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Managing Livestock Feed Cost Risks Using Futures and Options

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Abstract

The costs of corn- and soybean-based feeds compose a substantial proportion of the variable costs faced by both mainstream and emergent confined livestock producers. This research develops a method to provide a joint distribution of prices of corn and soybean meal at a future time. Black's 1976 option model and stochastic volatility jump diffusion (SVJD) model are compared in volatility forecasting performance. In general, SVJD is superior to Black's model, though their performance is both commodity-specific and forecasting horizon specific. The price forecast can assist livestock producers to assess different feed procurement strategies in terms of the distribution of costs projected for each strategy.

Keywords: feed cost risks, option pricing models, price forecast, risk management, volatility forecast

Introduction

Livestock producers face many uncertainties: the cost of feed, the efficiency of animal growth, the price of livestock and livestock products, weather shocks, and so on. These sources of volatility create substantial financial risk for an individual producer. During recent years the structure of many livestock sectors (e.g. cattle, poultry, hogs, and dairy) has experienced profound changes with rapid emergence of larger operations and the thinning of traditional output markets, while the structure of other livestock sectors (aquaculture, meat goats, and other ‘novelty’ animals) is still in its formative stages and features highly specialized or regional output markets. Both trends make mitigating output price risk problematic in many livestock sectors.

The costs of corn- and soybean-based feeds compose substantial proportions of the variable costs faced by both mainstream (e.g. hogs, beef, dairy and poultry) and emergent (e.g meat goats and aquaculture) confined animal producers (see Table 1 for an overview). For instance, the feed costs consist of 40-50%, 55-65%, and 20-30% of the variable costs in hog finishing, broiler production, and milk production, respectively. Moreover, feed costs are among the most volatile of all input costs. Because individual farm operators have exploited economies of scale to reduce unit costs and remain competitive at the cost of enterprise diversification, downside risk from feed costs can be fatal to an otherwise efficient farm operator. Hence, the ability to forecast the distribution of the prices of corn and soybean meal at a future time and to be aware of the possible price risk is more crucial than ever.

Because of the prominence of feed costs in determining the variable costs of operation and because of the prominence of corn- and soybean-based products in livestock feeds, the objective of this research is to develop a method to provide a joint distribution of prices of corn and soybean meal at a future time so that the price forecast can assist livestock producers to assess different feed procurement strategies in terms of the distribution of costs projected for each strategy.

In the remaining of this paper, a bivariate distribution is first constructed for corn and soybean meal prices. To forecast the price distribution, a key issue is to forecast volatility. The performance of Black's 1976 option model and a stochastic volatility jump diffusion (SVJD) model is compared in forecasting volatility. The results show that the SVJD model generally is superior to Black's model, though relative performance is both commodity-specific and forecasting horizon specific. Finally, Monte Carlo simulations are performed to create price forecasts.

Distribution of Feed Price

The prices of corn and soybean meal are assumed to follow a bivariate lognormal distribution. This random walk with drift model is a widely accepted setting for commodity price movement.

$$\ln(P_{t,i}) = \ln(P_{t-1,i}) + \mu_i + \varepsilon_{t,i} \tag{1}$$

where ln denotes natural logarithm; $P_{t,i}$ is the cash price of commodity i at time t; i denotes the commodity type (i.e., corn or soybean meal); μ_i is a drift term; and $\varepsilon_{t,i}$ is an innovation term at time t. The drift term captures intrinsic forces driving price movements; the innovation term represents random shocks due to price uncertainty. Equation (1) can be compactly expressed in vector form:

$$\ln(P_t) = \ln(P_{t-1}) + \mu + \varepsilon_t \tag{2}$$

where P_t is a price vector of the two commodities at time t; μ is a vector of drifts; and ε_t is an innovation vector at time t with $\text{cov}(\varepsilon_{t+m}, \varepsilon_{t+n}) = 0$ for $m \neq n$:

$$(3)\quad P_t = \begin{bmatrix} P_{t,c} \\ P_{t,s} \end{bmatrix}, \mu = \begin{bmatrix} \mu_{t,c} \\ \mu_{t,s} \end{bmatrix}, \varepsilon_t = \begin{bmatrix} \varepsilon_{t,c} \\ \varepsilon_{t,s} \end{bmatrix} \sim N(0, \boldsymbol{\Omega}_t), \text{ and } \boldsymbol{\Omega}_t = \begin{bmatrix} \sigma^2_{t,c} & \sigma_{t,c,s} \\ \sigma_{t,c,s} & \sigma^2_{t,s} \end{bmatrix}.$$

where the subscripts c and s denote corn and soybean meal, respectively. $\boldsymbol{\Omega}_t$ is the covariance matrix.

To forecast the P_{t+h} at time t, equation(2) is added up from time t to $t+h$.

$$(4)\qquad \ln(P_{t+h}) = \ln(P_t) + h\mu + \sum_{j=1}^{h} \varepsilon_{t+j},$$

where h is the forecasting horizon. From equation (4),

$$(5)\qquad P_{t+h} = e^{\ln(P_t) + h\mu + \sum_{j=1}^{h} \varepsilon_{t+j}}.$$

According to the efficient market hypothesis (Fama, 1970 and 1991), the observed futures price of a commodity is the unbiased point estimate for the commodity cash price at the futures expiration date. Therefore, the price of a futures contract, whose price is observed at time t and whose delivery is at time $t+h$, can be used as the expectation for cash price at time $t+h$. If a futures contract with expiration date of $t+h$ is not available, an artificial futures price will be constructed by linear interpolation of two nearby futures. Thus,

$$(6)\qquad F_{t,t+h} = E(P_{t+h}) = E(e^{\ln(P_t) + h\mu + \sum_{j=1}^{h} \varepsilon_{t+j}}).$$

where $E(\cdot)$ denotes the expectation at time t. Because $\ln(P_t)$ in equation (2) is normally distributed, $\ln(P_{t+h})$ is also normal by the property that sum of normal distributions is still a normal distribution. Then, it follows:

$$
\begin{aligned}
(7) \qquad F_{t,t+h} = E(P_{t+h}) &= E(e^{\ln(P_t)+h\mu+\sum_{j=1}^{h}\varepsilon_{t+j}}) \\
&= e^{E[\ln(P_t)+h\mu+\sum_{j=1}^{h}\varepsilon_{t+j}]+\frac{1}{2}\text{var}[\ln(P_t)+h\mu+\sum_{j=1}^{h}\varepsilon_{t+j}]} \\
&= e^{\ln(P_t)+h\mu+\frac{1}{2}diag(\Omega_{t,t+h})}.
\end{aligned}
$$

Where the 2×2 matrix $\mathbf{\Omega}_{t,t+h}$ is var$(\sum_{j=1}^{h}\varepsilon_{t+j})$. Therefore, the drift term μ can be derived by:

$$
(8) \qquad \mu = \frac{\ln(F_{t,t+h}) - \ln(P_t) - \frac{1}{2}diag(\mathbf{\Omega}_{t,t+h})}{h}
$$

Except for the covariance matrix $\mathbf{\Omega}_{t,t+h}$, all other items on the right hand side of equation (8) are known at time t. If $\mathbf{\Omega}_{t,t+h}$ is obtained, the drift term μ can be calculated. Further, the bivariate lognormal normal price distribution at time $t+h$ will be generated via Monte Carlo simulation of equation (5), and the mean and variance of the price distribution are explicitly expressed as:

$$
E(P_{t+h}) = F_{t,t+h},
$$

$$
\begin{aligned}
\text{var}(P_{t+h}) &= e^{2[\ln(P_t)+h\mu]+diag(\mathbf{\Omega}_{t,t+h})}(e^{diag(\mathbf{\Omega}_{t,t+h})} - 1). \\
&= (F_{t,t+h})^2[e^{diag(\mathbf{\Omega}_{t,t+h})} - 1].
\end{aligned}
$$

Therefore, the key issue of forecasting the price distribution is to derive the matrix $\mathbf{\Omega}_{t,t+h}$. The off-diagonal term (i.e., the covariance term) will be derived from historical covariance:

$$
(9) \qquad \sigma_{t,t+h,c,s} = \rho_{c,s}\sqrt{\text{var}(\sum_{j=1}^{h}\varepsilon_{t+j,c})}\sqrt{\text{var}(\sum_{j=1}^{h}\varepsilon_{t+j,s})}
$$

where $\sigma_{t,t+h,c,s}$ is the forecasted covariance for corn and soybean meal and $\rho_{c,s}$ is the historical correlation of the return series of corn and soybean meal. The return series are defined as the continuously compounded rates of return:

$$R_{t,i} = \ln(P_{t,i}) - \ln(P_{t-1,i}). \tag{10}$$

Now, the forecasting issue boils down to forecasting volatility terms in the covariance matrix $\Omega_{t,t+h}$.

Volatility Forecasting

Numerous studies have been put forth on forecasting volatility. There are two mainstreams: time series methods and implied volatility. Time series methods include historical methods, ARMA class models and ARCH class models. GARCH models among others are the most popular time series methods in forecasting volatility. Because time series methods use the realized volatility data to forecast volatility, they are sometimes called back-looking or retrospective methods. Because volatility is a key factor when pricing options and it is unobservable, many turn to options data and assume a particular option pricing model to estimate an implied volatility. Options, if rationally priced, contain all the market information about the future probability distribution. Therefore, the implied volatility method is forward-looking or perspective. Intrinsically, an appropriate option pricing model is the key to the implied volatility method.

Poon and Granger (2003) review 93 published papers written in the past two decades that forecasted volatility in financial markets and compare volatility forecasting results across different asset classes and markets in different regions. 39 of the 93 papers made comparisons of the historical methods and GARCH class models. 22 found the historical methods better than GARCH models, while 17 found GARCH superior to the historical methods. For papers comparing the historical methods and implied volatility method, 8 out of 34 found the historical methods better than implied volatility, while the other 26 found the reverse. In 18 papers that compared GARCH and implied volatility, 17 found implied volatility is superior to GARCH.[1] Hence, they conclude implied volatility provides the best forecasting with the historical method and GARCH roughly equal.

Shao and Roe (2001, 2003) conducted research on forecasting the distribution of net revenue in hog finishing, where the series studied included hog, corn and soybean meal prices. In their study, implied volatility derived using Black's (1976) option pricing model was found to be superior to historical volatility and GARCH-based volatility in predicting futures volatility.

The geometric Brownian motion (GBM) assumption girding Black's model, however, has been regarded as unsatisfactory by many researchers. Empirical evidence clearly indicates that many underlying return series display negative skewness and excess kurtosis features (see a review in Bates, 1996b) that are not captured by GBM. In addition, while volatility of the underlying process is assumed to be constant in Black's model, implied volatilities from Black's model often vary with the strike price and maturity of the options (e.g. Rubinstein, 1985 and 1994). A great deal of research in the option pricing literature has extended the GBM setting to better explain option prices. Alternative approaches include jump models (Merton, 1976; Bates, 1991), stochastic volatility models (Wiggins, 1987; Hull and White, 1987; Stein and Stein, 1991; Heston, 1993) and stochastic volatility jump diffusion models (Bates, 1996a and 2000; Bakshi, Cao and Chen, 1997; Scott, 1997), among others.

In this research, we compare the performance of Black's model and a stochastic volatility jump diffusion model in volatility forecasting. A stochastic volatility jump-diffusion (SVJD) model is selected because it possesses substantial qualitative improvements over GBM process. First, the SVJD model allows for discrete jumps due to impacts of new information. Second, it allows the volatility of the price process to change over time. In addition, there have been several articles testing the SVJD model and other option pricing models and the results uniformly favor SVJD. But to our knowledge, there is little research that tests the SVJD model by using agricultural commodity futures and options data or using a SVJD model to forecast volatility.

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[1] Note that Black-Scholes (1973) and Black's (1976) models are the most often used option pricing models for deriving implied volatility.

The forecast of the diagonal terms in the covariance matrix $\mathbf{\Omega}_{t,t+h}$ is defined as:

$$\sqrt{\text{var}(\sum_{j=1}^{h} \varepsilon_{t+j,i})} \equiv \widehat{\sigma}_{t,t+h,i} = IV_{t,i}\sqrt{\frac{h}{52}} \tag{11}$$

where $IV_{t,i}$ is the implied volatility at time t for commodity i; h is the forecasting horizon expressed as weeks, since implied volatility is an annualized estimate. Note that the implied volatility from options on futures is the volatility of futures. It is used as a proxy for the volatility of the cash price series in the absence of exchange traded options on commodity cash prices.

Geometric Brownian Motion Process

In Black's model, the price movement of commodity futures follows a geometric Brownian motion:

$$\frac{dF}{F} = \mu dt + \sigma dZ \tag{12}$$

where F is futures price; Z is a standard Brownian motion with $dZ \sim N(0, dt)$; μ is the expected rate of return on futures; and σ is the annualized volatility of the futures price.

From Ito's lemma, the equation of motion of the logarithm of futures price is:

$$d\ln F = (\mu - \frac{\sigma^2}{2})dt + \sigma dZ. \tag{13}$$

The discrete-time version of (13) is:

$$\ln F_{t+\tau} - \ln F_t \sim N((\mu - \frac{\sigma^2}{2})\tau, \sigma^2\tau). \tag{14}$$

The implied volatility is the σ that can best match the option model determined price with the observed market price. The closed-form option pricing formula can be found in Black (1976).

Stochastic Volatility Jump Diffusion Process

A stochastic volatility jump diffusion process increases flexibility as compared to the geometric Brownian motion by incorporating jumps and movement of volatilities:

$$\begin{aligned} \frac{dF}{F} &= (\mu - \lambda\bar{k})dt + \sqrt{V}dZ + kdq \\ dV &= (\alpha - \beta V)dt + \sigma_v\sqrt{V}dZ_v \\ cov(dZ, dZ_v) &= \rho dt \\ prob(dq = 1) &= \lambda dt, \quad \ln(1+k) \sim N(\ln(1+\bar{k}) - \frac{1}{2}\delta^2, \delta^2) \end{aligned} \tag{15}$$

where:

μ is the rate of return of the futures price;

λ is the annual frequency of jumps;

k is the random percentage of price change conditional on a jump occurring that is lognormally, identically, and independently distributed over time, with unconditional mean $\bar{k}$;

q is a Poisson counter with intensity of λ;

V is the variance term conditional on no jump occurring;

σ_v is the volatility of volatility;

Z and Z_v are each a standard Brownian motion;

ρ is correlation of the two standard Brownian motions, i.e. cov$(dZ, dZ_v) = \rho dt$;

q and k are uncorrelated with each other or with Z and Z_v.

In a representative agent production economy, risk neutral processes of futures price are given by

$$
\begin{aligned}
\frac{dF}{F} &= -\lambda^* \bar{k}^* dt + \sqrt{V} dZ^* + k^* dq^* \\
dV &= (\alpha - \beta^* V) dt + \sigma_v \sqrt{V} dZ_v^* \\
cov(dZ^*, dZ_v^*) &= \rho dt \\
prob(dq^* = 1) &= \lambda^* dt, \quad \ln(1+k^*) \sim N(\ln(1+\bar{k}^*) - \frac{1}{2}\delta^2, \delta^2)
\end{aligned}
\tag{16}
$$

where β^* and α/β^* are the speed of adjustment and long-run mean of the variance. The parameters α, σ_v, δ, and ρ in the risk-neutral process are the same as in the actual processes. A closed-form option formula can be found in Bates (1996a) and Bakshi, Cao and Chen (1997).

To derive the of logarithm of futures price, note that there are two independent uncertainty sources in this model: one is the Brownian motion, and the other is the jump component with a Poisson process. In order to derive the differential, we need to derive the differential of Brownian motion and Poisson motion respectively. The Brownian motion is continuous and Ito's lemma can be used. The jump process, however, is not continuous and hence Ito's lemma is not applicable. So Ito's lemma for semi-martingales is used here.

$$
d(\ln F) = (-\lambda^* \bar{k}^* - \frac{1}{2} V) dt + \sqrt{V} dZ^* + \ln(1+k^*) dq^*. \tag{17}
$$

The discrete-time version of the risk-neutral SVJD processes is:

$$\ln F_{t+\tau} - \ln F_t \sim \left[(-\lambda^* \bar{k}^* - \frac{1}{2}V_t + (\ln(1+k^*) - \frac{1}{2}\delta^2)\lambda^*)\tau, (V_t + \delta^2\lambda^*)\tau\right]. \tag{18}$$

Equation (18) shows that in the stochastic volatility jump diffusion process, in addition to variance term (V_t), the variation of mean jump size and jump frequency (i.e. δ and λ) also monotonically affect the variation of the continuously compounded rate of return. In fact, $\delta^2\lambda^*$ is the instantaneous conditional return variance per year attributable to jumps in the risk-neutral processes. Therefore, the implied volatility, IV_t, is $\sqrt{V_t + \delta^2\lambda^*}$ rather than $\sqrt{V_t}$ alone.

Data

Three years of intradaily transactions data for call options[2] on corn and soybean meal futures and for the underlying futures traded on the Chicago Board of Trade (CBOT) were used. The data consist of the time and price of every transaction for the period of January 2001 to December 2003. The CBOT corn futures contracts are available for March, May, July, September, and December expiration dates. The CBOT soybean meal futures contracts are available for January, March, May, July, August, September, October, and December expiration dates. American-style options are traded on all the contracts. The total sample consists of 18 corn futures contracts and 28 soybean meal futures contracts.

Several filters are applied to construct the synchronous futures and futures options prices. First, weekly data rather than daily data are used in order to reduce computational burden and to avoid the microstructure issues such as the day-of-the-week effect and limits of daily price change. Wednesday (or Tuesday if Wednesday is not available) is selected as having the fewest trading holidays. Second, options transactions are matched with the nearest underlying futures within 4 seconds for corn and 60 seconds for soybean meal, since transactions on soybean meal futures and options are much less liquid than those on corn futures and options. If no matching futures price is obtained within the lapsed time, this option observation is discarded. Third, the options with time-to-maturity of less than 10 trading days are deleted to avoid maturity effects. Fourth, corn options with price less than 2.5 cents and soybean meal options with price less than one dollar are deleted. Fifth, options with price lower than their intrinsic value (i.e., Call$<$[Futures-Strike]) are deleted to eliminate the observations with arbitrage opportunity. The resulting data set includes 8,995 and 4,300 Wednesday observations for corn and soybean meal, respectively. The average daily numbers of options matched are 59.2 and 28.1 for corn and soybean meal, respectively.

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[2] Call options are selected because they are more liquid than put options and therefore can represent the very liquid contracts.

The cash price data are Wednesday Toledo No.2 yellow corn price ($/lb) and Tuesday central Illinois truck delivered 48 percent soybean meal ($/bu).[3] Wednesday 3-month Eurodollar deposits rates are used for the risk-free discount rate.

Estimation Method

Besides the exogenous variables obtained from the data set, the two option pricing models require different parameters as inputs. For Black's option pricing model, the only unobservable input is the volatility term, σ; for the SVJD option pricing model, besides the volatility, inputs also include seven unobservable structural parameters Φ=(λ^*, $\bar{k}^*$, δ, α, β^*, σ_v, and ρ).

In principle, econometric methods can be applied to estimate the parameters since the stochastic processes are known. However, the requirement of a very long time series of futures prices makes this approach inconvenient. Alternatively, a very practical approach is to calculate the implied parameters using the market option prices and observable inputs in the option pricing formulae. Specifically, the implied parameters in the option pricing formulae are obtained by minimizing the sum of squared pricing errors of all options for each day in the sample data set.

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[3] The central Illinois truck delivered 48 percent soybean meal prices were only reported on Tuesdays and used as a proxy for transacted prices of the following Wednesdays.

$$SSE(t) = \min_{v(t),\Phi} \sum_{j=1}^{N_t} [C_j - \widehat{C}_j(v(t), \Phi)]^2 \tag{19}$$

where N_t is the number of options used for date t; C_j is the j-th observed market option price on date t; $\widehat{C}_j$ is the model determined option price with observed exogenous inputs; $v(t)$ is σ_t in Black's model and $\sqrt{V_t}$ in SVJD model for date t; Φ is the vector of structural parameters for the SVJD model. For Black's model there are no structural parameters. Then, the volatility term and structural parameters can be obtained by non-linear least square estimation.

This procedure can result in an estimate of implied volatility and the structural parameters for each day. As discussed in Bates (1991), it is potentially inconsistent with the assumption of constant parameters when deriving the option pricing models, because the implied parameters are not constrained to be constant over time; but a chronology of parameter estimates and some stylized facts for future specification of more complicated dynamic models could thereby be generated through this estimation procedure. The objective function in equation (19) for implied parameter estimation has been used by several others including Bates(1991), Bakshi, Cao and Chen (1997), Hilliard and Reis (1999), and Koekebakker and Lien (2004).

Model Performance Evaluation

Performance of the two option pricing models, Black's model and the SVJD model, is evaluated by three criteria: in-sample pricing fitness, out-of-sample prediction, and volatility forecasting accuracy. The root mean square errors (RMSEs) are compared for the two models.

In-Sample Pricing Fitness

Table 2 reports the summary statistics for the in-sample pricing fitness of Black's model and the SVJD model for corn futures options. First, the in-sample root mean square errors are considerably lower under the SVJD model (Mean=0.71) than under Black's model (Mean=1.09).

Second, the parameter estimates for the SVJD model indicate that the jumps and mean-reverting stochastic volatility are both important. Though the mean level of jump size ($\bar{k}^*$) is quite low, the jump frequency (λ^*) is significant. This may be due to the relative brevity of the sample period, which makes it hard to detect a salient jump pattern. The long run mean (median) of mean-reverting volatility process, which is measured by $\sqrt{\alpha/\beta^*}$, is 0.2423 (0.2285).

Third, the implied volatilities from Black's model and the SVJD model are very close. This finding is consistent with Bakshi, Cao and Chen (1997). They explained that option prices are sensitive to the volatility input and thus even small differences in volatility can result in significantly different pricing results. Note that the maximum implied volatility from Black's model is very high (1.00, the upper bound set for performing estimation), which occurs for July 31st, 2002. The implied volatility from the SVJD model is 0.2966 and the structural parameter estimates are λ^*=0.5479, $\bar{k}^*$=0.0389, δ=0.0589, α=0.7325, β^*=1.9058, σ_v=0.4575, and ρ=-0.5173. The jump size is positive, and the long-run mean and volatility of variance are both significantly higher than their mean levels. The unrealistically high implied volatility from Black's model is because the volatility term is the only parameter that determines the option prices and mere volatility can barely capture a sudden change of the expectation of the exogenous variables, while the SVJD model exhibits an advantage by adjusting its structural parameters timely.

Summary statistics for the in-sample pricing fitness of the Black's model and the SVJD model for soybean meal futures options are shown in Table 3. The SVJD model for soybean meal data still gives smaller RMSEs, but the improvement over Black's model is less impressive than for corn. One reason may be that soybean meal price is less volatile than corn, so that SVJD model does not have the ideal circumstances for exhibiting its advantages. Overall, the in-sample fit of the SVJD model is better than that of Black's model.

Out-of-Sample Pricing performance

One may argue that there might be an overfit problem because the SVJD model has more parameters than Black's model. Therefore, out-of-sample testing is performed. Specifically, the previous day's (Tuesday's) data are used to estimate the volatility and parameters, and then Tuesday's estimates and Wednesday's data are used to predict Wednesday's option prices based on the two models, separately. Then we subtract the model-determined price from its observed counterpart to compute the pricing error. This procedure is repeated for every call and each day in the data sample, to obtain the average root mean squared pricing errors and their associated standard deviations.

Note that this procedure does not constitute a true out-of-sample test in the usual sense, since Wednesday's volatility and structural parameters are assumed to be unchanged from Tuesday's. However, the out-of-sample testing here is pricing out-of-sample options rather than forecasting options prices. The latter involves not only an estimate for the volatility and structural parameters but a forecast for the exogenous variables such as price of the underlying asset and instantaneous interest rate. Therefore, our the testing is equivalent to testing the stability of parameters. This procedure is consistent with previous approaches in the literature (e.g. Bakshi, Cao and Chen, 1997; Hilliard and Reis, 1999).

For corn options, in 121 Wednesdays out of 150 Wednesdays in our sample, the SVJD model gives smaller root mean squared pricing errors than Black's model. For soybean meal, in 120 Wednesdays out of 151 Wednesdays, the SVJD model gives smaller root mean squared pricing errors. The means and standard deviations of the RMSEs are shown in Table 4. Tuesday parameter estimates are not reported because they are similar to their Wednesday's counterparts. As in in-sample testing, the improvement of the SVJD model over Black's model is more impressive for corn options than for soybean meal options.

Volatility Forecast Performance

The third and most important performance test compares the forecasted volatilities from the two models with the realized volatilities.

The realized (*ex post*) volatility is defined as:

$$\sigma_{t,t+h,i} = \sqrt{\sum_{j=1}^{h} R_{t+j,i}^2} \tag{20}$$

where $R_{t+j,i}$ is the rate of return at time $t+j$ for commodity i, as defined in equation (10); h is forecasting horizon. Therefore, $\sigma_{t,t+h,i}$, the realized volatility at time t over the forecasting horizon h for commodity i, can be derived using observed cash price series of commodity i from t to $t+h$.

One criterion for testing performance is the root mean square error of the forecasted volatilities. In addition, a regression of the realized volatility on an intercept and the implied volatility is performed:

$$\sigma_{t,t+h,i} = \alpha + \beta\widehat{\sigma}_{t,t+h,i} + \epsilon_t. \tag{21}$$

The coefficients of determination (R-square) are used to evaluate forecast performance. Another testable implication of the unbiasedness hypothesis is that $\{\alpha, \beta\} = \{0, 1\}$ in equation

(21). If $\alpha = 0$ and $\beta > 1$, or $\alpha > 0$ and $\beta \geq 1$, implied volatility is downwardly biased; if $\alpha > 0$ and $\beta < 1$, implied volatility under-forecasts low volatility and over-forecasts high volatility.

Table 5 gives the volatility forecast performance of Black's and the SVJD models for corn data. Seven forecasting horizons are chosen: h=1, 2, 4, 8, 12 , 26, and 52 weeks. SVJD provides smaller root mean squared volatility forecasting errors than Black's model across all 7 forecasting horizons. As an illustration, Figure 1 shows the comparison of implied volatility with realized volatility for h=12 weeks. The spike in implied volatilities from Black's model is on July 31st, 2002, on which date the Black's annualized implied volatility is 1.

The regression analysis results for corn are reported in Table 6. The coefficients of determination from the SVJD model are consistently higher than those from Black's model, and the highest two are for h=12 and h=8. For implied volatilities from the SVJD model, the estimates of intercept (α) are not significantly different from zero and the estimates of slope (β) are not significantly different from one for h=1, 2, 4, 8, and 12 weeks. For h=1 and h=2, SVJD yields upwardly biased volatility estimates; for h=4 and h=8, implied volatility over-forecasts low volatility and under-forecasts high volatility; and for h=12, implied volatility under-forecasts low volatility and over-forecasts high volatility. For h=12, h=8 and h=14, the F-test cannot reject that $\{\alpha, \beta\}$ = $\{0,\ 1\}$ and therefore the implied volatilities are unbiased forecasts of the realized volatilities. Overall, the SVJD model performs best for h=12, h=8, and h=4.

For soybean meal, the root mean squared errors of volatility forecast are shown in Table 7 and results of the regression analysis are shown in Table 8. The SVJD model does not perform significantly better than Black's model for soybean meal data.

Feed Price Forecast

Basis risk needs to be taken into account before simulating the future price distribution, because the volatilities recovered from options data are of futures prices. Basis is the difference between the local cash price and the closing futures price, so it is region-specific.

$$B_{T,i} = P_{T,i} - F_{T,i}, \tag{22}$$

where $B_{T,i}$ is the local basis at time T of commodity i; $P_{T,i}$ is local cash price at time T; and $F_{T,i}$ is the closing futures price at time T.

From equation (22),

$$E(P_{t+h,i}) = F_{t,t+h,i} + E(B_{t+h,i}) \tag{23}$$

and

$$\text{var}(P_{t+h,i}) = \text{var}(F_{t,t+h,i}) + \text{var}(B_{t+h,i}) + 2\text{cov}(F_{t+h,i}, B_{t+h,i}) \tag{24}$$

where the mean and variance of basis, i.e. $E(B_{t+h,i})$ and $\text{var}(B_{t+h,i})$, are derived from five years of historical futures and cash price data. The covariance between closing futures price and basis is set to zero since historical data show no significant correlation between them. Basis risk is assumed to have zero correlation between the two commodities.

Therefore, the prices and variances of futures contracts should be adjusted by basis to provide the realistic expectations and variances of future local cash prices. We choose the expiration days of futures as the dates to forecast, $t+h$, and go back forecasting horizons h to find the settlement futures prices, $F_{t,t+h}$, and then adjust the mean and variance of cash price P_{t+h} by incorporating basis risk to equation (5).

As an illustration, the forecast horizons are set as 12 weeks and 26 weeks. For corn, there are 14 price forecasts of 12 week horizons. In 10 out of the 14 forecasts, the realized cash prices fall in the intervals one standard deviation from the point estimate. In 14 out of the 14 forecasts, the realized cash prices fall in the 95 percent prediction interval. For horizons of 26 weeks, in 11 out of 12 cases, the realized cash prices fall within one standard deviation; and in all 12 cases, the realized cash prices fall in the 95 percent prediction intervals.

For soybean meal, there are 22 price forecasts of 12 week horizons. In 11 cases, the realized cash prices fall in the intervals one standard deviation from the point estimate; and in 20 cases, the realized cash prices fall in the 95 percent prediction intervals. For horizons of 26 weeks, in 12 out of 20 cases, the realized cash prices fall in the one standard deviation intervals; and in 17 of 20 cases, the realized cash prices fall in the 95 percent prediction intervals. The soybean meal price forecast is not as accurate as the corn forecast, possibly because the basis between soybean meal cash price and settlement futures price is very volatile. Sometimes it can be over 30 dollars.

Summary and Discussion

Being aware of the potential risk in feed price movement and the ability to forecast the distribution of feed cost is crucial for running a modern livestock operation. This study provides a method to forecast the feed price at a future time, where an important issue is to forecast the volatility of future returns. In the literature, implied volatility method based on Black's option model generally performs better than time series models, because forward-looking information is used when using options to recover the volatility of the underlying asset. A stochastic volatility jump diffusion model is selected to compare with Black's model because it possesses substantial qualitative improvements over the geometric Brownian motion process. Our results indicate the performance of Black's model and the SVJD model varies across forecasting horizons and commodities. But overall, SVJD is superior to Black's model. One limitation in this paper is that only one local corn cash price series and one local soybean meal cash price series are used for empirical analysis. Collecting more cash price series of different regions will allow us to examine if the performance of the two models are location-specific.

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Table 1: Corn- and Soybean-based Feed Costs as a Percent of Total Variable Costs

Enterprise	*% Variable Costs**
Feeder Pig Production	40 – 50
Hog Finishing	40 – 50
Beef Finishing	15 – 30
Broiler Production	55 – 65
Layer Production	40 – 50
Meat Goat Production	5 – 15
Milk Production	20 – 30
Ewe and Lamb production	30 – 40
Lamb Finishing	10 – 20
Aquaculture	30 – 60

* Compiled from various Land Grant universities’ enterprise budgets.

Table 2: Summary Statistics of Estimation for Corn Data

	Mean	Std	Median	Max	Min
RMSE$_{Black}$	1.0859	0.4668	1.0687	3.1021	0.2453
RMSE$_{SVJD}$	0.7123	0.3189	0.6353	2.2066	0.1956
IV_{Black}	0.2419	0.0720	0.2397	1.0000	0.1769
$\sqrt{V}_{SVJD}$	0.2289	0.0585	0.2172	0.3607	0.1063
IV_{SVJD}	0.2397	0.0550	0.2265	0.3615	0.1063
λ^*	0.6261	0.0988	0.6056	0.9789	0.3767
$\bar{k}^*$	-0.0237	0.0623	-0.0380	0.3011	-0.1711
δ	0.0775	0.0293	0.0757	0.1957	0.0010
α	0.1207	0.0727	0.1061	0.7325	0.0124
β^*	2.0554	0.1015	2.0314	2.7999	1.8896
σ_v	0.3837	0.0489	0.3800	0.5583	0.2077
ρ	-0.5787	0.0624	-0.5798	-0.4299	-0.7344

Note: RMSE$_{Black}$ and RMSE$_{SVJD}$ are root mean squared pricing errors of Black’s model and SVJD model respectively; the unit of the RMSEs is cent since soybean meal option prices are in cents; IV_{Black} is implied volatility from Black’s model; $\sqrt{V}_{SVJD}$ is the volatility in SVJD model conditional on no jump occurring; IV_{SVJD} is the implied volatility from SVJD model; and λ^*, $\bar{k}^*$, δ, α, β^*, σ_v, and ρ are structural parameters in SVJD model.

Table 3: Summary Statistics of Estimation for Soybean Meal Data

	Mean	Std	Median	Max	Min
RMSE$_{Black}$	0.5737	0.2680	0.5196	1.5185	0.1499
RMSE$_{SVJD}$	0.4100	0.2032	0.3528	1.2601	0.1127
IV_{Black}	0.2175	0.0331	0.2090	0.3044	0.1614
$\sqrt{V}_{SVJD}$	0.2181	0.0382	0.2080	0.3291	0.1574
IV_{SVJD}	0.2257	0.0371	0.2166	0.3321	0.1674
λ^*	0.5911	0.0790	0.5839	0.8786	0.2668
k^*	-0.0300	0.0385	-0.0396	0.1664	-0.1423
δ	0.0698	0.0186	0.0682	0.1420	0.0007
α	0.0982	0.0335	0.0884	0.2087	0.0172
β^*	2.0418	0.0398	2.0381	2.1754	1.9252
σ_v	0.3804	0.0199	0.3798	0.4878	0.2837
ρ	-0.5620	0.0237	-0.5635	-0.4597	-0.6387

Note: RMSE$_{Black}$ and RMSE$_{SVJD}$ are root mean squared pricing errors of Black's model and SVJD model respectively; the unit of the RMSEs is dollar since soybean meal option prices are in dollars; IV_{Black} is implied volatility from Black's model; $\sqrt{V}_{SVJD}$ is the volatility in SVJD model conditional on no jump occurring; IV_{SVJD} is the implied volatility from SVJD model; and λ^*, k^*, δ, α, β^*, σ_v, and ρ are structural parameters in SVJD model.

Table 4: Out-of-sample Average Root Mean Squared Errors

	Black	SVJD
Corn	1.1604	0.8904
	(0.4951)	(0.3959)
Soybean Meal	0.6220	0.4980
	(0.3050)	(0.2703)

Note: Numbers in parentheses are standard deviations.

Table 5: Root Mean Squared Errors of Volatility Forecast for Corn Data

Horizon	Black	SVJD
1	0.0275	0.0250
2	0.0327	0.0288
4	0.0368	0.0322
8	0.0441	0.0358
12	0.0494	0.0381
26	0.0617	0.0524
52	0.0851	0.0737

Table 6: Regression Analysis for Corn Data

Horizon	Black			SVJD		
	α	β	R^2	α	β	R^2
1	0.0197	0.1556	0.0039	-0.0077	0.9807	0.0910
	(0.0071)	(0.2024)		(0.0086)	(0.2531)	
2	0.0311	0.1807	0.0073	-0.0056	0.9613	0.1210
	(0.0085)	(0.1717)		(0.0102)	(0.2116)	
4	0.0311	0.4416	0.0609	-0.0110	1.0787	0.2118
	(0.0099)	(0.1416)		(0.0116)	(0.1699)	
8	0.0532	0.3979	0.0709	-0.0065	1.0366	0.2808
	(0.0116)	(0.1176)		(0.0131)	(0.1355)	
12	0.0711	0.3668	0.0756	0.0006	0.9827	0.3166
	(0.0127)	(0.1047)		(0.0139)	(0.1179)	
26	0.1310	0.2539	0.0656	0.1005	0.4362	0.1129
	(0.0140)	(0.0782)		(0.0174)	(0.9982)	
52	0.2655	-0.0567	0.0118	0.2864	-0.1442	0.0445
	(0.0107)	(0.0424)		(0.0134)	(0.0546)	

Note: Numbers in parentheses are standard deviations.

Table 7: Root Mean Squared Errors of Volatility Forecast for Soybean Meal Data

Horizon	Black	SVJD
1	0.02097	0.02106
2	0.02284	0.02283
4	0.02318	0.02279
8	0.02474	0.02392
12	0.02689	0.02628
26	0.03883	0.03771
52	0.11213	0.10738

Table 8: Regression Analysis for Soybean Meal Data

Horizon	Black			SVJD		
	α	β	R^2	α	β	R^2
1	-0.0059	1.0406	0.0505	-0.0074	1.0519	0.0664
	(0.0111)	(0.3674)		(0.0101)	(0.3211)	
2	-0.0026	1.0134	0.0752	-0.0029	0.9843	0.0913
	(0.0124)	(0.2892)		(0.0112)	(0.2527)	
4	-0.0042	1.0835	0.1522	-0.0040	1.0438	0.1818
	(0.0126)	(0.2081)		(0.0113)	(0.1802)	
8	0.0152	0.8767	0.1769	0.0159	0.8397	0.2088
	(0.0132)	(0.1539)		(0.0118)	(0.1330)	
12	0.0093	0.9850	0.2644	0.0190	0.8605	0.2597
	(0.0140)	(0.1337)		(0.0129)	(0.1182)	
26	0.0534	0.7834	0.2340	0.0712	0.6443	0.2037
	(0.0178)	(0.1153)		(0.0166)	(0.1037)	
52	0.0799	0.9332	0.1008	0.0861	0.8734	0.1136
	(0.0495)	(0.2269)		(0.0450)	(0.1986)	

Note: Numbers in parentheses are standard deviations.

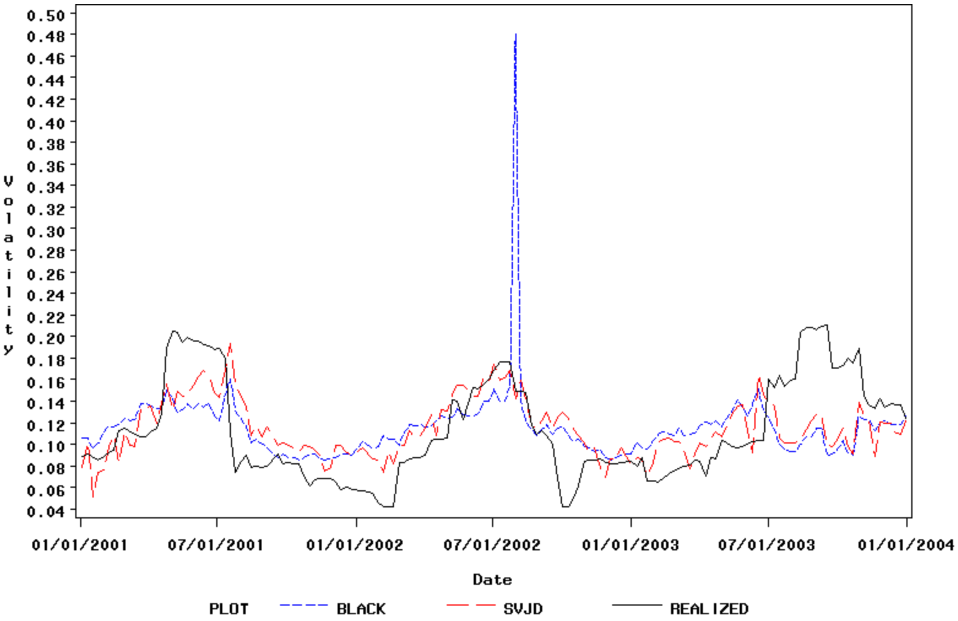

Figure 1: Volatility Forecast Performance for H=12