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Welfare Decomposition in Computable General Equilibrium Models: A Path Independent Decomposition of the Equivalent Variation

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1. Introduction

Computing the welfare effects of policy reforms is one of the main tasks for applied economists. In this respect, Computable General Equilibrium (CGE) models are often employed since one of their attractive features is that they capture all distortions in an economy (e.g., Harberger, 1964). In addition to computing these welfare effects, one critical challenge for the applied economist is to explain them. To date, several studies have proposed ways of attributing changes in welfare to sources corresponding to the alleviation, or exacerbation, of existing market imperfections and distortions. In general, they extend the pioneer decomposition proposed by Harberger (1971).

All these decompositions are based on first order approximations of underlying CGE specifications. Without doubt, these decompositions are extremely useful in order to understand some of the economic mechanisms at work. However, these locally based decompositions may have a rather poor explanatory power from a empirical standpoint in the case of "multiple and large shock" experiments. In other words, linearized representation of multiple non linear relations may be prone to erroneous conclusions (Hertel *et al.,* 1992). In order to cope with this potential empirical issue, current practice is to split the original experiment into many smaller ones, so as to update welfare elasticities in the decomposition equation. While this seems at first to be reasonable, this solution leads to another problem: the welfare decomposition becomes path dependent (Fane and Ahammad, 2003). The consequence is that the decomposition is not unique and thus may be of little value in explaining the welfare impacts of any given policy experiment.

In order to illustrate the point, let us consider one small country which applies a tariff and a quota on imports of a given sector. We assume that initially the quota is binding while the tariff is not. Let us then contemplate a radical policy reform with the removal of both instruments. In order to explain the resulting welfare effects, if this reform is split by first considering the removal of the tariff, then no allocation effects will result from this first step. Consequently the welfare effects of the complete scenario will be attributed to the quota removal implemented in the second step. On the other hand, if the reform is split by first considering the removal of quotas, then market allocations will change and the tariff will become binding at the end of this first step. In the second step of the implementation of the reform (i.e., the removal of the tariffs), it will appear that tariff removal has some welfare effects. This example illustrates that stepwise decomposition of welfare effects is path dependent.

The principle objective in this paper is to propose a new approach for decomposing welfare effects of CGE models. The main idea is to develop Taylor series approximations to CGE specifications rather than relying on first order marginal conditions only. This idea has already been presented in Harberger (1971) but, to our knowledge, has never been exploited in empirical analysis. In this way, we want to avoid the arbitrary sharing out of policy experiments. In other words, the path independency results from the absence of the definition of paths.

The paper is organized as follows. Section 2 introduces the notations of our CGE model that is maintained in a form simple enough to enable the focus to be on the properties of the approach. Section 3 presents the usual decomposition (hereafter labeled first order decomposition) of the Equivalent Variation (EV) measure in this framework. Section 4 offers the derivation of our alternative decomposition (hereafter labeled Taylor based decomposition) and compares it with the former. Finally, concluding comments are made.

2. Notation

Similarly to Fane and Ahammad (2003), we consider a static open economy model with only one representative consumer, I goods (indexed by i) and mono product activities and F primary factors of production (indexed by f). Producers are assumed to maximize their profit subject to constant returns to scale production technologies. Likewise, the representative consumer maximizes utility subject to budget constraints. His income is given by the primary factor returns and the net product of specific taxes/subsidies on production, primary factor use, consumption and trade. Primary factors of production are perfectly mobile between activities and are in fixed supply. Perfect competition prevails in all markets. Finally, we assume an uncompensated setting where transfers from/to abroad are fixed, that domestic and foreign goods are perfect substitutes, and that our economy is potentially large in world markets. Mathematically, such a model is represented by the following eleven equations:

$$X_{i,f} = X_{f,i}\left(Y_i, W + tf\right) \tag{1}$$

$$P_i.Y_i = \sum_{f=1}^{F}\left(W_f + tf_{i,f}\right).X_{i,f} \tag{2}$$

$$C_i = C_i\left(Q, R\right) \tag{3}$$

$$R = E\left(Q, U\right) \tag{4}$$

$$PW_i = PW_i\left(M, z\right) \tag{5}$$

$$P_i = PW_i + tm_i - ty_i \tag{6}$$

$$Q_i = PW_i + tm_i + tc_i \tag{7}$$

$$\sum_i X_{i,f} = x_f \tag{8}$$

$$C_i = Y_i + M_i \tag{9}$$

$$R = \sum_{f=1}^{F} W_f.x_f + \sum_{f=1}^{F}\sum_{i=1}^{I} tf_{i,f}.X_{i,f} + \sum_{i=1}^{I} ty_i.Y_i + tc_i.C_i + tm_i.M_i + b \tag{10}$$

$$b = \sum_{i=1}^{I} PW_i.M_i \tag{11}$$

with the following notations for the endogenous variables (always written with upper case letters):
$X_{i,f}$ the use of primary factor f by activity i, W_f the market price of primary factor f, P_i the producer price of good i, Y_i the domestic production of good i, C_i the domestic consumption of good i, Q_i the consumer price of good i, R the total income or expenditure, U the consumer utility, PW_i the world price of good i, M_i the trade volume of good i, and the following notations for the exogenous variables (always written with lower case letters):
$tf_{i,f}$ the specific tax on primary factor f used by activity i, ty_i the specific tax on production i, tc_i the specific tax on consumption i, tm_i the specific tax on trade i, b the balance of trade deficit and x_f the fixed endowment of factor.

Equations (1) and (2) form the production block and include the primary factor derived demands and the zero profit conditions. Equations (3) and (4) form the consumption block and include the final demands as well as the expenditure function, which implicitly defines utility. Equation (5) summarizes the behavior of foreign agents. Equations (6) and (7) form the price block where all taxes on good are introduced. Equations (8) and (9) are the market equilibrium conditions. Finally, equations (10) and (11) are the macro-economic conditions (the budget constraint and the balance of payments).

3. "First order" decomposition of welfare

The first order decomposition of EV usually starts from the total differentiation of the income equation (10) around the initial point, then subtracts $\sum_{i=1}^{I} C_i.dQ_i$ from both sides. By arranging terms, we get:

$$\begin{aligned} dR - \sum_{i=1}^{I} C_i.dQ_i = db + \sum_{f=1}^{F} W_f.dx_f + \sum_{f=1}^{F}\sum_{i=1}^{I} tf_{i,f}.dX_{i,f} + \sum_{i=1}^{I} ty_i.dY_i + tc_i.dC_i + tm_i.dM_i \\ + \sum_{f=1}^{F} dW_f.x_f + \sum_{f=1}^{F}\sum_{i=1}^{I} dtf_{i,f}.X_{i,f} + \sum_{i=1}^{I} dty_i.Y_i + (dtc_i - dQ_i).C_i + dtm_i.M_i \end{aligned} \tag{12}$$

The left hand side of this equation simply gives the change of real income (measured with initial consumption quantities). The first two elements of the right hand side represent exogenous flows of income to the economy (from the rest of the world or from an increase of primary factor endowment). The four following terms measure the alleviation, or exacerbation, of existing distortions. Other terms are much more difficult to interpret but can be greatly simplified by totally differentiating the zero profit conditions (equation 2) and summing over all production sectors to get:

$$\sum_{i=1}^{I} dP_i.Y_i = \sum_{f=1}^{F} dW_f.x_f + \sum_{i=1}^{I}\sum_{f=1}^{F} dtf_{i,f}.X_{i,f} \tag{13}$$

We are now in a good position to simplify the terms on the right hand side of equation (12). If we make use of equation (13), then use the first order differentiation of price equations (6) and (7), and finally make use of the product market equilibrium equation (9), equation (12) becomes:

$$\begin{aligned} dR - \sum_{i=1}^{I} C_i.dQ_i = db + \sum_{f=1}^{F} W_f.dx_f + \sum_{f=1}^{F}\sum_{i=1}^{I} tf_{i,f}.dX_{i,f} + \sum_{i=1}^{I} ty_i.dY_i + tc_i.dC_i + tm_i.dM_i \\ - \sum_{i=1}^{I} dPW_i.M_i \end{aligned} \tag{14}$$

The last new term reduces to classical terms of trade effects. Finally, EV is related to the real income just decomposed in the following manner:

$$EV = E(P^0, U^1) - E(P^0, U^0) \tag{15}$$

where superscript 0 refers to the initial situation and superscript 1 to the final situation. At this stage, let us recall that the current practice is to split the whole experiment into smaller ones. Let us assume that there are K sub experiments (indexed by k). For each sub-experiment, we have:

$$dEV^k = dE\left(P^0, U^k\right) = \frac{\partial E\left(P^0, U^k\right)}{\partial U} dU^k \tag{16}$$

Let us now totally differentiate the equation (4) for the same sub experiment:

$$dR^k = \frac{\partial E\left(P^k, U^k\right)}{\partial P} dP^k + \frac{\partial E\left(P^k, U^k\right)}{\partial U} dU^k = C^k.dP^k + \frac{\partial E\left(P^k, U^k\right)}{\partial U} dU^k \tag{17}$$

The second equality is satisfied only locally because the derivative of the expenditure function with respect to price gives the Hicksian demand function and not the Marshallian demand function. Combining equations (16) and (17) and summing over all sub experiments, we finally obtain:

$$\begin{aligned} EV &= \sum_{k=1}^{K} dEV^k = \sum_{k=1}^{K} \frac{\dfrac{\partial E\left(P^0, U^k\right)}{\partial U}}{\dfrac{\partial E\left(P^k, U^k\right)}{\partial U}} . \left(dR^k - C^k.dP^k\right) \\ &= \sum_{k=1}^{K} \frac{\dfrac{\partial E\left(P^0, U^k\right)}{\partial U}}{\dfrac{\partial E\left(P^k, U^k\right)}{\partial U}} . \left(\begin{aligned} & db^k + \sum_{f=1}^{F} W_f^k.dx_f^k + \sum_{f=1}^{F}\sum_{i=1}^{I} tf_{i,f}^k.dX_{i,f}^k \\ & + \sum_{i=1}^{I} ty_i^k.dY_i^k + tc_i^k.dC_i^k + tm_i^k.dM_i^k - dPW_i^k.M_i^k \end{aligned} \right) \end{aligned} \tag{18}$$

This expression is very similar to the one obtained by Fane and Ahammad (2003; equation 29). Our main issue here is that this decomposition is based on local approximations and thus may be empirically poor for experiments including large changes, unless many sub experiments are contemplated. In addition, the decomposition may not be unique, and thus may not really facilitate the interpretation of welfare effects.

4. Taylor-based decomposition of welfare

Like Harberger (1971) and Weitzman (1988), we start our procedure by developing Taylor series approximations to the EV and to the direct utility function:

$$EV = E\left(P^0, U^1\right) - E\left(P^0, U^0\right) = \frac{\partial E\left(P^0, U^0\right)}{\partial U}.\Delta U + 0.5.\frac{\partial^2 E\left(P^0, U^0\right)}{\partial U^2}.(\Delta U)^2 + O(\Delta U) \tag{19}$$

$$\begin{aligned} \Delta U &= \sum_{i=1}^{I} \frac{\partial U}{\partial C_i}.\Delta C_i + 0.5.\sum_{i=1}^{I}\sum_{j=1}^{I} \frac{\partial^2 U}{\partial C_i \partial C_j}.\Delta C_i.\Delta C_j + O(\Delta C) \\ &= \sum_{i=1}^{I} \frac{\partial U}{\partial C_i}.\Delta C_i + 0.5.\sum_{i=1}^{I} \Delta \frac{\partial U}{\partial C_i}.\Delta C_i + O(\Delta C) \end{aligned} \tag{20}$$

where $O(\Delta U)$ and $O(\Delta C)$ represent all polynomial terms of the third order or higher. Again following Harberger (1971), the total differentiation of the first order conditions of the utility maximization program gives:

$$\Delta \frac{\partial U}{\partial C_i} = \Delta\lambda.Q_i + \lambda.\Delta Q_i + \Delta\lambda.\Delta Q_i \tag{21}$$

with λ the marginal utility of income. Substituting (21) and (20) into (19), we get:

$$\begin{aligned} EV &= \sum_{i=1}^{I} Q_i.\Delta C_i + 0.5.\sum_{i=1}^{I} \Delta Q_i.\Delta C_i \\ &\quad + 0.5.\Delta\lambda.\left(\sum_{i=1}^{I} Q_i.\Delta C_i.+\sum_{i=1}^{I} \Delta Q_i.\Delta C_i\right) \\ &\quad + 0.5.\frac{\partial^2 E\left(P^0,U^0\right)}{\partial U^2}.(\Delta U)^2 + O(\Delta U,\Delta C) \end{aligned} \tag{22}$$

Harberger (1971) suggests using the first two terms only. But equation (22) makes clear that even with homothetic preferences (so that the last two terms of the last line disappear), this does not strictly correspond to EV. Terms remain that involve changes in the marginal utility of income. These terms generalize to N commodities the difference between EV and the consumer surplus derived by Boadway and Bruce (1984, p. 218) in a single commodity context. We now concentrate on the first two terms. Using the total differentiation of equations (6), (7) and (9), we can express them as follows:

$$\begin{aligned} \sum_{i=1}^{I} Q_i.\Delta C_i + 0.5.\sum_{i=1}^{I} \Delta Q_i.\Delta C_i &= \sum_{i=1}^{I} tc_i.\Delta C_i + 0.5.\sum_{i=1}^{I} \Delta tc_i.\Delta C_i \\ &\quad + \sum_{i=1}^{I} ty_i.\Delta Y_i + 0.5.\sum_{i=1}^{I} \Delta ty_i.\Delta Y_i \\ &\quad + \sum_{i=1}^{I} tm_i.\Delta M_i + 0.5.\sum_{i=1}^{I} \Delta tm_i.\Delta M_i \\ &\quad + \sum_{i=1}^{I} P_i.\Delta Y_i + 0.5.\sum_{i=1}^{I} \Delta P_i.\Delta Y_i \\ &\quad + \sum_{i=1}^{I} PW_i.\Delta M_i + 0.5.\sum_{i=1}^{I} \Delta PW_i.\Delta M_i \end{aligned} \tag{23}$$

It is already possible to see that the first three lines of the right hand side represent changes in one flow multiplied by the corresponding average tax. The purpose of subsequent derivations is to show that the terms of the last two lines can also be expressed in this form. In this respect, we first totally differentiate equation (11), and rearrange terms to get:

$$\sum_{i=1}^{I} PW_i.\Delta M_i + 0.5.\sum_{i=1}^{I} \Delta PW_i.\Delta M_i = \Delta b - \sum_{i=1}^{I} \Delta PW_i.M_i - 0.5.\sum_{i=1}^{I} \Delta PW_i.\Delta M_i \tag{24}$$

Finally, in order to simplify the terms involving changes in producer prices and domestic productions (penultimate line of equation 23), we again use a Taylor series approximation to the primal production technologies, in the same manner as we did for preferences:

$$
\begin{aligned}
\Delta Y_i &= \sum_{f=1}^{F} \frac{\partial Y_i}{\partial X_{i,f}}.\Delta X_{i,f} + 0.5.\sum_{f=1}^{F}\sum_{g=1}^{F} \frac{\partial^2 Y_i}{\partial X_{i,f}\partial X_{i,g}}.\Delta X_{i,f}.\Delta X_{i,g} + O(\Delta X) \\
&= \sum_{f=1}^{F} \frac{\partial Y_i}{\partial X_{i,f}}.\Delta X_{i,f} + 0.5.\sum_{f=1}^{F} \Delta \frac{\partial Y_i}{\partial X_{i,f}}.\Delta X_{i,f} + O(\Delta X)
\end{aligned}
\tag{25}
$$

We then totally differentiate the first order conditions of the profit maximization program which allow us to simplify (25) as follows:

$$
P_i.\Delta Y_i.\frac{1}{1-0.5.\dfrac{\Delta P_i}{P_i}} = \sum_{f=1}^{F}\left(W_f + tf_{i,f}\right).\Delta X_{i,f} + 0.5.\left(\Delta W_f + \Delta tf_{i,f}\right).\Delta X_{i,f} + 0(\Delta X) \tag{26}
$$

The left hand side of this last equation can also be expressed using another Taylor series approximation, such that we obtain:

$$
\begin{aligned}
\sum_{i=1}^{I} P_i.\Delta Y_i. + 0.5.\Delta P_i.\Delta Y_i &= \sum_{f=1}^{F}\left(W_f + tf_{i,f}\right).\Delta X_{i,f} + 0.5.\left(\Delta W_f + \Delta tf_{i,f}\right).\Delta X_{i,f} \\
&\quad - 0.25.\sum_{i=1}^{I} \frac{\Delta P_i.\Delta P_i.\Delta Y_i}{.P_i} + 0(\Delta P, \Delta X)
\end{aligned}
\tag{27}
$$

Finally we get our EV decomposition by substituting equations (23), (24) and (27) into equation (22):

$$
\begin{aligned}
EV = {} & \Delta b \\
& + \sum_{f=1}^{F} W_f.\Delta x_f + 0.5.\sum_{f=1}^{F} \Delta W_f.\Delta x_f \\
& + \sum_{f=1}^{F}\sum_{i=1}^{I} tf_{i,f}.\Delta X_{i,f} + 0.5.\sum_{f=1}^{F}\sum_{i=1}^{I} \Delta tf_{i,f}.\Delta X_{i,f} \\
& + \sum_{i=1}^{I} ty_i.\Delta Y_i + 0.5.\sum_{i=1}^{I} \Delta ty_i.\Delta Y_i \\
& + \sum_{i=1}^{I} tc_i.\Delta C_i + 0.5.\sum_{i=1}^{I} \Delta tc_i.\Delta C_i \\
& + \sum_{i=1}^{I} tm_i.\Delta M_i + 0.5.\sum_{i=1}^{I} \Delta tm_i.\Delta M_i \\
& - \sum_{i=1}^{I} M_i.\Delta PW_i - 0.5.\sum_{i=1}^{I} \Delta M_i.\Delta PW_i \\
& + 0.5.\Delta\lambda.\left(\sum_{i=1}^{I} Q_i.\Delta C_i. + \sum_{i=1}^{I} \Delta Q_i.\Delta C_i\right) \\
& + 0.5.\frac{\partial^2 E\left(P^0, U^0\right)}{\partial U^2}.\left(\Delta U\right)^2 - 0.25.\sum_{i=1}^{I} \frac{\Delta P_i.\Delta P_i.\Delta Y_i}{P_i} \\
& + O\left(\Delta U, \Delta C, \Delta P, \Delta X\right)
\end{aligned}
\tag{28}
$$

This expression must be compared to the expression (18) which gives the EV decomposition in the usual first order approach.[1]

Concluding comments

This paper proposes a new decomposition of welfare effects which is path independent, so that the explanation of welfare impacts of a given policy reform package is unique. In this new decomposition, the evolution of the marginal utility of income enters as an additional term rather than a multiplicative term. This is now more easily understood: we can divide the decomposition of EV between a Marshallian part (corresponding to the consumer surplus and observable) and a Hicksian one (not directly observable). The Marshallian parts are indeed very similar across the two decompositions, with all distortions evaluated at the average (Taylor based) rather than the initial (first order) level. Graphically, this can be seen as computing triangles rather than rectangles. Thus our decomposition is a generalization of the current one and still allows for the explanation of welfare effects by the alleviation, or exacerbation, of existing (and eventually new) distortions.

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[1] One may remark that the expression (28) was obtained using all CGE equations with the exception of the income equation (equation 10) while the expression (18) makes use of all CGE equations, with the exception of the balance of payments condition (equation 11). The omission of one CGE equation has absolutely no incidence on the solution of a CGE model (due to the Walras' law) and hence on the computation of welfare.

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