

The World's Largest Open Access Agricultural & Applied Economics Digital Library

This document is discoverable and free to researchers across the globe due to the work of AgEcon Search.

Help ensure our sustainability.

Give to AgEcon Search

AgEcon Search http://ageconsearch.umn.edu aesearch@umn.edu

Papers downloaded from **AgEcon Search** may be used for non-commercial purposes and personal study only. No other use, including posting to another Internet site, is permitted without permission from the copyright owner (not AgEcon Search), or as allowed under the provisions of Fair Use, U.S. Copyright Act, Title 17 U.S.C.

Realized Volatility in the Agricultural Futures Market

Yuanfang Wang and Matthew C. Roberts*

Selected Paper prepared for presentation at the American Agricultural Economics Association Annual Meeting, Providence, Rhode Island, July 24-27, 2005

Correspondence Author: Matthew Roberts The Ohio State University 318 Agricultural Administration Building 2120 Fyffe Road Columbus, Ohio 43210 Tel: (614) 688-8686 Email: Roberts.628@osu.edu

Copyright 2005 by Yuanfang Wang and Matthew C. Roberts. All rights reserved. Reader may make verbatim copies of this document for non-commercial purposes by any means, provided that this copyright notice appears on all such copies.

^{*}Graduate student and Assistant Professor, Department of Agricultural, Environmental and Development Economics, The Ohio State University, Columbus, OH 43210.

Realized Volatility in the Agricultural Futures Market

1 Introduction

Efficient estimation of market volatility is very important to financial research. However, because asset price volatility is not directly observable, much effort has been devoted to extracting volatility from other observable market activities. One common approach, which a voluminous literature has employed, is to estimate the latent volatility using time-varying volatility models. These time series models fall into one of two categories, the ARCH family, first introduced by Engel (1982), and the stochastic volatility (SV) family, which traces its roots to Clark (1973).

Estimating ARCH models is relatively easy since ARCH models have closed-form likelihood functions in spite of variance being unobserved. However, it is often difficult to draw sharp distinctions between competing ARCH models. Andersen, Bollerslev, Diebold, and Labys (2001, ABDL, hereafter) claim that the existence of multiple competing models suggests misspecification and the robustness of the volatility measures based on these models is uncertain. Compared with ARCH models, stochastic volatility models are relatively difficult to estimate since closed-form likelihood functions don't exist for stochastic volatility models.

An alternative approach to measure market volatility is a model-free estimator, which uses simpler techniques to provide estimates of the ex post realized volatility. The daily squared return constitutes an unbiased estimator for the latent volatility factor. However, Andersen and Bollerslev (1998b) show that it is also a very noisy volatility estimator and

does not provide reliable inferences regarding the underlying latent volatility in daily samples.

Most recently, the availability of intraday financial databases has had an important impact on research in financial market volatility. These intraday data, also called tick-bytick data, have been actively recorded in several exchanges, such as Chicago Board of Trade (CBOT), New York Stock Exchange (NYSE), American Stock Exchange (AMEX) or the National Association of Security Dealers Automated Quotation system (NASDAQ). Traditional financial databases usually provide daily or weekly data, but intraday data give much more information about the market and its associated characteristics. The availability of these new datasets has shed new light on the modeling of volatility. Taylor and Xu (1997) and Andersen and Bollerslev (1997, 1998a) provide thorough descriptions of intraday data and intraday volatility. They show that high frequency intraday returns contain valuable information for the measurement of volatility at the daily level.

Based on these earlier studies, Andersen and Bollerslev (1998b) introduce a new and complementary volatility measure, termed realized volatility. Realized volatility estimates volatility by summing squared intraday returns. Andersen, Bollerslev, Diebold and Ebens (2001, ABDE, hereafter) show that volatility estimates so constructed are close to the underlying integrated volatility. Thus, the volatility of a price process can be treated as an observable process.

Most of the recent research focuses on describing the distributional properties of realized volatility and modeling realized volatility. For example, ABDL (2000) study daily volatility of DM/\$ and Yen/\$ exchange rates; Ebens (1999) analyzes the Dow Jones Industrial Average (DJIA) index; ABDE (2001) examine 30 individual stocks; Areal and

Taylor (2002) study the distributional properties of FTSE-100 futures price; Thomakos and Wang (2003) consider four futures contracts: the Deutsche Mark, the S&P 500 index, US Bonds and the Eurodollar. Although these findings provide support for the realized volatility approach, there is an obvious void in the existing literature. Exchange and stock markets have drawn most researchers' attention. Although there are several studies on futures market, no papers have explored the properties of realized volatility in nonfinancial asset prices.

The purpose of this paper is to address the above issue. The other aim is to compare distributional properties of realized volatilities of non-financial futures with the ones obtained on currency markets and financial futures markets. This study has five sections. The next section explains some theoretical background of realized volatility, describes the data and discusses the choice of sampling interval. Section three introduces existing studies on distributional properties of realized volatilities and presents the results for soybean futures data. In section four, time series models are fitted to realized volatility measures and soybean futures daily returns. The final section concludes.

2 Realized Volatility Measurement

2.1 Theory

The idea of using higher frequency data to generate measures of lower frequency volatility traces its origin to French, Schwert, and Stambaugh (1987), Schwert (1989, 1990a) and Schwert and Seguin (1991). They construct monthly realized equity price volatilities by using squared daily returns. Schwert (1990b) use the standard deviations of intraday returns to study volatility. Schwert (1998) constructs daily stock market

volatilities relying on 15-minute returns. These studies lack formal justification and theoretical underpinnings for such measures. It was not until Andersen and Bollerslev (1998b) that realized volatility was formalized.

Let $p_{n,t}$ denote the time *n* logarithmic price at day *t*, where n = 1,...,N, and t = 1,...,T. Assume it follows a continuous-time stochastic volatility diffusion,

$$dp_{n,t} = \sigma_{n,t} dW_{n,t} \tag{1}$$

where $W_{n,t}$ denotes a standard Brownian motion. The discretely observed time series of returns with *n* observations per day, or a return horizon of 1/n, is defined by

$$r_{n,t} \equiv p_{n,t} - p_{n-1,t}$$
 (2)

Given the sample path of variance, $\{\sigma_{n,t}\}_{n=1,\dots,N;t=1,\dots,T}$, then the variance of daily returns is

$$\sigma_t^2 \equiv \int_1^N \sigma_{n,t}^2 dn \tag{3}$$

And the sum of intraday squared returns, the realized volatility, is defined as

$$\sigma_{rv}^{2} = \sum_{n=1}^{N} r_{n,t}^{2}$$
(4)

Using the theory of quadratic variation, Andersen and Bollerslev (1998b) show that the quadratic variation of the returns in (4) converges to the integrated volatility of (3) almost surely for all *t* as the sampling frequency of the returns increases, or $n \rightarrow \infty$,

$$\lim_{n \to \infty} \sum_{n=1}^{N} r_{n,t}^2 = \int_1^\infty \sigma_{n,t}^2 dn = \sigma_t^2$$
(5)

It follows, therefore, that by using intradaily returns, nonparametric, model-free estimates of volatility can be constructed.

2.2 Data Source and Construction

Intraday Chicago Board of Trade soybean futures prices were obtained from the Futures Industry Institute. The data are time and sales transaction prices, recorded by the exchange. The full sample consists of 4,949,175 high frequency prices from January 2, 1990 through July 31, 2001 for all futures maturities. From 1990 to 1999, the data specify the transaction time to an accuracy of one second-- for the final two years, only to an accuracy of one minute. The price record covers the full CBOT floor trading from 9:30 a.m. to 1:15 p.m. (Central Standard Time).

Dates on which there was a span of at least 25 minutes without trades were omitted. Since most trading activity is usually concentrated in the contract nearest to delivery, the calculation of the returns is based on the nearby futures contract over consecutive intervals. However, returns are calculated from the second nearby contract when the nearby contract is in the delivery month. In order to calculate a continuous sequence of futures returns, an interpolation method is employed. Specifically, returns are calculated using the last recorded logarithmic price before and the consecutive price after each fiveminute mark. This interpolated average is weighted linearly by the inverse relative distance to each time mark. The first return for the trading day is deleted since it is an overnight return. For example, the returns from 9:30 a.m. to 9:35 a.m. are deleted when calculating 5-min returns.

All in all, these corrections result in a sample of 2909 days. One normal trading day consists of 44 intraday 5-minute returns or 23 intraday 10-minute returns or 15 intraday 15-minute returns and so on. The realized volatility is calculated according to equation (4).

2.3 Selecting Time Interval

The usefulness of realized volatility computed from high-frequency data depends on sampling frequency. The theory suggests that realized volatility is effectively an errorfree volatility measure provided that returns are sampled sufficiently frequently. However, market microstructure effects prevent sampling too frequently. Consequently, a problem arises: a reasonable choice of sampling frequency may not be simply the highest available. It may be some value that can balance the microstructure frictions and the measurement errors. Most of the existing studies focus on using 5-minute returns to obtain the daily realized volatility.

ABDL (1999) develop a simple graphical diagnostic, the volatility signature plot. They propose that microstructure bias tends to manifest itself as sampling frequency increases by distorting the average realized volatility. Thus, plotting average realized volatility against sampling frequency may be useful in selecting the optimal frequency. Figure 1 shows the volatility signature plots. From this graph, average realized volatility remains stable as sampling frequency increases up to approximately 30-minute returns. Theoretically, the selection of 30-minute interval represents a reasonable tradeoff between the need of sampling at high frequencies and the cost of market microstructure biases. However, in practice, the open outcry session of soybean futures only spans from 9:30 am to 1:15 pm. The total pit time is 225 minutes. If 30-minute returns are used to formalize the realized volatility, there are only 7 observations for each trading day. Thus, the realized volatility estimates may have measurement errors due to few observations.

ABDL (1999) also suggest that high frequency return autocorrelations provide complementary information for constructing realized volatility intervals. The reason is

straightforward. Tick data are not regularly time-spaced. However, in order to calculate continuous returns series, interpolation methods are necessary for constructing regularly time-spaced data. ABDL (2001) conclude that irregular spacing of the data induces negative autocorrelation in the fixed-interval return series. Bid-ask bounce effects may exaggerate the spurious negative dependence. Thus, negative serial correlations signify the existence of measurement errors.

Since one of the purposes of this study is to compare the properties of realized volatility of grain futures market with those in extant literature, where 5-minute intervals are frequently used, the remainder of this study will focus on 5-minute return series. To check whether this is the proper frequency for computing realized volatility for the current dataset, autocorrelations of 5-minute returns are investigated.

Ebens (1999) provides an approach to determine the negative effect that spurious correlations may induce. Applying the standard MA(q) model to the return series,

$$r_{n,t} = \xi_{n,t} + \theta_{1,t}\xi_{n-1,t} + \theta_{2,t}\xi_{n-2,t} + \dots + \theta_{n,t}\xi_{n-q,t}$$
(6)

where $\xi_{n,t}$ is a white noise process, Ebens (1999) shows that the relationship between realized volatility and daily volatility can be expressed as,

$$E(\sigma_{rv}^{2}) = (1 + \sum_{i=1}^{q} \theta_{i,t}^{2})\sigma_{t}^{2}$$
(7)

where σ_t^2 denotes actual daily volatility.

From equation (7), it is obvious that spurious dependences between returns will result in larger estimates of the actual volatility. Given the fact that the first two sample autocorrelations are significant judged by the 95% confidence interval, applying an MA(2) model to soybean data obtains that $\hat{\theta}_1 = -0.0240$ and $\hat{\theta}_2 = -0.0177$. Equation (7) reveals that the measurement error induced by serial correlation is only 0.0009. In other words, the realized volatility estimate scales up the actual volatility by 1.0009. The order of magnitude is small and thus can be ignored. Hence, 5-minute return series are used in place of 30-minute returns for the remaining of this study.

2.4 Price Limits

Futures markets use price limits as one of the regulation tools to guarantee market integrity. Price limits restrict transaction prices to lie between a symmetric range around the previous day's settlement price. The origin of such limits can be traced to the desire of authorities to reduce the default risk and lower the margin requirement. The Chicago Board of Trade formally applied daily limits in 1925. No trade can take place outside of the limit bounds. For some futures contracts, however, the price limits may be expanded or removed after the contract is locked limit. Also, limits are lifted 2 business days before the delivery month. Advocates of price limits believe that price limits decrease price volatility, reduce default risks and margin requirement, and do not interfere with trading activity. On the other hand, critics claim that price limits create higher volatility levels on subsequent days, prevent prices from reaching the equilibrium level and interfere with trading activities. There has been a large amount of empirical research related to price limits. However, empirical research does not provide conclusive support for either position.

Hall and Kofman (2001) claim that price limits affect market participants' expectation and decisions. Traders adjust their trading behavior by revising their order flow. Correspondingly, the volatility of prices is affected. As revealed in previous

sections, the effectiveness of realized volatility depends on its ability to capture the real price discovery process. If price limits affect the underlying generating process for the equilibrium prices or delay price discovery, realized volatility may be a biased estimator of the market volatility. Furthermore, while some of the markets previously studied in the realized volatility literature do have trading limits in place, such as the S&P 500, they are much less frequently invoked than in physical commodity markets. When the equilibrium price moves beyond the trading limits, trading ceases. Since no trades are recorded during these moves, trading limits naturally bias realized volatility downwards. Consequently, price limits should be taken into account in modeling realized volatility.

To investigate whether price limits may lower the effectiveness of realized volatility, price limit days must be identified at first. Unfortunately, the dataset used in this study does not provide information about price limit days.

A procedure is used in this article to mitigate the inadequacy of data. Three types of dates are defined as follows: (1) Touch days. A touch day occurs when the price limit is touched, but prices do not close at the limit. These are characterized by three conditions:

$$|H_t-C_{t-1}| = \Delta_t \text{ or } |L_t-C_{t-1}| = \Delta_t; \Delta_t >= 30 \text{ cents}; Mod(\Delta_t, 5) = 0$$

where H_t and L_t are high and low prices at day t; C_{t-1} is the closing price at day t-1; Δ_t is the price change which must be greater than 30 cents; Mod denotes the modulus. (2) Closing days. On closing days, prices close at the limit price. They are identified using the three conditions for touch days plus the following condition:

$$C_t = H_t$$
 or $C_t = L_t$

(3) Limit move days. Limit move days occur when all trades occur at the limit move price. It satisfies all of the characteristics of closing days, plus

$$O_t = C_t$$

where O_t denotes the opening price at day t.

Based on the above definitions, 34 touch days, 11 closing days, and 0 limit move days are found in the current sample. The ratios of three different types of days to the total number of observations are 1.17%, 0.38%, 0% respectively.

To further check the effect of price limits on realized volatility, the relationship between realized volatility and conditional variances of the GARCH model is checked using the following equation,

$$\sigma_{rv}^{2} = a_{0} + a_{1}\sigma_{GARCH}^{2} + a_{2}D_{touch} + a_{3}D_{touch}\sigma_{GARCH}^{2} + \varepsilon$$
(8)

where σ_{GARCH}^2 denotes conditional variances estimated from the GARCH (1,1) model; D_{touch} denotes dummy variables, $D_{touch}=1$ if touch day occurs and $D_{touch}=0$ otherwise. The estimates of a_1 , a_2 and a_3 are 0.402, 1.247 and -0.1 respectively. The first two estimates are highly significant and the estimate for a_3 is not significant. The regression results for closing days are similar to touch days except the fact that the estimate for the parameter a_3 , -0.432, is significant. The positive sign of a_1 indicates that realized volatility and the conditional variances of the GARCH model have positive relationship. However, the negative sign of a_3 reveals that the GARCH model may have some information that realized volatility does not have.

Given the fact that detailed information on price limits are insufficient and the proportion of price limit days in the whole sample is small, price limits are not considered in the rest of this essay. However, the simple analysis conducted in this section demonstrates the worthiness to study price limits. From the regression results in this section, the effect of price limits on realized volatility in commodity markets is interesting for future research.

3 Distributions of Realized Volatility and Returns

3.1 Introduction

Distributions of market volatilities have been studied extensively by researchers, traders and regulators. Clark (1973) uses a stochastic process to describe cotton futures volatility. He proposes that the distribution for daily variance may be described as lognormal and that daily returns standardized by daily variance are conditionally normal. Given these two properties, cotton futures returns have been modeled by a leptokurtic mixture distribution with fat tails. The distributional assumptions advocated by Clark are named as the Mixture-of-Distributions-Hypothesis (MDH). Several later studies concentrate on testing this hypothesis. However, as pointed out by Areal and Taylor (2002), "empirical investigation of Clark's conjectures using daily returns has limited potential to provide decisive conclusions because daily volatility is then an unobservable latent variable." Thus, the emergence of intraday realized volatility provides a promising way for modeling volatility distributions since daily volatility can be treated as observed by summing intraday returns sufficiently frequently.

Although previous studies on realized volatility provide support for the realized volatility approach, there is an apparent void in the existing literature, that is, no papers have studied futures contracts of physical commodities. Agricultural futures markets, especially grain futures, are an important component of the CBOT. The trading of agricultural futures contracts has some distinct properties. For example, according to the

regulations of the Chicago Board of Trade (CBOT) the normal pit hours for financial and equity futures are from 7:20 a.m. to 2:00 p.m. and 7:20 a.m. to 3:15 p.m. respectively. By contrast, the market for CBOT grain futures contracts opens at 9:30 a.m. and closes at 1:15 p.m. For financial and equity futures, macroeconomic news play an important role in trading activities, and most news is reported at 8:30 a.m. Thus, U.S. macroeconomic news is announced during the trading hours of the financial and equity futures. The important reports for the grain markets are released when trading is closed. Andersen and Bollerslev (1998a) demonstrate that macroeconomic announcements have an important effect on daily volatility. Different timing may result in different effects on the distributional properties of the realized volatility. Thus, measuring the realized volatility of agricultural futures contract will be a good extension of and compliment to the existing studies.

3.2 Distributional Properties of Volatilities

3.2.1 Unconditional Distributions

In time series applications, volatility can be described by variance, standard deviation or logarithmic variance/standard deviation. In previous sections, σ_{rv}^2 is used to present realized volatility. To avoid any confusion caused by this notation, σ_{rv}^2 stands for realized variance hereafter. σ_{rv} and log(σ_{rv}) denote the realized standard deviation and logarithmic realized standard deviation, respectively. Table 1 provides summary statistics of the unconditional distributions of the realized variances, standard deviations and logarithmic standard deviations. The first column of the first panel in Table 1 provides mean, variance, skewness and kurtosis of the daily realized variance. The standard deviation indicates that the realized daily volatilities fluctuate significantly through time. Moreover, it is obvious that the distribution is right skewed with the coefficient 2.9108. Another obvious result is that the distribution is extremely leptokurtic with the kurtosis coefficient 16.6940.

The summary statistics of the standard deviations also indicate that the distribution has fatter tails than the normal distribution and are skewed to the right. But both values are reduced, with the skewness coefficient 1.2613 and kurtosis coefficient 5.6652. Taken together, although it is a more easily explainable volatility proxy than the variance, the standard deviation still retains non-Gaussian properties.

Also shown in Table 1 are the distributional characteristics for the realized logarithmic standard deviation. The sample skewness and kurtosis coefficient are 0.0410 and 3.1106 respectively. Both numbers suggest the Normal distribution is a close approximation to the logarithmic standard deviation. The results obtained for the logarithmic standard deviation contrast sharply with those of realized variance and standard deviation.

The second panel of Table 1 displays the Jarque-Bera test statistic. The small p-values resulted from the realized variance and standard deviation series and the larger value from the logarithmic series reveal same information as discussed above.

In summary, the unconditional distributional characteristics of the realized variance, realized standard deviation and the logarithmic standard deviation for the soybean futures data are consistent with the findings from ABDL (2001) for the exchange rates, ABDE (2001) for the Dow Jones stocks and Thomakos and Wang (2003) for equity futures.

3.2.2 Temporal Dependence

The existence of volatility clustering at different frequencies has been extensively documented in the finance literature. This high degree of volatility persistence suggests that financial market volatility is highly predictable. Previous studies usually rely on ARCH models to estimate variance, which is a latent process. As realized volatility is a direct measure of actual market volatility, it provides straightforward descriptions of the conditional dependence in volatility.

Figure 2 displays the time series plots of the realized variance, realized standard deviation and the logarithmic standard deviation. The basic properties are in line with those implied by ARCH effects. The three volatility measures seem positively serially correlated and the strong persistence is evident for all three series. The visual impression of the strong clustering effect is confirmed by the highly significant Ljung-Box test statistics and small p-values reported in the upper panel of Table 2. The null hypothesis of no serial correlation is overwhelmingly rejected for all three series.

Figure 3 provides autocorrelation functions for the realized variance, standard deviation and the logarithmic standard deviation. Except reinforcing the persistent correlations of the three series, Figure 3 also indicates that autocorrelations of realized volatilities tend to exhibit slow, hyperbolic decay. For the variance series, the autocorrelation starts around 0.5 and decay very slowly to about 0.04 at lag 90. After that, the autocorrelations tend to fall in the 95% confidence interval. At the 200 day offset, the autocorrelation is 0.0311. Similarly, the realized standard deviation begins around 0.53 and decreases to about 0.04 at the 100-day displacement. However, at some lags after 100, autocorrelations fall out of the 95% confidence band. At lag 200, the autocorrelation is

0.0724, out of the confidence interval. Moreover, the decaying rate of the standard deviation is a little bit slower than that of the variance process.

The correlogram of the logarithmic standard deviation tells a quite different story compared with the first two series. First, the autocorrelations are systematically greater than the 95% confidence level. There is no point where the series tends to fall in the 95% interval. Second, it is obvious that the realized logarithmic standard deviation decays more slowly than the realized variance and standard deviation. Finally, even at a lag of 200, where the correlation value is 0.115, the series is still well above the 95% critical value.

The results outlined above are different from the findings of Ebens (1999), in which both the variance and standard deviation have autocorrelations above the 95% bands. The pattern of the standard deviation is similar to the logarithmic variance instead of the variance.

The low first-order autocorrelations of the three series may indicate that they do not exhibit unit-roots. However, the slow decay may suggest the presence of a unit root. The Augmented Dickey-Fuller test, allowing for a constant with 10 lagged difference terms, routinely and soundly rejects the unit-root hypothesis. As shown in the lower panel of Table 2, the test statistics are -7.0267, -7.1378 and -7.6654 for three volatility measures and the 1% and 5% critical values are -3.4324 and -2.8623. Based on these results, the autocorrelations of the three series are best characterized by long memory processes. This assertion is consistent with the existing findings in exchange rates, stock index, and equity futures.

3.3 Distributions of Standardized Returns

The daily returns series is constructed by taking the first difference of the last recorded prices for the whole sample. Then, this series is standardized to a mean zero process. The following analysis is based on the transformed returns, which are denoted as r_t . The first column of Table 3 presents a summary of the unstandardized returns. Consistent with previous findings, the unconditional distributions of the futures returns are approximately symmetric but highly leptokurtic, with the sample skewness -0.0341 and the sample kurtosis 5.5787.

In time series applications, the daily returns are always described as: $r_t = \sigma_t \eta_t$, where $\eta_t \sim iid(0,1)$ and σ_t is the conditional standard deviation of returns. If a time series model, ARCH or Stochastic Volatility model, is correctly specified, the standardized returns, r_t / σ_t , should account for the tail thickness. However, it is well known that the standardized returns from ARCH models still display large kurtosis. The second column of Table 3 provides the sample moments of the standardized returns. Specifically, this series is calculated by dividing the unstardardized returns by estimates of standard deviations of Normal-GARCH (1,1) model. It is evident that the distribution appears fattailed, although the value of the sample kurtosis has decreased to 4.6241. Moreover, the skewness coefficient is positive, which indicates a right-skewed curve.

In contrast, the diagnostic statistics in the third column of Table 3 verifies that the distribution of the realized volatility standardized daily returns is more close to a standard normal. The coefficient of kurtosis has decreased to 3.7122, although it is still slightly leptokurtic.

In summary, the values of mean, standard deviation, and skewness for the three volatility measures are close. The differences in the sample kurtosis are significant. The result from the realized volatility standardized returns stands in sharp contrast to the unstandardized returns and GARCH standardized returns. The findings about the standardized returns largely correspond with extant literature. However, an interesting phenomenon is that the kurtosis coefficient of the standardized returns differs in value for different markets. For example, Ebens (1999) finds that the kurtosis estimate for DJIA is 2.75; ABDL (2000) determine the coefficients are 2.406 and 2.414 for DM/\$ and Yen/\$ respectively. Those values indicate that the distributions appear platykurtic. Thomakos and Wang (2003) find that the sample kurtosis coefficients for DM, E-Dollar and S&P futures are 3.0147, 3.1579, and 3.0521 and only T-bonds futures is less than 3.

All in all, except differences in magnitude with previous findings, the results in 3.2 and 3.3 confirm that the Mixture-of-Distributions-Hypothesis (MDH) advocated by Clark is valid in the soybean futures market.

3.4 Other Properties

3.4.1 ARCH-M Effect

In some financial applications, the expected return on an asset is related to the expected asset risk (volatility). This relationship has been widely explored in portfolio theory, for example, the Markowitz Portfolio analysis and the Capital Asset Pricing Model. Many argue that an increase in volatility results in an increase in the expected rate of return. Engel, Lilien and Robbins (1987) propose the ARCH in mean (ARCH-M) model, where the conditional variance is included into the mean equation. Two variants

of the ARCH-M specification use the conditional standard deviation and the log variance (standard deviation) in place of the conditional variance.

Figure 4 displays the relationships between daily returns, r_i , and three different volatility measures. The straight line in each graph is obtained by regressing volatility on daily returns using least squares. Also shown in Figure 4 are the scatter plots of returns and three volatility measures. It reveals the non-linear relationship between returns and volatility. The three graphs have the similar pattern. The linear relationship between current daily returns and volatilities is not obvious. The coefficients of multiple determination, R^2 , are 0.0014, 0.0009 and 0.0005 for three regression lines respectively. These diagnostic statistics confirm that impression. Thus, the ARCH-M effect can be ignored for current data.

3.4.2 Asymmetric Volatility

In 3.4.1, the relationship between volatility and current returns was investigated. In time series applications, the asymmetric response of volatility to past returns is also of interest. French, Schwert and Stambaugh (1987) and Schwert (1990b) find that stock volatility is negatively related to stock returns. Nelson (1991) argues that for equities, negative returns are followed by higher volatilities than positive returns of the same magnitude. This phenomenon is known as "leverage effect". In most applications, leverage effects have been probed for stock returns. For futures data, since the leverage hypothesis can not be applied¹, the term "asymmetric volatility" is used to describe an asymmetry in the relation between volatility and returns. The Threshold ARCH

¹ See Schwert (1990b) for details about two hypotheses of the leverage effects.

(TARCH) model is introduced independently by Zakoian (1990) and Glosten, Jagannathan and Runkle (1991) to describe asymmetric volatility. The following regression models are based upon the specification for the conditional variance of TARCH models. They are used to investigate the asymmetric effects on the realized variance, the standard deviation and the log standard deviation.

$$\sigma_{r_{v,t}}^2 = \overline{\omega} + \alpha \sigma_{r_{v,t-1}}^2 + \beta r_{t-1}^2 + \gamma r_{t-1}^2 d_{t-1} + \varepsilon_t$$
(9a)

$$\sigma_{rv,t} = \varpi + \alpha \sigma_{rv,t-1} + \beta r_{t-1} + \gamma r_{t-1} d_{t-1} + \varepsilon_t$$
(9b)

$$\log(\sigma_{rv,t}) = \varpi + \alpha \log(\sigma_{rv,t-1}) + \beta r_{t-1} + \gamma r_{t-1} d_{t-1} + \varepsilon_t$$
(9c)

where $d_t = 1$ if $r_{t-1} < 0$ and 0 otherwise.

In models (9a), (9b) and (9c), good news ($r_{t-1} > 0$), and bad news ($r_{t-1} < 0$), have different effects on the volatility. Good news has an impact of β , while bad news has an impact of ($\beta + \gamma$). If $\gamma \neq 0$, asymmetric volatility exists.

Table 4 reports the regression estimates with their standard errors. All estimates are significant at 1% level. The β coefficients are positive and the γ coefficients are negative for all three volatility measures. Thus, these results support the existence of asymmetric volatility. This parallels the findings of Thomakos and Wang (2003) for Deutsche Mark, Eurodollar and T-bonds futures contracts. And Ebens (1999) and ABDE (2001) all point toward the presence of asymmetries in stock returns. However, although statistically significant for all three volatility measures, the γ coefficients are smaller in absolute magnitude.

4 Modeling Realized Volatility and Daily Returns

4.1 ARMA and ARFIMA Models

From the stylized facts depicted above, the long memory characteristics of realized volatilities, realized standard deviations and logarithmic standard deviations are obvious. This is a well-known fact in existing literature. Related work includes ABDL (1999), Ebens (1999), Areal and Taylor (2002) and Thomakos and Wang (2003), among others. Given this fact, it is possible that standard parsimonious ARMA models may not account for the high order autocorrelations. Consequently, a fractionally integrated ARMA (ARFIMA) model may be applied to model the realized volatility series. In contrast to ARMA models, which explain the typical exponentially decaying autocorrelations, ARFIMA models capture the stylized fact of long memory processes, which have slower decay of the autocorrelations. Granger and Joyeux (1980) introduced the ARFIMA process. The general representation of the ARFIMA (p, d, q) model is,

$$\phi(L_p)(1-L)^d y_t = \phi(L_q)\xi_t \tag{10}$$

where L is the lag operator; d < 1 is the integration parameter; p and q are order

parameters; $\phi(L_p) = \sum_{i=1}^{p} \phi_i L^i$ and $\varphi(L_q) = \sum_{i=1}^{q} \varphi_i L^i$. This representation includes AR and MA processes as two special cases. When d = 1, equation (10) reduces to the ARIMA

4.2 ARMA and ARFIMA Model Estimation Results

model.

Table 5 reports the estimation results of ARMA (5,4) and ARFIMA (5,d,4) models.² These two models are selected as representatives of two types of competing families.

² All of the models (including ARFIMA and GARCH families) were estimated in OX. The corresponding packages include Arfima 1.0 and GARCH 3.0 (Doornik and Ooms (1998), Laurent and Peters (2002), and Doornik (2002)).

Various models have been tested based on log likelihood values and Q-tests. ARMA (5,4) and ARFIMA (5,d,4) balance the requirements of model fitting and parsimony.

It is evident that the estimate for the fractional integration parameter is highly significant. The value of this parameter is 0.4593, which is consistent with the findings of ABDL (1999). ABDL (1999) suggest that the estimate of the fractional integration coefficient tends to be in the neighborhood of 0.4 for many realized volatilities.

Compared with the ARMA (5,4) model, the ARFIMA (5,4) model has several advantages. First, most estimates are statistically significant, which indicates good in-sample fitting. Second, the Q-test results are better. The p-values at Lag 35, 45, 55 and 65 are 0.006, 0.020, 0.060, 0.050 for the ARMA (5,4) model. And the corresponding values for the ARFIMA (5,4) model are 0.023, 0.065, 0.163 and 0.110. Finally, less obvious, the log likelihood represents a small improvement.

The higher order models described above are not coincidences. For example, Thomakos and Wang (2003) fit an ARFIMA (5,d,5) model to D/M, E-dollar, S&P 500 and T-bonds futures data. However, Ebens (1999) finds that ARFIMA (0,d,0) model is good enough to capture a long memory process for DJIA data. Since the history of realized volatility is relatively short, it is still uncertain to conclude whether the higherorder model reveals any special properties of futures data.

4.3 GARCH Models

Numerous findings have demonstrated the persistence of volatility in financial markets. Engle (1982) introduces ARCH model to capture this stylized fact. Bollerslev (1986) generalizes it to the GARCH case. The GARCH (p,q) model is in the form of,

$$y_t = c + \varepsilon_t \tag{11}$$
$$\varepsilon_t = \sigma_t \eta_t$$

$$\sigma_t^2 = \omega + \sum_{i=1}^p \alpha_i \varepsilon_{t-1}^2 + \sum_{i=1}^q \beta_i \sigma_{t-1}^2$$

where $\eta_t \sim iid(0,1)$.

GARCH family has many extensions. Before choosing any model specifications, it is important to guarantee that candidate models can account for the facts outlined in Section 3. Based on the results in the previous section, FIGARCH and FIEGARCH models are possible model specifications for the soybean data. FIGARCH-M model is also considered here to check whether this parametric model reflect the same pattern as the realized volatility approach.

 $\sum_{i=1}^{p} \alpha_i + \sum_{i=1}^{q} \beta_i = 1$. They name this type of models as integrated GARCH (IGARCH) models. As integrated in mean processes, a shock persists in the future variance for IGARCH processes. Baillie, Bollerslev, Mikkelsen (1996) extend IGARCH models to accommodate fractional integration. The new model is titled as FIGARCH. The variance equation of a FIGARCH (p,d,q) model is,

Engle and Bollerslev (1986) consider a special class of GARCH models, in which

$$\sigma_t^2 = \omega [1 - \beta(L)]^{-1} + \{1 - [(1 - \beta(L))]^{-1} \phi(L) (1 - L)^d] \varepsilon_t^2$$
(12)

where *L* is the lag operator; d < 1 is the integration parameter; $\alpha(L) = \sum_{i=1}^{p} \alpha_i L^i$,

$$\beta(L) = \sum_{i=1}^{q} \beta_i L^i$$
, and $\phi(L) = (1 - \alpha(L) - \beta(L))(1 - L)^{-1}$.

The FIGARCH-M model has the same variance equation as the FIGARCH model specified above. The only difference is that the mean equation becomes,

$$y_t = c + \sigma_t^2 + \varepsilon_t \tag{13}$$

Nelson (1991) proposes the EGARCH model to accommodate the leverage effect and the asymmetry in the conditional variance. Bollerslev and Mikkelsen (1996) refine the EGARCH model as follows,

$$\ln \sigma_t^2 = \omega + [1 - \beta(L)]^{-1} [1 + \alpha(L)] g(\eta_{t-1})$$
(14)

where $g(\eta_t) = \theta \eta_t + \gamma [|\eta_t| - E |\eta_t|]$.

Combing the idea of fractional integration with EGARCH type of model, Bollerslev and Mikkelsen (1996) introduce the FIEGARCH model. The variance equation of FIEGARCH (p,d,q) is specified as follows,

$$\ln \sigma_t^2 = \omega + \phi(L)^{-1} (1 - L)^{-d} [1 + \alpha(L)] g(\eta_{t-1})$$
(15)

4.4 Model Estimation Results

For the soybean futures returns, $y_t = r_t$ and the constant *c* is excluded from the mean equation since the daily returns has been demeaned prior to estimation. Table 6 reports the estimation results of FIGARCH (1,d,1), FIGARCH-M (1,d,1), and FIEGARCH (0,d,1) models. For FIGARCH and FIGARCH-M models, the ARCH innovations η_t are the Student-t density; for FIEGARCH models, the innovations are the generalized error distribution (GED).

Several regularities emerge from the estimates presented in Table 6. First, the parameter estimates of d are highly significant for all models. For FIGARCH (1,d,1) and

FIGARCH-M (1,d,1)models, the estimates are 0.4571 and 0.4585 respectively. Again, this magnitude is consistent with the assertion of ABDL (1999). For the FIEGARCH model, the value of *d* is much higher, 0.5650. This is in line with the one reported by Ebens (1999), who finds d = 0.585 for the FIEGARCH model for the DJIA data.

Second, consistent with the findings of previous sections, the ARCH-M effect is not obvious and thus can be ignored. This point is confirmed through comparison between the FIGARCH (1,d,1) and the FIGARCH-M (1,d,1) models. There are no significant differences in goodness-of-fit for both models as implied by several test statistics.

Finally, as expected, the estimates of the two additional parameters in the FIEGARCH model are highly significant. And the sign of θ is positive, which indicates that a positive correlation between the past return and subsequent volatility. This point is consistent with the result in previous sections. Compared with the FIGARCH model, the FIEGARCH is a more promising GARCH specification for characterizing daily returns and volatilities. The addition of two parameters is not only highly significant from the log likelihood ratios but is also preferred from AIC and SC perspective.

All in all, the results from parametric models largely correspond with the findings in previous nonparametric analysis based on the realized volatilities. And the values of estimates are highly consistent with extant literature.

5 Conclusion

The use of intraday returns as a direct measure of market volatility is a relatively new field. Previous studies of realized volatility generally focus on equities or exchange rates.

The point of this study is not to offer a new theoretical approach for modeling volatility, instead, it is to investigate the properties of realized volatility in the grain futures market.

The results indicate that realized volatility based on 5-minute returns largely correspond with existing literature. Specifically, the properties of the realized variance, the standard deviation and the log standard deviation have quite similar patterns as those observed in stock market or exchange rate market, although there are some discrepancies in magnitude. The findings of three volatility measures confirm that the Mixture-of-Distributions-Hypothesis (MDH) advocated by Clark (1973) is valid in the soybean futures market.

In contrast, the standardized daily returns display some different properties compared with stock and exchange rate data. The asymmetric effect exists and the news impact curves are more steeply sloped to the right of the origin, which indicates that a positive correlation between the past return and subsequent volatility.

The long memory characteristics of realized volatilities, especially for the logarithmic standard deviations are obvious. Thus an ARFIMA model is used to describe the volatility process. And the results indicate that the ARFIMA model is better than the corresponding ARMA model.

Moreover, the parametric GARCH models, FIGARCH, FIGARCH-M and FIEGARCH, reflect the patterns described by nonparametric analysis. The implication of this conclusion is that the existing time series models can provide good in sample fits and may result in good forecasts since the validity and usefulness of realized volatility has been thoroughly explored.

	Realized	Realized Std.	Logarithmic Std.
	Variance	Deviation	Deviation
Mean	0.8628	0.8686	-0.2071
Std. Deviation	0.7340	0.3294	0.3628
Skewness	2.9108	1.2613	0.0410
Kurtosis	16.6940	5.6652	3.1106
Jarque-Bera Test	26837.5600	1632.2530	2.2991
P-Value	0.0000	0.0000	0.3168

 Table 1: Summary Statistics of Daily Realized Variance, Realized Standard Deviation and Logarithmic Standard Deviation

	Realized Varia	nce	Realized Std. I	Deviation	Logarithmic Std. Deviation	
Q-Test						
& P-Value						
Lag 1	709.31	0.0000	834.18	0.0000	870.59	0.0000
Lag 5	2540.10	0.0000	3247.40	0.0000	3484.70	0.0000
Lag 15	5197.80	0.0000	69996.7	0.0000	7677.00	0.0000
Lag 20	5910.80	0.0000	8097.20	0.0000	8997.00	0.0000
Lag 25	6419.80	0.0000	8995.50	0.0000	10171.00	0.0000
Lag 30	6740.50	0.0000	9625.20	0.0000	11065.00	0.0000
Lag 35	6945.50	0.0000	10094.00	0.0000	11799.00	0.0000
ADF Test*						
& P-Value	-7.6654	0.0000	-7.1378	0.0000	-7.0267	0.0000

* The 1% and 5% critical values for the ADF test are -3.4324 and -2.8623.

Table 2: Ljung-Box and Augmented Dickey-Fuller Test Statistics

	r_t^*	r_t / σ_{GARCH}	r_t / σ_{rv}
Mean	0.0000	0.0130	-0.0085
Std. Deviation	1.1942	1.0000	1.2634
Skewness	-0.0341	0.0015	-0.0497
Kurtosis	5.5787	4.6241	3.7122

* The return series is multiplied by 100.

Table 3: Descriptive Statistics for Daily Returns

Estimates and Standard Errors*								
	α	τ	6	χ	ļ	3	Ŷ	/
Variance	0.4380	0.0256	0.4403	0.0282	0.0489	0.0110	-0.0332	0.0125
Std. Deviatio n	0.4057	0.0200	0.4863	0.0221	0.0627	0.0118	-0.0930	0.0177
Log Std. Deviatio n	-0.1391	0.0112	0.5072	0.0214	0.0569	0.0101	-0.0848	0.0161

The table reports the ordinary least squares regression for the model (9). Standard errors are based on Newey-West heteroscedasticity and autocorrelation consistent estimators. * All estimates are significant at 1% level.

Table 4: News	Impact Function	Estimates
---------------	-----------------	-----------

	ARMA		ARFIMA	A
d-parameter			***0.4593	0.0283
AR-1	***-1.0170	0.2335	***-0.2675	0.0769
AR-2	***0.6735	0.2023	***1.0272	0.0776
AR-3	***1.1362	0.1941	***0.2916	0.0665
AR-4	0.1532	0.2261	***-0.7616	0.0743
AR-5	***-0.0626	0.0306	***-0.0936	0.0312
MA-1	1.2801	0.2322	0.0695	0.0586
MA-2	-0.1898	0.2586	***-1.1006	0.0562
MA-3	*-0.9004	0.1013	*-0.1174	0.0618
MA-4	-0.2391	0.1853	***0.8260	0.0582
Q-test & P-value				
Lag 35	47.400	0.006	41.000	0.023
Lag 45	55.504	0.020	48.425	0.065
Lag 55	61.754	0.060	54.218	0.163
Lag 65	74.516	0.050	68.163	0.110
Log likelihood		-386.822		-383.156

*, **, *** significant at 10%, 5% and 1%.

Table 5. ARMA and ARTIMA Model Estimate	Table 5	5: ARMA	and A	ARFIMA	Model	Estimates
-----------------------------------------	---------	---------	-------	--------	-------	-----------

	FIGARCH (1,	d,1)##	FIGARCH-M	(1,d,1)	FIEGARCH	(0,d,1)
ω	**0.1101	0.0475	**0.1094	0.0470	0.1304	0.1136
α	*0.1313	0.0675	*0.1305	0.0668		
β	***0.5688	0.1168	***0.5694	0.1157	***0.7728	0.0926
d-Figarch	***0.4571	0.0906	***0.4585	0.0904	***0.5650	0.0809
ARCH-in-mean			-0.0096	0.0139		
heta					***0.0406	0.0131
γ					***0.1089	0.0221
AIC		3.0311		3.0316		3.0276
SC		3.0413		3.0439		3.0399
Log-likelihood		-4402.162		-4401.930		-4396.144
Skewness		-0.0078		-0.0090		-0.0492
Kurtosis		4.7133		4.7148		4.5396
Q-test & P-Value#						
Lag 10	11.975	0.287	11.993	0.285	11.921	0.290
Lag 15	16.132	0.373	16.093	0.376	15.854	0.392
Lag 20	21.669	0.359	21.647	0.360	21.581	0.364

*, **, *** significant at 15%, 5% and 1%.# Q-test for the standardized residuals.

#The ARCH innovations η_t are conditional on the student-t distribution for the FIGARCH and FIGARCH-M models and the innovations are conditional on GED for the FIEGARCH model.

Table 6: FIGARCH, FIGARCH-M and FIEGARCH Estimation Results



Figure 1: Volatility Signature Plot for Different Time Intervals

Realized Variance



Realized Std. Deviation

Log Std. Deviation



Figure 2: Time Series Plots of Daily Realized Volatilities



Figure 3: Autocorrelation Functions of Daily Realized Volatilities







Figure 4: Realized Volatilities Vs. Daily Returns

Bibliography

- Andersen, T. G. and Bollerslev, T. (1997), "Intraday Periodicity and Volatility Persistence in Financial Markets," *Journal of Empirical Finance*, 4, 115-158
- Andersen, T. G. and Bollerslev, T. (1998a), "Deutsche Mark-Dollar Volatility: Intraday Activity Patterns, Macroeconomic Announcements, and Longer Run Dependencies," *Journal of Finance*, 53, 219-265
- Andersen, T. G. and Bollerslev, T. (1998b), "Answering the Skeptics: Yes, Standard Volatility Models Do Provide Accurate Forecasts," *International Economic Review*, 39, 885-905
- Andersen, T. G., Bollerslev, T., Diebold, F. X., and Ebens, H. (2001), "The Distribution of Realized Stock Return Volatility," *Journal of Financial Economics*, 61, 43-76
- Andersen, T. G., Bollerslev, T., Diebold, F. X., and Labys, P. (1999), "Understanding, Optimizing, Using and Forecasting Realized Volatility and Correlation," NBER Working Paper
- Andersen, T. G., Bollerslev, T., Diebold, F. X., and Labys, P. (2000), "Exchange Rate Returns Standardized by Realized Volatility are (Nearly) Gaussian," *Multinational Finance Journal*, 4, 159-179
- Andersen, T. G., Bollerslev, T., Diebold, F. X., and Labys, P. (2001), "The Distribution of Realized Exchange Return Volatility," *Journal of American Statistical Association*, 96, 42-55
- Areal Nelson M.P.C. and Taylor, S. J. (2002), "The Realized Volatility of FTSE-100 Futures Prices," *Journal of Futures Markets*, 22, 627-648
- Baillie, R. T., Bollerslev, T., and Mikkelsen, H. O. (1996), "Fractionally Integrated Generalized Autoregressive Conditional Heteroskedasticity," *Journal of Econometrics*, 74, 3-30
- Bollerslev, T. (1986), "A Generalized Autoregressive Conditional Heteroskedasticity," Journal of Econometrics, 31, 307-327
- Bollerslev, T. and Mikkelsen, H. O., (1996), "Modeling and Pricing Long-Memory in Stock Market Volatility," *Journal of Econometrics*, 73, 151-184
- Clark, P. K. (1973), "A Subordinated Stochastic Process Model with Finite Variance for Speculative Prices," *Econometrica*, 41, 135-155

Doornik, J.A. (2002), Object-Oriented Matrix Programming Using Ox, 3rd ed. London:

Timberlake Consultants Press and Oxford: www.nuff.ox.ac.uk/Users/Doornik

- Doornik, J. A. and Ooms, M (1998), "A Package for Estimating, Forecasting and Simulating ARFIMA Models: ARFIMA Package 1.01 for OX," Online Tutorial
- Ebens, H. (1999), "Realized Stock Volatility," Working Paper No. 420, Department of Economics, Johns Hopkins University
- Engle, R. F. (1982), "Autoregressive Conditional Heteroskedasticity with Estimates of the Variance of United Kingdom Inflation," *Econometrica*, 50, 987-1007
- Engle, R. F. and Bollerslev, T. (1986), "Modeling the Persistence of Conditional Variances," *Econometric Reviews*, 5, 1-87
- Engel, R. F., Lilien, D. M., and Robins, R. F. (1987), "Estimating Time Varying Risk Premia in the Term Structure: the ARCH-M Model," *Econometrica*, 55, 391-407
- French, K. R., Schwert, G. W., and Stambaugh, R. F. (1987), "Expected Stock Returns and Volatility," *Journal of Financial Economics*, 19, 3-29
- Glosten, L. R., Jagannathan, R. and Runkle, D. (1991), "Relationship between the Expected Value and the Volatility of the Nominal excess Return on Stocks," *Mimeo*, Northwestern University
- Granger, C. W. J., and Joyeux, R. (1980), "An Introduction to Long Memory Time Series and Fractional Differencing," *Journal of Time Series Analysis*, 1, 15-30
- Hall, A. D. and Kofman, P. (2001), "Limits To Linear Price Behavior: Futures Prices Regulated by Limits," *Journal of Futures Markets*, 21, 463-488
- Laurent, S. L. and Peters, J. P. (2002), "A Tutorial for G@RCH 2.3, a Complete Package for Estimating and Forecasting ARCH Models", Online Tutorial
- Nelson, D. B. (1991), "Conditional Heteroskedasticity in Asset Returns: a New Approach," *Econometrica*, 59, 347-370
- Schwert, G. W. (1989), "Why Does Stock Market Volatility Change over Time?" *Journal* of Finance, 44, 1115-1153
- Schwert, G. W. (1990a), "Stock Market Volatility," *Financial Analysts Journal*, 46, 23-34
- Schwert, G. W. (1990b), "Stock Volatility and the Crash of '87," *Review of Financial Studies*, 3, 77-102
- Schwert, G. W. (1998), "Stock Market Volatility: Ten Years after the Crash," Brookings

Wharton on Financial Services, 1998. Washington, DC: Brookings Institution

- Schwert, G. W. and Seguin, P. J. (1991), "Heteroskedasticity in Stock Returns," *Journal* of Finance, 45, 1129-1155
- Taylor, S. J., and Xu, X. (1997), "The Incremental Volatility Information in One Million Foreign Exchange Quotations," *Journal of Empirical Finance*, 4, 317-340
- Thomakos, D. D. and Wang, T. (2003), "Realized Volatility in the Futures Markets," *Journal of Empirical Finance*, 10, 321-353

Zakoian, J-M. (1990), "Threshold Heteroskedastic Model," Mimeo, INSEE, Paris