix and the other elements of γ. The rest of γ is treated similarly with the truncation only relevant for this subset. Draws from the truncated MVN were accomplished by drawing from the untruncated distribution and discarding draws that were not within the region R.

A total of 12,000 Gibbs draws were generated from four separate chains. Each chain was 3500 draws long with the first 500 discarded to remove dependence on initial starting values (standard GMM estimates were used for that purpose). Convergence was checked by confirming the posterior means of the four separate chains were statistically equivalent. For example, the posterior means and medians of the sample-average TE measures did not differ by more than 0.5% in any case across the chains. We impose the restriction $\beta_0 = 0$ in (2.8), in order to identify the coefficients of the firm-specific dummies, β_{f0}, $, \forall f$. We must also restrict the allocative inefficiency parameters to achieve identification; this is done by setting $k_{nft} = 1, \forall t$ for one n.

Consistent joint estimation of this system using GMM (which is a part of the formula for the posterior mode of our Bayesian estimator) requires that the model satisfy the moment conditions $E(v_{ft} \,|\, \mathbf{z}_{ft}) = 0$, where $\mathbf{z}_{ft}$ is a vector of instruments. The Hansen (1982) J test of overidentifying restrictions is used to determine the validity of the instrument set that is used to estimate our distance system using GMM. A variety of instrument sets are examined. We fail to reject the null hypothesis that the moment conditions are satisfied when we employ the following instruments: an intercept, the plant-level dummies, firm-level dummies, the log of the real node price of electricity, a variable measuring the relative hydrologic conditions, W, W^2, W^3, W^4, t, t^2, t^3, the interaction of the run-of-river dummy with monthly dummies, the interaction of the run-of-river dummy with yearly dummies, the interaction of the run-of-river dummy with output, the interaction of time and firm-level dummies, log output times the run-of-river dummy, log output, and log output squared. In constructing the instruments, one plant-level dummy and one firm-level dummy are eliminated. We allow for heteroskedasticity and autocorrelation of unknown form by computing the consistent covariance matrix following Newey and West (1987) with 10 monthly lags. Based on the J test, we easily accept the null hypothesis of the validity of the overidentifying conditions with a p-value typically from .56–.58 with 184 degrees of freedom.

For the duality between input prices and quantities to be valid, the input shadow distance function must be monotonically increasing in inputs and monotonically decreasing in outputs. Our estimated model satisfies the required monotonicity properties for inputs and outputs for at least 95% and 99% of the data points, respectively (since that condition was imposed in estimation by restricting draws to the region R).

4. Data and Results

4.1. Data

Our initial sample is a rotated and unbalanced panel, consisting of 21 Chilean hydroelectric power generation plants, observed monthly for a maximum of 141 data points per plant spanning April 1986 to December 1997. The monthly frequency is designed to capture the considerable variation in the country's hydrologic conditions throughout the year.

Table 1 lists the 21 plants for which we initially have observations, their controlling firm, year of initial service, type of hydro generation, and MW capacity. The six controlling firms really constitute four—Endesa, Gener, Colbún, and Pilmaiquén—since Endesa owns Pehuenche and Pangue.

Plants 15, 18, and 19 rotated into our sample near its end and we observed them for only the last 17, 6, and 14 months, respectively. Since they were still in the break-in phase of operation, we dropped these three plants, reducing our data set to 18 plants. We also dropped plants 11 and 14 because they produced large violations of monotonicity conditions for nearly all observations during preliminary estimation of the distance system. There are good reasons for this kind of behavior. Plant 14 was publicly-owned until 1995 and not subject to the same market incentives as the retained plants. Plant 11 is located at Lago Laja and as such is used to store water for the entire system, sometimes for a number of years, if drought conditions are forecast. This constraint on water release and generation has caused it to consistently violate the regularity condition for outputs. Thus, our final sample consists of the remaining 16 plants (owned by 3 firms) with a total of 1935 monthly observations.

The remaining panel was unbalanced for three reasons. The first reason is staggered initial in-rotation of 6 plants following their construction. Second, there is limited out-rotation by 5 plants, eliminating just 29 monthly observations in total where out-rotation is caused by plant-specific mechanical problems. The longest period of out-rotation was by plant 2 for 18 months beginning the second year of operation. The third (minor) reason for the unbalanced panel was that two observations were judged to be incorrectly recorded and were dropped: plant 4 in month 46 and plant 9 in month 45.

At the plant level, we record the output quantity (Q), the price per GWh of output, and the price and quantity of three inputs—labor (L), capital (K), and water (E), which we also refer to as energy. All the quantities and prices have been normalized by their means before taking their logarithms. Prices are all in real terms. Full details of the data set can be found in Atkinson and Halabí(2004). We arbitrarily normalize k_{Kft} to 1 for all plants and time periods.

To briefly characterize the industry structure and trends, the rank order of firms which own the plants in our sample, as measured by generation in 1997, is Endesa, Gener, and Pilmaiquén. Electricity output declines about 60% in each case as one moves to the next firm in the ordering. Total labor utilization has fallen steadily from 1990 and capital investment has risen steadily through 1995. However, water utilization has been far more variable. For example, water usage (and as a result, output) declined dramatically during the years 1994–96.

4.2. Empirical Results

Although the plant-specific allocative inefficiency measures all vary by year, the empirical estimates are exceedingly stable over time. Even given the flexible specification of the k_{nft} in equation (2.12), the estimates are almost constant over time. Table 2 presents all twelve posterior medians for annual estimates of plant 9's k_{Lt}, along with the upper and lower limits of marginal Bayesian 90% posterior density regions. The posterior median moves from 0.84 to 0.86 over the twelve year period. Since this plant has the largest change over time of any plant, this table presents clear evidence of the stability of these estimates over time. Given this, all remaining results are presented for only the final year of the sample.

Tables 3 and 4 present the posterior medians for the allocative inefficiency measures for energy and labor, both measured relative to capital, for all sixteen plants. Starting with energy (table 3), we see that relative to capital eight plants are under-utilizing energy and eight are over-utilizing energy. Plants 1 and 5 show such severe under-utilization, indicating a shortage of water relative to capital. Both are run-of-river plants; plant 5, in particular, had a drought on its river during large parts of the data period. Since all five reservoir plants and three run-of-river plants over-utilized energy, while eight run-of-river plants under-utilized energy, the distinction between over- and under-utilization of energy is not solely based on the type of plant. We note that these allocative inefficiency parameters are generally estimated quite accurately; most 90% posterior density regions are quite small as a percentage of the posterior median.

Moving to labor and the results shown in table 4, we find consistent over-utilization of labor, with $k_{Lt} < 1$ for thirteen plants. Only three plants under-utilize labor and none of these have estimated posterior medians far above one. Again, all the estimates are quite precise statistically. The estimated magnitude of over-utilization of labor is in keeping with the conventional wisdom that these plants all had utilized considerably too much labor in the period before 1990 and that many plants still employ excess labor today. The variety of point estimates in tables 3 and 4 and the narrowness of the posterior density regions relative to the magnitude of the posterior medians demonstrates that our estimation method is capable of accurately estimating plant-specific allocative inefficiencies.

In order to visually summarize our plant-specific estimates of k_{Lft} and k_{Eft}, we present their estimated marginal posterior distributions in Figures 1-8. The pdfs for the k_{Eft} are presented in Figures 1-4 and those for the k_{Lft} are shown in Figures 5-8. The 16 plants are grouped into four separate figures for capital and four for labor to minimize overlap of the distribution and reveal more detail. Thus, more posterior distributions overlap than it appears by examining the figures individually; however, inspection of the figures does reveal that we can statistically distinguish the values of the k_{Lft} and k_{Eft} between plants in the majority of comparisons.

These figures show clearly the value of the Bayesian approach in allowing the full posterior distribution of the parameters to be estimated. From Figure 1, we can clearly see the enormous inefficiency of plant 5 in terms of energy usage. From Figures 2-4 we see that the k_{Eft} parameters tend to be more precisely estimated in cases of relative over-utilization of energy ($k_{Eft} < 1$) compared to plants with under-utilization. In general, Figures 5-8 convey the same message with respect to labor over- and under-utilization.

Average technical efficiency scores, reported in table 5, show an enormous range of efficiencies. Plant 20, a small run-of-river plant, is the most efficient, defining the frontier in all twelve years (only year 12 is displayed in table 5). Eleven of the plants have median TE scores below 0.50, with the lowest being 0.18 (shared by plants 7 and 12). Interestingly, the larger plants (see table 1 for the MW capacity of each plant) seem to be less efficient than the smaller ones. Again, we find the estimates to be very precise, with 90% posterior density regions small enough to statistically differentiate the technical efficiency of most of the plants from each other. For example, plants 20 and 21 rank as the top two plants in terms of TE, plants 2 and 3 are tied for third place, while plant 4 is clearly by itself in fifth place. Overall, a very unambiguous ranking of all plants could be developed from these estimates. Figures 9-12 show the posterior distributions of the TE scores for all the plants except plant 20 (which is just a point mass at 1.00). Again, one can see that the estimates are generally quite precise so that plants can be differentiated in terms of technical efficiency. Unlike in the cases of the allocative inefficiency measures, these estimates are more uniform in their dispersion regardless of the magnitude of the median point estimate.

Table 6 displays the most recent period estimates of the posterior means for PC, TC, and EC for each plant. TC must, by definition, be the same for all plants since they are all on or under the same frontier and we measure TC in relation to $\mathbf{x}_t^*$ which places all plants on the same ray from the origin. That is, TC is measured as the percent movement in the frontier at the point of allocative efficiency. TC is estimated to be 3% per year in this final period (the change from year 11 to year 12). EC varies across plants from -6% to 13%, an enormous range. Seven of the sixteen plants are gaining ground on the efficiency leader (positive EC) and five are just keeping pace (EC equal to 0), leaving four plants falling behind. There appears to be no correlation between EC and TE scores. The two plants with the highest estimated EC measures (plants 2 and 3) have high TE scores. However, plant 21, which has the second highest TE score, has a negative EC measure. PC is the sum of EC and TC, so it reflects the same findings as EC. While not shown, posterior density regions are small relative to the posterior means.

By contrast, estimation of the distance system with only industry-wide allocative efficiency parameters yields posterior mean estimates of .071 and 3.273 for labor and energy, respectively, for the last year of the sample. Their standard deviations are .0027 and .0869, respectively. These industry-wide estimates appear to suffer from aggregation bias (i.e., they are not near the weighted average of the plant-specific values) and clearly mask important inter-plant differences that are only observable when plant-specifc allocative efficiency parameters are estimated. Figures 13 and 14 present the posterior distributions of the industry-wide allocative efficiency measures for energy and labor, which can be compared to the plant-specific posterior distributions reported previously.

5. Conclusions

Random effects and non-Bayesian parametric approaches have dominated the literature on the estimation of allocative and technical efficiency. While the choice of functional form and the treatment of endogeneity are problematic with the random effects approach, we avoid specifying the distribution of the errors and can easily treat endogeneity with our parametric approach. Our Bayesian MCMC parametric method allows us to compute time-varying, plant-specific allocative inefficiency measures utilizing a limited information instrumental variable approach, which is analogous to Bayesian GMM with instruments. The allocative efficiency parameters are jointly estimated with the structural parameters of a translog distance function and its associated price equations. Our approach should allow for much richer investigation of the magnitude, precision, and distribution of allocative inefficiencies of plants or firms within an industry than could be achieved previously.

In our application, a panel of Chilean hydroelectric power plants do not appear to become more allocatively efficient over time by learning as they go. However, technical efficiency does stay constant or improve over time, as evidenced by zero or positive efficiency change for twelve of the sixteen plants in our sample. The production frontier is being pushed outward at an estimated annual rate of 3% at the end of our sample. Most importantly, we found considerable differences in the location and precision of estimated allocative inefficiency measures across the plants in our sample, and found that industry-wide measures mask economically significant plant-specific heterogeneity.

Future work estimating plant or firm-specific allocative inefficiency might focus on examination of how a plant's or firm's attempt to solve optimization problems different from (2.2) would impact the various estimated efficiency measures and the importance of dynamic adjustment on measured allocative inefficiency.

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Table 1: Hydroelectric Power Plants, Dec. 1997

Plant ID	Plant	Utility	Year	Type	MW
1	Alfalfal	Gener	1991	run-of-river	160
2	Maitenes	Gener	1923	run-of-river	31
3	Queltehues	Gener	1928	run-of-river	41
4	Volcan	Gener	1944	run-of-river	13
5	Los Molles	Endesa	1952	run-of-river	16
6	Sauzal & Sauzalito	Endesa	1948	run-of-river	86
7	Rapel	Endesa	1968	reservoir	350
8	Canutillar	Endesa	1990	reservoir	145
9	Cipreses	Endesa	1955	reservoir	101
10	Isla	Endesa	1963	run-of-river	68
11	El Toro	Endesa	1973	reservoir	400
12	Abanico	Endesa	1948	run-of-river	136
13	Antuco	Endesa	1981	reservoir	300
14	Colbún	Colbún	1985	reservoir	490
15	San Ignacio	Colbún	1996	run-of-river	37
16	Pehuenche	Pehuenche	1991	reservoir	500
17	Curillinque	Pehuenche	1993	run-of-river	85
18	Loma Alta	Pehuenche	1997	run-of-river	38
19	Pangue	Pangue	1996	reservoir	450
20	Pilmaiquén	Pilmaiquén	1944	run-of-river	39
21	Pullinque	Pilmaiquén	1962	run-of-river	49

Note : provided by the ELDC

Table 2: Plant 9 Allocative Inefficiency for Labor Over Time

Year	lower limit	median	upper limit
1	0.77	0.84	0.92
2	0.77	0.84	0.92
3	0.78	0.84	0.92
4	0.78	0.84	0.92
5	0.78	0.84	0.93
6	0.78	0.85	0.93
7	0.78	0.85	0.93
8	0.79	0.85	0.94
9	0.79	0.85	0.94
10	0.79	0.86	0.94
10	0.79	0.86	0.95
12	0.80	0.86	0.95

Note: Measured relative to capital. Lower and upper limits are to symmetric 90% posterior density regions.

Table 3: Year 12 Plant-Specific Allocative Inefficiency for Energy

Plant	lower limit	median	upper limit
1	4.51	5.09	5.81
2	0.95	1.07	1.22
3	1.56	1.78	2.07
4	1.49	1.78	2.13
5	20.21	23.91	28.13
6	1.44	1.64	1.87
7	0.23	0.25	0.27
8	0.39	0.43	0.48
9	0.83	0.90	0.97
10	0.98	1.09	1.22
12	0.76	0.86	0.95
13	0.31	0.33	0.36
16	0.28	0.31	0.34
17	2.62	3.42	4.38
20	0.23	0.28	0.35
21	0.17	0.21	0.27

Note: Measured relative to capital. Lower and upper limits are to symmetric 90% posterior density regions.

Table 4: Year 12 Plant-Specific Allocative Inefficiency for Labor

Plant	lower limit	median	upper limit
1	0.11	0.12	0.13
2	0.09	0.10	0.11
3	0.17	0.18	0.20
4	1.12	1.24	1.35
5	0.35	0.37	0.40
6	0.12	0.13	0.15
7	0.18	0.22	0.27
8	0.84	0.94	1.08
9	0.80	0.86	0.95
10	0.60	0.65	0.70
12	1.06	1.17	1.29
13	0.43	0.47	0.52
16	0.31	0.34	0.38
17	1.01	1.16	1.38
20	0.06	0.07	0.07
21	0.10	0.12	0.13

Note: Measured relative to capital. Lower and upper limits are to symmetric 90% posterior density regions.

Table 5: Year 12 Plant-Specific Technical Efficiencies

Plant	lower limit	median TE	upper limit
1	0.28	0.29	0.31
2	0.69	0.73	0.77
3	0.64	0.68	0.72
4	0.49	0.53	0.56
5	0.28	0.30	0.32
6	0.24	0.26	0.27
7	0.17	0.18	0.19
8	0.25	0.26	0.28
9	0.37	0.40	0.42
10	0.38	0.40	0.43
12	0.17	0.18	0.19
13	0.21	0.22	0.24
16	0.20	0.22	0.23
17	0.26	0.28	0.31
20	1.00	1.00	1.00
21	0.82	0.86	0.89

Note: Lower and upper limits are to symmetric 90% posterior density regions.

Table 6: Year 12 Plant-Specific Posterior Means for PC, TC, and EC

Plant	PC	TC	EC
1	0.04	0.03	0.01
2	0.14	0.03	0.11
3	0.16	0.03	0.13
4	0.06	0.03	0.03
5	0.05	0.03	0.02
6	0.03	0.03	0.00
7	0.03	0.03	0.00
8	0.02	0.03	-0.01
9	0.03	0.03	0.00
10	0.05	0.03	0.02
12	0.03	0.03	0.00
13	0.05	0.03	0.01
16	0.01	0.03	-0.02
17	-0.03	0.03	-0.06
20	0.03	0.03	0.00
21	0.01	0.03	-0.02

Figure 1. Plant−Specific Allocative Inefficiency for Energy

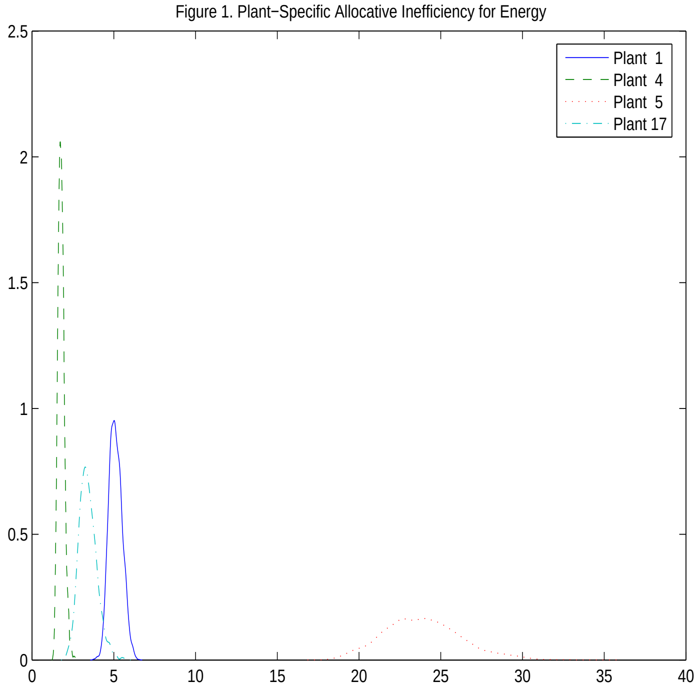


Figure 2. Plant−Specific Allocative Inefficiency for Energy

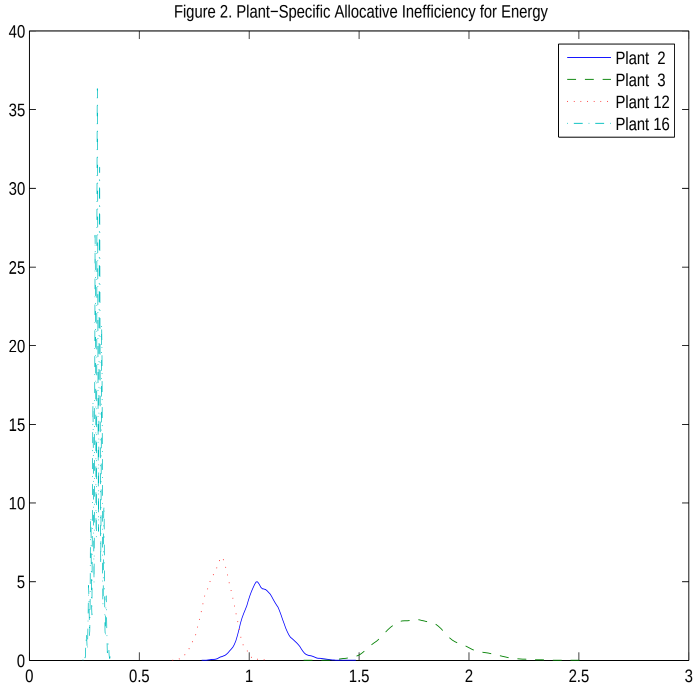


Figure 3. Plant−Specific Allocative Inefficiency for Energy

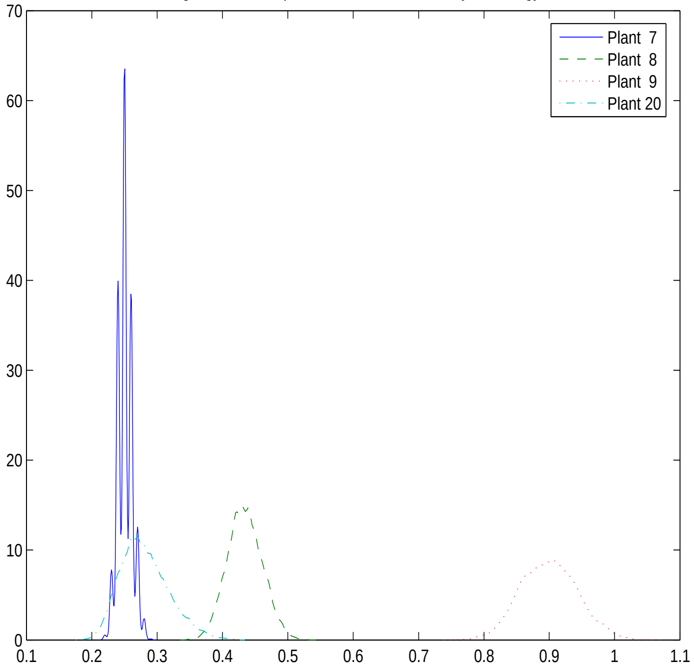
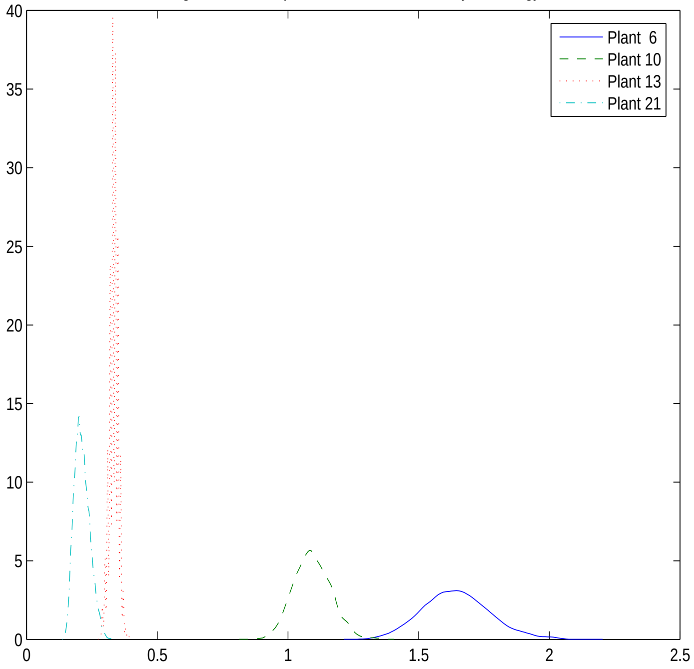


Figure 4. Plant−Specific Allocative Inefficiency for Energy



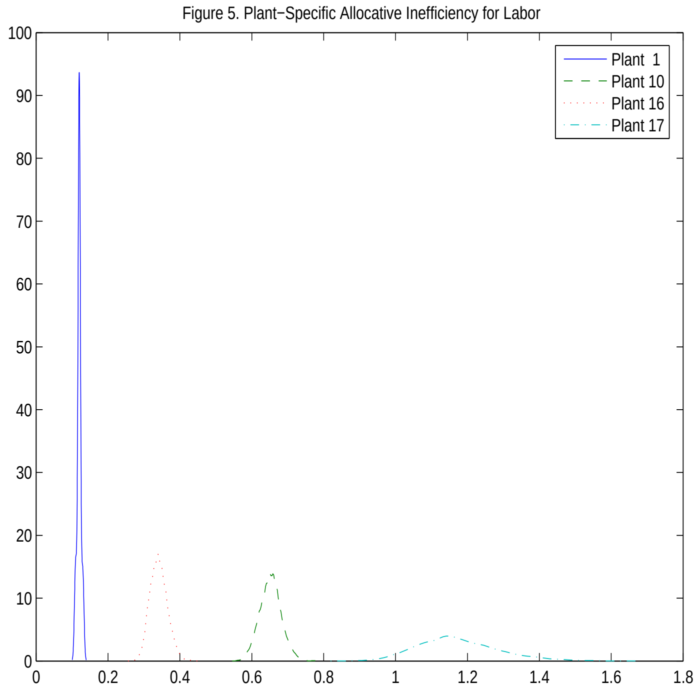


Figure 5. Plant−Specific Allocative Inefficiency for Labor
Plant 1
Plant 10
Plant 16
Plant 17
100
90
80
70
60
50
40
30
20
10
0
0
0.2
0.4
0.6
0.8
1
1.2
1.4
1.6
1.8

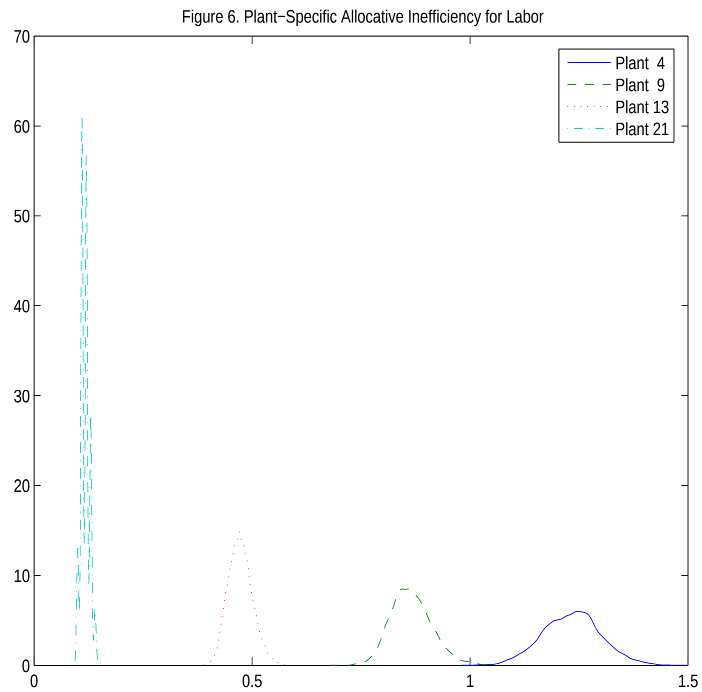


Figure 6. Plant−Specific Allocative Inefficiency for Labor
Plant 4
Plant 9
Plant 13
Plant 21
70
60
50
40
30
20
10
0
0
0.5
1
1.5

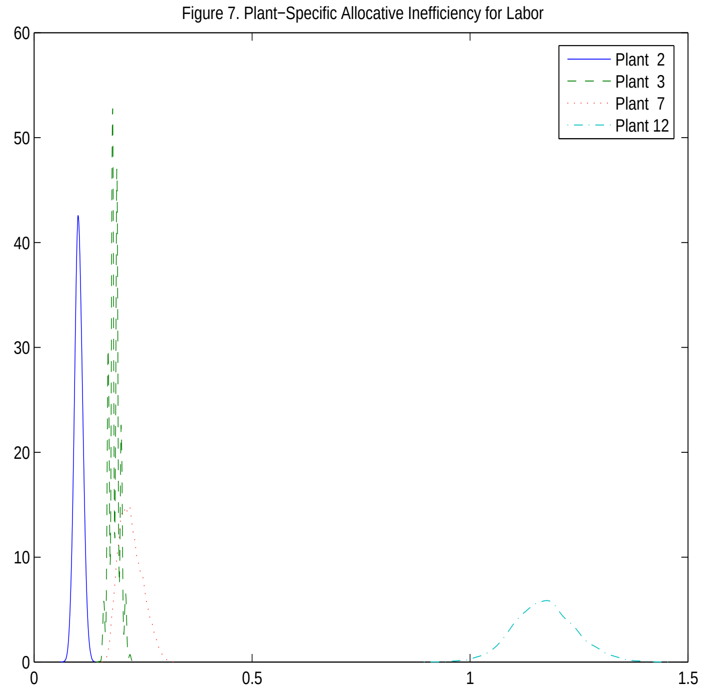


Figure 7. Plant−Specific Allocative Inefficiency for Labor
Plant 2
Plant 3
Plant 7
Plant 12
60
50
40
30
20
10
0
0
0.5
1
1.5

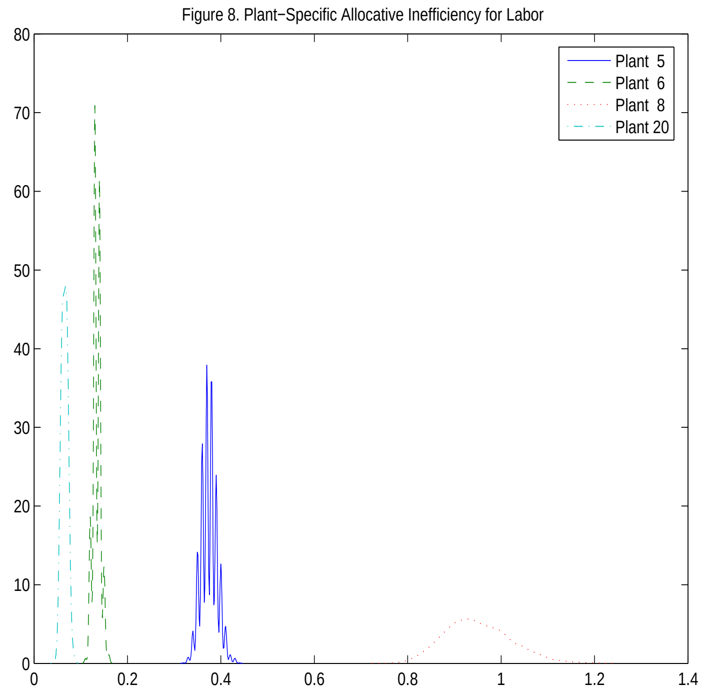


Figure 8. Plant−Specific Allocative Inefficiency for Labor
Plant 5
Plant 6
Plant 8
Plant 20
80
70
60
50
40
30
20
10
0
0
0.2
0.4
0.6
0.8
1
1.2
1.4

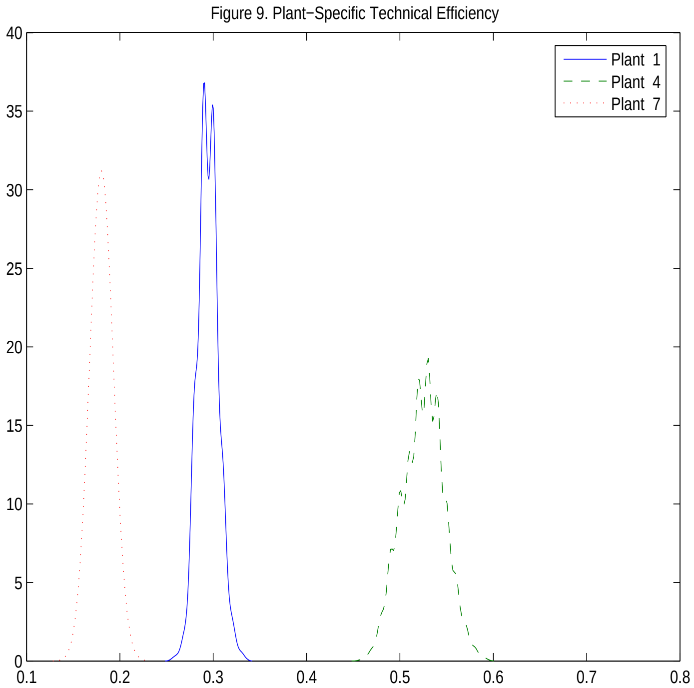


Figure 9. Plant−Specific Technical Efficiency
Plant 1
Plant 4
Plant 7
0
5
10
15
20
25
30
35
40
0.1
0.2
0.3
0.4
0.5
0.6
0.7
0.8

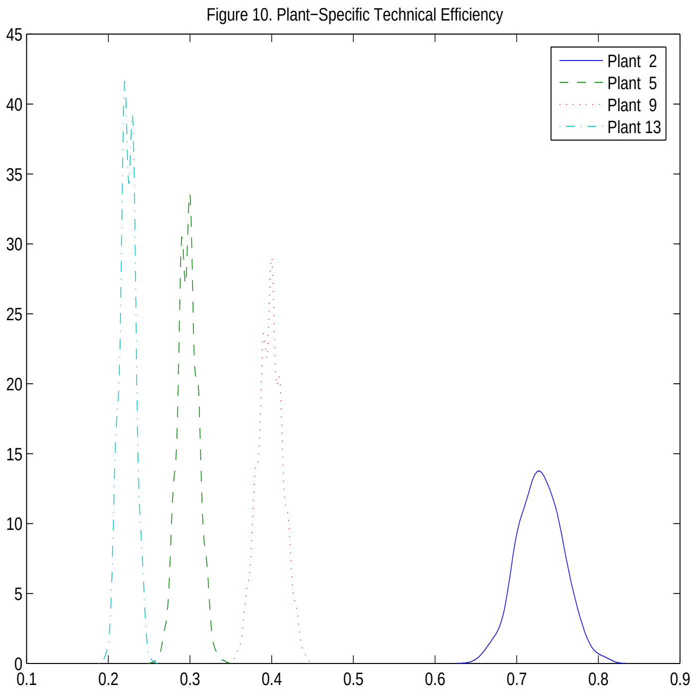


Figure 10. Plant−Specific Technical Efficiency
Plant 2
Plant 5
Plant 9
Plant 13
0
5
10
15
20
25
30
35
40
45
0.1
0.2
0.3
0.4
0.5
0.6
0.7
0.8
0.9

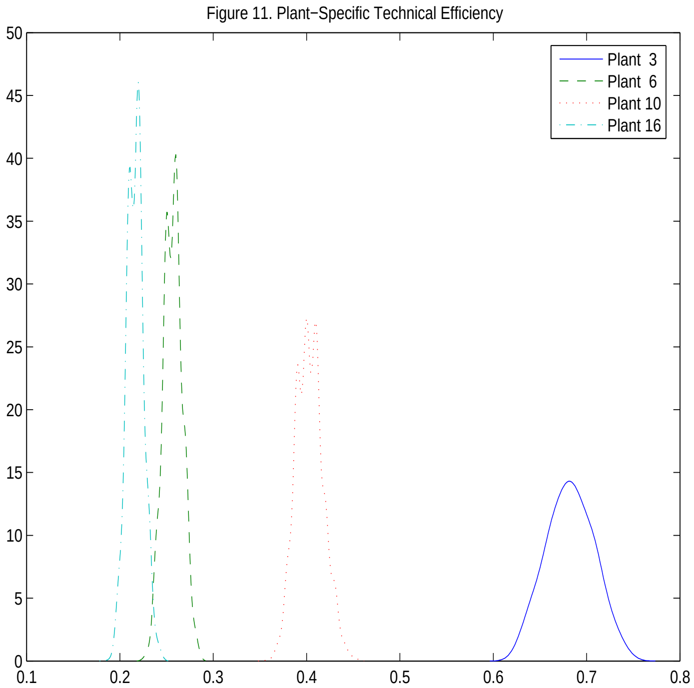


Figure 11. Plant−Specific Technical Efficiency
Plant 3
Plant 6
Plant 10
Plant 16
0.1
0.2
0.3
0.4
0.5
0.6
0.7
0.8
0
5
10
15
20
25
30
35
40
45
50

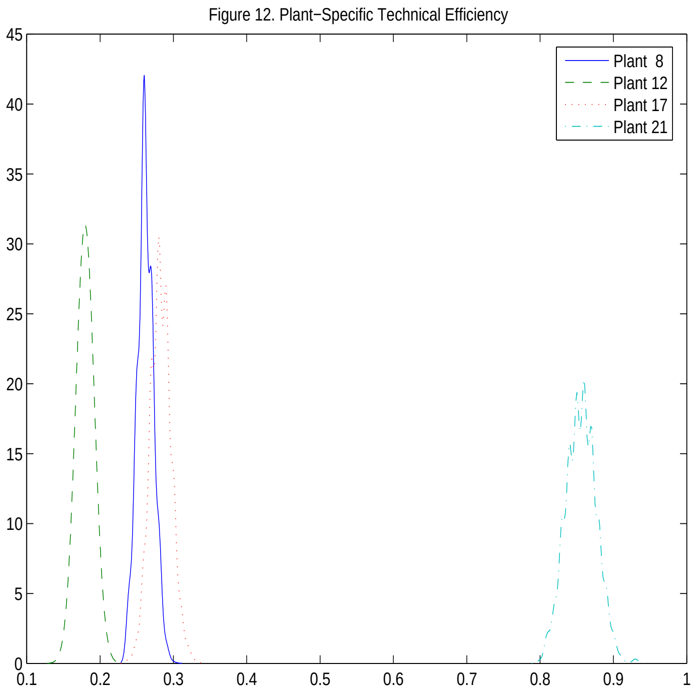


Figure 12. Plant−Specific Technical Efficiency
Plant 8
Plant 12
Plant 17
Plant 21
0.1
0.2
0.3
0.4
0.5
0.6
0.7
0.8
0.9
1
0
5
10
15
20
25
30
35
40
45

Figure 13. Industry−Wide Allocative Inefficiency for Energy

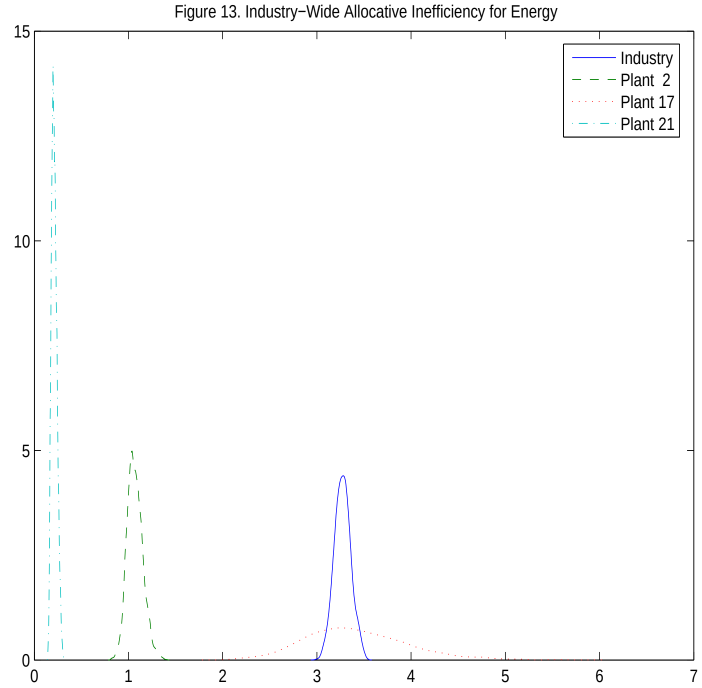


Figure 14. Industry−Wide Allocative Inefficiency for Labor