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HUNGARY LOW-CARBON PRINCIPLES AND SUSTAINABILITY RELATIONS OF THE RUBIK'S CUBE LAYER BY LAYER SOLUTION METHOD

Low-Carbon elméletek és fenntarthatósági kapcsolatok elemzése a rubik kocka sorról-sorra történő kirakási módszerén keresztül

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Abstact

The various sustainability logics can be synchronised with the 3×3×3 Rubik's Cube's solution algorithms, and the relations of the cube's sides define a planning strategy that provides a new scientific approach for renewable investment planning. We theoretically evaluated the various solution processes, and paralell sustainable investment planning levels following the solution levels and stages of the cube. After these various level-evaluations, we made „low-carbon interpretation" summaries. To show the various states of the cube, and to attach an explanation to the low-carbon interpretations, we used the Online Ruwix Cube Solver program. By solving the cube, we imitated the process of project development, meaning the road from complete disorder to the state of complete order. The complete state of equilibrium for Rubik's Cube is the solved state. It's not coincidence, that when someone sees a cube in disorder, their first idea is to solve it, since the desired state is the cube which has only single-colour sides. Rubik's Cube has inherent harmony even in its colour setting, the choice of colours by the developer was intended, and the neighboring logic of colours is not the work of coincidence. Without the mistification of the cube, we can state that it alredy has an inherent and colorful harmony even in its visual appeal, that makes us suggest a seamless logic and perfect logic supports its construction. During the theoretical process analysis, the goal of demonstrating the various rotations was to show what kinds of cube interactions are supposed behind the advancement from state to state, meaning which cubes'/attributes' effects on each other we have to analyse during the rotation process. We didn't define the exact locations and interactions for these during the research, but the division of the process to phases did happen, and we also synced the solution phases to the mechanisms of project development.

Összefoglaló

A 3x3x3 Rubik kocka egyes kirakó algoritmusaival a fenntarthatósági elvek szinkronizálhatók, a kocka oldalainak kapcsolatrendszere olyan térszemléletet és tervezési stratégiát ír le, amely új tudományos szemléletet ad a beruházás tervezés folyamatában. A kirakási folyamatok és az azzal parallel beruház tervezési szinteket teoretikusan, a kocka egyes kirakási szintjei, állomásai szerint folyamatértékeltem. Az egyes színt-vizsgálatokat követően „Low-carbon interpretációkat", a kirakási lépésekhez illeszkedő projekttervezési összefoglalókat

készítettünk. A kocka egyes állapotainak és kirakási szintjeinek ábrázolásához, valamint a low-carbon értelmezések magyarázatához az Online Ruwix Cube Solver programot használtuk fel.
A kocka kirakásával a projektfejlesztés folyamatát imitáltuk, tehát a rendezetlenségi állapotból a teljes rendezettség állapotába való eljutás útvonalát. A Rubik kocka egyensúlyi kockaállapota a teljesen kirakott Rubik kocka. Nem véletlen, ha valaki meglát egy összekevert Rubik kockát, azonnal szeretné megoldani, kirakni, mivel a kívánt vagy vágyott állapot, a színre kirakott kocka állapot. A Rubik kocka színösszeállításában is hordozza a harmóniát, a kocka színeinek kiválasztása a feltaláló részéről tudatosan történt, a színek egymásmellettisége szintén nem a véletlen műve. A kocka misztifikálása nélkül kijelenthető, hogy a kocka már látványában is magában hordozza azt a színgazdag harmóniát, mely révén tökéletes egyensúlyt és hibátlan logikát feltételezünk a konstrukcióban. A teoretikus folyamatértékelés során, a forgatások bemutatásának célja annak szemléltetése, hogy egyes állapotokba való eljutás milyen kocka interakciókat feltételez, tehát mely kockák/tulajdonságok egymásra hatását kell vizsgálnunk a forgatási folyamat alapján. Ezek pontos helyét és interakcióit jelen kutatás során nem határoztuk meg, de a folyamat fázisokra történő felosztása megtörtént, illetve a kirakási szakaszok és projektfejlesztés mechanizmusainak összevetését elvégeztük. A párhuzamok egyértelműen igazolták, hogy a két logikai művelet egymást erősen támogathatja. A folyamatértékelés alapján bebizonyosodott, hogy a 3x3x3 Rubik kocka egyes kirakó algoritmusaival a fenntarthatósági elvek szinkronizálhatók, a kocka oldalainak kapcsolatrendszere olyan térszemléletet és tervezési stratégiát ír le, amely új tudományos szemléletet ad a beruházástervezés folyamatának.

Key words:
3×3×3 Rubik's Cube's solution, sustainable planning, three-dimensional modelling, Rubik's Cube logic, low-carbon interpretations, layer-by layer methodolody, Rubik software applications.
Kulcsszavak:
3×3×3 Rubik kocka megoldás, fenntartható tervezés, háromdimenziós modellezés, Rubik logika, low-carbon interpretációk, layer-by layer módszertan, Rubik szoftver alkalmazások
JEL: C7

Introduction

In 1980, Ernő Rubik wrote that the cube seems to be alive, as it comes into life while you rotate it in your hands. Rubik's Cube has three rows and three columns, and this can also have a symbolic, or even mystical meaning (Rubik, 1987). If we look at the attributes of the various blocks, the 3×3×3 cube's sides, it's almost immediately obvious that in case of each side, we have system elements, or specific small cubes (mid cubes, edge cubes, and cornercubes) which hide a specific meaning, and keep this meaning in them, regardless of where we rotate them in the system. According to Ernő Rubik, the number „three", through its special meaning, is even able to model life itself. It's able to show the relationship of man and nature, the process of creation, care and destruction, and the relations of cooperation between our resource systems (Rubik, 1981). We may think that the solution to the „mystical cube game" problem may properly portray the biggest question of one of today's hardest problems – the proper and effective use of energy. Nowadays, the entire energy consumption system seems like a huge puzzle, where we don't seem to be able to find the correct pieces. However, we suggest that the 3×3×3 Rubik's Cube's solution method may help us find the various pieces' relations, the relevant inclusion of system attributes in both a 2D and 3D interpretable manner, therefore, it may give correct pointers on interpreting the supply and demand sides of energy consumption. (Fogarassy, 2014).

One of the most widely known and most used method of solving Rubik's Cube is the „layer by layer" method, but we must also note that it's the basis for the more advanced methods like Fridrich, Corner first, etc. The gist of the method is to complete the cube during the solution process row by row. That means that at first, we form a colour cross on the first row, then insert the correct corners, then comes the middle row, and finally, the lower middle cube goes into its position, followed by the lower cornercubes (Fogarassy, 2012).

Most amateurs use the layer by layer method, since this is the easiest to learn, and this is one of the few that has both a professionally based algorithm, and introduction guides. All other advanced solution methods began from this one. We introduced the process of solution according to the outline provided by the www.rubikkocka.hu official website. However, in the current document, we also included UNFCCC's basic development theories, namely „Low-Emission and low-carbon Development Strategies" (LEDS) – which has close ties to basic sustainability criteria – for the official solution method cited in this document. We made the assumption that since the Rubik's Cube's number „three" offers indirect answers to many of our world's currently unsolved questions though it's mystical logic, it's correct to also assume that those who can complete the cube can think „Rubically" in general, or more specifically, about the questions of strategic planning and economic equilibrium search. In the next part of this document, you can find the methodical steps on solving the cube, which can be taken as a compilation theory during strategic development following the solution of the cube, usable for f.e. the advancement from fossilized to renewable energy support systems.

Materials and methods

Process evaluation of layer by layer solution method for 3×3×3 Rubik's Cube

The layer by layer method is fundamentally a structured arrangement system, which defines cornerstones, stages to the process of completion (white cross, second row, yellow cross, etc.), where even though these stages can be achieved by different routes, or one might say that everyone does it to their own personal leisure, it is technically impossible to advance to the next stage without going through the various stages and phases. In the case of sustainability principles and low-carbon development concepts (Clapp et. al. (2010), the abidement by the steps of development phase to phase has importance, because even though the circumstances and the makings may define different routes to equilibrium search, the arrangement logic must be the same, wherever we search for the equilibrium points – be it Hungary, or China, etc. we relied on the methodical guideline of the www.rubikkocka.hu official website, and the solution designs of Singmaster (1980) during the defining of the row by row solution phases. However, because of the low-carbon methodology correspondences, the process which is demonstrated and interpreted in this document differs greatly from these guides. To illustrate the various stages and different solution levels of the cube, we used the Online Ruwix Cube Solver program.

Discussion and results

White cross, multi-level syncing of starting criteria

The special characteristic of the *layer by layer* method is that it always considers the white side as the starting side, and the white mid cube (the cube which only ever has one colour) as the starting point. Naturally, any colour can be the starting point of the solution process, meaning the same rotation logic can be used starting from any level without any changes. Therefore, after we have our white mid cube, as a first step, we find all the four edge cubes (edge cubes are the ones with two colours) which have white as one of their colours. We rotate these one by one next to the white mid cube (Ajay, 2011). The other cubes may be rotated anywhere for now, let's consider them grey! If all white cubes are in place, let's position them by rotating the white side to match at least two above the same colour mid cube! Therefore, it is a general demand for at least two (or optimally all four) elements to be positioned correctly on the bottom side as well, as seen on Illustration 1. This is the first step in the process of the cube's solution, also known as „White Cross".

Illustration 1.: White cross with matching edge cubes on the side
Source: Fogarassy, 2014

It is extremely important for the White Cross to be oriented on the starting side, while the mid cubes match on all sides traversely. If the white edge cubes don't take this position, we can't proceed with the solution according to the method. Bringing the white edge cubes up to the starting point can be done in various ways from various positions, but all follow the same logical sequence. Usually, we have to bring up the bottom row's edge cubes to the starting side. The process of rotating from bottom to top can be seen on Illustration 2. The two different cases show to different cube states. On the upper part of Illustration 2 (1) we do a 180º rotation on the top row to bring the cube to its place from the bottom. On the lower part (2) we do a 90º rotation upward, followed by another 90º rotation of the right column upwards. This is how the white-green edge cube goes to its place.

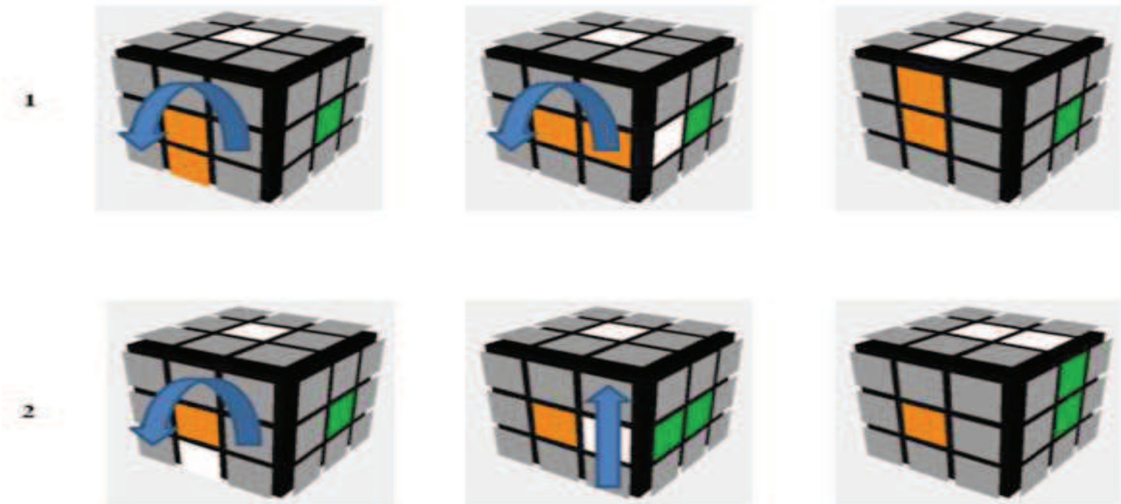

Illustration 2.: Rotating edge cube to its position from bottom row
Source: Fogarassy, 2014

If the white is lodged between two completed edges, we use the rotation seen on Illustration 2. At first sight, this brings the edge cube to the incorrect position, but from here, we can easily relocate it to its proper position.

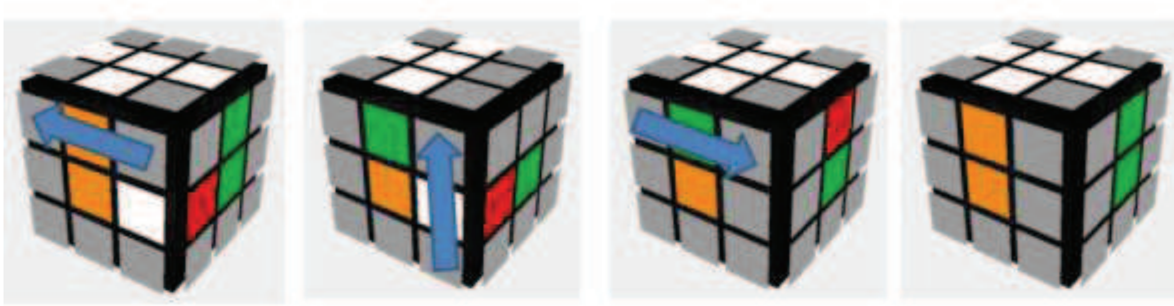

Illustration 3.: Rotating edge cube to its position from mid row
Source: Fogarassy, 2014.

If by the time we make the cross, only two cubes match the mid cube, we can exchange the other two sides by finding the pieces we want to switch, and rotate that side two times, thereby positioning the white on the bottom. After this, we rotate the cube to its own colour, then rotate this side two times. Now, we have the cubes which were in the wrong position on the bottom. Afterwards, we arrange this cube to its own colour, and rotate this side two times, meaning 180º (Illustration 2, upper part)! This method works even if two neighboring cubes have to be switched, or if two opposing ones need to be exchanged. If all four colours are in place (white and edge cubes match the four colour mid cubes, as seen on Illustration 1), we can move on to the next step, which is the solution of white corners. However, let's first view what this phase means in the process of search of sustainability.

„LOW-CARBON INTERPRETATION" NO1:
Our objective system is defined by defining the boundary conditions of the starting state (or the Input side), and the complete or partial system rearrangement (fossilized energy provision system's complete or partial change). This is where we define the development program itself, the condition framework, the boundaries of the project or task. We define what kinds of correspondence systems have an impact on the creation of our process, project, or concept (Molnár (1994). This will be ou white mid cube, which will mean the unchangeable objective system, meaning the fixpoint of our starting state. In our case, according to professional opinions, we can define Energy rationalisation as our fixpoint. We also need four comparison points, which have a strong impact on the project environment. These can be the 2D interpretations of the strategic subconnection, the basic technological requirement, the financing requirements, and the basic market positioning. These attributes which correlate with the various edge cubes and fixed attributes of points of impact (orange, blue, red, green mid cubes) give the starting 2D attributes of the development.

Example: If we change the energy supply system immediately and completely to the new, cleaner technology (strategy 1), or weI wait until the life cycle of the current technology runs out (strategy 2), then I have two different stratefic goals. In version 1, I induce an immediate and final intervention with decisive costs, while in version 2, the exchange of fossilised energy supply systems will happen gradually, take a longer time, and distributes the cost of the investment in a longer timeframe. The causality of this process is what should be examined. If we don't sync the operation criteria of the „old, outdated", and the „new, clean" technologies, the solution of the cube, and the continued sustainable planning of the project can't advance In this case, the next step of the project can't be completed, or if it continues, it will take a wrong turn in development. Therefore, it is not enough to define the starting basis (solution of white side) with regards only to the obvious facts, which fundamentally define the starting criteria, we also have to sync it to the fixpoints of the next level. We can interpret this in practise as the white side (or basics of the project) also being solvable while they're not in sync with the first row, or the fixpoints of the second planning level, the mid cubes (orange, blue, red, green). This project/cube state can be seen on Illustration 4. From this state, the project won't be sustainable, and is doomed to fail.

Illustration 4.: Incorrect solution of white side, meaning starting point of project designed incorrectly
Source: Fogarassy, 2014

Algorithms of solving white corners, search for equilibrium at starting state

After making the White Cross, the next step is to organise the corners to their respective positions (Illustration 5). If this is done correctly, the corners match the colours of the sides. Cornercubes are the ones that have three colours (f.e. white, orange, green). The cube has 8 of these altogether, therefore, our task is to rotate the cornercubes that have white colour to the corners of the White Cross.

Illustration 5.: Correct positions of white corners, and solution of first row
Source: Fogarassy, 2014

First, we have to find the four cornercubes, then put them into their correct positions using algorithms (rotation combinations) (a) and (b). Both rotation combinations (a) and (b) needs the White Cross to be positioned facing upwards. We have the easiest solutions if the bottom row has white cornercubes. First, let's see what colours we can find next to the white colour. Let's place this colour as close as we can to its own mid cube, by rotating the bottom row. This cornercube's now positioned left or right to the mid cube. We take the bottom row towards the way it's aligned, then match the top row as well. To finish the rotation, we rotate the bottom row back, and the top row back as well. The two rotation combinations can be seen on Illustrations 6 and 7.

(a) The cornercube's white is oriented towards the right. We rearrange it to the white front.

Illustration 6.: Right-oriented cornercube's rotation to correct position from bottom row
Source: Fogarassy, 2014

(b) The cornercube's white is oriented towards the left. We rearrange it to the white front.

Illustration 7.: Left-oriented cornercube's rotation to correct position from bottom row
Source: Fogarassy, 2014

Doing solution (b) is simple, as seen on Illustration 7. We merely have to rotate the cornercube „out of the way", then replace it with the cube, to which's place we want to move it. After rotating the cornercube backwards, we rotate the now neighboring white edge cubes (right column) and cornercubes back to the top row, rotating the corner to its final position.

(c) Solution if the white colour of the cornercube faces downward

The most complicated position at first look if the cornercubes face downward wih the white colour. In this case, the colour can be rotated upwards to the starting side with a 180º rotation of the right column, after which we can easily arrange the edge cubes to match it (Illustration

8). If the cornercube is in the wrong upwards position, it has to be rotated to the bottom row, and we have to apply one of the previous rotations. We may use different combinations of the previously introduced rotations, depending on personal depth perception, and simple skillfulness (left-handed, right-handed).
If there are no more white coloured cubes in the bottom row, we've completed our starting white side. But we must be cautious, since one of the cube's sides can be completed even if the cornercubes seem in place, but don't match sideways. The cornercube might be in position while the white side is facing outwards. Neither of these positions are suitable for proceeding with the second row, since the misplaced cubes can't be rotated into their positions ideally in either case.
(d) If the cornercube is on top, but is not orientated correctly, we use multiple versions
Let's turn the cube, so that the cornercube faces us from the right side, then rotate the right side of the cube to face us. This time, our cornercube went to the bottom row. Let's rotate the bottom row counter-clockwise, meaning backwards, and the right side to face away from us. With this process, we result in one of (a), (b) or (c) combinations, where we can put the cornercube into its proper position!

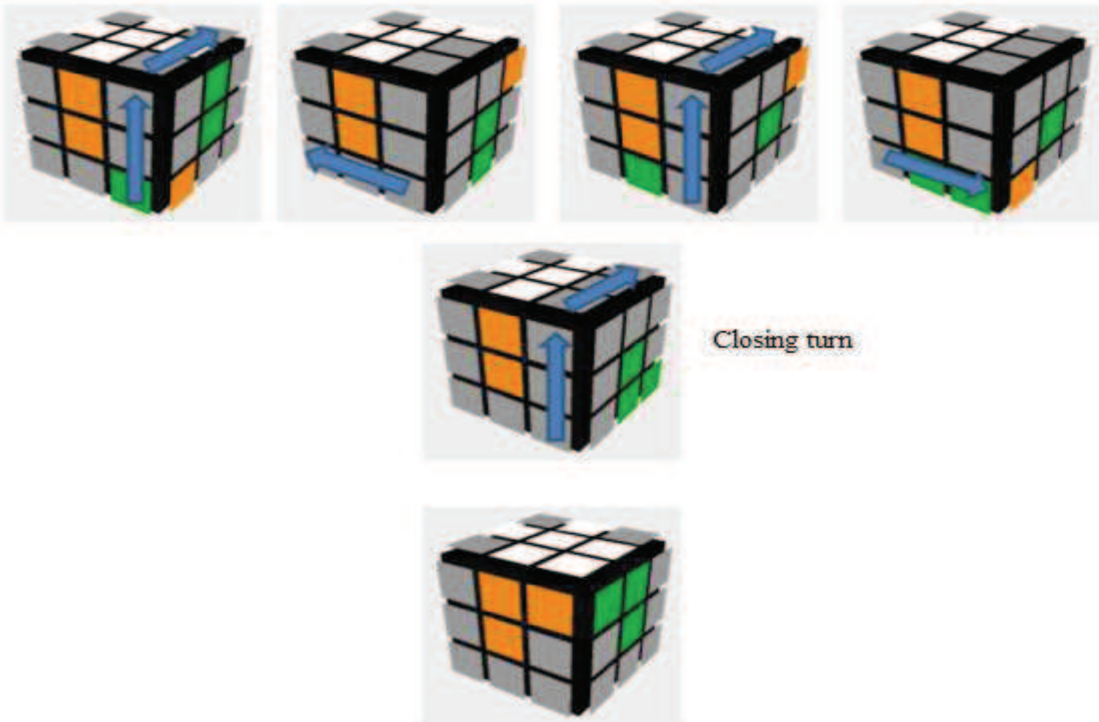

Illustration 8.: Rotating downward facing cornercube to its place
Source: Fogarassy, 2014

„LOW-CARBON INTERPRETATION" NO2:
The goal is to define the project's sustainable development course, and the finalisation of the fixpoints of the starting state. Syncing the definitive criteria and definition of the correspondence systems can be done with the cornercube defining the three attributes at once. All attributes are independent, but the process of their sync can be realised via the shortest route, and the most effective way. It's important to note that the cornercube in the top row can also be positioned with the white colour facing outwards. This can be seen on Illustration 9.

Illustration 9.: Top row cornercube in place, but facing outwards
Source: Fogarassy, 2014

This is also a position from where the solution can't be continued with the second row, since later, the cubes in wrong positions won't be rotatable to their correct positions. This shows us that we can also find project attributes in the process of project development, which seem to be in place at first glance, but isn't in a state of equilibrium. We can't develop our program further, or if we continue to try, the project will take a turn for the worst. In the present cycle of project development (and solution), the search for this starting point of equilibrium is underway.
The state of equilibrium we're searching for is called a Nash equilibrium. Writing the function during the process of project development's phase of planning of the first layer can be used for f.e. defining regulation policies and financing policies.
In the case of Nash equilibrium, the strategies of the various players are the optimal replies to others' strategies, so there aren't any players who want to break this status quo by choosing new, different cooperative strategies. The game will not be stable if it's not in the Nash equilibrium point, because there is always at least one player in this case, to whom his strategy does not mean the best answer in the given situation, and therefore, he will be interested in looking for a new strategy for himself (Harsányi, 1995).

In case of cooperative games, the state of equilibrium can be stable even if a strategy combination isn't Nash equilibrium, if the players agree to choose it ().

By the definition for the Nash equilibrium:

At the equilibrium point of a $J = (n, S, (\varphi_i)_{i=1}^{n})$ n-member game or strategy, we classify a $(x_1^*, \ldots\ldots, x_n^*) \in S$ point (strategic n), where

$$\varphi_i(x_1^*, \ldots, x_{i-1,}^* x_i^*, x_{i+1,}^*) \geq \varphi_i(x_1^*, \ldots, x_{i-1,}^* x_i, x_{i+1,}^*)$$

holds not strictly for every $i = 1, \ldots\ldots, n$ player. Therefore, the point of equilibrium is called a Nash equilibrium. Following the completion of the first layer, only the connection with a Nash equilibrium can be further developed, meaning that we can only rotate the cube further from this position. The first layer always correlates with the second layer's mid cube, and can only be the same color. The true point of equilibrium for the first layer, and the mid cube is what we may call a Nash equilibrium (Szidarovszki, 1978).

Example: The syncing of technology developments connected to the objective system, and the boundary conditions of monetary effectiveness may happen directly, or indirectly (by making it abide by the regulation conditions – standards, norms), with the use of a rotation that has impact on three attributes. A good example to this would be how american standards aren't applicable to european user environments, meaning that here, the principle of preferring local acquisition over global acquisition means a sustainable and proper point of equilibrium.

Solution of mid row by rotating edge cubes to position (using 3 algorithms)

It is obvious, as seen on Illustration 10, that after completion of the first row, the mid cubes will also be in position, which makes our next task the correct positioning of the side edge cubes. Comparing the first rows' solution algorithms to our next ones, we have to say that we need to implement longer rotation sequences, which assumes 7 rotations for repositioning each edge cube (Demaine et al. (2011). Interesting though, that the solution of the mid row can be much more easily automatised (f.e. with a software application). Using heuristic algorithms doesn't cause a problem here, we can give a fixed algorithm for every state, we only have to decide which to implement first.

Illustration 10.: Two rows solved by positioning edge cubes
Source: Fogarassy, 2014

Therefore, by positioning the edge cubes, our second row is complete. There are three (a), (b), (c) possible positions for the edge cubes, which have the following solutions:
In case of solutions (a) and (b), we need an edge cube on the bottom side of the cube next to the yellow mid cube, which has no yellow colour. The reason for this is that edge cubes which don't have yellow, all belong to the mid row. If we find the edge cubes which belong in the mid row, we can match them to their respective colours one by one, meaning rotating them right below their mid cubes. If we hold this side to face us, we have to look at what's the edge cube's other colour. The matching colour will either be to the right (Illustration 11) or left (Illustration 12).
The colours of the mid cube and the bottom cube will match, and in the next step, we'll look at where our edge cube is missing from. (That colour must be either to our right, or our left!) We rotate the bottom row away from the colour of the mid cube which matches the colour of our edge cube! After realising where we have to rotate our edge cube, we turn that side to face us, and re-rotate the edge cube to its original position. This leaves us with two white cubes, which we rotate back to the white side!
If we look at the cube now, we can see that the cornercube on the opposite side (which has white in it) was matched with its edge cube (meaning the one we originally picked out). From this position, we have an easy task, we simply position the cornercube to its place (as was written in the previous, white cornercube's positioning part).

a) Process of rotating from the right (Illustration 20)

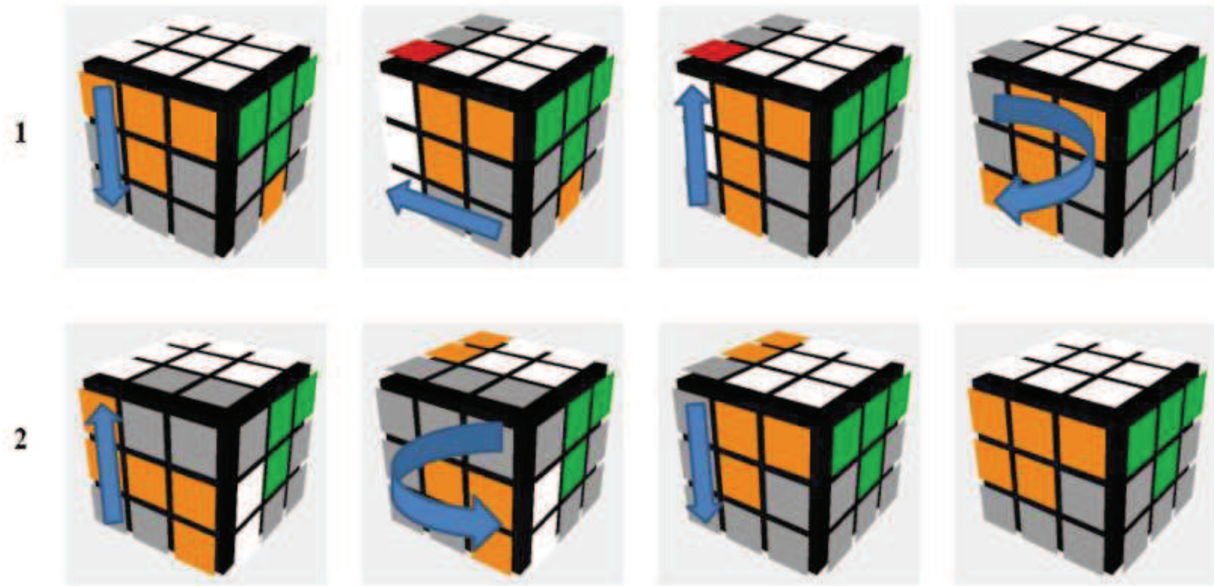

Illustration 11.: Rotating edge cube to its place, if the missing cube faces rightward
Source: Fogarassy, 2014

b) Process of rotating from the left (Illustration 12)

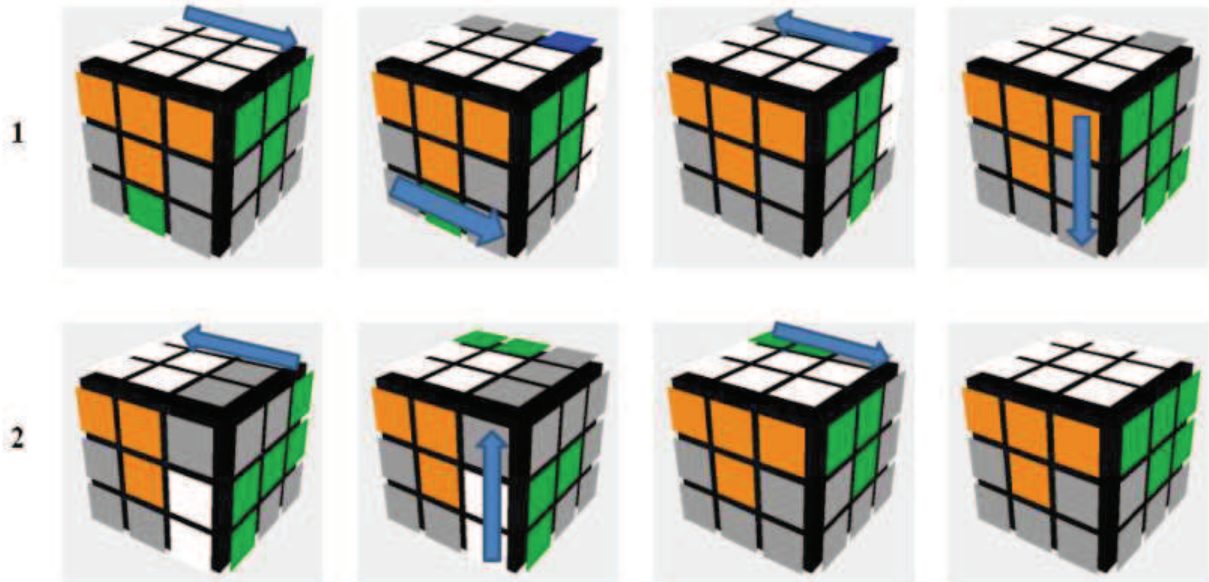

Illustration 12.: Rotating edge cube to its place, if the missing cube faces leftward
Source: Fogarassy, 2014

c) Edge cube is in the second row, but in wrong position or orientation (Illustration 13)
Using solution (c) might be required because, even though the edge cube is in position, it's f.e. in a wrong orientation colour-wise. In this case, we have to go through either solution (a) or (b), with which we achieve that our edge cube, which was previously in the mid row either positioned or orientated wrong, is now in the bottom row, from where we can rotate it back into its proper position using either algorithm (a) or (b).

„LOW-CARBON INTERPRETATION" NO3:

During the process of project planning, our goal with positioning the mid row's edge cubes is to further arrange the correspondence systems of the various attribute sets which have an impact, and find the various points of equilibrium defined by the attributes directly influencing each other, meaning the attributes inherent in the edge cube's two colours, and the matching coloured opposite edge cube, which is paired with a different colour. Without syncing the variables indirectly affecting each other, and the attributes they represent, the state of equilibrium isn't optimal (since more than one state or point of equilibrium is present). This state can be defined by the previously introduced multi-variable continuous functions:

Let φ_i be two objective's payoff function, and $\boldsymbol{u}_1$, $\boldsymbol{u}_2$, vectors be strategic vectors, by which we can define a two-person game of infinite kind, with at least two points of equilibrium (Molnár – Szidarovszky, 2011):

$$\varphi_i(\boldsymbol{u}) = \varphi_i(\boldsymbol{u}_1, \boldsymbol{u}_2,)$$

The main reason of multiple points of equilibrium is that the cross-affecting attributes can be optimised multiple ways (we can optimise the edge cube, or its represented attributes to both the left and the right, but this is only a stable equilibrium if we can continue the solution of the cube). The cube's wrong state of equilibrium can be seen on Illustration 13.

Illustration 13.: Rotating edge cube to position, if the missing cube faces rightward
Source: Fogarassy, 2014

Example: We can directly sync the most economical technological solutions to high quality and innovation, but if the effect of market changes on financing system (change in interest rate), risks of foreign currency, and global effects are disregarded, the project can't be realised, or only with major redesign and changes (no innovation, or lower quality).

Algorithm of Yellow Cross, and tuning Output side

Rotating the „Yellow Cross" is the most important phase prior to the solution of the cube. With this rotation, we start to sync the white and yellow sides. By the time we finish the rotation, the yellow coloured edge cubes are on the front side facing outwards. In the case of the „Yellow Cross", it's not important for the yellow edge cubes to be colour matched, meaning their sides don't have to match the colours of the various mid cubes (Illustration 14.).

Illustration 14.: Yellow Cross
Source: Fogarassy, 2014

After the repositioning, we hold the two, not colour matched cube parts to face us rightward (Illustration 15), in a way that the yellow mid cube faces upward! We rotate a block of 6 cubes from the bottom upwards, making sure the side that faces us contain exactly two columns of white (excluding the left column)! We remake this into an inverted L shape (Illustration 15, upper part, last cube). This is done by rotating the top row clockwise, repositioning the two whites in the right column to the bottom, and finally rotating the top row clockwise.
As we get our inverted L, we take the mid column (the L's vertical line) to the bottom, then turn the cube to make the white side face upwards, then we rotate the missing corner from the left, and turn the completed column down.
a) If two neighboring edge cubes are in the wrong position, rotation sequence is as follows:

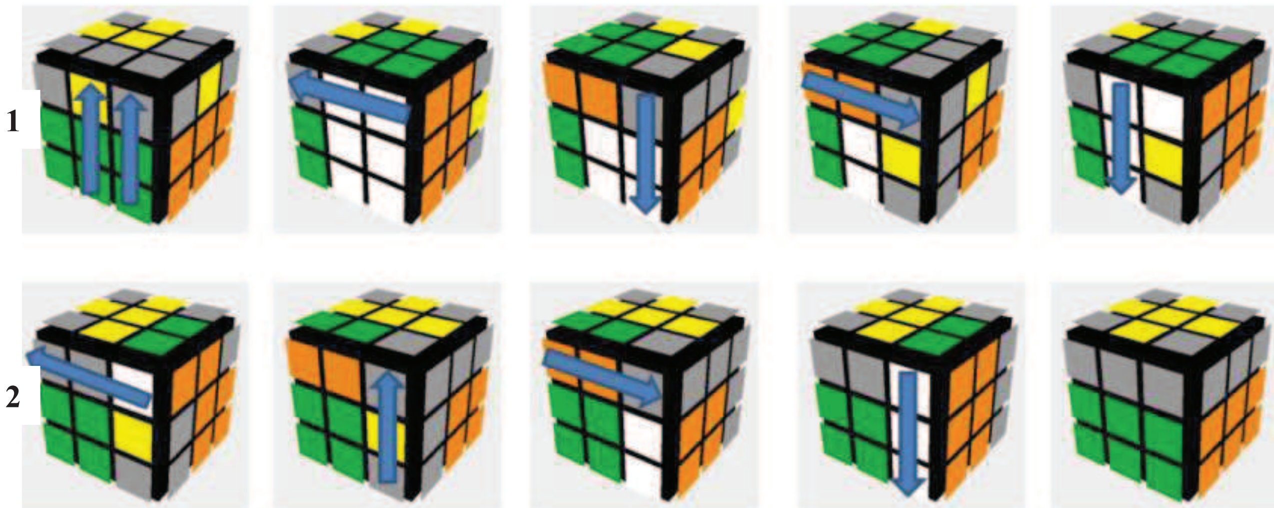

Illustration 15.: Repositioning edge cubes on yellow side
Source: Fogarassy, 2014.

b) If we find the edge cubes on opposite sides, the rotation sequence is as follows:

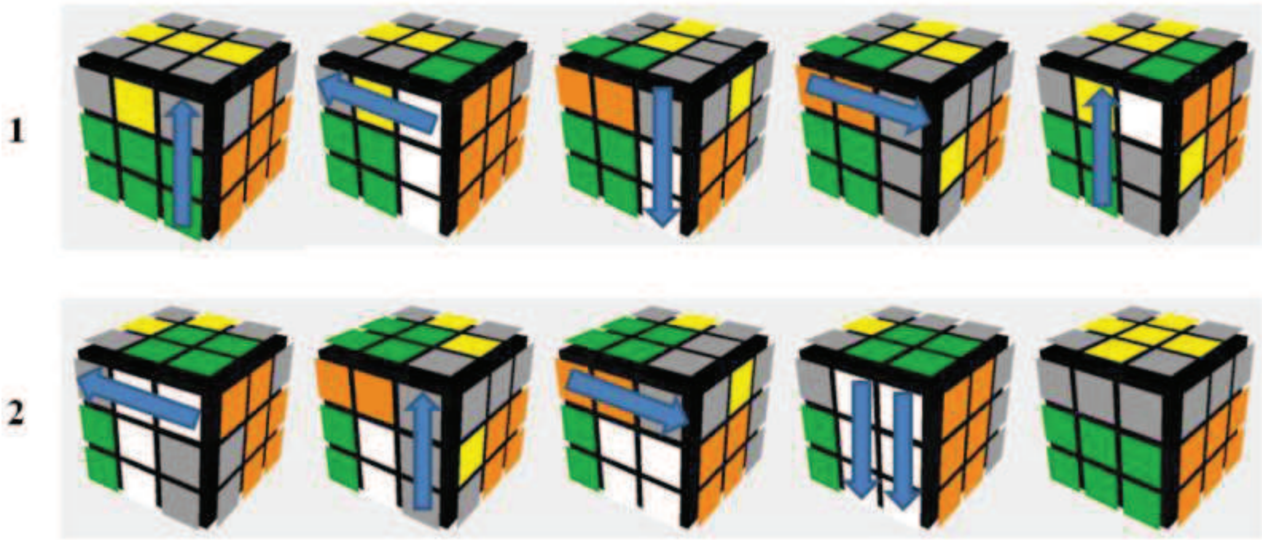

Illustration 16.: Repositioning edge cubes on yellow side, if they're on opposite sides
Source: Fogarassy, 2014.

The process of solution is as follows: we hold the one of the two wrongly positioned cubes in front of us, and the other opposite to it, as seeen in version (b) (Illustration 16). We bring a white column up on the right column, rotate the top row (clockwise), and bring the remaining two whites (the right column) down. We rotate the top row counter-clockwise, and by rotating the mid column backwards, we bring up three whites. In this case, we get an inverted L. This has to be completed into a block of six. This can be done by rotating the top row (clockwise), bringing up two whites to the right column by rotating it backwards, then rotating the top row counter-clockwise. The completed block of six has to be rotated back to the other three whites downwards.

c) The front side has no yellow edge cubes

We might not find an edge cube with yellow on the front side. In this case, we follow either algorithm (a) or (b), which results in one or two edge cubes being positioned on the front side. After this, we use the rotation algorithms of either (a) or (b) to reposition the edges.

„LOW-CARBON INTERPRETATION" NO4:

Basically, the solution of the Yellow Cross is the syncing of the Output expectances (yellow side) and the Input side (white side), including all details of the development objective system. The goal here is primarily syncing the trends of Input and Output indirectly. This indirect syncing is important, because this phase still offers opportunities for some corrections, or the modification of smaller, flexible attributes, depending on how the points of equilibrium are sorted. The indirect assortion is possible due to disregarding the top row's sync with the mid cubes during the solution of Yellow Cross, which means they're not colour matched by the time we finish the rotation phase. After the solution of the mid row, the yellow edge cubes might be in various positions in the top row. If (excluding the yellow mid cube) we can't find any yellow coloured cubes on the front side (Illustration 17, state „D"), the repositioning takes more time, since we have to apply an algorithm, which doesn't help us advance in the solution, only rearrangement. After this rearrangement happens, we can begin using the selected algorithm. The above mentioned circumstance clearly illustrates that we may find a state, where the sealing side of the cube is not as assorted as expected, because no edge cubes are in their proper position. This can be said about project development as well, since there might be times when we have to rearrange the project outputs compared to what the expected outputs originally were. This can easily happen, since during actualisation, we can face situations when the realisation of a development or investment is late months, or even years, which is enough time for the economic environment (market, regulations) to generate new changes related to requirements. One of the more defining moments of the economic rearrangement process of the 2010's was the phenomenon which caused failed „giga-developments" not only in Hungary, but all around the entire world (f.e. chinese ghost-towns, failed european ethanole and bio-diesel factories, etc.). Therefore, on the field of actual usefulness, the Yellow Cross can have high expectations of being put to the spotlight.

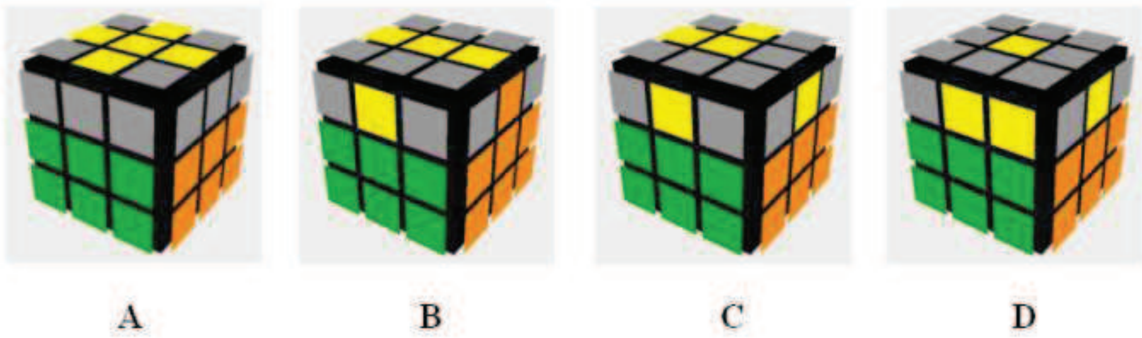

Illustration 17.: Possible positions of edge cubes after arranging mid row
Source: Fogarassy, 2014

Example: The possible changing of flexible technology requirements compared to the planned order is possible in this phase, without changing the Output criteria, or the points of equilibrium. A similar variable might be f.e. the inclusion of handlable changes in tax and other financial requirements. we basically assume that a well-planned and long-term predictable economic environment may result in Output criteria, which are close to the originally planned business requirements, therefore, they have no need of rearrangement to new states of equilibrium. Following the cube's logic, if the Yellow Cross is on the front side immediately after the solution of the mid row, the solution of the cube is quite simple, since the only remaining task is to rotate the cornercubes to their respective positions. This state can be assumed during project development if the Output expectations of the project form the Yellow Cross, which means the project or investment can be completed without changes (Illustration 17, state „C"). If the finishing phase is like Illustration 18's „B" or „C" states, the project must be rearranged into a new state of equilibrium, for which a moderate intervention is advisable. If, however, the „D" cube state defines the state of project development, meaning not a single Output expectation is as they were in the project planning assumed they would be, a major rearrangement of the state of equilibrium, and serious re-planning is necessary, which is usually time-consuming (and also needs one-two additional algorithms), which can halt the project's finishing phase.

Positioning yellow cornercubes, and arranging sustainability criteria to finished state

In this rotation sequence, we move all four yellow cornercubes in place, making sure that the yellow top row isn't colour matched with the row beneath it.

Illustration 18.: Independent solution of yellow side
Source: Fogarassy, 2014

A multitude of various possibilities/algorithms were developed for this rotation in the last few years, and listing these would be too time-consuming, not to mention, needless. For us to be able to rotate the cornercubes, it's sufficient to define an easier combination, which can be repeated multiple times, therefore resulting in the solution of the yellow side from any given starting state.

On Illustration 18, we can see a case when only two cornercubes are in the wrong place, with them having the yellow colours on the same side. The cube must always be held in a way that the two cornercubes to be rotated face rightwards. We also have to be mindful to have the side which has the yellow colours of the cornercubes we want to rotate facing upwards. As a start, let's rotate the right column downwards, then rotate the top row (clockwise). After this, let's rotate the left side backwards, the top row again (clockwise), then rotate the left column downwards, after which comes the top row twice (clockwise). As a finish, we rotate the left column upwards. This process must be repeated for the right side as well. In case that two neighboring cornercubes have the yellow colours on opposing sides, we also use this algorithm, but hold the cube in a way that the yellow side faces upwards, and the cubes we want to rotate face rightwards. In any other possible scenario, we can rotate the yellow cornercubes to their place in two steps.

We also use this rotation combination in case of three cornercubes being oriented wrongly, meaning facing outwards from the front side. We start the combination with the „wrong" cornercube which is closest to the one that's in the correct place. As a result of this rotation, the next cornercube also gets placed in its position, or faces the front side with the yellow colour. Therefore, we get a state similar to that of Illustration 20, or a different one where two „wrong" cornercubes are neighboring, meaning on the same side. Using the rotation combination seen on Illustration 19 from this state, we can easily do the rotations, correcting the cornercubes.

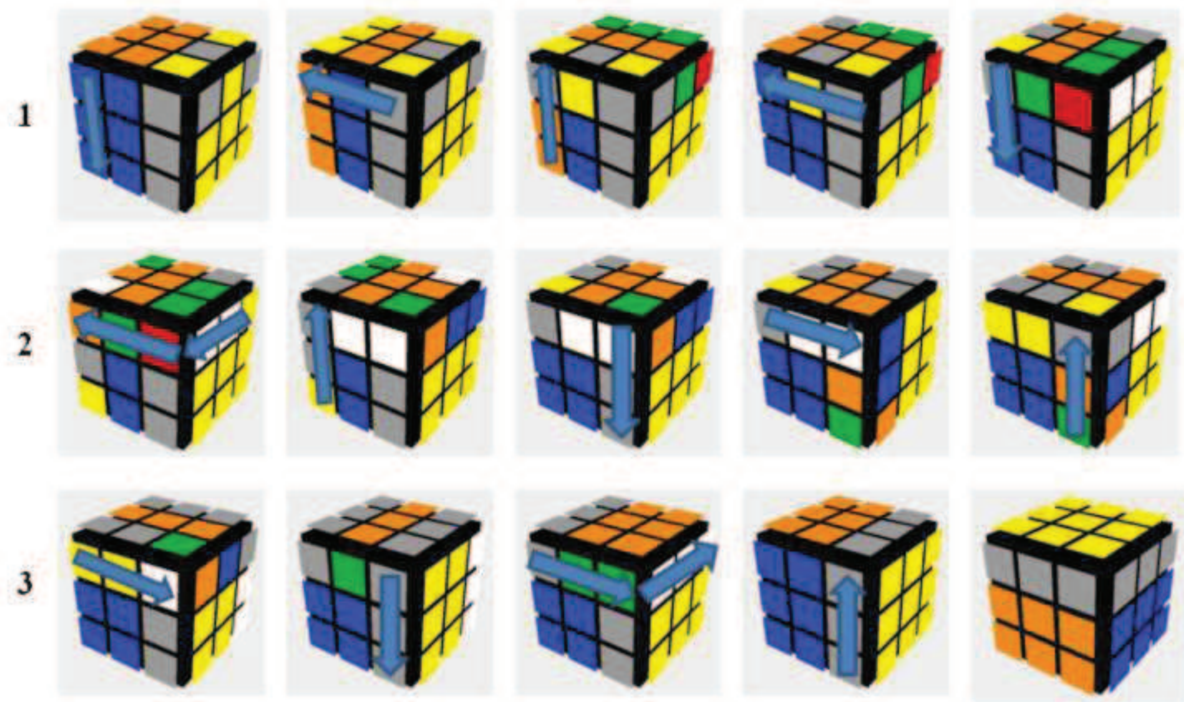

Illustration 19.: Positioning yellow cornercubes, providing sustainability requirements
Source: Fogarassy, 2014

„LOW-CARBON INTERPRETATION" NO5:

After the bottom (yellow) side's cornercubes are in place, we can continue with arranging Output requirements. By completing the Yellow Cross, we can put the system in a state of equilibrium that means clear criteria to the „consumer" side, or affiliates, political decision makers. Finalising the attributes of the Output side is done by arranging the cornercubes to their proper positions. I assume that one of the keys for sustainable business strategies is if the project or development abides by market conditions in a way that they're arranged by at least four strategic objective systems. This can be done easily with the help of the four yellow cornercubes. These have a total of 12 inherent attributes, which is a very big subset, in terms of the cube. With the various sides of the cube, we can define a total of 54 attributes, out of which 3 are inherent in each cornercube respectively. This means that this single rotation algorithm defines the orderedness of the system attributes by 22%. Though the multi-dimension problem solution theory for Rubik's Cube will be introduced in the next chapter, this simple correspondence shows that there are some system elements (cubes/attributes) which have a strong impact on the state of equilibrium of the entire status space with their various positions. The search for points of equilibrium using Game Theory solutions shown in the process of specialised literature can be necessary in this case as well, if the cornercubes are not in their proper positions. The search for points of equilibrium related to project development can be imagined during actualisation can be imagined as searching for the states of equilibrium of the cornercubes' inherent attributes (3 in total) in the status space. This can be defined as a function as follows:

Let φ_i be payoff functions optimising three objective statuses, while vectors $\boldsymbol{u}_1$, $\boldsymbol{u}_2$, $\boldsymbol{u}_3$ strategic vectors, abd we cab define a three-person game of infinite kind, with at least three different points of equilibrium, where the appropriate strategy vectors, $\boldsymbol{u} = (\boldsymbol{u}_i)_{i=1}^{3}$.

$$\varphi_i(\boldsymbol{u}) = \varphi_i(\boldsymbol{u}_1, \boldsymbol{u}_2, \boldsymbol{u}_3,)$$

Example: The „possible changing of flexible technology requirements compared to the planned order is possible in this phase, without changing the Output criteria, or the points of equilibrium" mentioned in *Example No4* can be expanded with the fact that neighboring attributes with a direct influence (three sides of cornercube) have finalised cooperation strategies. Implementing the technological change, and the corrected financing construction which follows it can be as such. These attributes define the project's „shelf-life", meaning its sustainability in a changing economical environment. We have to know that economical points of equilibrium, meaning attributes that have an impact on business sustainability are both ever- and swift-changing. During the planning of investments, or making business plans, this is a factor which is hard to balance, which means that the investments related to mandatory sustainability criteria (enviro-protection, renewable energy production, climate-friendly, etc.) may quickly get into an impossible objective state. This is one thing that the use of the sustainability algorithm of project planning based on Rubik's Cube may help with.
During the rotation sequence, few connections change, which signifies that the optimalisation of cross-effecting correspondences needs a short time interval, and not much work, but the above mentioned intensive sorting effect makes the execution very important.

Linking top and bottom row with edge swap, strict sync of Input/Output variables

In this rotation sequence, we have to move all yellow edge cubes to their various positions. This is the state of the cube, for which everyone can see that their cube is in harmony, and only a very minor step is between them and their objective, success. The first phase of harmonically sortinging yellow and white sides can be seen on Illustration 20.

Illustration 20.: Sorting yellow and white sides by main attributes in status space
Source: Fogarassy, 2014.

Similarly to what's been said at White Cross, we can either position either two, or all four edge cubes by rotating the yellow side during the solution. If we move two edge cubes, they can either be neighboring, or opposite of each other. We use the same algorithm for both cases, but if the cubes which are to be swapped are opposide of each other, we have to do the rotation sequence twice.

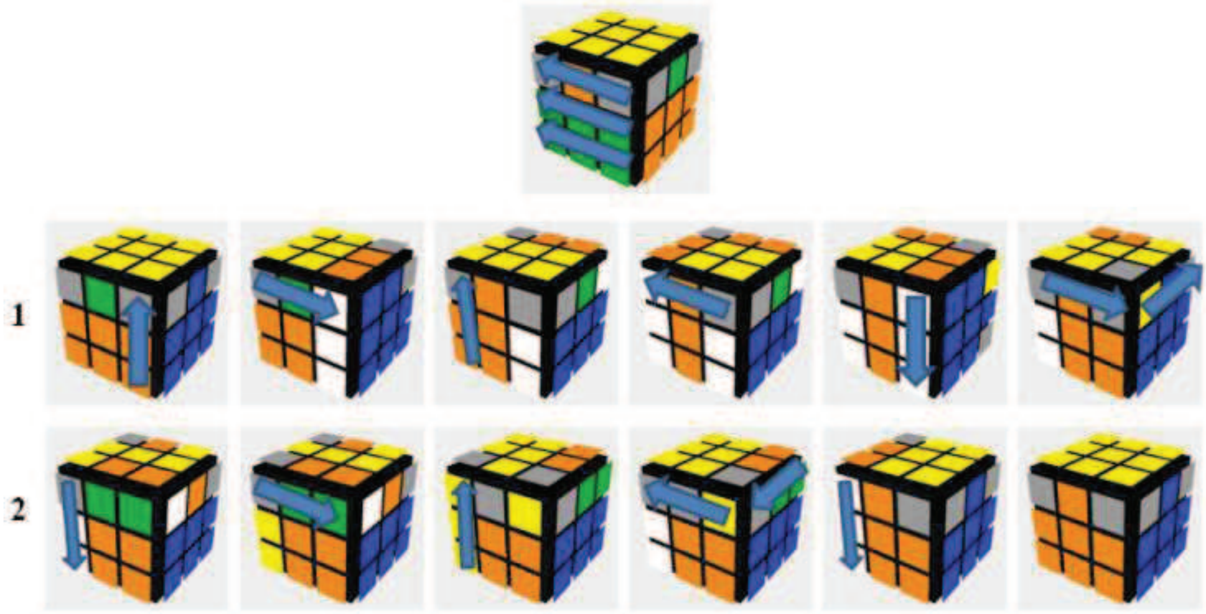

Illustration 21.: Positioning sealing side's yellow edge cubes
Source: Fogarassy, 2014

During the positioning of the edge cubes, we have to keep the two cubes which we want to swap opposite of each other, and to our left side. Now, let's rotate the right column upwards, then its top row (counter-clockwise), followed by the rotation of the left column upwards, and its top row (clockwise). After this, we have three white cubes in front of us to the right, let's rotate these to the bottom row (Illustration 21, upper part). Now, let's make two rotations on the top row (counter-clockwise), then rotate the left column downwards. Rotating the top row (counter-clockwise), and the left column backwards brings the edge cube back in front of us, and the left column will have two white cubes (Illustration 22 lower part), to which we can arrange the third by rotating the top row twice (clockwise). The last step is moving this finished white column back to the other white cubes by rotating them downward.

„LOW-CARBON INTERPRETATION" NO6:
Linking the Input (white) and Output (yellow) sides it the goal of the rotations. During the process of equilibrium search, we're talking about the strict syncing of the most important Input and Output requirements. By rotating the yellow side's edge cubes to their proper place, the strategic fixpoints (meaning the four definitive mid cubes), and the input variables of the Input side form a direct, non-changeable connatcion with the Output variables, requirements. Practically, we finish the whole process/planning/development with this edge swap.
Example: the edge swap shows us how all the Input and Output attributes important for the planning of the project are finalised. Such a case can be if the political requirement system of the Input side is finalised in regards to the program's realisation Output. During the project's evolution, we can handle changes or fixation of „corruption factors" or global variables in a similar manner.

Corner swap, defining the final state of equilibrium for system attributes
Corner swap is the final phase of the solution of the cube, and the definition of the final state of equilibrium for the system attributes (Illustration 22).

Illustration 22.: The cube is in state of equilibrium
Source: Fogarassy, 2014

The state of the cube in this phase is well-known – either three corners are in the wrong place, or all four of them. Solving three corners leads us directly to the solution of the fourth, which means this doesn't need further learning. If we don't want to learn more, faster solution algorithms, it's sufficient to know a single algorithm, for this phase, since using this multiple times will lead to the cornercubes being positioned in their proper place.

If we have a cornercube which is positioned properly, we begin by holding it to our left, and starting the task on the right column. Let's rotate the front yellow row twice (clockwise), by which we bring a white row up, and rotate the right column backwards twice as well, making an L (Illustration 23, upper part). Now, let's rotate the front row once (clockwise), and the left column downwards (Illustration 23, upper part, fourth cube), finally restoring the L by rotating the front row again (counter-clockwise. Now, we can make this L into an I, by rotating the right column backwards twice. Now, let's rotate the front row once (clockwise), followed by rotating the left column upwards. As a finishing touch, we only have to rotate the front row once (counter-clockwise), which puts white together with white, yellow with yellow, and continue to repeat this rotation sequence until all the cornercubes are in place. If two cornercubes weren't in place, we do it twice, if three, we do it three times. We know multiple algorithms which can deliver the cornercubes to their „destinations" from various positions faster. Obviously, knowing and using these may shorten the time required for solution.

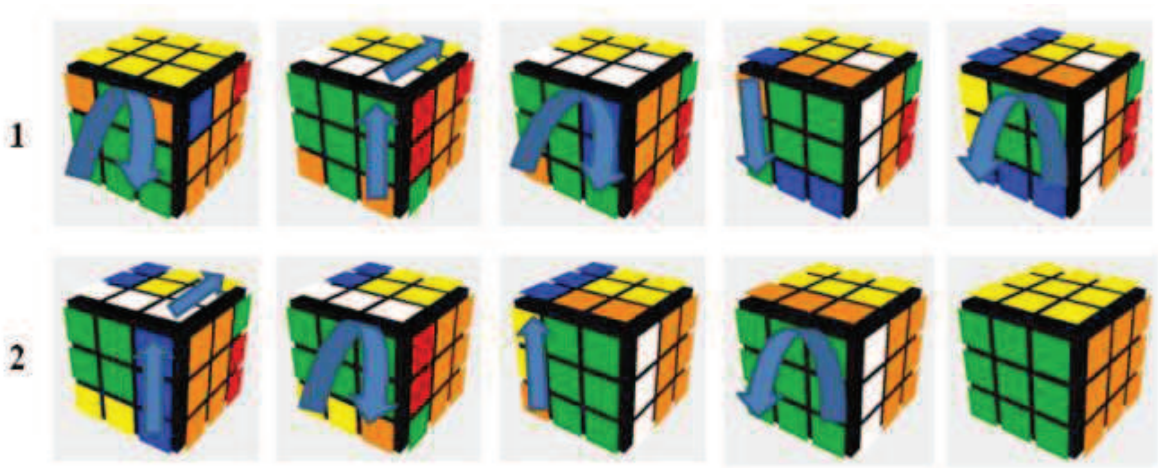

Illustration 23.: Swapping coenerscubes
Source: Fogarassy, 2014

„LOW-CARBON" INTERPRETATION NO7:

The goal of the rotation sequence is to define sustainability criteria, and to set the final state of equilibrium. During the cornercube swap, the rotations have the characteristic of comparing and checking all the attributes inherent in the Input side and the cube side. The edge swap is done for at least three different sides, but usually, the swap of all four cornercubes happens

with edge swaps. By modeling the little details of the project planning or development, we can say that the analysis system gets a finalised frame by these edge swaps. Via the cornercubes which have three inherent attributes, four times three, totaling twelve relevant attributes get into a final state of equilibrium, which is perhaps the most important rotation sequence in the entire solution process. During the project planning using Rubik's Cube, we can call this process of searching for the final state of equilibrium the abidement by sustainability criteria. As we can see in the above mentioned rotations, the point of equilibrium for the Output side (Yellow Cross, solving yellow corners) can be done during the solution process multiple times, but the 3D assortment only means the abidement by sustainability criteria, if the cornercube swaps are done.
Searching for the points of equilibrium/sustainability optimum of sealing cornercubes: one of the most important values, the final harmony of the development project or strategy is given by the rotation combination based on syncing three different attributes. Without this, there's no final coordination between Input and Output sides, meaning the flexibility of the entire system drops significantly, since it didn't adapt requirements which mean the „shelf-life", or capability to adapt to the various possible changes of the system attributes.

In light of the above mentioned, we can define three different strategy programs during the process of low-carbon strategy planning:

A. The existence of a technologically sufficient planning option (to avoid over-planning and obsoletion)
B. Optimalisation of liquidity and financial sustainability is met (safe self-suffieience and revenue for at least 10 years).
C. Avoiding detrimental project effects on the relevant product areas (functionally self-sufficient system).

Mathematically defining the above mentioned goals is no easy task, furthermore, writing the Game Theory payoff functions after this also requires the definition of specialised requirement systems.
Our task can f.e. be written as a three-person game, where $\boldsymbol{u}_1$, $\boldsymbol{u}_2$, $\boldsymbol{u}_3$, are the strategy vectors, and $\boldsymbol{u} = (\boldsymbol{u}_i)_{i=1}^{3}$ is the simultaneous strategic vector. This means:

$$\varphi_i(\boldsymbol{u}) = \varphi_i(\boldsymbol{u}_1, \boldsymbol{u}_2, \boldsymbol{u}_3,) = c^T_{i1}\,\boldsymbol{u}_1 + \boldsymbol{c}^T_{i2} + \boldsymbol{u}_2 + \boldsymbol{c}^T_{i3} + \boldsymbol{u}_3 = \boldsymbol{c}^T_i + \boldsymbol{u}$$

are the objective functions and strategy vectors, therefore

$$\boldsymbol{A}_1\,\boldsymbol{u}_1 + \boldsymbol{A}_2\boldsymbol{u}_2 + \boldsymbol{A}_3\boldsymbol{u}_3 \geq \boldsymbol{b}$$

requirement holds true for them. In this case, the coefficients will be the vectors and matrixes derived from our previous model coefficients.

Example: Finding the final acceptable planning option (from both a financial and technological point of view) is a good example of this (using a technological solution which offers realistic return), since if this can't be realised, the development might even be detrimental to society. However, if the sustainability criteria are met, f.e. the European Union shouldn't have the (quite common) cases, where if financing is cancelled for various development environments, it makes (in the best scenario) the related activities falter (f.e. waste collection systems, waste management), or (in the worst scenario) the entire product path falls apart (f.e. enterpreneur incubation programs, or R&D programs).

Conclusions

Summarising evaluation of process analysis

The processes of project planning and development based on the row by row solution of the 3×3×3 Rubik's Cube show us the correspondece of the sustainable use and correspondence systems of the resources around us, which makes building our development and strategy concepts around this advisable in the future. The process regulation based on the solution process of Rubik's Cube is a swift, effective and low-cost protocol, furthermore, the demonstrated process analysis showed us that if it's not disregarded, the criteria of long-term (sustainable) operation are met, which means that we may suppose (with a high probability) that the result of the entire process won't be detrimental to society.

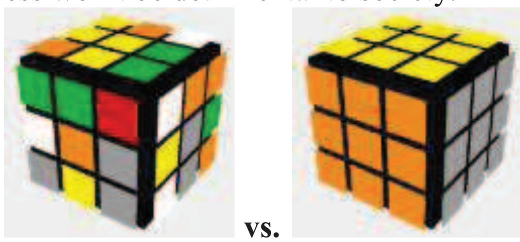

Illustration 24.: Cube in entropic and equilibric states
Source: Fogarassy, 2014

By solving the cube, we imitated the process of project development, meaning the road from complete disorder to the state of complete order. The complete state of equilibrium for Rubik's Cube is the solved state. It's not coincidence, that when someone sees a cube in disorder, their first idea is to solve it, since the desired state is the cube which has only single-colour sides (Illustration 24). Rubik's Cube has inherent harmony even in its colour setting, as we have alredy mentioned, the choice of colours by the developer was intended, and the neighboring logic of colours is not the work of coincidence. Without the mistification of the cube, we can state that it alredy has an inherent and colorful harmony even in its visual appeal, that makes us suggest a seamless logic and perfect logic supports its construction. During the theoretical process analysis, the goal of demonstrating the various rotations was to show what kinds of cube interactions are supposed behind the advancement from state to state, meaning which cubes'/attributes' effects on each other we have to analyse during the rotation process. We didn't define the exact locations and interactions for these during the research, but the division of the process to phases did happen, and we also synced the solution phases to the mechanisms of project development. The correspondences verified that the two logical processes may support each other. During the process evaluation, we proved that sustainability criteria can be synced to some solution algorithms of the 3×3×3 Rubik's Cube, and the correspondence systems of the cube's various sides defines a 3D perception and planning strategy which shows the process of investment development from a new scientific perspective.

In Chart 1, we summarised the various definition levels which mean defineable intervals in the process of project development as well, and in places where we deemed it necessary, we also portrayed correcpondences of the search for states of equilibrium using Game Theory methods, which can be put into a state of equilibrium with project attributes inherent in the various colours or phases – for the sake of sustainability.

Chart 1.: Evaluation of modeling process, and results

CUBE INTERPRETATIONS (number of rotation algorithm)	LEVEL OF MODEL DEVELOPMENT	/LOW-CARBON/ PROJECT ATTRIBUTE IN QUESTION	CORRELATION WITH GAME THEORY
NO1	INPUT	„White cross" – defining the starting criteria	A stage defineable by an n-person zero sum game of infinite kind.
NO2	INPUT	„White corner" – defining the sustainable development routes, equilibrium-search, non-cooperative optimum	According to functions on Nash-equilibrium, non-cooperative strategy, defineable by games of finite kind.
NO3	MID CUBE	„Mid row" – anchoring of relation points, achieving equilibrium, arranging two-dimensional attributes, positioning fixpoint	Positioning edge cubes is possible with conflict alleviation methods. Fixpoint positioning is advised to be done with zero sum game.
NO4	MID CUBE	„Yellow cross" – indirect synchronising of input/output sides	Defineable by oligopolistic games of finite kind, or method of equal compromise.
NO5	OUTPUT	„Yellow corner" – interpretation of sustainability attributes during the arrangement of outputs	Defineable by three-person game of infinite kind, needs Nash-equilibrium.
NO6	OUTPUT	„Yellow side edge-switch" – strict synchronising of input/output sides	Defineable by zero sum game, conflict alleviation method, and cooperative strategy.
NO7	OUTPUT	„Corner switch" – the phase of setting the final balance, achieving equilibrium, finalising sustainability attributes	Oligopolistic games by functions based on either cooperative equilibrium strategy or Nash-equilibrium. Cooperative strategy.

Source: Fogarassy, 2014

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