APPLICATION OF MATHEMATICAL PROGRAMMING MODELS SIMULATING COMPETITIVE MARKET EQUILIBRIUM FOR AGRICULTURAL POLICY AND PLANNING ANALYSIS

David E. Kunkel, Leonardo A. Gonzales and Mario H. Hiwatig*

In this paper, recent theoretical advances of modeling are amplified and applied to a sample linear programming problem to demonstrate its usefulness for policy analysis. Policymakers and planners in agriculture are looking for analyses that will aid them in appraising what future levels of production may be expected at alternative price levels. This requires knowledge of what changes result from different supply-demand balances due to possible input scarcities, changes in productivity under alternative technologies and changes in consumer's preferences.

The objectives of this paper are to (1) restate the theoretical underpinning that are useful for this analysis by specifying the complete model; (2) to demonstrate how the model can be solved using linear programming techniques; and, (3) demonstrate how the model can be used in policy analysis.

Review of Related Literature

Lately, sectoral planning models and problems of spatial equilibrium programming models under competitive conditions have been receiving an increasing emphasis by economists and planners. Evidently, this great emphasis stems from two factors: the intervention of governments in economic development and the emergence of high speed computers to facilitate large scale applications (22).

According to Bassoco and Norton (1), the principal aim of sectoral models is to simulate market conditions that approximate a perfectly competitive market. This is in contrast to the adding-up approach that has characterized the construction of supply response model on the U.S. for the past decades (15).

---

*David E. Kunkel is Agricultural Economist with the Economic Research Service, U.S. Dept. of Agriculture stationed in the Philippines. Leonardo A. Gonzales and Mario H. Hiwatig are Economists with the Bureau of Agricultural Economics, Dept. of Agriculture, Philippines. Assistance was also provided by Josefina Deoquino and Garibaldi Leonardo of the Bureau of Agricultural Economics.

The authors acknowledge the help and criticism from Jerry Sharples and Jose Encarnacion, Jr., in particular, as well as comments from B. McCarl, C. Edwards and H. Fuchs. Funding for this research was provided by the National Science Development Board and the U.S. Agency for International Development, Philippines, respectively.

Samuelson (16) initiated the classic methodology when he demonstrated that in spatially separated markets with fixed transportation costs between the markets, there exists an objective functions (defined by him as "net social payoff"), such that the conditions for its being a maximum coincide with well known conditions for competitive market equilibrium. Years later, Smith (18) further developed the concept and provided a dual interpretation of Samuelson's analysis by showing that under identical situations to that discussed by Samuelson, the minimization of "economic rent" will lead towards price competitive equilibrium.

Since then, a flurry of extensions and applications of this modeling effort gained recognition. It was extended to multiproduct (22) cases with given independent demand functions (8, 11) to the maximization of a given social function defined by Takayama and Judge (19, 20, 21) as the sum of the producer's and consumer's surplus.

The most recent application of this type of study, where multiple products and various inputs of the agricultural sector were programmed to simulate competitive market equilibrium has been in Mexico (4), Portugal (6) and the Philippines (13).

Except for the Philippine study, the theoretical framework has been cast in terms of demand and supply as a function of product output. Since supply functions are derived, a much better understanding of the implication and limitation of this approach are obtained by specifying the full theoretical model in terms of production function and input supply curves.

Theoretical Framework

The agricultural sector is most easily characterized as having a downward sloping demand curve for its products; and upward sloping supply curve for inputs; and a technological process by which the inputs are transformed into outputs. Since agriculture, in general, is also characterized as having a large number of producers producing relatively homogenous products, it meets the conditions for competitive equilibrium.

The following set of relationships are assumed for the purposes of this analysis:

Product Demand

Product demand is represented by inverse demand functions without cross price elasticities. While this assumption is fairly strict, it facilitates analysis. Ways of relaxing this assumption are discussed later. Mathematically:

(1) $P_i = f_i (Y_i^o)$ $i = 1, \ldots n$

Where: P_i is the price of the commodity;

Y_i^o is the quantity demanded; and,

f_i is a concave function.

Production Processes

Production processes are represented by a convex production function or series of production functions if there is more than one production process.

(2) $Y_{ik} = Q_{ik} (X_{111} \ldots X_{ijk})$ $i = 1, \ldots n$; $j = 1, \ldots m$; $k = 1, \ldots r$

Where: Y_{ik} is the amount produced by the kth production process;

Q_{ik} is a convex function; and,

X_{ijk} is the amount of the jth resources used in production of product i by process k.

Product Market Clearing Equation - the amount consumed is equal to the amount produced.

(3) $Y_i^o = \sum_{k=1}^{r} Y_{ik}$ $i = 1, \ldots n$

Input Market Clearing Equation - insures that input use is equal to its supply.

(4) $X_{jo} = \sum_{i=1}^{n} \sum_{k=1}^{r} X_{ijk}$ $j = 1, \ldots m$

Input Supply - Input supply is also represented by inverse supply functions without cross price elasticities and as convex functions:

(5) $w_j = h_j (X_{jo})$ $j = 1, \ldots m$

Maximizing the following augmented Lagrangian function will result in a competitive equilibrium (following the logic of Takayama and Judge (20).

(6) $$Z = \sum_{i=1}^{n} \left[\int_{0}^{y_k^{o}} P_i \, d_{yi} + \lambda_{ik} (Y_{ik} - Q_{ik} (X_{ijk})) + \mu_i (Y_{io} - \sum_{k=1}^{r} Y_{ik}) \right] - \sum_{j=1}^{m} \left[\gamma_j \left(\sum_{i=1}^{n} \sum_{k=1}^{r} X_{ijk} - X_{jo} \right) + \int_{0}^{x_{jo}^{*}} w_j dw_{jo} \right]$$

(7) $$\frac{\partial Z}{\partial Y_i^o} = P_i + \mu_i = 0 \qquad \text{e.g. } P_i = - \mu_i$$

(8) $$\frac{\partial Z}{\partial Y_{ik}} = \lambda_{ik} - \mu_i = 0 \qquad \text{e.g. } \mu_i = \lambda_{ik}$$

$$\therefore P_i = - \lambda_{ik}$$

(9) $$\frac{\partial Z}{\partial X_{ijk}} = - \lambda_{ik} \, Q'_{ik} (X_{yk}) - \gamma_j = 0$$

(10) $$\frac{\partial Z}{\partial X_j^o} = + \gamma_j - w_j = 0 \qquad \text{e.g. } \gamma_j = w_j$$

The solution of equation (7) through (9) results in the following relationships:

(11) $$P_i Q'_{ik} (X_{ijk}) = W_j \qquad \begin{array}{l} i = 1, \ldots n \\ j = 1, \ldots m \\ k = 1, \ldots r \end{array}$$

Equation 11 shows that marginal revenue product of all resources used in each production process is equal to resource cost. This is a sufficient condition for competitive equilibrium to exist. Thus, the objective function simulates competitive market equilibrium.

Uses and Limitation of this Approach

The theoretical model presented can be used to look at supply-demand balances by using a comparative statics approach. This is done by making changes in supplies of inputs, demand for output or changes in technology. The model then provides an idea of the direction and magnitude of changes. However, the programming framework imposes some rather strict conditions via equation (11) for the fact that the marginal revenue product of each resource used in the production of each product must be equal to resource cost. Thus, an error in the specification of any part of the model affects the whole results. Because of this characteristic, the validation of model results against a base period becomes very important.

Another limitation arises when either input supply or product demand is specified as a fixed constraint. The case of fixed resources in particular has raised questions about the validity of using this approach for predictive purposes (Encarnacion (7)). In this case, equation (10) drops out and only shadow prices of resources which are binding are generated. Given that the rest of the model is correctly specified, a shadow price which is the same as the market price will only be generated if the resource level specified is consistant with that market equilibrium. Any error is estimation then will be reflected throughout the model.

For resources which are truly fixed in nature and for which good estimates exist this presents only a minor problem. However, for resources for which there are not good estimates and for which a price is known, it is better to provide a fixed price.

The same argument holds for producing a fixed product demand. Only in this case, the first term of the objective function drops out and it becomes a minimization problem. Without input supply curves and only fixed constraints or input costs, this then becomes the usual approach for interregional competition models. Transportation activities are only additional production activities. Since, in this case, both marginal value products, μ_i in equation (7) and shadow prices are generated internally, the probability of either of these being consistent with market prices is greatly reduced.

For policy purposes, the "comparative statics" approach is useful with this sectoral programming model. The comparative statics approach consists of three steps: (1) model the sector for a base period such that the solution approximates the real world, (2) introduce a parameter change as dictated by the policy question being asked and compute a new solution, and (3) analyze the differences between the two short-run static solutions. Parameter changes fall into three categories, a shift in input supply, a shift in product demand or a change in technology. Any policy question that can be translated into one of these three categories can be analyzed with the sector programming model.

EXAMPLE PROBLEM

An example problem is presented to demonstrate the application of the above theory and procedures. Standard linear programming methods are used rather than quadratic programming because (a) linear programming may be more manageable for large models, and (b) demand and supply functions that are not quadratic may be modeled. Grid linearization techniques (described by Duloy and Norton, 1973, pp. 311-316) are used to approximate smooth supply and demand curves. See also Kunkel, (12). For approaches using linear programming without demand, see Singh and Day (17).

The short-run demand curve for products are shown in Figure A1 and the short-run supply curves for resources are shown in Figure A2. Each curve is approximated by 6 steps and entered into the linear programming tableau by adding the increment in quality multiplied by the price of that segment to the previous segment. Demand for domestic rise is assumed to have an upper limit on price (import price) and a lower limit (export price).

In this example, four solutions are obtained to demonstrate how the model can be used for policy analyses. The first is a base solution obtained from the coefficients shown in Table A1. The matrix consists of 11 constraints and 34 activities. Note that one could increase the number of steps in the supply and demand function without increasing the number of constraints. The second solution shows the results of an increase in the labor supply by 50 percent. The third solution shows the results of 100 percent increase in the demand for meat and the fourth solution demonstrates the results of a rise in the productivity of palay by 15 percent with cash inputs increased by 17 percent.

Results

Equilibrium results are shown for each of the three alternatives in Table 1. Result consists of shadow prices (cost of inputs and prices of products sold), activity levels, welfare (the maximum value of the objective function divided into its consumer and producer surplus components) and value added by each production activity. Shadow prices, activity levels and the value of the objective function are read directly off the computer printout.

The solution levels of the objective function and the consumer and producer surplus are meaningless because of the arbitrary method used to define the first step of the demand and supply curves. Some economic interpretation, however, can be given to shifts in relative magnitudes of producer and consumer surplus between two solutions.

TABLE 1. SAMPLE MODEL RESULTS

ITEM	UNITS	SOLUTION			
		Base	Increase Labor Supply	Increase Meat Demand	Productivity Shift of Palay
Shadow Price					
Labor	₱/manday	7.24	5.00	9.00	6.00
Capital	Interest rate	1.12	1.12	1.12	1.16
Rice	₱ per kilo	1.75	1.57	1.89	1.65
Meat	₱ per kilo	8.16	7.66	8.55	8.02
Eggs	₱ per kilo	.34	.35	.41	.37
Palay	₱ per cavan	40.40	36.20	43.60	38.03
Activity Level					
Rice sales	Mil. kilo	4031	4500	4000	4500
Meat sales	Mil. kilo	80	80	160	80
Egg sales	Mil. eggs	1250	1750	1250	1750
Labor used	Mil. Man days	364	410	379	362
Capital used	Mil. peso	4958	5564	5393	5647
Layers	Mil. birds	4.88	6.84	4.88	6.84
Broilers	Mil. birds	22.23	21.88	45.35	21.88
Palay	1,000 has.	3546	3985	3679	3465
Milling	Mil. cavans	177.30	199.20	183.00	199.20
Welfare					
CJ	Mil. pesos	6211	6612	6387	6456
Consumers Surplus	Mil. pesos	4784	5699	5005	5267
Producers Surplus	Mil. pesos	1427	913	1383	1189
Value Added (Cost of resources used)					
Palay	Mil. pesos	7153	7208	8024	7576
Milling	Mil. pesos	603	606	677	653
Layers	Mil. pesos	141	184	147	199
Broilers	Mil. pesos	293	285	603	297
Total		8190	8282	9454	8725
Income					
Labor	Mil. pesos	2635	2050	3411	2172
Interest	Mil. pesos	595	668	647	904
Capital	Mil. pesos	4958	5564	5393	5647
Total	Mil. pesos	8188	8282	9451	8723

Value added by a sector is defined as gross sales of the sector minus the cost of inputs from outside the sector. In this problem, no inputs come from outside the sector--it is a closed economy model. Thus, total value added by the sector modeled here is equal to total gross sales.

Value added can also be computed for each production activity. In this closed economy model, value added by an activity is defined as gross sales from the given activity minus the cost of intermediate inputs used by that given activity, e.g. palay, is an intermediate input to rice milling. In all cases, the competitive solution dictates that value added by each activity is equal to the cost of primary inputs (i.e., labor and capital) in the activity. Note that total cost always equals total value added (total revenue). Note also that the sum of the producer-consumer surplus in the three solutions bears little relation to the total value added or total revenue.

The base solution shows that palay production is by far the most important source of income for the sector (see value added by palay) with much smaller income generated by rice milling, broilers and at the low end, layers.

By shifting the labor supply function to the right and making labor more abundant (solution 2) the price of labor in equilibrium went down and the interest rate remained at base year level, while the largest product price decrease was labor-intensive palay. The quantity of palay produced increased while capital-intensive meat production remained constant (because of the discrete stepped nature of the demand function) and egg production increased. Labor income decreased even though the labor use increased indicating an inelastic demand for labor. Income to capital increased. From these results, one could compute cross-price elasticities of products and resources. Since the only change behind solution 2 is a decrease in the price of one resource (labor), it is reasonable for total value added to increase. More abundant resources also increase the well-being of society as measured by the sum of producer-consumer surplus. Consumer gain while producers lose because of the inelasticity of demand for the agricultural products--especially rice.

The third solution shows the impact of an expansion in meat demand. Relative to the base solution, the price of meat and labor use increase most (capital price stays constant because we are on the same step), while the price of rice increases by a small amount to reflect less availability for direct consumption. Thus, the shift in meat demand raises the returns to the owners of capital, but causes a very small increase in consumer-producer surplus. The price of labor and amount of labor use increases resulting in higher labor income. Value added increases most in percentage terms for broilers since they produce only meat. Value added from all other production activities also increase because prices increase and the MVP and AVP increase.

The fourth solution illustrates the effect of an increase in the productivity of a major product, namely, palay, after allowing for a corresponding increment in the use of cash inputs. As expected, the optimum solution obtained indicates a substantial increase in the price of capital. Labor price, on the other hand, is reduced although the amount employed did not differ materially from the base solution. The respective prices of palay and rice decreased which favored the consumer sector most. This shift in productivity does not indicate gains to producers neither does it provide prospects of higher income shares to the labor force. However, it does result in a significant rise in the income of capital owners. Total value added increase quite substantially over the base year with most of the increase coming from palay production and palay processing activity.

CONCLUSIONS

The authors think the sector model described in this paper is a valuable tool for policy analysis. The example problem shows that one can trace the impact of a change in one variable throughout the whole sector giving a more complete picture of a policy change. Second, some producer and consumer implications can be drawn from comparative static analyses. Third, by skillfully using grid linearization techniques to define steps on the supply and demand functions, linear programming can be used effectively to simulate a static competitive equilibrium.

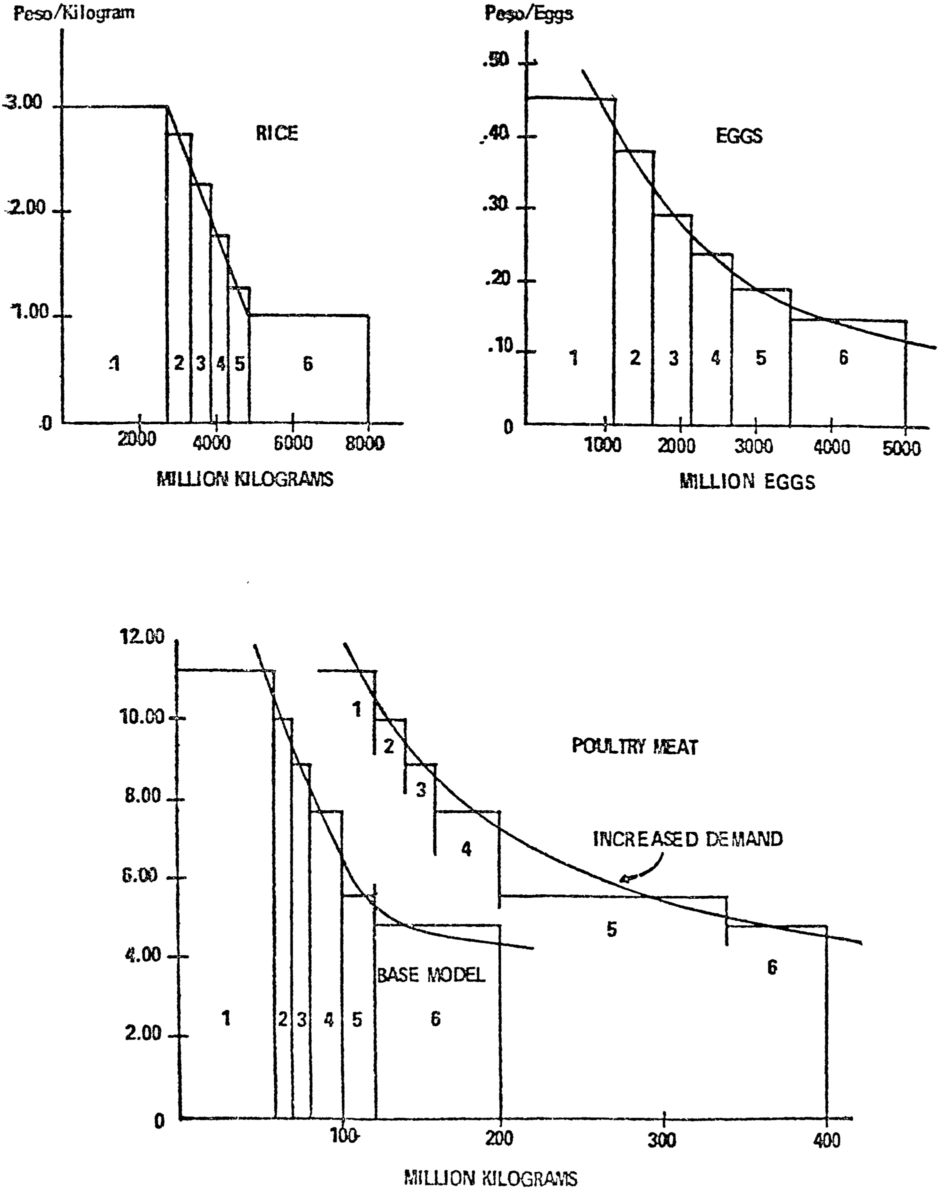
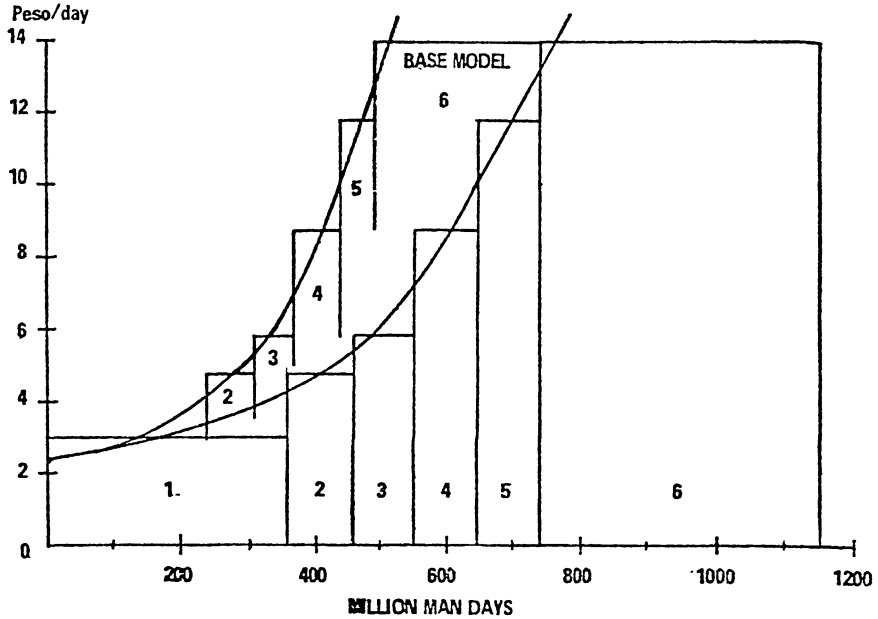


FIGURE A1. PRODUCT DEMAND



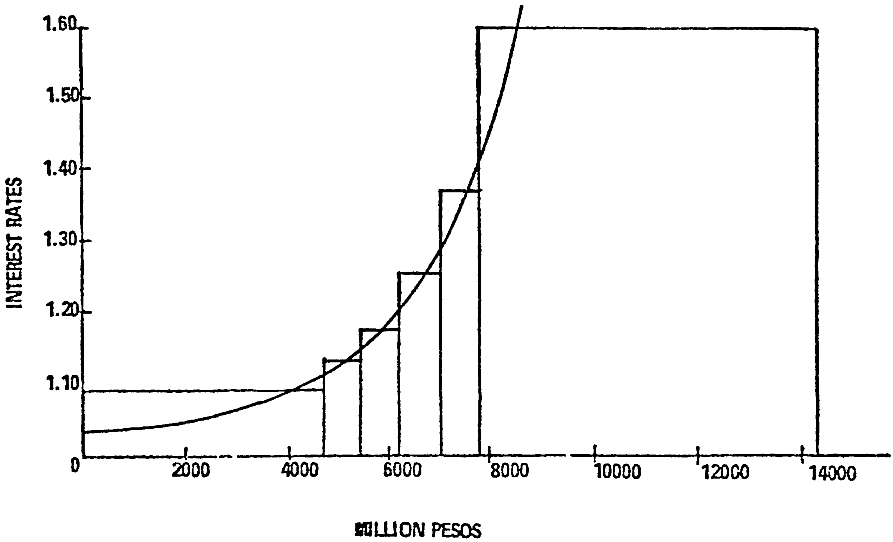


FIGURE A2. RESOURCE SUPPLIES

TABLE A1. LINEAR PROGRAMMING TABLEAU OF THE SAMPLE MODEL

ROWS \ COLS.		UNITS	LABOR SUPPLY						CASH INPUT SUPPLY						RICE DEMAND					
			1	2	3	4	5	6	1	2	3	4	5	6	1	2	3	4	5	6
Objective		Million Pesos	-715	-1046	-1403	-2059	-2694	-6494	-5212	-6044	-6905	-7826	-8836	-20122	9000	10375	11500	12375	13000	16000
Labor		Million man days	-238	-305	-364	-437	-490	-761												
Inputs		Million pesos							-4826	-5569	-6311	-7054	-7796	-14850						
Rice		Million kilos													3000	3500	4000	4500	5000	8000
Meat		Million kilos																		
Eggs		Million eggs																		
Palay		Million cavans																		
Convex Combination	Labor		1	1	1	1	1	1												
	Inputs								1	1	1	1	1	1						
	Rice														1	1	1	1	1	1
	Meat																			
	Eggs																			

TABLE A1. LINEAR PROGRAMMING TABLEAU OF THE SAMPLE MODEL (Cont'd)

COLS. / ROWS	MEAT DEMAND						EGG DEMAND						PRODUCTION ACTIVITIES				TYPE OF CONS-TRAINTS	RHS
	1	2	3	3	5	6	1	2	3	4	5	6	POULTRY	PALAY	MILLING	BROILERS		
Objective	672	768	856	1000	1122	1496	562	750	900	1025	1125	1350						
Labor													.90	.093	.16	.072	≦	0
Inputs													20.0	1.20	2.0	11.30	≦	0
Rice													42.80		25.0	8.60	≦	0
Meat	60	70	80	100	120	200							-.63			-3.46	≦	0
Eggs							1250	1750	2250	2750	3500	5000	-256.0				≦	0
Palay														-.05	1.0		≦	0
Convex combination																		
Labor																	≦	1
Inputs																	≦	1
Rice																	≦	1
Meat	1	1	1	1	1	1											≦	1
Eggs							1	1	1	1	1	1					≦	1

APPENDIX

Welfare Implications

This paper has not taken up the question of how adequate the maximization of consumer and producer surplus is as a measure of welfare since the primary objective is how well market equilibrium prices and quantities are simulated. For further discussion of this subject, the reader should refer to Currie, et.al. Harberger and Mishan, among others.

The welfare implications are obtained by taking the first and last terms of equation (6) and manipulating by the addition and subtraction of total revenue.

$$\text{(A1)} \qquad \sum_{i=1}^{n} \left[\int_{o}^{Yi^{o*}} Pi\, dYi^{o} - Pi^{*}\, Yi^{o*} \right] - \left[\sum_{j=1}^{m} \int_{o}^{Xi^{*}} Pw_{j}\, dX^{o} \right] + \sum_{i=1}^{n} Pi\, Xi^{o*}$$

where: $Pi^{*}Yi^{o*}$ is the equilibrium price and quantity of output.

The first part is easily recognized as consumer surplus (CS)

$$\text{(A2)} \qquad CS = \sum_{i=1}^{n} \left[\int_{o}^{Yi^{o*}} Pi\, dYi^{o} - Pi^{*}\, Yi^{o*} \right]$$

or graphically, the shaded area Pi^{*} AP in figure 1.

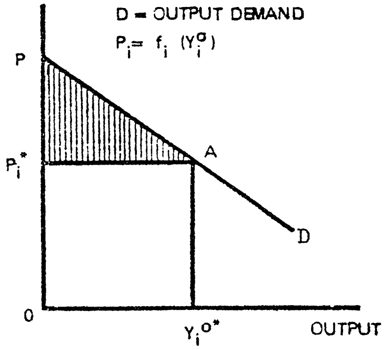


FIGURE 1

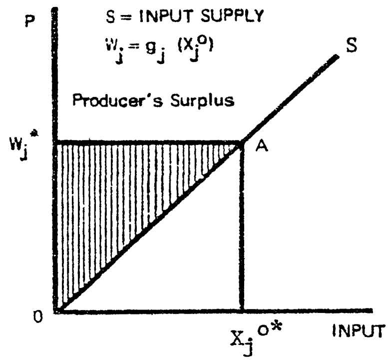


FIGURE 2

It is not obvious that the second part is producers' surplus. To obtain producers' surplus, it is necessary to remember one of the conditions for competitive equilibrium is that total revenue is equal to total cost or

$$\sum_{i=1}^{n} Pi^* \; Yi^{o*} = \sum_{j=1}^{m} w_j^* \; X_j^{o*}$$

where: $w_j^* \; X_j^{o*}$ are the equilibrium prices and quantity of inputs.

Thus

$$\text{(A3)} \qquad PS = \sum_{j=1}^{m} [w_i^* \; X_j^{o*} - \int_{o}^{X_j^{o*}} w_j \; dX_j^{o}]$$

While this is not the usual formulation it has considerable meaning.

Graphically, in figure 2, the producer's surplus is the area OAw_j^* while the area OAX_j^{o*} is the return to owners of the variable inputs.

REFERENCES

(1) Bassoco, Luz Maria and Roger D. Norton, "A Quantitative Agricultural Planning Methodology", paper presented to the Sixth International Conference on Input-Output Techniques, Vienna, April 1974, IBRD Bank Staff Paper No. 180.

(2) Currie, John M., John A. Murphy and Andre Schmitz, "The Concept of Economic Surplus and Its Use in Economic Analysis", The Economic Journal, December 1971, pp. 741-799.

(3) Duloy, John H. and Roger D. Norton, "Prices and Incomes in Linear Programming Models", Discussion Paper No. 3, Development Research Center, International Bank for Reconstruction and Development, November 1974 (forthcoming AJAE).

(4) ______________. "CHAC, A Programming Model of Mexican Agriculture," Chapter IV in L. Goreax and A. Manne eds., Multi-Level Planning: Case Studies in Mexico, North Holland Publishing Co. 1973.

(5) ______________. "Competitive and Non Competitive Demand Structures in Linear Programming." Paper presented at the Meeting of the Economic Society, Boulder, Colorado, August 1971.

(6) Egbert, Alvin C. and Hyung M. Kim. "Analysis of Aggregation Errors in Linear Programming Planning Models." American Journal of Agricultural Economics 57 (1975): 292-301.

(7) Encarnacion, Jose Jr., "On Predicting Subsectoral Outputs and Prices via an Optimizing Model." Discussion Paper No. 75-17, U.P. School of Economics, November 1975.

(8) Fox, K.A. "A Spatial Equilibrium Model of the Livestock Feed Economy in the U.S." Econometric 21(1953): 547-566.

(9) Hadley, G., Nonlinear and Dynamic Programming, Addison-Wesley Publishing Co., 1964.

(10) Harberger, Arnold C., "Three Basic Postulates for Applied Welfare Economics: An Interpretive Essay", J. of Econ. Lit. Vol. IX, No. 3, September 1971, pp. 785-797.

(11) Heady, E. O. and A. C. Egbert. "Programming Regional Adjustments in Grain Production to Eliminate Surpluses" J. of Farm Economics 41(1959): 718-733.

(12) Kunkel, David E., "A Programming Model of Philippine Agriculture," Project ADAM Working Paper, Bureau of Agricultural Economics, Quezon City, May 1, 1974.

(13) ______________. "A Mathematical Planning and Policy Analysis Model for Philippine Agriculture." Paper presented at the Third Agricultural Policy conference, October 15-17, 1975. Continuing Education Center, U.P., Los Banos, College, Laguna.

(14) Mishan, E. J., "What is Producer's Surplus?" The Amer. Econ. Rev. Vol. 58, No. 5-1, pp. 1269-1282, December 1968.

(15) Sharples, Jerry A. "The Supply Function in Agriculture Revisited, The Representative Farms Approach to Estimating of Supply Response." The American Economic Review Proceedings 14(1969): 168-188.

(16) Samuelson, Paul A. "Spatial Price Equilibrium and Linear Programming." Amer. Econ. Rev. 42(1952): 283-303.

(17) Singh, Inderjit and Richard H. Day, "Factor Utilization and Substitution in Economic Development: A Green Revolution Case Study" Economics and Sociology Occasional Paper No. 138, January 1973, Dept. of Agrl. Econ. and Rural Sociology, The Ohio State University.

(18) Smith, V.L. "Minimization of Economic Rent in Spatial Price Equilibrium." Review of Econ. Study 30(1963): 24-31.

(19) Takayama T. and G.G. Judge. "An Interregional Activity Analysis Model for the Agricultural Sector." Journal of Farm Economics 46(1964): 349-365.

(20) ______________. Spatial and Temporal Price and Allocation Models. Amsterdam: North Holland Publishing Company, 1971.

(21) ______________. "Spatial Equilibrium and Quadratic Programming." Journal of Farm Economics 46(1964): 67-93

(22) Yaron, D. "Application of Mathematical Programming to National Planning in Agriculture." The Farm Economist 10(1963): 256-264.

(23) Yaron Dan, Yakir Plessner and Earl O. Heady, "Competitive Equilibrium - Application of Mathematical Programming." Canadian Journal of Agricultural Economics. 13(1965): 65-79.