A GLOBALLY FLEXIBLE, QUADRATIC ALMOST IDEAL DEMAND SYSTEM: AN APPLICATION FOR MEAT DEMAND IN TAIWAN

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ABSTRACT

A new demand system, combining the quadratic almost ideal demand system and the Fourier expenditure system, is introduced. An application for meat consumption in Taiwan indicates that the new demand system fits the data well and that the restriction to the usual specifications, such as locally flexible functional form, linear Engel curve, and both, are rejected.

Key words: the Q-AIDS, Fourier flexible form, meat demand, Taiwan.
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1. Introduction

Functional form specification is one of the most concurrent issues in empirical studies of consumer demand and has been popularly used to approximate either indirect utility or expenditure functions. Many of the commonly used demand systems, such as the translog (Christensen, et al., 1975) and the almost ideal demand system (AIDS) by Deaton and Muellbauer (1980), utilized Taylor's series to form a second order approximation for either the indirect utility function or the cost function, which satisfies the defining property within a finite domain of prices and income. A sharp critique on Taylor's approximation or so-called locally flexible functional forms (Barnett, 1983) can be remedied by the Fourier approximation, which is proved not local, but uniform or global (Chalfant, 1983).

The purpose of this study is to introduce a globally flexible, quadratic almost ideal demand system (GF-QAIDS) by incorporation of the Fourier series suggested by Gallant (1981, 1982, 1984) into a quadratic Engel curve (Banks, et al., 1997). This new model is globally flexible, possesses all properties from the AIDS, and is also consistent with the observed expenditure patterns of meat consumption data in Taiwan.

An application for meat demand in Taiwan is applied in this study. The

GF-QAIDS model is estimated including beef, chicken, aquatic products (fish, in short), and pork to cover the period 1962-2000. The remainder of this study is organized as follows: In the next section, a GF-QAIDS will be discussed and derived. In the third section, meat consumption data in Taiwan will be employed, and the empirical results are presented in section 4. Summary and further studies will conclude this paper in section 5.

2. The Globally Flexible, Quadratic Almost Ideal Demand System

In this section, the QAIDS and the Fourier cost function is described, following Chalfant (1987) and Banks et al. (1997). The Fourier series approach suggested by Gallant (1981, 1982, and 1984) is adopted by modifying the translog price index, $\ln a(p)$, in the usual specification of the almost ideal demand systems.

Modifying the indirect utility function of the QAIDS proposed by Banks et al. (1997) and replacing the translog price aggregator using the Fourier form (Chalfant, 1987), the cost function (in natural logarithm) of the GF-QAIDS can be obtained by the duality theorem as:

$$\ln c(u,p) = \ln a(p) + \frac{b(p)}{[\ln V]^{-1} - \delta(p)}, \qquad (1)$$

where $\ln a(p)$ is approximated by the Fourier flexible function; $b(p)$ and $\delta(p)$ is specified as in Banks et al. (1997). Namely,

$$\ln a(p) = u_0 + \mathbf{b}'\ln\mathbf{p} + \frac{1}{2}(\ln\mathbf{p})'\mathbf{C}(\ln\mathbf{p}) + \sum_{\alpha=1}^{A}\left\{u_{0\alpha} + 2\sum_{j=1}^{J}\left[u_{j\alpha}\cos(j\cdot\lambda\cdot\mathbf{k}'_\alpha\ln\mathbf{p}) + v_{j\alpha}\sin(j\cdot\lambda\cdot\mathbf{k}'_\alpha\ln\mathbf{p})\right]\right\} \quad (2)$$

$$b(p) = \prod_k p_k^{\beta_k} = \exp(\beta'\ln\mathbf{p}) \quad (3)$$

$$\delta(p) = \sum_k \delta_k \ln p_k = \delta'\ln\mathbf{p} \quad (4)$$

where $\ln\mathbf{p} = (\ln p_1, \ln p_2, \cdots, \ln p_n)'$ is a (n×1) column vector of prices in natural logarithm. Equation 2 is originally in translog form (in both the AIDS and the QAIDS), which is modified by the Fourier flexible function in this study. Equations 3 and 4 are identical to those specifications in Banks et al. (1997), which are the Cobb-Douglas price aggregators. In addition, regularity conditions are satisfied provided by $\mathbf{b}'\mathbf{i} = 1$, and $\delta'\mathbf{i} = \beta'\mathbf{i} = 0$.

Applying Shephard's lemma to its cost function, the GF-QAIDS in share form can be expressed as (in vectors):[2]

$$\mathbf{w} = \mathbf{b} + \mathbf{C}\cdot\ln\mathbf{p} - 2\lambda\sum_{\alpha=1}^{A}\sum_{j=1}^{J}\left[j\left(u_{j\alpha}\sin(j\lambda\mathbf{k}'_\alpha\ln\mathbf{p}) - v_{j\alpha}\cos(j\lambda\mathbf{k}'_\alpha\ln\mathbf{p})\right)\right]\mathbf{k}_\alpha + \beta\cdot\left[\ln X - \ln a(p)\right] + \delta\cdot\frac{\left[\ln X - \ln a(p)\right]^2}{b(p)} \quad (5)$$

where $\mathbf{C} = -\lambda^2\sum_{\alpha=1}^{A}u_{0\alpha}\mathbf{k}_\alpha\mathbf{k}'_\alpha$, which follows Chalfant (1987). Parameters to be estimated are **b**, $u_{0\alpha}$, $u_{j\alpha}$, $v_{j\alpha}$, β, δ, $\forall\, j, \alpha$. sin and cos are mathematical operators of sine and cosine, respectively. λ and $\mathbf{k}_\alpha\ \forall\alpha$ are deterministic coefficient and vectors, respectively, which will be specified later.

2 The derivation of the GF-QAIDS is available from the author upon request.

Compared with the usual specifications, the GF-QAIDS is more generalized and nest some other specifications, such as the AIDS (Deaton and Muellbauer, 1980), the GF-AIDS (Chalfant, 1987) and the QAIDS (Banks, et al., 1997). Therefore, the hypotheses testing for model specifications can be summarized as:

H_0^1: the GFAIDS, $\delta = 0$;

H_0^2: the QAIDS, $u_{j\alpha} = 0 \quad \forall j, \alpha$, and $v_{j\alpha} = 0 \quad \forall j, \alpha$;

H_0^3: the AIDS, $\delta = 0$, $u_{j\alpha} = 0 \quad \forall j, \alpha$, and $v_{j\alpha} = 0 \quad \forall j, \alpha$;

Against the alternative hypothesis H_a: the GF-QAIDS.

The expenditure elasticity is provided by:

$$E_i = 1 + \frac{1}{w_i} \cdot \frac{\partial w_i}{\partial \ln X}, \tag{6}$$

where $\dfrac{\partial w_i}{\partial \ln X} = \beta_i + 2 \cdot \delta_i \cdot \dfrac{\ln X - \ln a(p)}{b(p)}$.

The Marshallian price elasticities (E_{ij}^M) can be computed by:[3]

$$E_{ij}^M = -\bar{\delta}_{ij} + \frac{1}{w_i} \cdot \frac{\partial w_i}{\partial \ln p_j}, \tag{7}$$

where $\bar{\delta}_{ij} = \begin{cases} 0, & if \quad i \neq j \\ 1, & if \quad i = j \end{cases}$, and

$$\begin{aligned} \frac{\partial w_i}{\partial \ln p_j} = & -\lambda^2 \mathbf{K}_{ij} - 2\lambda^2 \sum_\alpha [u_{1\alpha} \cos(\lambda \mathbf{k}'_\alpha \ln \mathbf{p}) + v_{1\alpha} \sin(\lambda \mathbf{k}'_\alpha \ln \mathbf{p})]\, \mathbf{K}_{\alpha ij} \\ & - (\beta_i + 2\delta_i \frac{\ln X - \ln a(p)}{b(p)})[\frac{\partial \ln a(p)}{\partial \ln p_j}] - \delta_i \beta_j \frac{[\ln X - \ln a(p)]^2}{b(p)} \end{aligned} \tag{8}$$

where $\dfrac{\partial \ln a(p)}{\partial \ln \mathbf{p}} = \mathbf{b} + \mathbf{C} \cdot \ln \mathbf{p} - 2\lambda \sum_{\alpha=1}^{A} [u_{1\alpha} \sin(\lambda \mathbf{k}'_\alpha \ln \mathbf{p}) - v_{1\alpha} \cos(\lambda \mathbf{k}'_\alpha \ln \mathbf{p})] \mathbf{k}_\alpha$.

The Hicksian price elasticities (E_{ij}^H) can be calculated by using the Slutsky

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[3] Assume J=1 to derive the elasticities in order to match the empirical study.

equation, i.e.,

$$E_{ij}^{H} = E_{ij}^{M} + w_j E_i . \tag{9}$$

3. Data Sources and Descriptions

Annual data of meat consumption in Taiwan from 1962 to 2000, including beef, chicken, fish, and pork, are employed in this study. The dataset is taken from the Food Balance Sheet of the Council of Agriculture in Taiwan. Note that the meat consumption data were aggregated and derived from supply side, which would only be a proxy of actual consumption levels in demand analysis.

In this study, prices of the four meat items and expenditure on meat are explanatory variables. Since the consumer price indices (CPI) for meat or food were not available until 1981, the published CPI for all items is used to calculate the deflated prices.[4] In addition, the real expenditure is obtained by dividing the expenditure on meat by Stone's price index, $\ln P^S = \sum_{i=1}^{n} w_i \ln p_i$, (Chalfant, 1987). The descriptive statistics are presented in Table 1.

Figure 1 exhibits expenditure shares with respect to the total expenditure of the four meat items for the period 1962 to 2000, and shows that the expenditure shares of the meat items are different. Pork is the major meat consumed in Taiwan comprising over 50% of the total expenditure on meat; however, recently consumers have spent

[4] The CPI data are available from the Directorate-General of Budget, Accounting and Statistics, Executive Yuan, Taiwan or from their website: http://www.dgbas.gov.tw/dgbas03/bs8/dbase/data.htm.

less on pork and more on chicken. Shares of chicken were more than 10% during the early stages of this study but climbed to over 20% in the most recent years, which has surpassed expenditure shares of fish during the same time period. Clearly, shares of fish were second in consumption between 1962 and 1997 but declined after 1998. Fish reached its peak in 1987 with close to 40% of the meat expenditure. After 1997, the shares of fish declined. From Figure 1, the most stable trend is for beef which reached only 1-3% of the meat expenditure during the last 40 years.

In addition to these descriptive analyses, Figure 1 also shows implicitly a non-linear trend over time, especially for fish and chicken. This trend in expenditure shares might be a hint for fitting a quadratic Engel curve.

4. Estimation of the GF-QAIDS Model

Beef, chicken, fish, and pork are assumed to be weakly separable from other food items as well as non-food items. To estimate the GF-QAIDS, techniques of the iterative seemingly unrelated regression (ITSUR) are utilized in this study with the assumption of contemporaneous correlation among the error terms. In order to guarantee that sine and cosine functions in the Fourier flexible form behave properly, rescaling of prices is necessary to keep them within the $(0,2\pi)$ interval, as follows:

$$\ln p_{it} = \ln P_{it} - \min(\ln P_{it}) + \varepsilon \quad \forall i \tag{10}$$

where $\varepsilon = 10^{-5}$ is a small positive constant to ensure all the rescaled log prices are positive (Chalfant, 1987).

The scaling factor, λ, is then calculated by:

$$\lambda = 6/\max(\ln p_{it}) \qquad (11)$$

Note that the rescaling changes in units only and does not affect any calculation of estimated elasticities (Chalfant, 1987).

The multi-indexes used in the Fourier flexible forms are listed in Table 2. Using the econometric package Eviews 4.1, the parameter estimates are obtained by dropping the beef equation from the demand system due to linear dependency. From Table 3, more than half of the estimates are statistically different from zero with less than or equal to 0.10 significance level. Most of the parameters corresponding to expenditure terms are also significant.

In this study, we are interested in whether or not the GF-QAIDS is superior to its nested models. The Wald test statistics of the AIDS, the QAIDS, and the GF-AIDS models against the GF-QAIDS model are 186.75, 59.40, and 33.51, respectively. These Wald test statistics result in very small p-values which indicate the GF-QAIDS model fits the data well and its nested models are all rejected as a proper functional specification.

Table 4 shows the mean of the elasticities in the sample period where all of the elasticities are in correct signs. Specifically, most of the expenditure elasticities are above unity except for pork (0.88). This finding indicates that consumers in Taiwan may spend more on beef, chicken, or fish but less on pork with an increasing total expenditure on meat. Expenditure elasticities of beef, chicken, and fish are 1.19, 1.18, and 1.11, respectively; hence, the changes in consumption of these three meat items should be similar when expenditure on meat increases. Own-price elasticity of beef shows the most price-elastic with its own-price elasticity over unity (-1.40), whereas own-price elasticities of fish (-0.68) and pork (-0.76) are inelastic. This means that price changes of fish or pork would not change their quantity demanded significantly. The Marshallian cross-price elasticities present a mixed picture. Only beef and chicken as well as beef and pork have positive cross-price elasticities, implying they are substitutes whereas other combinations are complements.

Figures 3 and 4 present the expenditure and own-price elasticities of meat demand in Taiwan for the period 1962 to 2000. These two figures present intriguing results. The estimates of elasticities for pork are the most stable among the four meat items; its expenditure elasticity varies between 0.872 to 0.895 and its own-price elasticity changes between -1.098 to -0.378. Fish has a similar trend of own-price elasticity as pork with a range between -1.282 to -0.217. The pattern of expenditure elasticity of fish is similar to that of beef: both start with high expenditure elasticities then decline close to unity in the early 1990s and climb up again shortly after 1995.

In addition, expenditure elasticity for beef is more than that for fish for most of the sample period except for a few years in the early stages. Finally, both expenditure and own-price elasticities of chicken fluctuate the most with the expenditure elasticities ranging from 0.893 to 1.454 with a peak occurring in 1987. The own-price elasticities of chicken have a similar trend, however, the lowest own-price elasticity (-3.831) was in 1970, whereas the highest (0.819) was in 1987. The positive own-price elasticities violate the law of demand and are found for seven years during the period 1979 to 1988; nevertheless, the estimated elasticities are acceptable with correct signs in general.

5. Concluding Remarks

A new demand system is introduced in this paper combining the quadratic almost ideal demand system and the Fourier expenditure system. An application for meat consumption in Taiwan during the period 1962 to 2000 indicates that the new demand system fits the data well and that the restriction to the usual specifications, such as locally flexible functional form, linear Engel curve, and both, are rejected.

Further research will focus on the correction of any autocorrelation among the four meat series. In addition, volatile elasticities for chicken might indicate any structural change that occurred during the 1980s; therefore, incorporation of structural changes might shed some light on this study.

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Table 1. Descriptive Statistics of the Related Variables in the GF-QAIDS Model

Variable	Description	Mean	Std. Dev.	Maximum	Minimum
Dependent Variables					
WB	Budget share of beef	0.022	0.004	0.028	0.014
WC	Budget share of chicken	0.143	0.043	0.268	0.104
WF	Budget share of fish	0.302	0.046	0.393	0.209
WP	Budget share of pork	0.534	0.043	0.632	0.464
Explanatory Variables					
DPB	Deflated price of beef	1.941	0.753	4.095	1.045
DPC	Deflated price of chicken	1.588	0.653	2.575	0.897
DPF	Deflated price of fish	0.724	0.165	0.988	0.421
DPP	Deflated price of pork	1.536	0.255	2.101	1.084
X	Real expenditure on meat	91.625	32.384	135.236	39.394

Table 2. Multi-Indexes Used with the Globally Flexible, Quadratic Almost Ideal Demand System

k_1	k_2	k_3	k_4	k_5	k_6
0	0	0	1	1	1
0	1	1	0	-1	0
1	0	-1	-1	0	0
-1	-1	0	0	0	-1

Source: Chalfant, 1987.

Table 3. Parameter Estimates in the GF-QAIDS Model

Parameter	Coefficient	Std. Error	Parameter	Coefficient	Std. Error
b2	0.499***	0.123	u11	0.090	0.066
b3	-0.225	0.152	u12	-0.396**	0.149
b4	0.768***	0.199	u13	0.035	0.028
β2	-0.092**	0.037	u14	-0.002	0.002
β3	0.156***	0.046	u15	0.012	0.008
β4	-0.075*	0.039	u16	0.014	0.010
δ2	0.015***	0.003	v11	0.061**	0.023
δ3	-0.015***	0.003	v12	-0.087*	0.048
δ4	0.002	0.002	v13	0.044***	0.009
u01	-0.194	0.128	v14	0.004**	0.002
u02	0.723**	0.298	v15	0.004*	0.002
u03	-0.039	0.053	v16	0.005	0.003
u04	-0.020	0.016			
u05	0.002	0.021			
u06	-0.024	0.019			

*p<0.10;**p<0.05;***p<0.01.

Table 4. Estimated Elasticities of the Meat Demand in Taiwan, 1962-2000

Food Item[a]	Price of Beef	Price of Chicken	Price of Fish	Price of Pork	Expenditure
Beef	-1.400	2.045	-1.875	0.041	1.189
	(0.728)	(0.474)	(0.364)	(0.323)	(0.098)
Chicken	0.333	-1.013	-0.037	-0.468	1.184
	(0.094)	(1.277)	(0.835)	(0.789)	(0.180)
Fish	-0.137	-0.049	-0.684	-0.239	1.109
	(0.037)	(0.337)	(0.243)	(0.389)	(0.099)
Pork	0.008	-0.071	-0.064	-0.756	0.883
	(0.012)	(0.166)	(0.144)	(0.156)	(0.006)

[a] Standard deviations are in parentheses.

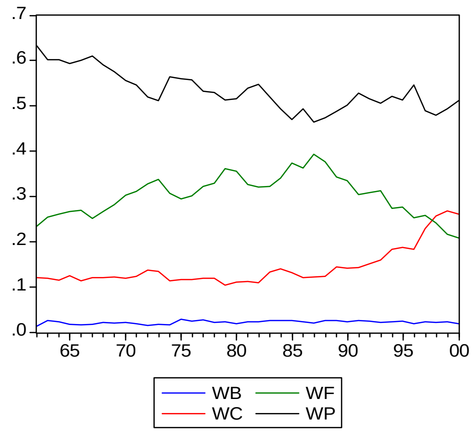

Figure 1 Expenditure shares of meat consumption in Taiwan, 1962-2000.
Note: WB: Beef; WC: Chicken; WF: Fish; WP: Pork.

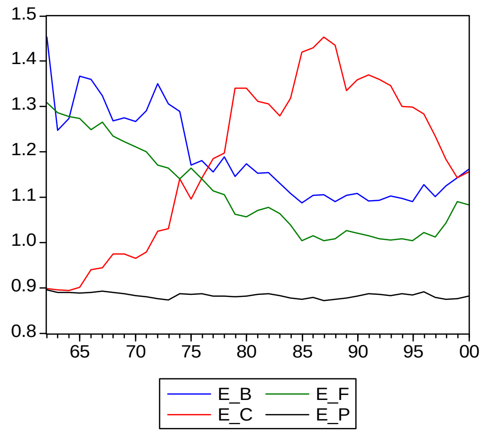

Figure 2. Expenditure Elasticities of Meat Demand in Taiwan, 1962-2000.
Note: E_B: Beef; E_C: Chicken; E_F: Fish; E_P: Pork.

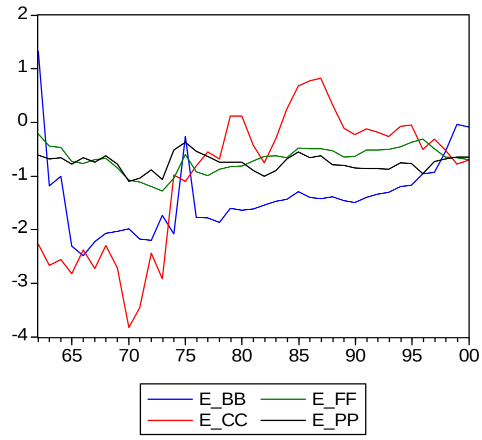

Figure 3. Own-Price Elasticities of Meat Demand in Taiwan, 1962-2000.
Note: E_BB: Beef; E_CC: Chicken; E_FF: Fish; E_PP: Pork.