Probability Distortion and Loss Aversion in Futures Hedging

Fabio Mattos

Philip Garcia

Joost M. E. Pennings *

* Fabio Mattos (fmattos@uiuc.edu) is research assistant, Philip Garcia is professor, and Joost Pennings is associate professor at the Department of Agricultural and Consumer Economics, University of Illinois at Urbana-Champaign

Probability Distortion and Loss Aversion in Futures Hedging

We analyze how the introduction of probability distortion and loss aversion in the standard hedging problem changes the optimal hedge ratio. Based on simulated cash and futures prices for soybeans, our results indicate that the optimal hedge changes considerably when probability distortion is considered. However, the impact of loss aversion on hedging decisions appears to be small, and it diminishes as loss aversion increases. Our findings suggest that probability distortion is a major driving force in hedging decisions, while loss aversion plays just a marginal role.

Keywords: hedging, probability distortion, loss aversion, risk aversion

INTRODUCTION

It is widely recognized that estimated hedge ratios differ from observed hedge ratios (Peck and Nahmias, 1989; Collins, 1997; Garcia and Leuthold, 2004). Hedging models traditionally adopt an expected utility framework to calculate hedge ratios. While these hedge ratios are generally tractable and easy to estimate, two problems arise here. First, the underlying assumptions may not be consistent with hedgers' decision context. There is an extensive literature showing how hedge ratios can change drastically as the assumptions about the decision context are relaxed, e.g. when transaction costs, alternative investments and downside risk measure are introduced (Lence, 1995 and 1996; Mattos, Garcia and Nelson, 2006).

A second issue with expected-utility hedge ratios is that empirical evidence shows that the expected utility framework frequently fails to explain decision making under risk (Schoemaker, 1982; Hirschleifer, 2001). Several studies have proposed alternatives to expected utility. The proposed theories try to account for several behavior patterns observed in laboratory and field experiments, e.g. individuals tend to evaluate prospects in isolation and make decisions in terms of gains and losses relative to a reference point. Prospect theory developed by Kahneman and Tversky (1979) and Tversky and Kahneman (1992) incorporates those behavioral dimensions and is possibly the most well-know construct. However, there has been very limited research investigating futures hedging in the framework of non-expected utility, i.e. it remains to be seen whether different choice models can yield hedge ratios more consistent with observed behavior.

The objective of this paper is to investigate hedging behavior in a non-expected utility framework. More specifically, we will examine the effects of probability distortion and loss aversion on futures hedging. Empirical evidence shows that individuals usually don't evaluate probabilities objectively, i.e. they tend to distort objective probabilities. Evidence from experiments also suggest that individuals are more sensitive to losses than to gains, which implies that people are more affected by a loss than by a gain of similar magnitude.

This study contributes to the literature by incorporating probability distortion and expanding the investigation of loss aversion in the analysis of futures hedging. In spite of the intuitive appeal of probability distortion and loss aversion, little research exists that investigates the extent to which

hedge ratios differ from the traditional procedure in their presence. This effort may refine the way in which producers preferences are modeled and help understand their hedging behavior.

DECISION MAKING UNDER RISK

Schoemaker (1982) argues that expected utility theory fails as a descriptive and predictive model because it does not recognize several psychological principles of judgment and choice. Research has shown that individuals don't structure problems and process information according to expected utility theory. Three sources of biases can generally be identified: heuristic simplification, self-deception, and emotions and self-control (Hirschleifer, 2001).

Heuristic simplifications arise when individuals focus on a subset of available information due to unconscious associations, and limited attention, memory and processing capacities. Common examples of this source of biases include – but are not limited to – availability heuristics (events that are easier to recall or relate to are judged to be more probable), narrow framing (the form of presentation of logically identical problems affect the agent's final decision), status quo bias (individuals prefer the alternative identified as status quo among a list of options), loss aversion, clustering illusion (agents perceive a random sequence of events as reflecting causal patterns), and conservatism (individuals might not change their beliefs as much as expected for a rational Bayesian when faced with new evidence).

Self-deception comes indirectly from cognitive constraints and it basically states that individuals tend to believe that they are better than they really are, which helps them make others believe in those qualities too. Two of the consequences commonly found are overconfidence and biased self-attribution. Overconfident individuals believe that their knowledge is more accurate than it really is, which implies overoptimism about their abilities to succeed. Biased self-attribution means that agents tend to attribute favorable outcomes to their own abilities and unfavorable outcomes to exogenous variables, which implies that people cannot easily learn with their own mistakes.

Finally, mood and emotions can make individuals choose alternatives that wouldn't be chosen if the decision was solely based on reason. Mood states generally affect abstract judgments relatively more than judgments for which there are concrete information. People in good mood are likely to be more optimistic than those in bad mood. Individuals also tend to focus more on ideas and facts which are reinforced by conversation and opinions, i.e. there is a self-reinforcement of ideas because people generally conform with the judgment and behavior of others. Further, empirical evidence suggests that in the decision to defer consumption discount rates can change under different circumstances: gains tend to be discounted more heavily than losses, choice framing (delay, advance in consumption) has a large effect on decision, time preference differs substantially in distinct domains, among others.

Those psychological dimensions are underlined by a human tendency to seek cognitive simplification (Schoemaker, 1982), and have several practical implications on how individuals make choices. First, individuals make decisions by comparing alternatives one dimension at a time, and not by assigning a separate level of utility to each one of them. Another implication is that evaluation strategies can vary with the complexity of the alternatives available to the

decision-maker. Isolation is a third implication and it means that alternatives are not considered in a comprehensive way. Individuals just tend to consider different options in isolation, even though they might be related. A fourth aspect is the importance of reference points, i.e. it's cognitively easier to consider alternatives by comparing them to a reference level than to consider them in absolute terms. A final point refers to probability judgment. In general individuals overweigh small probabilities and underweigh high probabilities, implying that there are subjective probabilities which relate non-linearly to objective (or stated) probabilities.

NON-EXPECTED UTILITY MODELS

Several studies have proposed alternatives to expected utility, and prospect theory (Kahneman and Tversky, 1979; Tversky and Kahneman, 1992) is possibly the most referred theory in this matter. Prospect theory differs from expected utility theory basically by assuming that decisions are made in terms of gains and losses rather than final wealth, individuals´ reactions to gains are different than to losses, and that agents use probability weights rather than objective probabilities when making decisions under risk. This choice model is based on a value function $V(x)$, which has two components: a two-step preference function $v(x)$ and a weighting function $w(F(x))$, where x is the argument of the value and utility functions, and $F(x)$ is the objective cumulative probability distribution of x.

$$V(x) = \int v(x) \cdot \frac{d}{dx} w(F(x)) dx \qquad (1)$$

Two-step preference function

A two-piece representation takes into account the fact that variations in the framing of alternatives systematically yield different preferences (framing effects), e.g. agents react differently to gains and losses. A loss aversion coefficient λ is incorporated to account for the fact that losses loom larger than gains. Finally, the two-piece preference function assumes that risk-averse behavior is observed in the domain of gains $(x > 0)$, and risk-seeking behavior is verified in the domain of losses $(x < 0)$. Risk-seeking assumption in the domain of losses has empirical support and comes from the idea that people dislike losses so much that they would be willing to take greater risks in order to make up for their losses.

$$v(x) = \begin{cases} U(x), & x \geq 0 \\ -\lambda \cdot U(-x), & x \leq 0 \end{cases} \qquad (2)$$

Several studies have explored this two-piece preference function in financial decision-making models. In general those studies have used market data to explain investors' behavior, look for evidence of a two-piece utility function, estimate the degree of loss aversion, or find the solution for optimal portfolio problems. The focus has usually been on equity and bond markets, and little attention has been paid to futures markets. For example, Barberis, Huang, and Santos (2001) look at the consumption-based model for the stock market assuming that investors are loss averse and derive utility not only from consumption but also from changes in their financial wealth.

Berkelaar, Kouwenberg, and Post (2004) use stock market data to investigate portfolio allocation problems assuming that investors are loss averse. Hwang and Satchell (2005) and Davies and Satchell (2005) use data on equity returns to examine admissible ranges for the parameters of a two-piece preference function.

However, there has been very limited research on the application of a two-piece representation on hedging problems. Albuquerque (1999) looks at currency hedging in the context of loss-averse firms, and he claims that loss aversion provides incentive to hedge against downside risk. Hence his focus is basically on which instrument – forward contracts or options – produces a better hedge against downside risk. Lien (2001) examines how the strategy of a short hedger is affected in the presence of loss aversion. His findings show that loss aversion has no effect on the optimal hedge ratio if markets are unbiased. But when markets are in contango or backwardation the optimal hedge ratio in the presence of loss aversion will differ from the minimum-variance hedge ratio.

Weighting function

The idea of weighting function comes from the empirical observation that people don't evaluate risky prospects as a linear function of the actual probabilities of different outcomes, but rather they evaluate risky alternatives using a non-linear weighting function. Kahneman and Tversky (1979, p.280) explain that "decision weights measure the impact of events on the desirability of prospects, and not merely the perceived likelihood of these events". And the explanation for giving lower or higher weights to such events lies on the psychological dimensions of decision making discussed earlier.

The weighting function $w(F(x))$ adopted in the current paper is given by equation 3 and is characterized by a unique parameter γ (Prelec, 1998):

$$w(F(x)) = \frac{1}{\exp(-\ln F(x))^{\gamma}} \tag{3}$$

where $F(x)$ is the objective cumulative probability distribution, and γ defines the curvature of the decision weight curve. The weighting function assumes equal probability distortion in the domains of gains and losses. Although there are other functional forms for the weighting function in the literature, we chose Prelec's one-parameter function because of its accuracy and parsimony to explain aggregate behavior. Gonzalez and Wu (1999) tested several functional forms and concluded that one-parameter weighting function provides an excellent and parsimonious fit to the median data.

This weighting function is an increasing function of probability $F(x)$ (Figure 1). Empirical estimates usually find that $0 < \gamma < 1$, implying that the weighting function is regressive and inverse s-shaped, i.e. first $w(F(x)) > F(x)$ (small probabilities are overweighed) and the function is concave, and then $w(F(x)) < F(x)$ (high probabilities are underweighed) and the function is convex. For $\gamma > 1$ the weighting function is s-shaped, which means that small

probabilities are underweighed and high probabilities are overweighed. In general this function is also asymmetrical in the sense that the inflection point – where the function intersects the diagonal given by $\gamma = 1$ – is at $F(x) = w(F(x)) = 0.37$.

Figure 1: Weighting function for three different values of γ
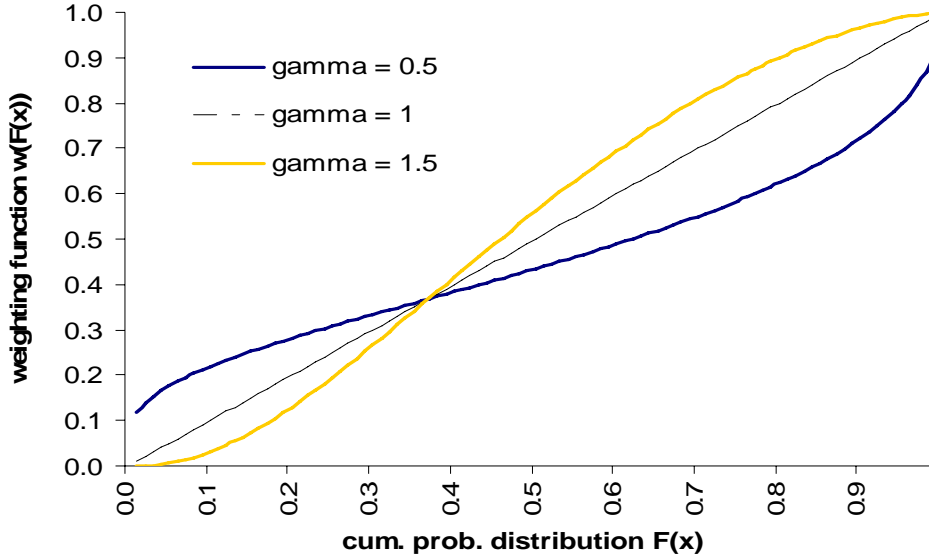
RESEARCH METHOD

The analysis is based on a soybean producer who takes a short position in the futures market to hedge stored soybeans. In a one-period model final wealth is given by:

$$W_1 = W_0 + \Delta p + h \cdot \Delta f \qquad (4)$$

where W_1 is end-of-period wealth, W_0 is beginning-of-period wealth, Δp is the cash price change, Δf is the futures price change, and h is the hedge ratio. In this model the hedge ratio is negative for short positions, and positive for long positions. Wealth change $W_1 - W_0$ is adopted as the argument of a constant absolute risk aversion two-piece utility function (equations 5 and 6):

$$W_1 - W_0 = \Delta p + h \cdot \Delta f \qquad (5)$$

$$E\left[v\left(W_1 - W_0\right)\right] \equiv V(\sigma, \mu) = \left(\mu_h - \frac{\theta_G}{2} \cdot \sigma_h^2\right) - \lambda \cdot \left(\mu_h + \frac{\theta_L}{2} \cdot \sigma_h^2\right) \qquad (6)$$

where $\mu_h = E\left(W_1 - W_0\right) = \mu_p + h \cdot \mu_f$, $\sigma_h^2 = Var\left(W_1 - W_0\right) = \sigma_c^2 + h^2 \sigma_f^2 + 2h\sigma_{cf}$, $\mu_c\left(\mu_f\right)$ is the mean of the cash (futures) price changes distribution, $\sigma_c^2\left(\sigma_f^2\right)$ is the variance of the cash (futures) price changes distribution, σ_{cf} is the covariance between cash and futures price changes, $\theta_G\left(\theta_L\right)$ is the risk aversion coefficient in the domain of gains (losses), and λ is the loss aversion coefficient.

The optimal hedge ratio is obtained through the maximization of (6) with respect to the hedge ratio:

$$\frac{\partial}{\partial h} E\left[v\left(W_1 - W_0\right)\right] = \frac{\partial V\left(\mu_h, \sigma_h\right)}{\partial \mu_h} \cdot \frac{\partial \mu_h}{\partial h} + \frac{\partial V\left(\mu_h, \sigma_h\right)}{\partial \sigma_h} \cdot \frac{\partial \sigma_h}{\partial h} = 0 \tag{7}$$

$$h = \frac{\left(1+\lambda\right) \cdot \mu_f}{\left(\theta_G - \lambda\theta_L\right) \cdot \sigma_f^2} - \frac{\sigma_{cf}}{\sigma_f^2} \tag{8}$$

The hedge ratio in equation 8 incorporates loss aversion in the speculative component, but no probability distortion is assumed at this point. In this situation it can be seen that in an unbiased futures market $\left(\mu_f = 0\right)$ the speculative component of the hedge ratio is equal to zero, and hence the optimal hedge ratio becomes the minimum-variance hedge ratio given by σ_{cf}/σ_f^2. This finding is consistent with Lien (2001), who found that loss aversion has no effect on the optimal hedge ratio if futures and cash markets are unbiased.

Now we introduce probability distortion in the hedging problem. The optimal hedge ratio can be derived in a similar fashion as the one in equation 8, except that now we have transformed probabilities – and not objective probabilities – in the calculation of means and variances. Thus the optimal hedge ratio with probability distortion given by equation 9:

$$h_p = \frac{\left(1+\lambda\right) \cdot \mu_{f,p}}{\left(\theta_G - \lambda\theta_L\right) \cdot \sigma_{f,p}^2} - \frac{\sigma_{cf,p}}{\sigma_{f,p}^2} \tag{9}$$

where h_p is the hedge ratio, $\mu_{f,p}$ and $\sigma_{f,p}^2$ are the mean and variance of the futures price changes distribution, and $\sigma_{cf,p}$ is the covariance between cash and futures price changes.

The distribution moments with probability distortion are calculated as:

$$\mu_{f,p} = \mu_f + \Delta\mu_f \tag{10}$$

$$\sigma_{f,p}^2 = \sigma_f^2 - \mu_f^2 + \left\{\mu_f + \Delta\mu_f\right\}^2 - 2\cdot\Delta\sigma_f^2 \tag{11}$$

$$\sigma_{cf,p} = \sigma_{cf} + \Delta\rho - \mu_f\Delta\mu_c - \mu_c\Delta\mu_f - \Delta\mu_f\Delta\mu_c \tag{12}$$

where

$$\Delta\mu_f = \int_{-\infty}^{+\infty} F\left(r_f\right)dr_f - \int_{-\infty}^{+\infty} w\left[F\left(r_f\right)\right]dr_f \tag{13}$$

$$\Delta\mu_c = \int_{-\infty}^{+\infty} F\left(r_c\right)dr_c - \int_{-\infty}^{+\infty} w\left[F\left(r_c\right)\right]dr_c \tag{14}$$

$$\Delta\sigma_f^2 = \int_{-\infty}^{+\infty} r_f \cdot F\left(r_f\right)dr_f - \int_{-\infty}^{+\infty} r_f \cdot w\left[F\left(r_f\right)\right]dr_f \tag{15}$$

$$\Delta\rho = \int_{-\infty}^{+\infty}\int_{-\infty}^{+\infty} F\left(r_f, r_c\right)dr_f dr_c - \int_{-\infty}^{+\infty}\int_{-\infty}^{+\infty} w\left[F\left(r_f, r_c\right)\right]dr_f dr_c \tag{16}$$

In equations 10 through 16, $r_c(r_f)$ is the cash (futures) price change, $F(r_c)$ and $F(r_f)$ are respectively the cumulative distribution functions of cash and futures price changes, $F(r_f, r_c)$ is the joint cumulative distribution function of cash and futures price changes, $w[F(r_c)]$ and $w[F(r_f)]$ are respectively the weighted cumulative distribution functions of cash and futures price changes, and $w[F(r_f, r_c)]$ is the weighted joint cumulative distribution function of cash and futures price changes. In the absence of probability distortion, $\Delta\mu_F = \Delta\mu_C = \Delta\sigma_F^2 = \Delta\rho = 0$ and consequently $\mu_{f,p} = \mu_f$, $\sigma_{f,p}^2 = \sigma_f^2$, and $\sigma_{CF,p} = \sigma_{CF}$.

The hedge ratio in equation 9 incorporates both probability distortion and loss aversion, and probability distortion is present in the speculative and hedging components. Here, the speculative component of the hedge ratio doesn't disappear when the futures market is unbiased, and hence loss aversion does have an effect on the optimal hedge ratio.

However, comparative statics suggest loss aversion has a diminishing effect on hedge ratios. Consider the partial derivatives in equations 17 and 18 with respect to the loss aversion coefficient. The sign of the first partial derivative is given by the sign of the expected change in futures prices $\mu_{F,p}$, since all other parameters are positive. But the sign of the second partial derivative depends on the sign of $\mu_{F,p}$ and $(\theta_G - \lambda\theta_L)$. By definition, and also consistent with empirical findings, we have $\lambda > 1$ and $\theta_G \leq \theta_L$, which implies $(\theta_G - \lambda\theta_L) < 0$. Hence, when $\mu_{F,p} > 0$ we have $\partial h_p / \partial\lambda > 0$ and $\partial^2 h_p / \partial\lambda^2 < 0$, i.e. there is a positive relationship between hedge ratio and loss aversion which becomes less positive as loss aversion increases. When $\mu_{F,p} < 0$ we have $\partial h_p / \partial\lambda < 0$ and $\partial^2 h_p / \partial\lambda^2 > 0$, and hence there is a negative relationship between hedge ratio and loss aversion which becomes less negative as loss aversion increases. Therefore, $\partial h_p / \partial\lambda$ tends to zero as the loss aversion coefficient λ becomes larger.

$$\frac{\partial h_p}{\partial\lambda} = \frac{\mu_{f,p}\sigma_{f,p}^2(\theta_G + \theta_L)}{\left[(\theta_G - \lambda\theta_L)\sigma_{f,p}^2\right]^2} \tag{17}$$

$$\frac{\partial^2 h_p}{\partial\lambda^2} = \frac{2\mu_{f,p}\theta_L(\theta_G + \theta_L)}{(\theta_G - \lambda\theta_L)^3\sigma_{f,p}^2} \tag{18}$$

These effects can be explained intuitively because the coefficients of loss and risk aversion just affect the speculative component of the hedge ratio in (9). Probability distortion, however, affects the moments of the distribution, influencing both speculative and hedging components of the hedge ratio. Partial derivatives of (9) with respect to γ yield (19) and (20). Signs of these expressions depend on the signs of the derivatives of the moments with respect to probability distortion, and also on the relative magnitude of those derivatives relative to each other.

$$\frac{\partial h_p}{\partial \gamma} = \frac{(1+\lambda)(\theta_G - \lambda\theta_L)\left(\sigma_{f,p}^2 \dfrac{\partial \mu_{f,p}}{\partial \gamma} - \mu_{f,p} \dfrac{\partial \sigma_{f,p}^2}{\partial \gamma}\right)}{\left[(\theta_G - \lambda\theta_L)\cdot\sigma_{f,p}^2\right]^2} - \frac{\left(\sigma_{f,p}^2 \dfrac{\partial \sigma_{cf,p}}{\partial \gamma} - \sigma_{cf,p}\dfrac{\partial \sigma_{f,p}^2}{\partial \gamma}\right)}{\left(\sigma_{f,p}^2\right)^2} \quad (19)$$

$$\frac{\partial^2 h_p}{\partial \gamma^2} = \frac{(1+\lambda)\mu_{f,p}}{(\theta_G - \lambda\theta_L)\left(\sigma_{f,p}^2\right)^2}\cdot\left[\frac{2}{(\theta_G - \lambda\theta_L)\sigma_{f,p}^2}\cdot\frac{\partial \sigma_{f,p}^2}{\partial \gamma} - \frac{\partial^2 \sigma_{f,p}^2}{\partial \gamma^2}\right] + \frac{\sigma_{cf,p}}{\left(\sigma_{f,p}^2\right)^3}\cdot\left[\frac{\partial^2 \sigma_{f,p}^2}{\partial \gamma^2} - 2\left(\frac{\partial \sigma_{f,p}^2}{\partial \gamma}\right)^2\right]$$

$$+\frac{1}{\left(\sigma_{f,p}^2\right)^2}\cdot\left[\frac{1+\lambda}{\theta_G - \lambda\theta_L}\left(\sigma_{f,p}^2\cdot\frac{\partial^2 \mu_{f,p}}{\partial \gamma^2} - \frac{2}{\theta_G - \lambda\theta_L}\cdot\frac{\partial \mu_{f,p}}{\partial \gamma}\right) - \frac{\partial^2 \sigma_{cf,p}}{\partial \gamma^2} + 2\frac{\partial \sigma_{f,p}^2}{\partial \gamma}\frac{\partial \sigma_{cf,p}}{\partial \gamma}\right]$$

$$(20)$$

SIMULATION DESIGN

Normal distributions are generated for soybean cash and futures price changes. Means and variances are based on real price changes occurred during the 1990-2004 period in Illinois. Consistent with most research on agricultural futures markets (Garcia and Leuthold, 2004), it is assumed that futures markets are unbiased. Simulated normal distributions for cash and futures price changes have, respectively, mean 0.037 and standard deviation 0.561, and mean zero and standard deviation 0.571. The correlation between cash and futures price changes is 0.90. Based on those distributions we introduce a weighting function and investigate how different levels of probability distortion affect the values for the mean, variance, and covariance expected by the hedger. Further, we discuss the impact of probability distortion and loss aversion on the hedge ratio adopted by the producer.

RESULTS

First we investigate how probability distortion change the distribution functions, and hence the moments of the distribution. When $0 < \gamma < 1$ small probabilities are overweighed and high probabilities are underweighed, yielding a probability distribution function (PDF) with more mass in the tails and less mass around the mean than the original PDF. The cumulative distribution function (CDF) with probability distortion has more mass in the domain of negative values and less mass in the domain of positive values than the original CDF (Figure 2). Alternatively, when $\gamma > 1$ small probabilities are underweighed and high probabilities are overweighed. In this situation probability distortion causes a decrease of the mass of the PDF in the tails and an increase around the mean, resulting in a CDF that is less dense in the domain of negative values and more density in the domain of positive values (Figure 3).

Figure 2: PDF and CDF of cash price changes without probability distortion (dark line) and with probability distortion (grey line, $\gamma = 0.5$)
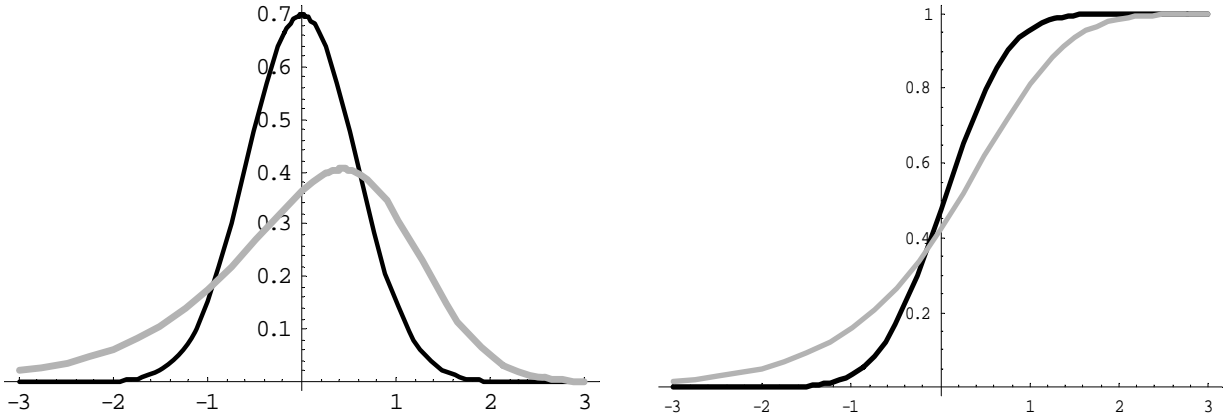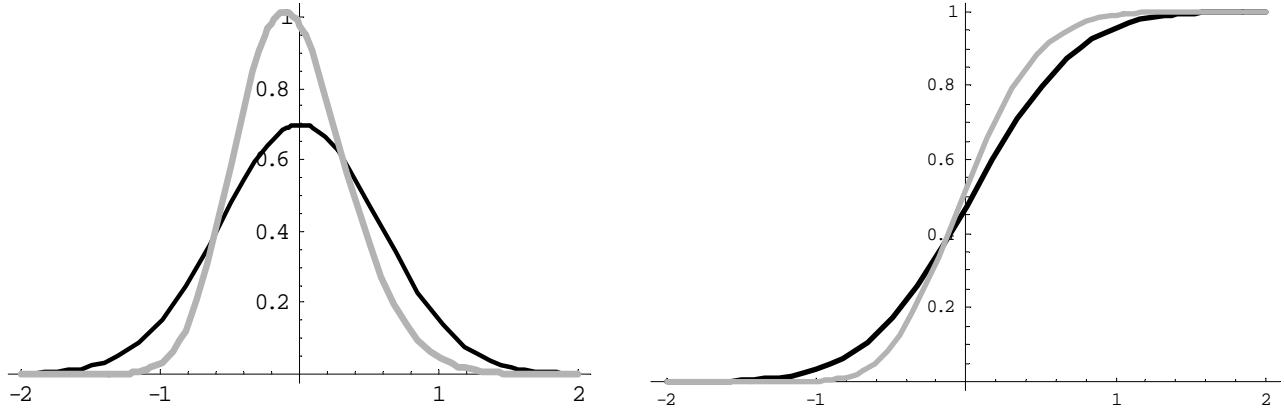

Figure 3: PDF and CDF of cash price changes without probability distortion (dark line) and with probability distortion (grey line, $\gamma = 1.5$)

The effect of probability distortion on the probability distribution impacts the expected change in cash and futures prices and its variance (Figure 4). When small probabilities are overweighed and high probabilities are underweighted ($0 < \gamma < 1$), the hedger expects the price changes to be greater than when γ is close to one, and smaller than when γ is close to zero. In addition, the variance of price changes is expected to be greater than the true variance for all values of γ in this interval. In contrast, when small probabilities are underweighed and high probabilities are overweighed ($\gamma > 1$), the hedger will consistently expect price changes to be smaller than the mean of the distribution, and the variance to be also smaller than the true variance. The correlation between cash and futures price changes is also affected by probability distortion (Figure 5). In the absence of probability distortion ($\gamma = 1$) the correlation coefficient is 0.90. However, the expected correlation coefficient decreases quickly even for small deviations from $\gamma = 1$. When γ reaches 0.6 expected correlation is zero. Similarly, when γ takes values greater than one, the expected correlation coefficient reaches zero when $\gamma = 1.6$.

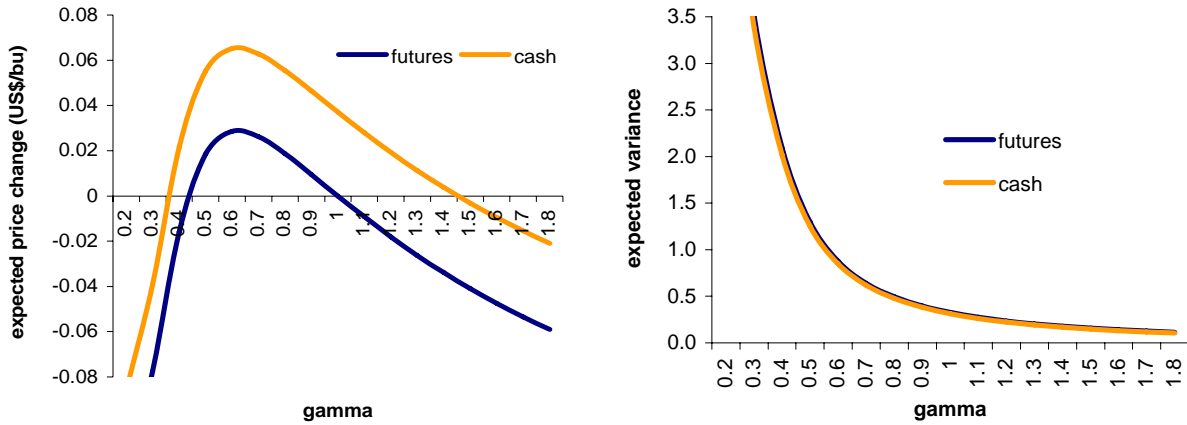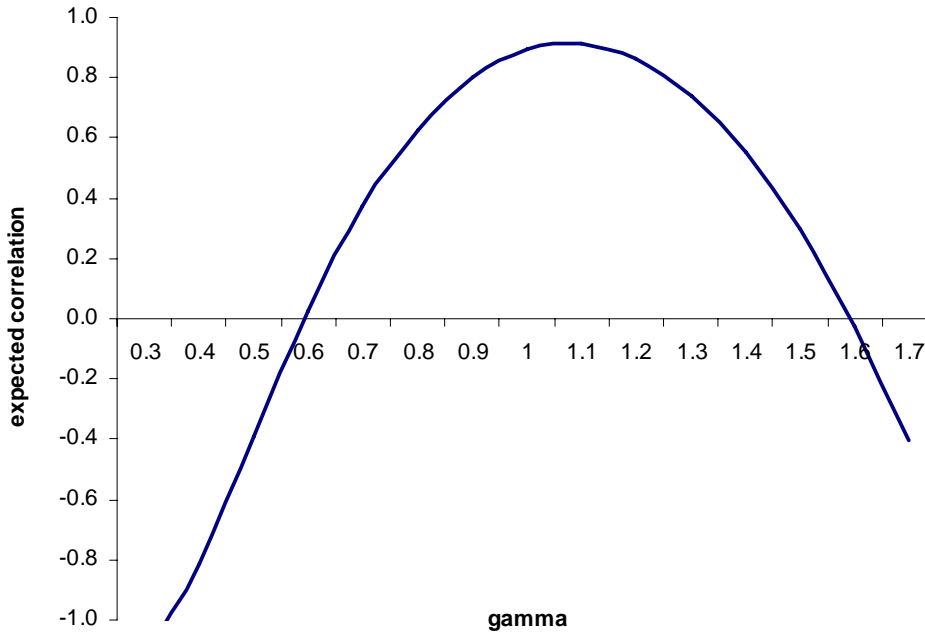**Figure 4:** Expected price changes and variance for different levels of probability distortion (γ)

Figure 5: Expected correlation of price changes for different levels of probability distortion (γ)

Because the moments of the distribution of cash and futures price changes are affected by probability distortion, so are the hedge ratios. Figure 6 shows the hedge ratio calculated for a risk aversion coefficient of 2 in the domain of gains and 2.5 in the domain of losses, and a loss aversion coefficient of 2. In the absence of probability distortion ($\gamma = 1$) the hedge ratio is –0.87, but it goes towards zero when γ deviates from one. As γ becomes smaller than one, the hedge ratio decreases and reaches zero at $\gamma = 0.6$. Similarly, the hedge ratio goes to zero as γ increases towards 1.6. Nevertheless, the effect of loss aversion and risk aversion on hedge ratios is small relative to the impact of probability distortion. Table 1 presents a comparative analysis of how changes in one parameter at a time affect the hedge ratio. The upper part of Table 1 shows the effect of a 10% increase in each parameter on the hedge ratio relative to the base scenario (which is represented in the shaded row on the top). A 10% increase in the loss (risk) aversion parameter

11

causes a reduction of 0.30% (0.40%) in the hedge ratio, with the level of probability distortion held constant. But when a 10% increase is applied to γ the hedge ratio increases by 11%[1]. Similarly, the bottom part of Table 1 shows that a decrease in loss and risk aversion has an impact on the hedge ratio which is smaller relative to a decrease of similar magnitude in γ.

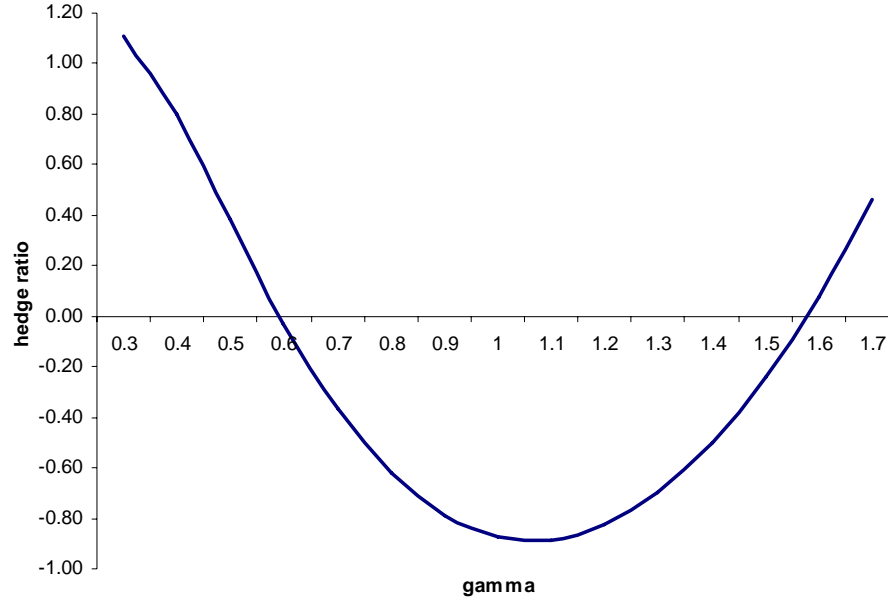**Figure 6:** Hedge ratio for different levels of probability distortion (γ)

Table 1: Effect of probability distortion and loss aversion on hedge ratios

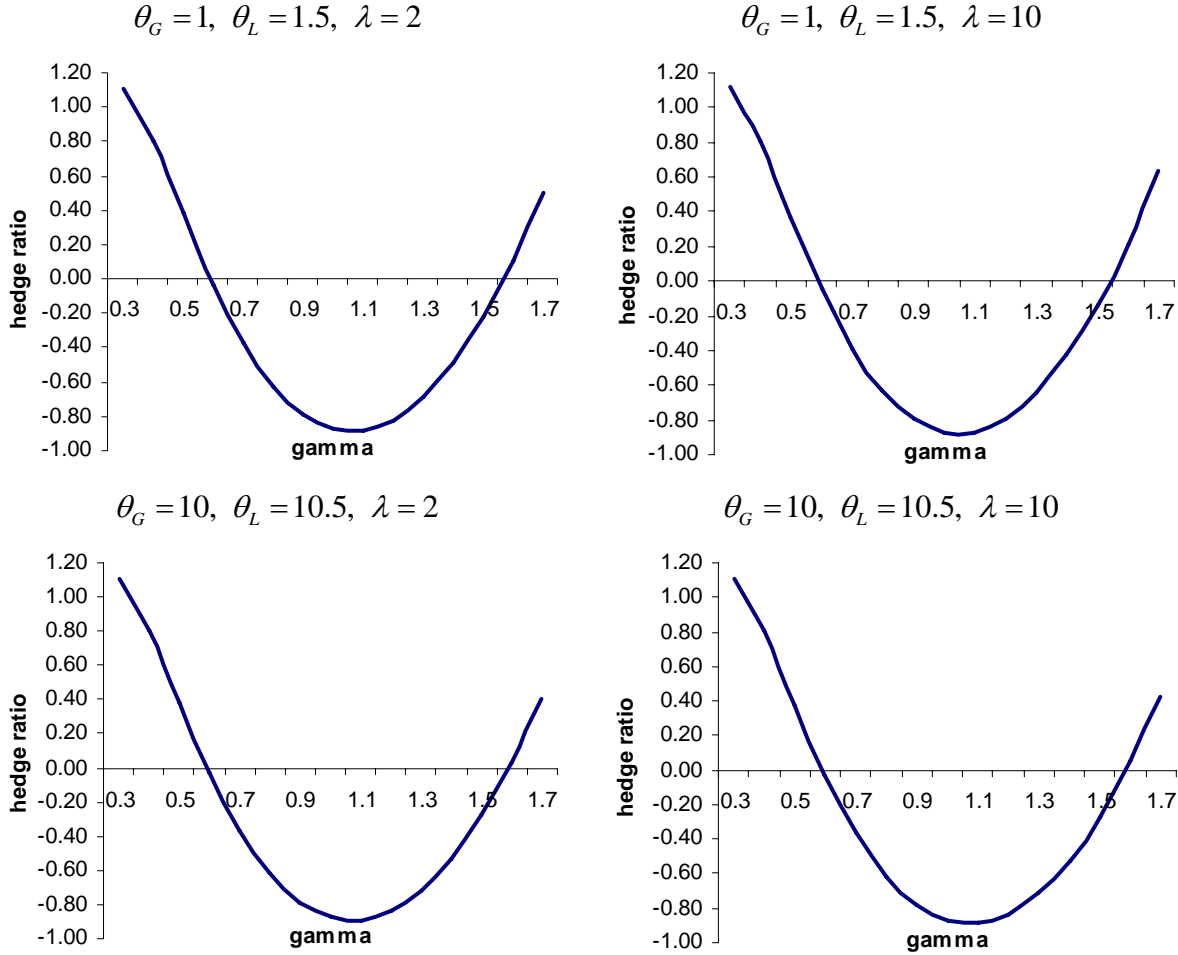| Risk aversion		Loss aversion	Probability	Hedge	
θ_G	θ_L	λ	distortion (γ)	ratio	
2	2.5	2	0.9	-0.7871	
2	2.5	**2.2 (+10%)**	0.9	-0.7869	**(− 0.30%)**
2.2 (+10%)	**2.75**	2	0.9	-0.7868	**(− 0.40%)**
2	2.5	2	**1.0 (+10%)**	-0.8747	**(+ 11%)**
2	2.5	2	0.9	-0.7871	
2	2.5	**1.001 (–50%)**	0.9	-0.7890	**(+ 0.24%)**
1 (–50%)	**1.25**	2	0.9	-0.7906	**(+ 0.44%)**
2	2.5	2	**0.5 (–44%)**	-0.3823	**(− 148%)**

Obs: percentage changes refer to the base scenario (shaded row)

Figure 7 presents further evidence of the large impact of probability distortion on hedge ratios relative to the effect of loss and risk aversion. Like in Figure 6 the graphs show how the hedge ratio changes as different levels of probability distortion are assumed, but now for several combinations of loss and risk aversion. As can be seen, the pattern is very similar in all

---

[1] There is some asymmetry in these effects, e.g. a 10% decrease in loss aversion implies an increase of 0.06% – and not 0.30% – in the hedge ratio, all else held constant.

simulations, and even for large changes in loss and risk aversion there is barely any impact on the hedge ratio. These empirical findings supports our theoretical analysis which suggested that the optimal hedge was rather insensitive to changes in loss and risk aversion.

Figure 7: Hedge ratio for different levels of probability distortion (γ), risk aversion (θ), and loss aversion (λ)

$$\theta_G = 1, \ \theta_L = 1.5, \ \lambda = 2$$

$$\theta_G = 1, \ \theta_L = 1.5, \ \lambda = 10$$

$$\theta_G = 10, \ \theta_L = 10.5, \ \lambda = 2$$

$$\theta_G = 10, \ \theta_L = 10.5, \ \lambda = 10$$

CONCLUSION

This paper investigates how probability distortion and loss aversion affect hedging decisions. The findings suggest both dimensions have an impact on hedge ratios, but probability distortion appears to be dominant. Probability distortion alone always affects optimal hedge ratios, while loss aversion by itself has an impact only when futures market is biased. Furthermore, even when loss aversion affects the hedge ratio, its impact is small and negatively related to the magnitude of loss aversion.

Consequently, changes in the hedge ratio seem to be driven by probability distortion or, more specifically, by the impact of probability distortion on the expected correlation between cash and futures price changes. Simulation results show that even a small degree of probability distortion

13

in the decision process can drive the optimal hedge ratio far away from the standard minimum-variance estimate. Moreover, it doesn't take much probability distortion to turn the optimal hedge ratio to zero, i.e. even little probability distortion is enough to make hedging less attractive for a producer.

The findings of this study have several implications. First, while expected utility-based hedge ratios are easier to calculate than non-expected utility-based hedge ratios, their results tend to differ dramatically when more realistic models are used. Second, loss aversion and risk aversion seem to have just a marginal influence on hedge ratios in a non-expected utility framework. Third, probability distortion appears to be a major driving force to determine hedge ratios under non-expected utility models. Hence, the results of this research seem to suggest that future research should focus on the determinants of probability distortion in choice models, which could shed more light on how individuals make hedging decisions.

REFERENCES

Albuquerque, R. (1999). Optimal Currency Hedging Under Loss Aversion. Working Paper, University of Rochester.

Barberis, N., Huang, M., and Santos, T. (2001). Prospect Theory and Asset Prices. *The Quarterly Journal of Economics* 116, 1-53.

Berkelaar, A.B., Kouwenberg, R., and Post, T. (2004). Optimal Portfolio Choice Under Loss Aversion. *The Review of Economics and Statistics* 86, 973-987.

Camerer, C.F., and Loewenstein, G. (2004). Behavioral Economics: Past, Present, Future. In: *Advances in Behavioral Economics*, edited by C.F. Camerer, G. Loewenstein, and M. Rabin, Princeton University Press.

Davies, G.B., and Satchell, S.E. (2005). Continuous Cumulative Prospect Theory and Individual Asset Allocation. Working Paper, University of Cambridge.

Garcia, P., and Leuthold, R.M. (2004). A selected review of agricultural commodity futures and options markets. *European Review of Agricultural Economics* 31, 235-272.

Gonzalez, G., and Wu, G. (1999). On the Shape of the Probability Weighting Function. *Cognitive Psychology* 38, 129-166.

Hirshleifer, D. (2001). Investor Psychology and Asset Pricing. *The Journal of Finance* 56, 1533-1597.

Hwang, S., and Satchell, S.E. (2005). The Magnitude of Loss Aversion Parameters in Financial Markets. Working Paper, University of Cambridge.

Kahneman, D., and Tversky, A. (1979). Prospect Theory: an Analysis of Decision Under Risk. *Econometrica* 47, 263-291.

Lence, S.H. (1995). The economic value of minimum-variance hedges. *American Journal of Agricultural Economics* 77, 353-364.

Lence, S.H. (1996). Relaxing the assumptions of minimum-variance hedging. *Journal of Agricultural and Resource Economics* 21, 39-55.

Lien, D. (2001). A Note on Loss Aversion and Futures Hedging. *The Journal of Futures Markets* 21, 681-692.

Mattos, F., Garcia, P., and Nelson, C. (2006). Relaxing Standard Hedging Assumptions in the Presence of Downside Risk. *Quarterly Review of Economics and Finance*, forthcoming.

Peck, A.E., and Nahmias, A.M. (1989). Hedging your advice: do portfolio models explain hedging? *Food Research Institute Studies* 21, 193-204.

Prelec, D. (1998). The Probability Weighting Function. *Econometrica* 66, 497-527.

Schoemaker, P.J.H. (1982). The Expected Utility Model: Its Variants, Purposes, Evidence, and Limitations. *Journal of Economic Literature* 20, 529-563.

Tversky, A., and Kahneman, D. (1992). Advances in Prospect Theory: Cumulative Representation of Uncertainty. *Journal of Risk and Uncertainty* 5, 297-323.