Time-Varying Multiproduct Hedge Ratio Estimation in the Soybean Complex:
A Simplified Approach

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Abstract

In developing optimal hedge ratios for the soybean processing margin, many authors have illustrated the importance of considering the interactions between the cash and futures prices for soybeans, soybean oil, and soybean meal. Conditional as well as time-varying hedge ratios have been examined, but in the case of multiproduct time-varying hedge ratios, the difficulty in estimation has been found to often outweigh any improvement in hedging effectiveness. This research examines the hedging effectiveness of the *Risk Metrics* procedure for estimating a time-varying covariance matrix for developing optimal hedge ratios for the soybean processing margin. The *Risk Metrics* method allows for a time-varying covariance matrix while being considerably easier to implement than multivariate GARCH (MGARCH) procedures. The *Risk Metrics* procedure has been advocated for use in developing Value-at-Risk estimates. While it provided considerable out-of-sample improvement in hedging effectiveness relative to a constant correlation MGARCH procedure, the *Risk Metrics* method provided only minimal improvement over a naïve (1-to-1) hedging strategy. However, this research does illustrate the potential for the *Risk Metrics* methodology as a viable alternative to MGARCH procedures in a multiproduct hedging context.

Introduction

Many optimal hedging studies have considered the appropriateness of both conditional (Myers and Thompson) as well as time-varying hedge ratios (Myers; Baillie and Myers; Kroner and Sultan; Park and Switzer; Tong; Bera, Garcia, and Roh). In developing optimal hedging strategies for the soybean processing margin, it is important to consider the interactions between soybean, soybean oil, and soybean meal cash and futures prices (Tzang and Leuthold; Fackler and McNew). In particular, Garcia, Roh, and Leuthold specifically examined the hedging effectiveness of simultaneously estimated, time-varying hedge ratios for the soybean processing margin.

Garcia, Roh, and Leuthold found that a time-varying covariance matrix was statistically appropriate for estimating hedge ratios for the soybean processing margin. Nevertheless, the multiproduct hedge ratios estimated using MGARCH techniques yielded little reduction in unhedged variance relative to a traditional constant variance/covariance estimation procedure. In part, this may be explained by their constant correlation MGARCH specification (Bollerslev) where the variances are allowed to change over time but the correlations between the various cash and futures prices are constant. This restrictive specification was needed in order to produce a positive definite variance/covariance matrix. Given the difficulty of implementing MGARCH estimation procedures as well as their relatively weak performance in a hedging framework, Garcia, Roh, and Leuthold questioned the value of using time-varying hedge ratios in hedging the risks of the soybean processing margin.

Recently, much attention has been given to the *Risk Metrics* methodology for estimating a variance/covariance matrix for a portfolio of assets for use in Value-at-Risk models.[1] Specifically, the *Risk Metrics* method uses an exponentially weighted moving average procedure for estimating variances and covariances for a multi-asset portfolio. This exponentially weighted

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[1] Value-at-Risk estimates determine the downside risk of a portfolio of assets with a given level of confidence and have received wide attention among financial economists and risk management practitioners. The research department of JP Morgan developed and advocated the *Risk Metrics* procedure for creating Value-at-Risk estimates for portfolios of financial assets. Due to the dissemination of the Risk Metrics method through the WWW, this method has become one of the more widely accepted procedures for calculating Value-at-Risk estimates.

moving average is specified such that only one parameter is estimated. Therefore, the exponentially moving average model captures the time-varying nature of multiple asset return variances while being considerably more parsimonious than multivariate GARCH (MGARCH) procedures. The exponentially weighted moving average is particularly desirable since it is difficult, if not sometimes impossible, to achieve well behaving MGARCH estimates when the number of assets in a portfolio is large such as the case with the soybean processing margin. Initial evidence exists as to the appropriateness of the *Risk Metrics* method for estimating time-varying variances and covariances for agricultural prices. Manfredo and Leuthold found that the *Risk Metrics* method provided superior Value-at-Risk estimates compared to those estimated using a constant correlation MGARCH procedure in the context of the cattle feeding margin.

The overall objective of this research is to estimate multiproduct time-varying hedge ratios for the soybean processing margin using the *Risk Metrics* procedure and to compare its hedging effectiveness relative to complex MGARCH procedures as well as a simplistic naïve model. If the *Risk Metrics* procedure produces superior hedging results relative to these alternative strategies, a new case can be made for the use of time-varying optimal hedge ratios in the soybean complex.

The following sections define the soybean processing margin as well as the theoretical hedging model. The data used for calculating the soybean processing margin, estimating the hedge ratios, and for testing hedging effectiveness are also explained. The *Risk Metrics* method for estimating a time-varying covariance matrix is formally presented along with alternative procedures for estimating time-varying hedge ratios. Finally, estimation results as well as insight into the hedging effectiveness of the *Risk Metrics* method relative to alternative models is presented.

Hedging Model for the Soybean Processing Margin

The soybean processing margin is defined as the difference between the prices of soybean meal and soybean oil (outputs) and raw soybeans (input). Soybean processors face considerable variability in this cash margin due to fluctuations in the cash prices of soybeans, soybean oil, and soybean meal. In managing the variability of their profit margin, soybean processors often employ the use of futures contracts to hedge these three cash prices simultaneously.

In developing a hedging framework for the soybean processing margin, a two-period decision framework is established following the procedures of Garcia, Roh, and Leuthold as well as Tzang and Leuthold. The first stage covers the production planning decision (two weeks) up to the point where raw soybeans are purchased in the cash market prior to crushing (t-3 to t-1). Period two (t-1 to t) represents the period of time when the soybeans are crushed/processed into meal and oil. It is assumed that one bushel of soybeans produces 48 pounds of soybean meal, 11 pounds of soybean oil, and approximately 1 pound of waste. Therefore, at the beginning of the production planning period (t-3), futures hedges are placed simultaneously in soybeans (long), soybean meal (short), and soybean meal (short). At t-1, soybeans are purchased in the cash

market, the respective long soybean position is liquidated, and crushing begins. At time period t, both the soybean meal and oil futures positions are liquidated and the soybean meal and oil is sold in the cash market. Taking this planning and production scenario into consideration, the hedged soybean processing margin at time t is:

$$(1) \quad R_t = S_{m,t} + S_{o,t} - S_{b,t-1} + b_{b,t-3}(F_{b,t-1} - F_{b,t-3}) - b_{m,t-3}(F_{m,t} - F_{m,t-3}) - b_{o,t-3}(F_{o,t} - F_{o,t-3}) - c$$

where S_b, S_m, S_o are the cash prices of soybeans, soybean meal, and soybean oil, F_b, F_m, F_o are the corresponding futures prices, b_b, b_m, b_o are the respective hedge ratios at time period t-3, and c is a fixed processing cost.[2] The return in (1) is expressed in per bushel terms by dividing soybean meal cash and futures prices ($/ton) by 2000 and multiplying by 48 pounds as well as dividing soybean oil cash and futures prices ($/cwt) by 100 and multiplying by 11 pounds.

A minimum variance hedging (MVH) framework is used in derivng the optimal hedging ratios for soybeans, soybean meal, and soybean oil. Considering the soybean processing margin as a portfolio of 3 unique cash prices and 3 futures positions, as well as assuming that the respective futures prices are unbiased, the variance of the hedged processing margin in (1) is:[3]

(2)
$$
\begin{aligned}
V(R) =\ & V(S_b) + V(S_m) + V(S_o) + b_b^2 V(F_b) + b_o^2 V(F_o) + b_m^2 V(F_m) - 2\,\mathrm{cov}(S_o, S_b) - 2\,\mathrm{cov}(S_m, S_b) \\
& + 2\,\mathrm{cov}(S_m, S_o) - 2 b_b\,\mathrm{cov}(F_b, S_b) + 2 b_b\,\mathrm{cov}(F_b, S_o) + 2 b_b\,\mathrm{cov}(F_b, S_m) + 2 b_o\,\mathrm{cov}(F_o, S_b) \\
& - 2 b_o\,\mathrm{cov}(F_o, S_o) - 2 b_o\,\mathrm{cov}(F_o, S_m) - 2 b_o b_b\,\mathrm{cov}(F_o, F_b) + 2 b_m\,\mathrm{cov}(F_m, S_b) - 2 b_m\,\mathrm{cov}(F_m, S_o) \\
& - 2 b_m\,\mathrm{cov}(F_m, S_m) - 2 b_m b_b\,\mathrm{cov}(F_m, F_b) + 2 b_m b_o\,\mathrm{cov}(F_m, F_o) \quad .
\end{aligned}
$$

Taking the partial derivative of (2) with respect to b_b, b_m, b_o, setting the derivatives equal to zero, and solving for b_b, b_m, b_o simultaneously using Cramer's Rule gives the respective hedge ratios at time t-3. Therefore, the hedge ratios for each of the three commodities are expressed as a function of the appropriate covariances (see Appendix A). In estimating time-varying hedge ratios, the respective time-varying variance and covariance terms must be estimated simultaneously.

Data

In estimating a conditional time-varying covariance matrix, conditional returns of the respective spot and futures prices must be generated. Given the production planning and subsequent hedging scenario developed, percent price changes (returns) for soybean spot and futures prices are defined as:

$$(3) \quad R_t | y_{t-3} = 100\ln(P_{t-1} / P_{t-3})$$

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[2] For the purposes of this study, c is assumed to be constant.
[3] Time subscripts are removed to conserve space.

while returns for both soybean meal and soybean oil spot and futures prices are defined as:

(4) $R_t|y_{t-3} = 100\ln(P_t/P_{t-3})$

where R_t is the return at time t and y_{t-3} represents the information available at time t-3 (Garcia, Roh, and Leuthold).

Weekly (Wednesday) price data are used in generating both the spot and futures returns defined in (3) and (4). When Wednesday prices are not available, Tuesday or Thursday prices are used. The source for both the spot and futures data is the *Technical Tools* database of cash and futures prices provided by the Office for Futures and Options Research. Soybean (No. 1 yellow), soybean meal (48%), and soybean oil cash prices are for Central Illinois while the corresponding futures contracts are traded on the Chicago Board of Trade. Futures prices are taken from the nearby futures contract with the rollover occurring on the last trading day prior to the contract month. In the cases where futures returns are generated between two prices from different contract months, the price in the expiring month (e.g., P_{t-3}) is replaced with the corresponding price from the nearby contract. The data for both cash and futures prices span the period from January 1985 through December 1999 for a total of 780 observations. The period from January 1985 to the end of December 1996 is used for model estimation and in-sample testing (621 observations) while the data from January 1997 through December 1999 (159 observations) are used for testing the out-of-sample hedging effectiveness of the alternative hedging procedures.

Methods

Time-varying variances and covariances can be estimated in several ways (e.g., MGARCH procedures). For purposes of this study, emphasis is placed on estimating a time-varying covariance matrix for the soybean processing margin using the *Risk Metrics* method. Unlike MGARCH methods, the *Risk Metrics* method relies on only one estimated parameter (δ). The variances and covariances are modeled as an exponentially weighted moving average. The model is likened to a constrained GARCH model where the estimated GARCH parameters are constrained to equal 1 (e.g., IGARCH). Therefore, the *Risk Metrics* specification is defined as:

(5)
$$s_{i,t}^2 = \delta s_{i,t-1}^2 + (1-\delta) R_{i,t-1}^2$$
$$s_{ij,t} = \delta s_{ij,t-1} + (1-\delta) R_{i,t} R_{j,t}$$

where $s_{i,t}^2$ is the conditional variance of the return of price i at time t, and $s_{ij,t}$ is the conditional variance between the return of price i and j where $i \neq j$. The *Risk Metrics* method as presented by the *Risk Metrics* research group JP Morgan is not a simultaneous estimation procedure. Through their research, *Risk Metrics* has advocated the use of a fixed, pre-determined decay parameter δ. They have found δ=.97 to be appropriate for monthly data while δ=.94 for daily data. They have not recommended a fixed decay factor for use with weekly data. However, a simultaneously determined covariance matrix can be and is estimated consistent with (5). This is done by replacing the covariance matrix in the log likelihood function of a general MGARCH

estimation procedure with a covariance matrix as defined in (5). This is easily accomplished using the procedures in the S+GARCH statistical programming package.

With the growing interest in Value-at-Risk estimates, the specification in (5) has become the standard in estimating time-varying variances and covariances for Value-at-Risk models. The claim of *Risk Metrics* is that the procedure has the ability to adequately capture the time-varying nature of asset returns without resorting to complex MGARCH procedures which are often too cumbersome to incorporate for portfolios containing many assets (e.g., the soybean processing margin). In fact, unless a parsimouneous MGARCH specficiation is used, such as the constant correlation MGARCH model (Bollerslev), the potential number of unique parameters to be estimated becomes unmanageable and it becomes difficult, if not impossible, to obtain a positive definite covariance matrix with GARCH parameters that are positive and stable.[4]

To compare the hedging performance of the *Risk Metrics* procedure to that of a more complex MGARCH procedure, a constant correlation MGARCH(1,1) model is specified. The constant correlation MGARCH model assumes that conditional correlations between return series are constant over time and that individual price return variances follow a univariate GARCH(1,1) process (Campbell, Lo, and MacKinlay; Bera, Garcia, and Roh, Garcia, Roh, and Leuthold). Therefore, the constant correlation MGARCH(1,1) model can be shown as:

$$\begin{aligned} \boldsymbol{s}^2_{ii,t} &= c_{ii,t} + \boldsymbol{a}_{ii,t} R^2_{ii,t-1} + \boldsymbol{b}_{ii,t} \boldsymbol{s}^2_{ii,t-1} \\ \boldsymbol{s}_{ij,t} &= \boldsymbol{r}_{ij} \boldsymbol{s}_{ii,t} \boldsymbol{s}_{jj,t} \end{aligned}$$

(6)

where $\boldsymbol{s}^2_{ii,t}$ is the conditional variance of asset i, $\boldsymbol{s}_{ij,t}$ is the conditional covariance between return i and j, $\boldsymbol{s}_{ii,t}$ is the conditional standard deviation of return i, and $\boldsymbol{r}_{ij,t}$ is the constant correlation between asset i and j ($i \neq j$). While the constant correlation assumption is often criticized as being too restrictive, it is a relatively parsimoneous specification relative to other MGARCH procedures and yields a positive definite covariance matrix provided that all GARCH parmeter estimates ($c_{ii}, \boldsymbol{a}_{ii}, and \boldsymbol{b}_{ii}$) are positive, $\boldsymbol{a}_{ii} + \boldsymbol{b}_{ii} < 1$, and the correlations between assets are between –1 and 1. In addition, this procedure has also been used in many empirical applications in the context of multiproduct risk management (Kroner and Sultan; Bera, Garcia, and Roh), and in particular has been used in estimating time-varying optimal hedge ratios for the soybean processing margin (Garcia, Roh, and Leuthold).

Both the *Risk Metrics* and constant correlation MGARCH(1,1) models are estimated using the BHHH algorithm in the S+GARCH statistical package. The covariance matrix estimated in-sample is used to develop the respective simultaneously determined hedge ratios and test their hedging effectiveness. In the out-of-sample period, both the *Risk Metrics* and constant correlation MGARCH models are updated each week using a growing sample size. The predictions of the relevant variances and covariances are used in estimating the the hedge ratios.

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[4] Campbell, Lo, and MacKinlay (pp. 490 - 493) provide an excellent discussion of various MGARCH procedures that help to reduce the number of parameters to be estimated as well as procedures that help to ensure a positive definite covariance matrix including VECH models, the BEKK model, and the constant correlation model.

In addition to the above simultaneously determined procedures, a more simplistic hedge ratio model is also specified. This model is consistent with traditional optimal hedge ratios such that

$$(7) \quad b_{i,t-3} = \frac{\text{cov}(S_i, F_i)}{V(F_i)}$$

where $\text{cov}(S_i, F_i)$ is the covariance between spot and futures returns for price i and $V(F_i)$ is the variance of the futures price for i. The variances and covariances from the *Risk Metrics* estimation procedure are used here. The hedge ratios defined in (7), while not explicitly considering all the interactions between the various cash and futures prices, are still time-varying and provide a simplistic alternative to the simultaneous specification.

However, the true benchmark to all of the above hedge ratio estimates is a naïve specification such that the hedge ratio is always equal to 1 (e.g., 1-to-1 hedging). A hedge ratio of 1 implies that soybean processors take equal but opposite positions in the corresponding futures contracts to that of the appropriate cash position. Hedging effectiveness is defined and measured as the percent reduction in the variance of the hedged margin relative to the unhedged margin such that

$$(8) \quad 1 - \frac{V(hedged)}{V(unhedged)} \quad .$$

Therefore, complex hedge ratios should be considered superior to the naïve hedging model if the percent reduction in variance in (8) is greater than that produced by the naïve model.

Results

Table 1 presents the parameter estimates for the in-sample period for both the *Risk Metrics* estimation procedure as well as the constant correlation MGARCH model. The estimated decay factor (8) for the *Risk Metrics* specification was 0.96544 which is very close to the pre-determined decay factor recommended by *Risk Metrics* for use with monthly data. In order to achieve GARCH parameters consistent with the conditions necessary for positive definiteness, it was necessary to also specify a structure for the means of the various price return series. Examination of the autocorrelation function and the partial autocorrelation function of the returns suggested a MA(2) process for the mean of the returns. This was the case even though the means of the various return series are very close to zero.

In table 2, the in-sample, average hedge ratios for all three commodities are quite similar and very close to 1. However, for the out-of-sample period (table 2), the average hedge ratios produced using the *Risk Metrics* simultaneous procedure for both soybeans and soybean oil were considerably smaller relative to the hedge ratio for soybean meal. The out-of-sample period (January 1997 to December 1999) was a period of declining prices of both cash and futures prices for soybeans, soybean meal, and soybean oil. The periods of September - October of 1997

as well as July – August of 1997 were particularly volatile for soybean meal cash prices. These windows realized large decreases in the price of cash soybean meal. Given that the price returns generated for soybean meal are in essence three-week returns, the resulting returns (negative) were quite large relative to the returns both beans and soybean oil. For this reason, given the time-varying nature of the *Risk Metrics* simultaneous procedure, the hedge ratios responded accordingly. Thus, the magnitude of the hedge ratios for soybeans and soybean oil were smaller relative to meal during these periods which is ultimately reflected in the size of the average hedge ratios in the out-of-sample period.

Table 3 presents the in-sample mean, variance, coefficient of variation, and the percent reduction of the hedged soybean processing margin variance relative to the unhedged variance. Table 4 also shows the same information for the out-of-sample period. In sample, the naïve hedging model provided the largest percent reduction relative to the unhedged position. However, all of the more complex models were not far behind. The percent reduction using the *Risk Metrics* simultaneous model was only slightly less (-.6783 percentage points) than that of the naïve model. Still, the naïve model had the smallest coefficient of variation at .2891.

More interesting is the performance of the alternative hedging procedures during the out-of-sample period. The *Risk Metrics* simultaneous procedure realized a reduction in variance of 45.22% relative to the unhedged margin. Of the four procedures tested, the *Risk Metrics* simultaneous procedure provided the best out-of-sample performance. However, it should be noted that the procedure only provided a reduction in variance of approximately 1.45% over the naïve model. The *Risk Metrics* simultaneous procedure did produce considerable improvement over the constant correlation MGARCH model during the out-of-sample period. The *Risk Metrics* simultaneous method yielded an improvement of approximately 4 percentage points over the MGARCH model. This is an important result given the difficulty in estimating MGARCH models, even restrictive models such as the constant correlation MGARCH model. In fact, well behaved MGARCH parameters that conformed to the conditions that all estimated parameters are positive could only be achieved by also modeling the mean of the various series as a MA(2) process. Determining this specification took considerable analysis and time. Thus, these results suggest that the *Risk Metrics* specification provides a reasonable and manageable specification of a time-varying covariance matrix for estimating multiproduct time-varying optimal hedge ratios.

Summary and Conclusions

This research focused on the estimation and measurement of the hedging effectiveness of simultaneously determined multiproduct time-varying hedge ratios in the soybean complex using the exponentially weighted moving average model of variances and covariances proposed by *Risk Metrics*. The hedging effectiveness of these hedge ratios were compared to a constant correlation MGARCH procedure which has been previously suggested for use in estimating time-varying hedge ratios. The hedging effectiveness of a simple naïve hedging model was also examined for comparative purposes. The hedging effectiveness of these various procedures were examined on both an in-sample as well as an out-of-sample basis.

It was found that the simultaneously determined hedge ratios using the *Risk Metrics* specification for a time-varying covariance matrix provided increased reduction in the variance of the soybean crushing margin relative to a more complex multivariate GARCH estimation procedure as well as a naïve (1-to-1) hedging model. While providing considerable out-of-sample improvement over the MGARCH model, the *Risk Metrics* approach only provided minimal improvement over the naïve model. These results are important in considering the appropriateness and practicality of complex hedging strategies in the soybean complex. While the risk and returns from the use of the *Risk Metrics* dominate the other procedures in a mean variance context, it is uncertain and probably not likely that any hedging improvement attained by using the procedure is economically meaningful. These findings are consistent with those of Collins. In an examination of several multivariate hedging procedures for the soybean complex, Collins found that that there was no statistically significant difference between the out-of-sample hedging effectiveness of a naïve hedging model and the other multivariate hedging procedures (e.g., Fackler and McNew; Tzang and Leuthold).

These conclusions, however, should not discount the potential of the *Risk Metrics* procedure for providing superior hedging performance for other return series and other multiproduct optimal hedge ratio scenarios. This research does illustrate the potential for the *Risk Metrics* methodology as a viable procedure for estimating time-varying hedge ratios, especially relative to complex MGARCH procedures that make time-varying hedge ratio estimation impractical in most situations. Future research should consider the hedging effectiveness of the *Risk Metrics* procedure relative to other, more flexible multivariate GARCH specifications than just the constant correlation MGARCH. Future research should also consider the hedging effectiveness of this procedure in other multiproduct as well as single product hedging situations.

References

Baillie, R.T., and R.J. Myers . "Bivariate GARCH Estimation of Optimal Commodity Futures Hedge." *Journal of Applied Econometrics.* 6(1991):109-124.

Bera, A.K., P. Garcia, P. and J-S. Roh. "Estimation of Time-Varying Hedge Ratios for Corn and Soybeans: BGARCH and Random Coefficient Approaches." *Sankhya: The Indian Journal of Statistics.* Series B, 59(1997):346-368.

Bollerslev, T. "Modeling the Coherence in Short-Run Nominal Exchange Rates: A Multivariate Generalized ARCH Model." *The Review of Economics and Statistics*. 31(1990):498-505.

Campbell, J.Y., A.W. Lo, and A.C. MacKinlay. *The Econometrics of Financial Markets*. New Jersey: Princeton University Press, 1997.

Collins, R.A. "The Risk Management Effectiveness of Multivariate Hedging Models in the U.S. Soy Complex." *Journal of Futures Markets.* 20(2000):189-204.

Fackler, P.L. and K.P. McNew. "Multiproduct Hedging: Theory, Estimation, and an Application." *Review of Agricultural Economics.* 15(1993):521-535.

Garcia, P., J-S Roh, and R.M. Leuthold. "Simultaneously Determined, Time-Varying Hedge Ratios in the Soybean Complex." *Applied Economics*. 27(1995):1127-1134.

Kroner, K.F. and J. Sultan. "Time-Varying Distributions and Dynamic Hedging with Foreign Currency Futures." *Journal of Financial and Quantitative Analysis.* 28(1993):535-551.

Manfredo, M.R., and R.M. Leuthold. "Market Risk Measurement and the Cattle Feeding Margin: An Application of Value-at-Risk," Office For Futures and Options Research Working Paper # 99-04, University of Illinois at Urbana-Champaign, 1999.

Myers, R.J. "Estimating Time-Varying Optimal Hedge Ratios on Futures Markets." *Journal of Futures Markets*. 11(1991):39-53.

Myers, R.J. and S.R. Thompson. "Generalized Optimal Hedge Ratio Estimation." *American Journal of Agricultural Economics*. 71(1989):858-67.

Park, T.H., and L.N. Switzer. "Bivariate GARCH Estimation of the Optimal Hedge Ratios for Stock Index Futures: A Note." *Journal of Futures Markets*, 15(1995):61-67.

Risk Metrics Group (1996-2000): *Risk Metrics Technical Documentation*, http://www.riskmetrics.com/

Tong, W.H.S. "An Examination of Dynamic Hedging." *Journal of International Money and Finance.* 15(1996):19-35.

Tzang, D. and R.M. Leuthold. "Hedge Ratios under Inherent Risk Reduction in a Commodity Complex." *The Journal of Futures Markets.* 10(1990):497-504.

Table 1. In-Sample (1985 – 1996) Estimates for Both Constant Correlation MGARCH(1,1) and *Risk Metrics*.

MGARCH

Parameters	Soybeans Cash	Soybean Oil Cash	Soybean Meal Cash	Soybeans Futures	Soybean Oil Futures	Soybean Meal Futures
μ_i	0.305	-0.098	0.547	0.095	-0.312	0.233
	$(0.160)^1$	(0.364)	(0.321)	(0.163)	(0.347)	(0.280)
ma(1)	0.853	0.873	0.916	0.849	0.859	0.876
	(0.030)	(0.020)	(0.020)	(0.032)	(0.020)	(0.022)
ma(2)	0.080	0.778	0.797	0.096	0.747	0.753
	(0.028)	(0.018)	(0.019)	(0.030)	(0.018)	(0.019)
c_i	0.370	1.787	0.0405	0.467	0.798	0.335
	(0.075)	(0.788)	(0.150)	(0.114)	(0.211)	(0.102)
$\alpha_{i,1}$	0.197	0.031	0.086	0.173	0.026	0.072
	(0.021)	(0.012)	(0.017)	(0.021)	(0.007)	(0.014)
$\beta_{i,1}$	0.767	0.804	0.873	0.765	0.896	0.885
	(0.021)	(0.079)	(0.025)	(0.029)	(0.023)	(0.021)
ρ_{Sb}	1.00					
	(-)					
ρ_{So}	0.130	1.00				
	(0.042)	(-)				
ρ_{Sm}	0.094	0.292	1.00			
	(0.044)	(0.037)	(-)			
ρ_{Fb}	0.944	0.122	0.092	1.00		
	(0.004)	(0.041)	(0.043)	(-)		
ρ_{Fo}	0.143	0.958	0.323	0.134	1.00	
	(0.042)	(0.003)	(0.037)	(0.042)	(-)	
ρ_{Fm}	0.121	0.332	0.902	0.128	0.364	1.00
	(0.043)	(0.035)	(0.007)	(0.041)	(0.035)	(-)
Risk Metrics[2]						
(1-λ)	0.03456					
	(0.0009)					
λ	0.96544					

[1]Standard errors in parenthesis

[2]The S+GARCH package estimates a version of *Risk Metrics* method where $s_{i,t}^2 = (1-\lambda)s_{i,t-1}^2 + \lambda R_{i,t-1}^2$ which is effectively no different than that specified in (5).

Table 2. In-Sample and Out-of-Sample Average Hedge Ratios for Soybeans, Soybean Meal, and Soybean Oil.

| Model | In-Sample (1985 - 1996) | | | Out of Sample (1997 - 1999) | | |
	Soybeans	Soybean Meal	Soybean Oil	Soybeans	Soybean Meal	Soybean Oil
Naïve	1	1	1	1	1	1
Risk Metrics (simultaneous)	0.9669	1.0465	0.9333	0.6347	0.9742	0.6625
Risk Metrics (cov/var)	0.9849	1.0353	0.9983	0.9847	1.0669	0.9629
Constant Correlation MGARCH	0.9660	1.0036	0.9741	1.0416	1.0869	0.9594

Table 3. In-Sample Hedging Effectiveness of the Soybean Processing Margin Using Alternative Hedge Ratio Estimation Procedures (1985 - 1996).

Model	Mean	Variance	Coefficient of Variation	Percent Reduction	Difference in Hedging Effectiveness from Naïve
Unhedged	0.7817	0.0981	0.4008	----	
Naïve	0.7727	0.0499	0.2891	49.1613	----
Risk Metrics (simultaneous)	0.7692	0.0506	0.2923	48.4830	-0.6783
Risk Metrics (cov/var)	0.7711	0.0502	0.2906	48.8393	-0.3221
Constant Correlation MGARCH	0.7741	0.0508	0.2912	48.2182	-0.9431

Table 4. Out-of-Sample Hedging Effectiveness of the Soybean Processing Margin Using Alternative Hedge Ratio Estimation Procedures (1997 - 1999).

Model	Mean	Variance	Coefficient of Variation	Percent Reduction	Difference in Hedging Effectiveness from Naïve
Unhedged	0.6701	0.1430	0.5644	----	
Naïve	0.6775	0.0804	0.4186	43.7653	----
Risk Metrics (simultaneous)	0.6801	0.0784	0.4116	45.2161	1.4508
Risk Metrics (cov/var)	0.6792	0.0795	0.4152	44.3922	0.6269
Constant Correlation MGARCH	0.6836	0.0841	0.4241	41.2162	-2.5491

Appendix A. Simultaneoulsy Determined Multiproduct Optimal Hedge Ratios for the Soybean Processing Margin.

$b_{b,t-3}^{*} =$

```
    -cFbSm*cFmFo*cFmFo  + cFbSb*cFmFo*cFmFo  - cFbSo*cFmFo*cFmFo  - cFbSb*vFo*vFm +
    cFbSo*vFo*vFm + cFbSm*vFo*vFm - cFoFb*cFoSo*vFm - cFoFb*cFoSm*vFm -
    cFoFb*cFmFo*cFmSb + cFoFb*cFmFo*cFmSo + cFoFb*cFmFo*cFmSm + cFmFb*cFmFo*cFoSo -
    cFmFb*cFmFo*cFoSb - cFmFb*cFmSm*vFo - cFmFb*cFmSo*vFo + cFmFb*cFmSb*vFo +
    cFmFb*cFmFo*cFoSm + cFoFb*cFoSb*vFm
```
_____
```
    (vFo*cFmFb*cFmFb - vFo*vFb*vFm + cFoFb*cFoFb*vFm + cFmFo*cFmFo*vFb -
    cFmFb*cFoFb*cFmFo - cFmFb*cFoFb*cFmFo)
```

$b_{m,t-3}^{*} =$

```
    (-cFmFb*cFoFb*cFoSo + cFmFb*cFoFb*cFoSb - cFmFb*cFbSb*vFo + cFmFb*cFbSo*vFo +
    cFmFb*cFbSm*vFo - cFmFb*cFoFb*cFoSm + cFmFo*vFb*cFoSo - cFmFo*vFb*cFoSb -
    cFmSm*vFb*vFo - cFmSo*vFb*vFo + cFbSb*cFoFb*cFmFo + cFmSb*vFb*vFo -
    cFbSo*cFoFb*cFmFo - cFbSm*cFoFb*cFmFo + cFmFo*vFb*cFoSm - cFoFb*cFoFb*cFmSb +
    cFoFb*cFoFb*cFmSo + cFoFb*cFoFb*cFmSm)
```
_____
```
    (vFo*cFmFb*cFmFb - vFo*vFb*vFm + cFoFb*cFoFb*vFm + cFmFo*cFmFo*vFb -
    cFmFb*cFoFb*cFmFo - cFmFb*cFoFb*cFmFo)
```

$b_{o,t-3}^{*} =$

```
    -(-cFoSb*vFb*vFm - cFmFb*cFoFb*cFmSb + cFoSo*vFb*vFm + cFoSb*cFmFb*cFmFb  +
    cFmFb*cFoFb*cFmSo - cFmFo*cFmFb*cFbSb + cFmFb*cFoFb*cFmSm - cFoFb*cFbSm*vFm -
    cFoFb*cFbSo*vFm + cFoFb*cFbSb*vFm + cFmFo*cFmFb*cFbSm + cFmFo*cFmFb*cFbSo -
    cFoSo*cFmFb*cFmFb  + cFmFo*vFb*cFmSb - cFmFo*vFb*cFmSo - cFmFo*vFb*cFmSm +
    cFoSm*vFb*vFm - cFoSm*cFmFb*cFmFb)
```
_____
```
    (vFo*cFmFb*cFmFb - vFo*vFb*vFm + cFoFb*cFoFb*vFm + cFmFo*cFmFo*vFb -
    cFmFb*cFoFb*cFmFo - cFmFb*cFoFb*cFmFo)
```

v = variance operator
c = covariance operator
Sb = soybean cash price
So = soybean oil price
Sm = soybean meal price
Fb = soybean price
Fo = soybean oil price
Fm = soybean meal price