AGRICULTURAL AND RESOURCE ECONOMICS
AGRAR- UND RESSOURCENÖKONOMIK

Discussion Paper
Diskussionspapier 97-01

Bayesian Analysis of a Japanese Meat Demand System: A Robust Likelihood Approach

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Abstract

This paper presents an application of Bayesian analysis to an AIDS model of Japanese meat demand extending previous approaches in three ways: (1) The methodology employed is robust with respect to the likelihood function but retains the generic, easily programmable character of algorithms offered by Monte Carlo Integration approaches based on the normal likelihood function. (2) In addition to inequality constraints, linear exact restrictions and stochastic prior information are subjected to a Bayesian posterior analysis of validity and incorporated into Bayesian point estimates of model parameters and elasticities. (3) In order to assess the influence of the prior density on posterior distributions of model parameters relative to the likelihood, a measure quantifying the "degree of prior influence" on the posterior is defined.

Zusammenfassung

Das Diskussionspapier stellt eine Bayes'sche Analyse eines AIDS Modells japanischer Fleischnachfrage vor, die eine Erweiterung früherer Ansätze in den folgenden drei Punkten darstellt:(1) Die verwendete Methode ist robust bezüglich der Likelihood Funktion, erhält dabei aber die Flexibilität und einfache Umsetzung von Algorithmen basierend auf Monte Carlo Integration und der Annahme der Normalverteilung. (2) Zusätzlich zu Ungleichheitsbedingungen werden exakte Restriktionen und stochastische a-priori Information einer Bayes'schen a-posteriori Analyse unterzogen und in die Bayes'sche Punktschätzung von Parametern und Elastizitäten einbezogen. (3) Ein Index zur Messung des Einflusses der a-priori Information auf die a-posteriori Verteilung der Modellparameter wird vorgestellt.

1 Introduction

In recent years, empirical economists have shown increasing interest in Bayesian methodology either to enforce and evaluate "objective" prior restrictions derived from economic theory that are difficult to implement using classical statistical techniques, or to formally incorporate "subjective" prior beliefs about model parameters in order to obtain defensible results in policy modeling work (Chalfant and White; Chalfant, Gray, and White; Hayes, Wahl, and Williams). Applications have been fostered by the development of generic, algorithmic approaches to Bayesian analysis based largely on normal likelihood functions and Monte Carlo integration of posterior distributions. Such approaches allow a flexible formulation of prior information, especially with regard to the use of inequality constraints, and also facilitate substantially the analysis of posterior distributions of the model parameters (Kloek and van Dijk; van Dijk and Kloek; Geweke 1986, 1989, 1991). Along the same lines, Heckelei, and Heckelei and Mittelhammer (1996a,b) have relaxed the normality assumption to allow Bayesian analysis of econometric models based on bootstrapped Regression Structure Likelihoods that are robust with respect to the underlying probability model.

This paper presents a Bayesian analysis of an Almost Ideal Demand System (AIDS) model of Japanese meat demand (originally analyzed by Wahl and Hayes) that in addition to substantive empirical results, extends previous approaches in three ways. First, in addition to inequality constraints, linear exact restrictions and stochastic prior information are subjected to a Bayesian posterior analysis of validity and incorporated into Bayesian point estimates of model parameters and elasticities. Second, the methodology used is robust with respect to the likelihood function but retains the generic, easily programmable character of Monte Carlo integration approaches usually based on the normal likelihood function. Finally, in order to assess the influence of the prior density on posterior distributions of model parameters relative to the likelihood, a measure is defined quantifying the "degree of prior influence" on the posterior.

The remainder of the paper is organized as follows. We first introduce the AIDS model of Japanese meat demand and present various types of prior information on the model parameters. Next, a description of the Bayesian bootstrap inferential methodology is given. The method is then applied to the meat demand model, and the different types of prior information are evaluated a posteriori. Finally, the economic results are interpreted and conclusions are drawn regarding the usefulness and limitations of the methodology.

2 The Japanese Meat Demand Model

The AIDS model developed by Deaton and Muellbauer (1980a,b), is now a widely used systems approach for modeling consumption behavior. It is consistent with the axioms of choice, allows perfect aggregation over consumers, and is capable of providing first-order approximations to any demand system. Moreover, properties of demand systems deduced from consumer choice theory - adding up, homogeneity, and symmetry conditions - can be straightforwardly imposed through linear restrictions on the parameters of the model. Consequently, AIDS has been used extensively to test hypotheses relating to the economic theory of the consumer.

Our Bayesian analysis utilizes the same linearized AIDS expenditure share specification of Japanese meat demand and data as was used by Wahl and Hayes. The share equations are

$$(1) \qquad w_i = \alpha_i + \sum_j g_{ij} \log p_j + \beta_i \log(E / P), \; i, j = 1, \ldots, m,$$

where w_i is the share of meat group expenditure allocated to meat product i, p_j is the price of meat product j, E is total expenditure on the meat group and $P = \exp(\sum_j w_j \log p_j)$ denotes Stone´s price index. Wahl and Hayes estimated this model using Japanese meat expenditure and price data on five different meat products: Wagyu beef, import quality (IQ) beef, pork, chicken, and fish. Their analysis focused on the question of whether prices are exogenous or endogenous, i.e., whether supply curves are perfectly elastic or upward sloping. They rejected the exogeneity hypothesis for all but one meat (chicken) and showed that ignoring simultaneous equation bias has a significant impact on parameter and elasticity estimates.

Following the findings of Wahl and Hayes, we perform a Bayesian analysis of the Japanese linearized AIDS system that takes the endogeneity of prices into account. Instruments used in the estimation procedure are in the form of ten principal components created from variables which are considered to be predetermined with respect to the supply of each of the five meats. The price and expenditure data, as well as observations on instruments, are for the period 1965-86 and are given in tables A1 and A2 of the appendix. For a detailed description of the data and instruments, see Wahl and Hayes.

3 Prior Information on Model Parameters

The prior information utilized in this study is a collection of theoretical restrictions, expert opinion, and empirical observations that place exact, inequality, and probabilistic restrictions on the admissible values of demand model parameters. Prior information used in Bayesian analyses is often differentiated on the basis of whether the information is "objective" or "subjective". In the context of this study, prior information considered to be "objective" includes exact and inequality restrictions on the parameters of the demand model derived from the neoclassical theory of the consumer. The purely "subjective" information consists of inequality restrictions that express a prior conjecture that net substitutability should exist among the demands for the various meat products in the model.

There is also prior information in the form of a prior distribution on own-price elasticities derived from past research on the demand for meat in the Korean and Taiwanese markets that, in our context, might best be described as containing elements of both objective and subjective information. The objectivity of the information derives from the fact that the prior elasticities are deduced from a reproducible economic analysis based on observed data and a widely accepted econometric approach. Its subjectivity derives from the professional judgments that were made by the original demand analysts in the selection of particular functional forms, commodity definitions, and types of variables included in the demand models, as well as from our decision to provisionally consider meat demand responses in the Korean and Taiwanese markets as providing informative guides to Japanese meat demand response.

3.1 Theoretical Restrictions

The neoclassical restrictions of additivity, homogeneity, and symmetry define linear exact restrictions on the parameters of the AIDS share equations. These restrictions are given by

(2) $$\sum_i \alpha_i = 1; \sum_i \gamma_{ij} = 0; \sum_i \beta_i = 0,$$

(3) $$\sum_j \gamma_{ij} = 0,$$

(4) $$\gamma_{ij} = \gamma_{ji} \ \forall_{i \neq j},$$

respectively. A prior belief that these neoclassical assumptions hold is tantamount to prior information stating that equations (2), (3), and (4) are jointly satisfied with prior probability 1.

Theoretical considerations relating to the concavity of the cost function and bounds on the admissible values of budget shares provides additional prior information in the form of inequality restrictions on functions of the AIDS parameters. Concavity of the cost function can be represented in terms of a prior probability of one that the eigenvalues of the Slutsky substitution matrix, **S**, are nonpositive. Equivalently, one can also check the signs of the eigenvalues of the elasticity of a substitution matrix whose typical (i,j)th entry is defined by $\sigma_{ij} = \varepsilon^*_{ij} / w_j$, where

$\varepsilon^*_{ij} = (\partial q^*_i / \partial p_j)(p_j / q^*_i)$ denotes a Hicksian price elasticity and q^*_i denotes the Hicksian demand for product i. For given values of the budget shares in Stone's price index, the substitution elasticities in the AIDS model are defined as

(5) $$\sigma_{ij} = 1 + \frac{\gamma_{ij}}{w_i w_j} - \frac{\delta_{ij}}{w_j} \quad \forall_{i,j},$$

where δ_{ij} is the Kronecker delta such that δ_{ij} = 1 if i=j and δ_{ij} = 0 otherwise. [See Chalfant, Gray, and White; for extensive discussion on the calculation of AIDS elasticities, refer to Green and Alston (1990, 1991), and Buse.] The prior information regarding concavity of the cost function then states that the eigenvalues of the substitution matrix, whose typical (i,j)th entry is σ_{ij}, are all nonpositive with prior probability 1.

Woodland pointed out that typical share model specifications, such as those based on a normal error distribution, do not account for the fact that budget shares must all be nonnegative and less than or equal to unity. Therefore, a more appropriate representation of the probability model is desirable and a prior probability of one should be assigned to the event that the vector of budget shares resides in the unit simplex. Inequality constraints on budget shares that jointly hold with prior probability 1 are then given by

(6) $$0 \le w_i \le 1, \; i = 1, \ldots, m.$$

3.2 Net Substitutability

Prior beliefs about substitutability or complementarity between certain commodities within a product group are often considerably strong. For example, within a food group such as meats, and in the context of U.S. tastes and preferences, it is generally expected that meat commodities would be net substitutes for one another. Unfortunately, most demand systems with high theoretical structure, like the AIDS, do not have sufficient parameter flexibility to enforce these prior beliefs globally via reparameterization, and classical statistical techniques for enforcing these beliefs locally via parametric constraints are generally cumbersome (if at all tractable). In contrast, within the Bayesian methodology net substitutability between meat commodities in the context of the AIDS model can be ensured by assigning prior probability 1 to the event that the following inequality constraints on the Hicksian price elasticities hold jointly:

$$(7) \qquad \varepsilon_{ij}^{*} = -\delta_{ij} + \frac{\gamma_{ij}}{w_i} + w_j \geq 0, \ \forall i \neq j.$$

3.3 Prior Elasticities

Oftentimes more informative prior information is available than merely the signs of certain functions of model parameters. Based on previous studies or expert opinion, it may be possible to construct proper prior probability distributions on model parameters that can be combined with observed data in order to broaden the base of information and narrow the uncertainty regarding demand response. However, there are often problems concerning the comparability of different types of information. For example, the current demand model being analyzed differs from previous demand models in terms of data periods, functional forms, and underlying theoretical assumptions. A review of publications relating to meat demand in Japan (for a survey, see Dyck) revealed that all have problems of comparability with the demand model employed here, one significant problem being that other studies do not treat Wagyu beef and IQ beef as separate commodities. Also, Bayesian elasticity estimates from the current AIDS model based on ignorance (uninformative or diffuse) priors (see tables 1 and 2) are generally well within the range of estimated elasticities obtained from past studies such that prior information based on these past analyses is generally not very informative. While there are specific elasticities from past studies that deviate substantially from current ignorance prior-based AIDS estimates, these earlier estimates tend to be statistically insignificant and highly unreliable.

In this study, we follow a different tack with regard to the use of prior elasticity information. In particular, we analyze prior own-price demand elasticities for pork and chicken from non-Japanese markets (obtained from Capps et al.) that refer to definitions of pork and chicken commodities and a time period of analysis (1962-1991) that are comparable to those utilized in our Japanese AIDS model and that are derived under the imposition of all neoclassical equality restrictions on parameters. The elasticities refer to South Korean and Taiwanese meat demand. We engage in a posterior analysis of the relative similarity of price response in Japan to price response in South Korea and/or Taiwan for these two meat commodities. To accomplish this, we formulate two different bivariate normally distributed prior distributions on the mean-level Marshallian own-price elasticities for pork and chicken having respective mean vectors (-0.6468, -0.4698) and (-0.9192, -0.2779), and a common diagonal covariance matrix equal to (0.019, 0.126). The means of the prior densities correspond precisely to the mean-level elasticities for South Korea and Taiwan reported in Capps et al. Variances of the elasticity estimates were not reported by Capps et al., and so prior variances are set equal to the posterior variances of the corresponding elasticities calculated from our AIDS model based on an ignorance prior. This approach can be interpreted as assigning an equal measure of imprecision to both the prior elasticity information and the purely data-based information relating to these elasticities. Marshallian price elasticities for the linearized AIDS model, which are reported in the results section and are needed for comparison with the South Korean and Taiwanese prior information, are calculated as

$$(8) \qquad \varepsilon_{ij} = -\delta_{ij} + \frac{\gamma_{ij}}{w_i} - \beta_i \frac{w_j}{w_i}.$$

4 Bayesian Bootstrapping of Reduced Form Mappings

Zellner, Bauwens, and van Dijk developed several posterior mappings of reduced form coefficients that allow for limited information Bayesian posterior analysis of the parameters of structural equations. In their approach, the posterior distributions of structural equation parameters must be analyzed via Monte Carlo integration based on random samples from the posterior distribution of reduced form coefficients. The posterior distribution of the reduced form coefficients is a matrix Student-*t* density if it can be assumed that the errors are normally distributed and a standard ignorance prior is specified for the reduced form parameters (Zellner, Bauwens, and van Dijk, p. 46).

Heckelei (Part 2, 1995), and Heckelei and Mittelhammer (1996b) introduced a robust and simplified alternative procedure for sampling from the reduced form posterior by demonstrating how a random sample from a posterior distribution based on a "Regressione Structure Likelihood" (RSL) could be obtained. In essence, the bootstrapped joint sampling distribution of the usual OLS location and scale estimators of reduced form parameters is used to form a distribution-robust representation of the likelihood function of the reduced form parameters, thereby rendering the specification of a parametric family for the likelihood function unnecessary and introducing robustness to the representation of the underlying error distribution (see also Heckelei and Mittelhammer 1996a for a discussion of the single-equation case). Heckelei (1995), and Heckelei and Mittelhammer (1996b) further extended the 2SLS-mapping of Zellner, Bauwens, and van Dijk to a "3SLS mapping" for the case where more than one structural equation is of interest.

The algorithm we use here to generate sample outcomes from a posterior distribution of reduced form parameters is based on the algorithm given in Heckelei and Mittelhammer (1996b). We present below a brief account of the theory and algorithm in sufficient detail so that the interested reader can reproduce the results reported here as well as adapt the procedure to his/her own applications. Proofs and further conceptual details are deferred to the references (i.e., Zellner, Bauwens, and van Dijk, Heckelei, and Heckelei and Mittelhammer (1996a,b)).

4.1 Robust Bayesian Bootstrapped 2SLS and 3SLS Mappings

Represent the AIDS model of Japanese meat demand in matrix notation [compare to equation (1)] as follows:

(9) $$W = \iota\alpha + Z\delta + U,$$

where **W** is an (n $\times$ m) matrix of budget shares, i is an (n X 1) vector of ones, a is a (1 X m) vector of unknown constants, **Z** is an (n X k) matrix of right-hand-side endogenous variables [containing observations on ln(P_i), where i = 1, ..., m and on ln(E/P) in the AIDS model], d is a (k X m) matrix of coefficients (the elements are the γ_{ij}'s and β_i's in the current application), and **U** is an (n X m) matrix of structural equation errors.

Let the reduced form representation of the right-hand-side endogenous variables be given by

(10) $$\mathbf{Z} = \mathbf{X}P + \mathbf{V},$$

where **V** is an (n x k) matrix of disturbance terms whose rows are independently distributed according to some multivariate probability distribution with mean vector zero and finite positive definite covariance matrix S, **X** is an (n x p) matrix of predetermined and/or exogenous variables, and P is a (p x k) matrix of reduced form coefficients. Let the posterior distribution of the parameter matrix P implied by (10) be given by p(P |**Z**). Then a 2 SLS mapping of the posterior distribution of the parameter matrix P into the posterior distribution of d, h(d| **Z**), based on an ignorance prior for the structural equation parameters, is defined in accordance with Zellner, Bauwens, and van Dijk (p. 54) as

(11) $$\mathbf{Z}_* = \{\iota\mathbf{X}P\} \text{ and } P \sim \mathbf{p}(P|\mathbf{Z}) \rightarrow \delta^{2SLS} = \left(\mathbf{Z}_*'\mathbf{Z}_*\right)^{-1}\mathbf{Z}_*' W \sim h(\delta|\mathbf{Z}).$$

For purposes of posterior inference, a sample outcome of d^{2SLS} from $h(d\,|\mathbf{Z})$ can be derived by first obtaining a sample outcome of P from $p(P\ |\mathbf{Z})$, then calculating $\mathbf{Z}_*$ based on the sample outcome of P, and then finally calculating $d^{\mathbf{2SLS}}$ as a function of the realized value of $\mathbf{Z}_*$**.**

A likelihood-robust Bayesian bootstrapped version of the preceding 2SLS mapping is obtained by utilizing Heckelei and Mittelhammer´s (1996b) Bayesian bootstrap multivariate regression (BBMR) technique for generating outcomes from a bootstrapped posterior distribution of P**,** but otherwise following the remainder of the calculations implied by (11). The BBMR procedure is based on a posterior distribution for P**,** say, $p(\Pi|\hat{\Pi}, S),$, that is defined by taking a regression structure likelihood (RSL), for P and S, $L(\Pi, \Sigma|\hat{\Pi}, S)$, weighting the RSL by the standard ignorance prior for S, and then integrating out S. The RSL is the likelihood for P and S implied by the probability distribution of the standard least squares estimators, $\hat{P} = (\mathbf{X}'\mathbf{X})^{-1}\mathbf{X}'\mathbf{W}$ and $n^{-1}S = n^{-1}(\mathbf{Y} - \mathbf{X}\hat{\Pi})'(\mathbf{Y} - \mathbf{X}\hat{\Pi}),$ of the reduced form parameters and, when used to define the posterior in (11), leads to a posterior distribution for d of the form $h(d|\hat{\Pi},\ S)$. In the event that the error terms $\mathbf{V}$ are multivariate normally distributed, the posterior distribution for the structural parameters based on the RSL is identical to the distribution implied by the Zellner, Bauwens, and van Dijk approach. More generally, the BBMR is robust within the entire class of elliptically contoured probability distributions (Heckelei and Mittelhammer, 1996b, p. 8), which includes the multivariate normal distribution as a special case. (See Johnson for a discussion of the class of elliptically contoured distributions.)

The specific steps in the BBMR algorithm for obtaining sample outcomes from $p(P\,|\,\hat{P},\ S)$ and $h(\delta|\hat{P},\ S)$, are as follows (see Heckelei and Mittelhammer 1996b, p. 11) :

Step 1 Obtain OLS estimates of P and the reduced form residuals as

$\hat{\Pi} = (\mathbf{X}'\mathbf{X})^{-1}\mathbf{X}'\mathbf{Z}$ and $\hat{\mathbf{V}} = (\mathbf{Z} - \mathbf{X}\hat{\Pi})$,

and calculate

(13) $$\mathbf{S}^{1/2} = (\hat{\mathbf{V}}'\hat{\mathbf{V}})^{1/2},$$

where the exponent 1/2 denotes the symmetric square root matrix[1].

Step 2 Generate N bootstrap random samples (i.e., sampling with replacement) of size n from $\hat{\mathbf{V}}_1, \ldots, \hat{\mathbf{V}}_n$, with the subscripts indicating the rows of the matrix $\hat{\mathbf{V}}$, resulting in the $(n \times m)$ matrices $\mathbf{V}_{*i}$, for i = 1, ..., N. Transform each bootstrapped matrix outcome as

(14) $$\mathbf{V}_{**i} = \mathbf{V}_{*i}\,\mathbf{S}^{-1/2}(\mathbf{S}\mathbf{S}_{*i}^{-1}\ \mathbf{S})^{1/2}$$

where $\mathbf{S}_{*i} = \mathbf{V}_{*i}'\mathbf{M}\mathbf{V}_{*i}$, and $\mathbf{M} = \mathbf{I} - \mathbf{X}(\mathbf{X}'\mathbf{X})^{-1}\mathbf{X}'$.

Step3 Generate N bootstrapped sample outcomes from the posterior distribution $L(d\,|\,\hat{\Pi},\ \mathbf{S})$ based on an ignorance prior and the bootstrapped outcomes of the RSL as

(15) $$\mathbf{P}_{*i} = \hat{\mathbf{P}} - (\mathbf{X}'\mathbf{X})^{-1}\mathbf{X}'\mathbf{V}_{**i},\ i = 1, \ldots, N.$$

Step 4 Insert the N outcomes from (15) into (11) to generate N bootstrapped outcomes from the posterior distribution of d^{2SLS}, yielding d_i^{2SLS}, i = 1, ... ,N.

Calculation of an iterated restricted 3SLS(R3SLS) mapping of reduced form parameters into structural equation coefficients that enforces linear restrictions of the form $\mathbf{R}\text{vec}(d) = \mathbf{r}$, (e.g., additivity, homogeneity, and/or symmetry constraints) can be accomplished by first calculating[2]

(16) $$d^{3SLS} = \left(\mathbf{Z}_{*b} - (\hat{W} \otimes \mathbf{I})^{-1}\mathbf{Z}_{*b}\right)^{-1}\mathbf{Z}_{*b}'(\hat{W} \otimes \mathbf{I})^{-1}\text{vec}(\mathbf{W})$$

where

$$\mathbf{Z}_{*b} = \mathbf{I}_m \otimes \mathbf{Z}_*,\ \hat{W} = (\mathbf{W} - \mathbf{Z}_* d^{3SLS})'(\mathbf{W} - \mathbf{Z}_* d^{3SLS})/n,\ \mathbf{Z}_* = \{i\ \mathbf{X}\ P\},$$

and Π_* represents an outcome from the BBMR algorithm described above.

Then the R3SLS mapping is calculated as

(17) $$\mathrm{d}^{R3SLS} = \mathrm{vec}(\mathrm{d}^{3SLS}) + \hat{\mathbf{C}}\mathbf{R}'\,(\mathbf{R}\hat{\mathbf{C}}\mathbf{R}'\,)^{-1}(\mathbf{r} - \mathbf{R}\,\mathrm{vec}(\mathrm{d}^{3SLS})),$$

where $\hat{\mathbf{C}} = \left[\mathbf{Z}'_{*b}(\hat{W} \otimes \mathbf{I})^{-1}\,\mathbf{Z}_{*b}\right]^{1}$, , and (16) and (17) are iteratively reapplied until convergence is achieved. The whole process is repeated for all N bootstrapped outcomes of Π_{*i} to obtain N outcomes of the restricted 3SLS mapping as d_i^{R3SLS}, i = 1, ..., N (see footnote 2).

Note that it is often the case in empirical analyses of demand systems that a complete model for determining market equilibrium prices and quantities is not specified, so that a reduced form representation of the right-hand-side endogenous variables is not explicitly determined. In these cases, (10) is to be interpreted as an equation expressing the right-hand-side variables of the equation system (9) in terms of instrumental variables contained in the matrix **X**. The preceding 2SLS and R3SLS mappings then can be interpreted as instrumental variable mappings.

4.2 Posterior Expectations Based on Reduced Form Mappings

The preceding mapping outcomes are based on an ignorance prior-bootstrapped posterior distribution representing only data-based information about the structural coefficients via the respective mappings of reduced form coefficients. By Bayes' theorem, the posterior distributions $\mathrm{h}(\mathrm{d}|\hat{\mathrm{P}},\ \mathrm{S})$ derived from these mappings are proportional to the product of the prior density $p(\mathrm{d})$ and the likelihood $\mathrm{L}(\mathrm{d}|\,\hat{\mathrm{P}},\ \mathbf{S})$ as

(18) $$\mathrm{h}(\delta|\hat{\Pi},\ \mathbf{S}) \propto \mathrm{p}(\delta)\mathrm{L}(\delta|\hat{\Pi},\mathrm{S}).$$

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1) Let l, **P** denote the vector of eigenvalues and the matrix of eigenvectors of a square matrix **A**. Then, the symmetric matrix square root of **A**, $\mathbf{A}^{1/2}$, can be calculated as $\mathbf{P}\mathrm{L}^{1/2}\mathbf{P}'$, with $\mathrm{L}^{1/2} = (\mathbf{P}'\mathbf{A}\mathbf{P})^{1/2}$, i.e., a diagonal matrix with the vector of square roots of the entries in l as the diagonal.

2) Compare to Judge et al. (p. 457) for restricted system estimation. Note that unrestricted 3SLS and 2SLS applied to each structural equation separately will yield identical results since the right-hand-side variables of the equations in (9) are the same for all equations. Thus, in the current context, there is no difference between posterior analyses based on an unrestricted 3SLS mapping and a 2SLS mapping.

When an ignorance prior is used, so that $p(\mathrm{d}) \propto c$, then the posterior distribution of d equals the normalized (to unit mass) likelihood function for d. Since to this point such an ignorance prior has been effectively assumed in the construction of both of the preceding mapping procedures, bootstrapped sample outcomes from the posterior distribution of d based on either of the aforementioned mappings can be equivalently interpreted as bootstrapped sample outcomes from a normalized likelihood for d.

Informative prior information can be incorporated into the calculation of posterior expectations of structural coefficients or functions thereof (means, variances, probabilities, and elasticities) by forming weighted averages of mapping outcomes, with values of the informative prior density providing the weights. In the case of 2SLS mappings, the expectation calculation, justified by laws of large numbers, is given by

$$\text{(19)} \qquad \mathrm{E}[\mathrm{g}(\mathrm{d})] \approx \frac{\sum_{i=1}^{N} \mathrm{g}(\mathrm{d}_i^{2SLS})\mathrm{p}(\mathrm{d}_i^{2SLS})}{\sum_{i=1}^{N} \mathrm{p}(\mathrm{d}_i^{2SLS})}.$$

Posterior expectations for R3SLS mappings are obtained analogously by simply replacing 2SLS mapping outcomes with those generated from the R3SLS mapping.

Much of the prior information on the AIDS parameters discussed previously is in the form of inequality restrictions on functions of the structural parameters. This type of prior information leads to a simplification in the preceding expectation calculation because the prior distribution $p(\delta)$ then only takes on a value of either zero (if the constraints are not satisfied) or one (if the constraints are satisfied). In this case, the posterior expectation of $g(\delta)$ is the simple average of all bootstrapped outcomes of $g(\delta^{2SLS})$ or $g(\delta^{R3SLS})$ that satisfy the constraints (compare to Chalfant, Gray and White, p. 483).

4.3 A Measure of Prior Influence

In evaluating prior information on d, it is of significant interest to assess the extent to which the prior density influences the posterior distribution. A prior on d that equals a positive constant over the entire support of the likelihood function leads to a posterior that is equal to the normalized (to unit mass) likelihood. This type of prior is referred to as an "ignorance prior" because it adds nothing to the information about d over what is contained in the data itself. The more the posterior deviates from the normalized likelihood, the more information the prior contains relative to the data. When sampling from the likelihood for δ (or equivalently, from a posterior based on an ignorance prior), this deviation can be measured by the index

$$(20) \qquad m_p = \frac{\left[\frac{1}{N}\sum_{i=1}^{N} p(d_i)\right]^2}{\frac{1}{N}\sum_{i=1}^{N} p(d_i)^2} = \frac{\overline{p(d)}^2}{\overline{p(d)^2}} \approx \frac{[E_L\, p(d)]^2}{E_L[p(d)^2]} = \mathbf{1} - \frac{\mathrm{var}_L(p(d))}{E_L[p(d)^2]}$$

where $E_L(\cdot)$ and $\mathrm{var}_L(\cdot)$ denote an expectation and a variance taken with respect to the normalized likelihood of the data.

The measure m_p has a number of useful properties. First, m_p is bounded over its domain of definition as $0 < m_p \leq 1$. It takes on a value of one if either an ignorance prior is employed or if the likelihood is degenerate on a particular parameter value that is in the support of the prior density. In either case, the prior is uninformative relative to the data information represented by the likelihood function. Second, whenever the prior is in the form of an indicator function, as in the case of inequality constraints, m_p equals the proportion of sampled likelihood outcomes that satisfies the constraints. Thus m_p represents (for large enough N) the ignorance-based posterior probability that the constraints are satisfied, and provides a measure of the reasonableness of the constraints as judged by the likelihood function (compare to P^* in Geweke 1986, pp. 131-32). Third, the measure is invariant to any arbitrary scaling of an improper (i.e., does not integrate to 1) prior density.

Finally, m_p is measuring a deviation of the posterior from the likelihood in the sense that it approximates (arbitrarily close depending on the sample size N) the expected prior density value with respect to the normalized likelihood, $E_L[p(d)]$, relative to the expected prior density value with respect to the posterior, $E_h[p(d)]$. To see this, note that

$$(21) \qquad E_L[p(d)] = \int p(d)L(d|\hat{P}, \mathbf{S})\, dd \approx \frac{1}{N}\sum_{i=1}^{N} p(d_i)$$

when the d_i are outcomes from $L(d|\hat{P}, \boldsymbol{S})$. Furthermore, $E_L[p(d)]$ in (21) is equal to the reciprocal of the normalizing constant of the posterior density of d such that

$$(22) \qquad E_h[p(d)] = \int p(d) \frac{p(d)L(d\,|\,\hat{P}, \mathbf{S})\, dd}{\int p(d)L(d\,|\,\hat{P}, \mathbf{S})\, dd} = \frac{E_L[p(d)^2]}{E_L[p(d)]} \approx \frac{\frac{1}{N}\sum_{i=1}^{N} p(d_i)^2}{\frac{1}{N}\sum_{i=1}^{N} p(d_i)}.$$

Dividing (21) by (22) yields m_p in (20), validating that m_p equals the ratio of the aforementioned two expectations. The more the prior density differentiates between outcomes from the likelihood function (i.e., the more it influences the shape of the posterior relative to the likelihood), the smaller is the value of (21) relative to (22), i.e., the smaller is the measure m_p.

5 Posterior Analysis of Prior Information in the Japanese Demand Model

In this section, we apply the likelihood-robust Bayesian bootstrap procedure to the Japanese meat AIDS demand system and assess the various types of prior information presented earlier. We begin with an examination of a base model generated via the 2SLS mapping that incorporates an ignorance prior with no restrictions on model parameters. Then we assess the validity of the neoclassical equality and inequality restrictions relative to the information contained in the data and present theoretically consistent estimates of coefficients and elasticities. Last, the support for net substitutability is assessed and the Korean and Taiwanese prior price elasticities are evaluated.

5.1 Base Model Using Ignorance Prior and 2SLS Mapping

The Bayesian point estimate that minimizes the expected value of any positive definite quadratic loss function is the mean of the posterior distribution (Judge et al., p. 135). Posterior variances measure the precision of the posterior information on model coefficients. To determine whether a bootstrap sample size is large enough to provide stable estimates of the posterior means, one can calculate numerical standard errors, $\hat{s}_j$, of the posterior mean estimates as

$$(23) \qquad \hat{\sigma}_j = \sqrt{\frac{\sum_{i=1}^{N}\left[\delta_{ij} - \bar{\delta}_j\right]^2 p(\delta_i)^2}{\left[\sum_{i=1}^{N} p(\delta_i)\right]^2}},$$

where δ_{ij} is the *i*th bootstrap outcome (here defined by either the 2SLS or R3SLS mappings), and $\bar{\delta}_j$ is the estimated posterior mean for the structural coefficient δ_j (Geweke 1989). This measure is analogous to the usual standard error of the estimate of a population mean, but through incorporating prior weights it accounts for the fact that one is not sampling directly from the posterior distribution, but from the likelihood.

Posterior means and standard deviations of the parameters, as well as numerical standard errors of the means for the 2SLS mapping under an ignorance prior, are presented in table 1 based on a bootstrap sample size of 5,000. Generally, the posterior standard deviations of the 2SLS mappings are larger than the respective coefficients, or are at least of the same order of magnitude. The data information on the coefficients is evidently not very precise and the supports of the marginal posterior distributions generally include positive and negative signs. If one were in a sampling theory context (i.e., for classical 2SLS estimation), the analyst would be led to conclude that most coefficients are not significantly different from zero. However, note that the posterior standard deviations reported here are Bayesian and do not refer to the sampling distribution of an estimator as do the standard errors of 2SLS coefficients. The numerical standard errors are quite low, so that the bootstrap sample size of 5,000 seems sufficient in this case to yield stable estimates of posterior means.

5.2 R3SLS Mapping With Neoclassical Restrictions Imposed

An R3SLS mapping that imposes homogeneity, symmetry, and additivity constraints on model parameters, but otherwise utilizes an ignorance prior on model parameters, was generated based on a bootstrap sample size of 5,000 (see table 1). Note because of the covariance matrix singularity inherent in demand systems based on budget shares, one equation (fish) was deleted in the R3SLS mapping calculations, and then the additivity condition was used to recover the coefficients of the deleted equation. The posterior means of the R3SLS mapping are of course consistent with the theoretical constraints and imposing the neoclassical constraints also increases the precision of the posterior information. The posterior standard deviations of the R3SLS mappings are - with one exception (γ_{33}) - lower than the respective standard deviations from the 2SLS mappings. Furthermore, the efficiency of the posterior mean calculations is improved, as indicated by the lower numerical standard errors, despite the unchanged bootstrap sample size of 5,000.

Table 1. **Posterior Means, posterior Standard Deviations, and Numerical Standard Errors of the Means: 2SLS and R3SLS Mappings with Ignorance Prior**

		2SLS Mappings			R3SLS Mappings		
		Posterior Mean	Posterior Std. Dev.	Numerical Std. Err. of the Mean	Posterior Mean	Posterior Std. Dev.	Numerical Std. Err. of the Mean
Wagyu Beef	α_1	0.056	0.423	0.00599	-0.086	0.257	0.00363
	γ_{11}	-0.060	0.029	0.00041	-0.041	0.027	0.00039
	γ_{12}	0.024	0.029	0.00041	0.01[illegible]	0.020	0.00029
	γ_{13}	-0.004	0.031	0.00044	0.029	0.025	0.00035
	γ_{14}	0.047	0.043	0.00061	0.02[illegible]	0.021	0.00029
	γ_{15}	0.000	0.052	0.00073	-0.026	0.033	0.00047
	β_1	0.000	0.106	0.00150	0.03[illegible]	0.064	0.00091
IQ Beef	α_2	-0.044	0.512	0.00724	-0.079	0.254	0.00360
	γ_{21}	0.023	0.036	0.00051	0.012	0.020	0.00029
	γ_{22}	0.004	0.037	0.00052	0.006	0.027	0.00038
	γ_{23}	-0.022	0.039	0.00055	-0.004	0.020	0.00029
	γ_{24}	-0.032	0.052	0.00073	-0.028	0.022	0.00031
	γ_{25}	0.020	0.063	0.00089	0.015	0.033	0.00046
	β_2	0.029	0.128	0.00182	0.038	0.064	0.00090
Pork	α_3	0.310	0.415	0.00588	0.485	0.275	0.00389
	γ_{31}	0.087	0.030	0.00043	0.029	0.025	0.00035
	γ_{32}	-0.014	0.030	0.00042	-0.004	0.020	0.00029
	γ_{33}	0.041	0.031	0.00044	0.036	0.031	0.00045
	γ_{34}	-0.027	0.043	0.00060	-0.033	0.024	0.00034
	γ_{35}	-0.083	0.050	0.00071	-0.028	0.035	0.00050
	β_3	-0.032	0.104	0.00147	-0.076	0.069	0.00098
Chicken	α_4	-0.011	0.427	0.00603	-0.004	0.374	0.00529
	γ_{41}	0.027	0.032	0.00045	0.026	0.021	0.00029
	γ_{42}	-0.034	0.032	0.00045	-0.028	0.022	0.00031
	γ_{43}	-0.041	0.032	0.00046	-0.033	0.024	0.00034
	γ_{44}	0.024	0.044	0.00063	0.031	0.034	0.00048
	γ_{45}	0.012	0.052	0.00074	0.004	0.044	0.00063
	β_4	0.030	0.107	0.00151	0.028	0.094	0.00133
Fish	α_5	0.646	0.755	0.01068	0.683	0.583	0.00825
	γ_{51}	-0.077	0.053	0.00075	-0.026	0.033	0.00047
	γ_{52}	0.020	0.055	0.00077	0.015	0.033	0.00046
	γ_{53}	0.027	0.057	0.00080	-0.028	0.035	0.00050
	γ_{54}	-0.010	0.078	0.00110	0.004	0.044	0.00063
	γ_{55}	0.047	0.092	0.00130	0.035	0.075	0.00106
	β_5	-0.016	0.189	0.00268	-0.025	0.146	0.00207

Moving from 2SLS to R3SLS mappings changes the values of the posterior means of the parameters substantially, and in some cases even the signs of the parameters change. Table 2 reports the implications regarding posterior means of Marshallian and Hicksian elasticities together with their respective posterior standard deviations for both the 2SLS and R3SLS mappings. Analogous to the parameters themselves, the information on Marshallian and Hicksian elasticities is more precise when homogeneity, symmetry, and additivity are imposed. Under both mappings, all Marshallian own-price elasticities are negative, as expected, and constraining the coefficients did not notably change the elasticity values - with the exception of the Wagyu beef share which shows less own-price sensitivity under the constraints. In addition, all own-price elasticities are within the ranges of estimation results from other studies, as reported and updated by Dyck. This is also true for the cross-price elasticities (insofar as previous studies were successful in producing significant cross-price elasticity estimates), so that no discrimination between the 2SLS and R3SLS mappings can be made on the basis of elasticity magnitudes. For the 2SLS mapping, substitution effects are not constrained to be symmetric, so that the corresponding Hicksian elasticities for certain pairs of commodities can have different signs (as in the case of Wagyu beef and fish); consequently, they are inconclusive regarding the question of substitutability or complementarity. Negative signs on Hicksian elasticities for beef-pork, beef-chicken, and pork-chicken indicate complementarity. For the R3SLS mapping, beef and pork are estimated to be net substitutes, but the respective Hicksian elasticities have high posterior standard deviations relative to their size and thus the substitute characterization is quite tenuous.

Table 2. Posterior Means of Marshallian and Substitution Elasticities and Their Posterior Standard Deviations (in parenthesis): 2SLS and R3SLS Mappings with Ignorance Prior

		2SLS-Mappings				R3SLS-Mappings			
		Marshallian Elasticities		Hicksian Elasticities		Marshallian Elasticities		Hicksian Elasticities	
Wagyu beef	Wagyu Beef	-2.01	(0.52)	-1.95	(0.49)	-1.58	(0.38)	-1.47	(0.37)
	IQ Beef	0.40	(0.46)	0.47	(0.49)	0.14	(0.26)	0.20	(0.27)
	Pork	-0.07	(0.49)	0.11	(0.52)	0.29	(0.29)	0.59	(0.33)
	Chicken	0.80	(0.60)	0.90	(0.73)	0.30	(0.23)	0.45	(0.28)
	Fish	0.01	(1.89)	0.58	(0.87)	-0.63	(0.92)	0.23	(0.44)
IQ Beef	Wagyu Beef	0.31	(0.58)	0.40	(0.55)	0.21	(0.51)	0.35	(0.47)
	IQ Beef	-0.96	(0.51)	-0.87	(0.55)	-0.91	(0.59)	-0.83	(0.62)
	Pork	-0.41	(0.55)	-0.14	(0.58)	-0.26	(0.49)	0.12	(0.46)
	Chicken	-0.53	(0.64)	-0.38	(0.78)	-0.74	(0.40)	-0.56	(0.50)
	Fish	0.04	(2.04)	0.88	(0.94)	-0.16	(1.58)	0.92	(0.75)
Pork	Wagyu Beef	0.48	(0.17)	0.53	(0.16)	0.17	(0.13)	0.21	(0.12)
	IQ Beef	-0.07	(0.15)	-0.01	(0.16)	0.00	(0.10)	0.03	(0.10)
	Pork	-0.75	(0.16)	-0.60	(0.17)	-0.75	(0.14)	-0.62	(0.15)
	Chicken	-0.12	(0.19)	-0.04	(0.23)	-0.13	(0.10)	-0.06	(0.12)
	Fish	-0.34	(0.59)	0.14	(0.27)	0.08	(0.36)	0.44	(0.17)
Chicken	Wagyu Beef	0.24	(0.32)	0.32	(0.30)	0.24	(0.23)	0.34	(0.21)
	IQ Beef	-0.34	(0.28)	-0.25	(0.30)	-0.30	(0.20)	-0.24	(0.22)
	Pork	-0.44	(0.30)	-0.20	(0.31)	-0.39	(0.27)	-0.13	(0.24)
	Chicken	-0.80	(0.36)	-0.67	(0.42)	-0.71	(0.27)	-0.59	(0.34)
	Fish	-0.05	(1.08)	0.70	(0.50)	-0.12	(0.98)	0.62	(0.45)
Fish	Wagyu Beef	-0.13	(0.10)	-0.07	(0.91)	-0.04	(0.06)	0.03	(0.06)
	IQ Beef	0.04	(0.09)	0.10	(0.94)	0.03	(0.05)	0.07	(0.06)
	Pork	0.05	(0.09)	0.23	(0.97)	-0.04	(0.06)	0.16	(0.06)
	Chicken	-0.01	(0.11)	0.09	(0.13)	0.01	(0.06)	0.11	(0.08)
	Fish	-0.90	(0.34)	-0.34	(0.16)	-0.92	(0.27)	-0.36	(0.13)

5.3 Homogeneity and Symmetry

The R3SLS mapping imposes homogeneity and symmetry, but does not allow for testing the reasonableness of these neoclassical restrictions with respect to the data. Therefore, a Bayesian posterior analysis of these equality constraints on the basis of the unrestricted 2SLS mapping is of some interest. The posterior means in table 1 certainly violate the restrictions in a point comparison sense, but in order to assess to what extent the posterior information contradicts or supports homogeneity and symmetry, one must examine the posterior distributions of appropriate functions of the parameters.

Figure 1: **Posterior distribution of the degree of homogeneity for the IQ-beef Equation: 2SLS mapping and ignorance prior**

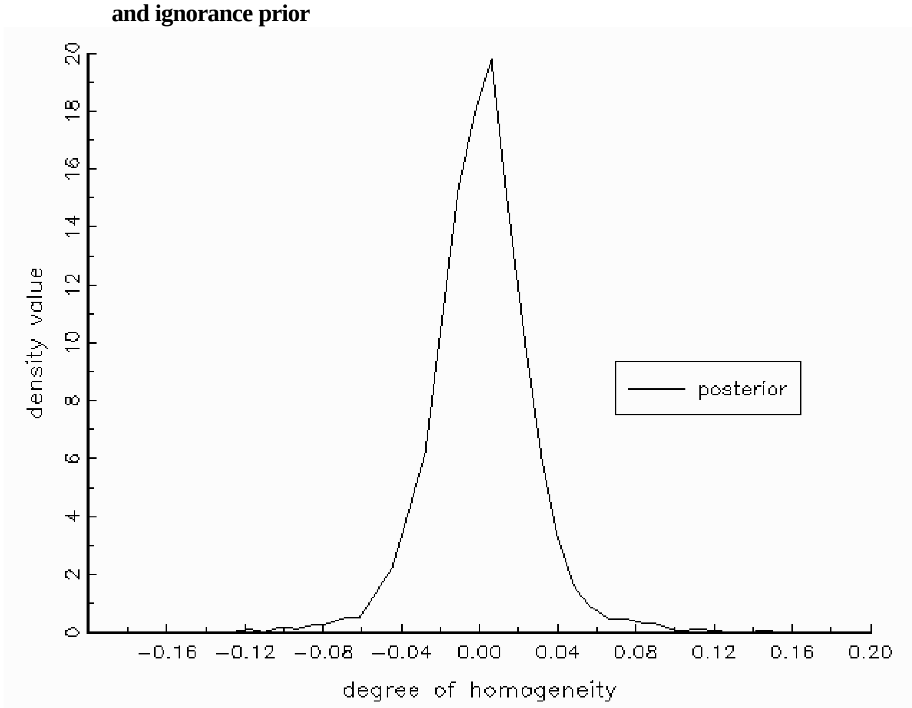
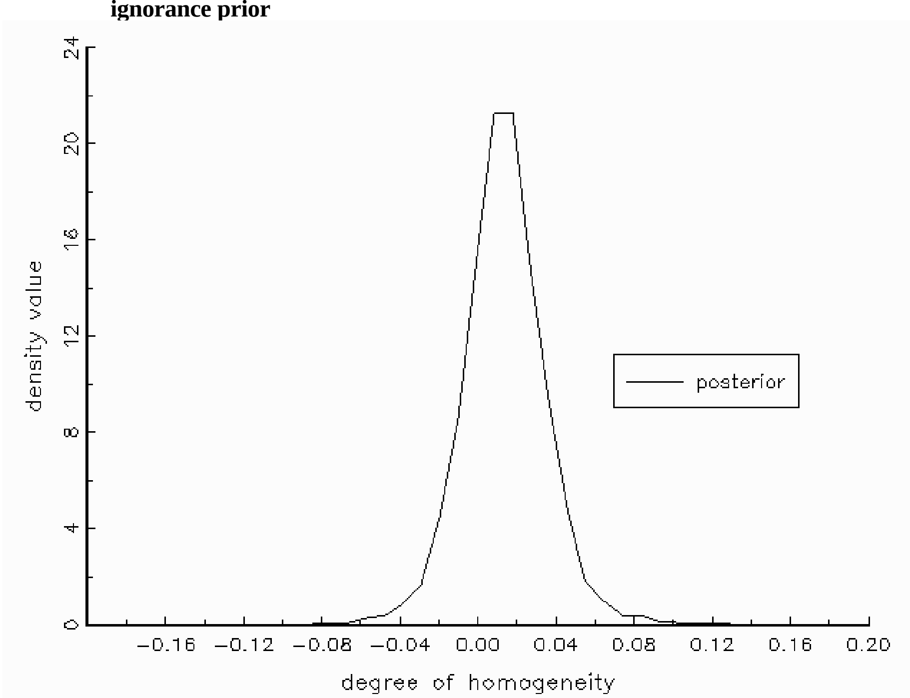


Figure 1 and 2 present posterior distributions of the degree of homogeneity, i.e., the sum of own-price and cross-price coefficients [see equation (3)], for the IQ beef and pork equations, respectively. In the case of the IQ beef equation, the distribution is very well centered on the theoretically expected value of zero and is nearly symmetric. For the pork equation, the distribution is also nearly symmetric but the highest posterior density lies somewhat to the right of the zero value. The other posterior distributions of the degrees of homogeneity are shaped similarly to the ones shown and the value of zero is always easily contained within the 95% highest posterior density (HPD) regions, defined as the region that contains 95% of the probability mass with all density values inside the region being no less than any density value outside the region.

If one were to adopt Lindley's hypothesis testing procedure for a simple versus composite hypothesis under an ignorance prior, homogeneity of degree zero would consequently not be rejected for all five equations at a significance level of 0.05 (Lindley, p. 58ff; Zellner, p. 298f). However, the use of significance levels conflicts somewhat with the Bayesian philosophy of examining the entire posterior distribution to evaluate all information available about a hypothesis rather than merely comparing test statistics to predetermined critical values. Even in sampling theory-based econometrics, it is increasingly the case that probability values (*P*-values) of test statistics are reported in addition to, or instead of, significance values, suggesting dissatisfaction with the "pure" testing procedures. The smaller the probability value, the more justification there is to reject the null hypothesis.

Figure 2: **Posterior distribution of the degree of homogeneity for the pork equation: 2 SLS-mapping and ignorance prior**



To obtain measures in the Bayesian context that serve a purpose similar to *P*-values in the sampling theory context, the analyst can start from Lindley's testing approach and measure the support of the posterior distribution for homogeneity of zero degree by the probability mass contained within the smallest HPD region that still contains the homogeneity value of zero. The smaller this region, the larger is the marginal significance level at which the null hypothesis could be rejected by Lindley´s procedure (or the closer is the zero value to the mode of the posterior), and - since we are still utilizing only ignorance priors - the stronger is the support from the data for the null hypothesis. The upper portion of table 3 reports the sizes of these smallest HPD regions for all five equations and they support the graphical impression that zero degrees of homogeneity are well inside the posterior for all share equations.

Table 3: Smallest HPD-Regions for Homogeneity and Single Symmetry Restrictions: 2SLS Mappings with Ignorance Prior

Constraint Type	Constraint Form	Smallest HPD Probability
Homogeneity		
	$\gamma_{11}+\gamma_{12}+...+\gamma_{15}=0$:	0.599
	$\gamma_{21}+\gamma_{22}+...+\gamma_{25}=0$:	0.175
	$\gamma_{31}+\gamma_{32}+...+\gamma_{35}=0$:	0.534
	$\gamma_{41}+\gamma_{42}+...+\gamma_{45}=0$:	0.174
	$\gamma_{51}+\gamma_{52}+...+\gamma_{55}=0$:	0.730
Symmetry		
	$\gamma_{12}-\gamma_{21}=0$:	0.564
	$\gamma_{13}-\gamma_{31}=0$:	0.957
	$\gamma_{14}-\gamma_{41}=0$:	0.737
	$\gamma_{15}-\gamma_{51}=0$:	0.925
	$\gamma_{23}-\gamma_{32}=0$:	0.542
	$\gamma_{24}-\gamma_{42}=0$:	0.441
	$\gamma_{25}-\gamma_{52}=0$:	0.491
	$\gamma_{34}-\gamma_{43}=0$:	0.411
	$\gamma_{35}-\gamma_{53}=0$:	0.792
	$\gamma_{45}-\gamma_{54}=0$:	0.825

Similarly, one can measure the support from the data for each symmetry restriction by deriving posterior distributions for differences in the relevant parameters in order to assess the degree to which they support a value of zero [see equation (4)]. All but one of these posteriors have smallest HPD regions containing zero that have probability mass smaller than 95% (lower portion of table 3).

The exception is the difference between the cross-price effects of Wagyu beef and pork, γ_{13} - γ_{31} (figure 3) for which the zero value is notably in the right tail of the distribution but with the smallest HPD region still quite close to 95%. As an example of a "well-behaved" symmetry difference, we show the posterior for γ_{34} - γ_{43} (pork and chicken) in Figure 4.

Classical tests performed by Hayes, Wahl, and Williams could not reject homogeneity and symmetry restrictions for the data set. Together with our preceding Bayesian posterior analysis based on the unrestricted 2SLS mappings, it seems largely appropriate to treat these restrictions as maintained hypotheses in our subsequent Bayesian analysis and to henceforth use R3SLS mappings to derive posterior distributions of coefficients and elasticities. However, we will continue to report some results for the unrestricted 2SLS mappings to show some interesting differences with regard to the evaluation of other prior information.

Figure 3: **Posterior distribution of the difference between the cross-price effects of Wagyu beef and pork: 2SLS mapping and ignorance prior**

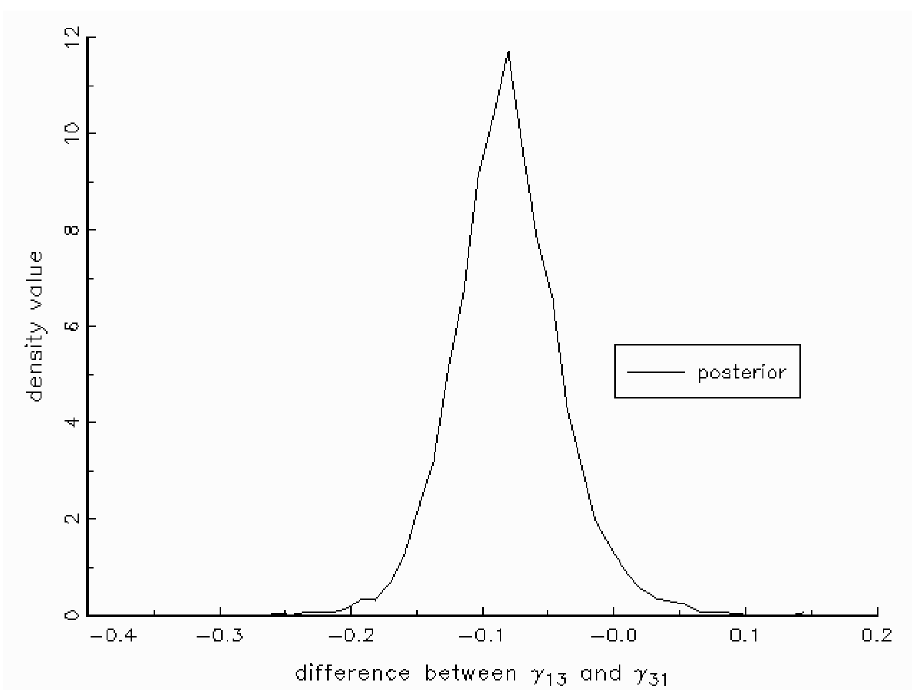
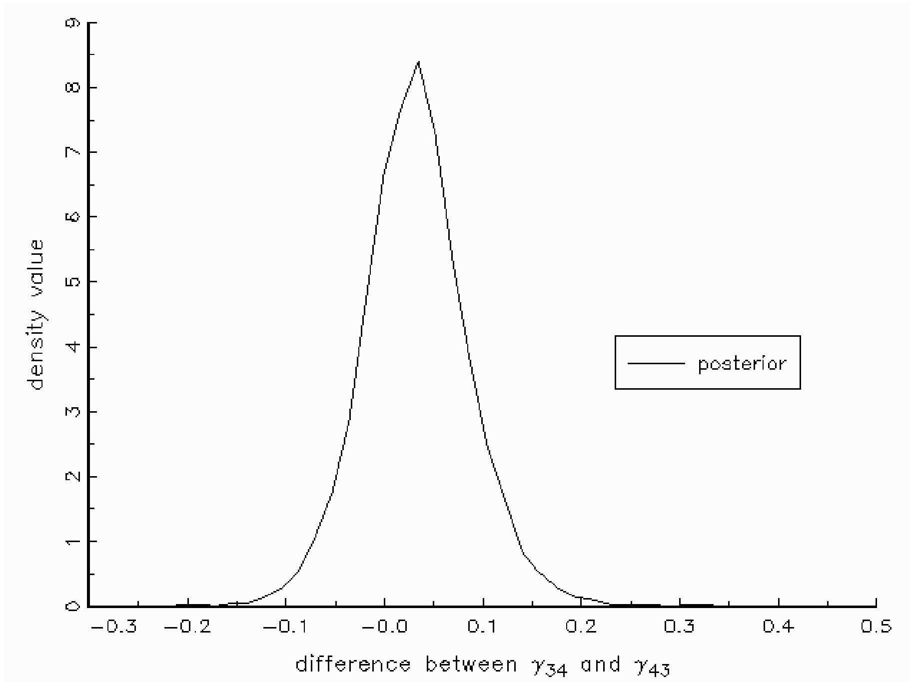


Figure 4: **Posterior distribution of the between the cross-price effects of pork and chicken: 2SLS mapping and ignorance prior**



5.4 Concavity and Consistent Budget Shares

The matrix of posterior means of the substitution elasticities [recall (5)] under an ignorance prior (table 2) has eigenvalues that are positive, and so the concavity restriction is violated by the point estimates derived from both mappings. Employing a prior distribution that assigns zero weight to positive eigenvalues yields estimates of substitution elasticities that satisfy concavity (table 4). The probability that the concavity constraints hold, as measured by the proportion of the bootstrapped sample outcomes satisfying the constraints is 0.0036 for the unrestricted (2SLS) mapping and 0.2802 for the restricted (R3SLS) mapping (see also table 5). Thus, the likelihood function with homogeneity, symmetry, and additivity imposed evaluates concavity as far more reasonable than does the unrestricted likelihood.

Consistency of budget shares is not in conflict with either type of mapping. All bootstrapped outcomes satisfy budget share restrictions to the unit simplex (see also table 5).

Table 4 presents Bayesian estimates of parameters as well as estimates of Marshallian and Hicksian elasticities based on R3SLS mappings that are consistent with all theoretical restrictions. Note that imposing a combination of equality constraints (homogeneity, symmetry, and additivity) and inequality constraints (concavity and consistent budget shares) in the Bayesian context would not have been tractable with a traditional analytical Bayesian analysis of simultaneous equation systems (Zellner).

Table 4. **Posterior Means, Posterior Standard Deviations, and Numerical Standard Errors of the Coefficients, and Posterior Means and Posterior Standard Deviations (in parentheses) of Marshallian and Substitution Elasticities: R3SLS Mapping with Concavity and Consistent Shares Imposed**

Share		Posterior Mean	Posterior Std. Dev.	Numerical Std. Err. of the Mean	Price	Marshallian Elasticities	Hicksian Elasticities
Wagyu Beef	α_1	-0.077	0.233	0.00624			
	γ_{11}	-0.044	0.026	0.00069	Wagyu Beef	-1.62 (0.36)	-1.51 (0.35)
	γ_{12}	0.016	0.016	0.00043	IQ Beef	0.19 (0.21)	0.26 (0.22)
	γ_{13}	0.030	0.023	0.00061	Pork	0.31 (0.26)	0.61 (0.31)
	γ_{14}	0.023	0.018	0.00047	Chicken	0.27 (0.19)	0.41 (0.24)
	γ_{15}	-0.025	0.032	0.00085	Fish	-0.60 (0.86)	0.24 (0.43)
	β_1	0.033	0.059	0.00157			
IQ Beef	α_2	-0.060	0.219	0.00586			
	γ_{21}	0.016	0.016	0.00043	Wagyu Beef	0.31 (0.40)	0.44 (0.37)
	γ_{22}	-0.002	0.020	0.00053	IQ Beef	-1.09 (0.43)	-1.01 (0.46)
	γ_{23}	-0.005	0.018	0.00047	Pork	-0.26 (0.42)	0.10 (0.40)
	γ_{24}	-0.027	0.018	0.00047	Chicken	-0.69 (0.32)	-0.52 (0.41)
	γ_{25}	0.018	0.028	0.00076	Fish	-0.02 (1.36)	1.00 (0.65)
	β_2	0.033	0.055	0.00147			
Pork	α_3	0.466	0.258	0.00690			
	γ_{31}	0.030	0.023	0.00061	Wagyu Beef	0.17 (0.12)	0.22 (0.11)
	γ_{32}	-0.005	0.018	0.00047	IQ Beef	-0.01 (0.08)	0.02 (0.09)
	γ_{33}	0.031	0.029	0.00077	Pork	-0.77 (0.12)	-0.64 (0.14)
	γ_{34}	-0.028	0.021	0.00057	Chicken	-0.10 (0.09)	-0.04 (0.10)
	γ_{35}	-0.029	0.034	0.00090	Fish	0.06 (0.34)	0.44 (0.17)
	β_3	-0.071	0.065	0.00173			
Chicken	α_4	0.065	0.312	0.00833			
	γ_{41}	0.023	0.018	0.00047	Wagyu Beef	0.23 (0.19)	0.31 (0.18)
	γ_{42}	-0.027	0.018	0.00047	IQ Beef	-0.28 (0.16)	-0.23 (0.18)
	γ_{43}	-0.028	0.021	0.00057	Pork	-0.30 (0.22)	-0.08 (0.21)
	γ_{44}	0.020	0.027	0.00072	Chicken	-0.81 (0.21)	-0.70 (0.27)
	γ_{45}	0.011	0.038	0.00101	Fish	0.06 (0.83)	0.69 (0.38)
	β_4	0.010	0.078	0.00209			
Fish	α_5	0.606	0.524	0.01399			
	γ_{51}	-0.025	0.032	0.00085	Wagyu Beef	-0.04 (0.06)	0.03 (0.05)
	γ_{52}	0.018	0.028	0.00076	IQ Beef	0.03 (0.04)	0.08 (0.05)
	γ_{53}	-0.029	0.034	0.00090	Pork	-0.05 (0.05)	0.15 (0.06)
	γ_{54}	0.011	0.038	0.00101	Chicken	0.02 (0.05)	0.12 (0.07)
	γ_{55}	0.024	0.069	0.00184	Fish	-0.95 (0.25)	-0.38 (0.12)
	β_5	-0.005	0.131	0.00351			

5.5 Net Substitutability

Comparing the Hicksian elasticities between tables 2 and 4 for the R3SLS mappings reveals that the imposition of the concavity restriction left all signs unaltered. In particular, according to the Hicksian elasticities, chicken remains a net complement to IQ beef and pork. For the 2SLS mappings (table 2), the IQ beef and pork are also estimated to be complementary goods, whereas Wagyu beef and fish show conflicting signs regarding their respective Hicksian elasticities. In order to evaluate formally the posterior support for net substitutability the posterior probabilities that the net substitute inequality restrictions hold [recall (7)] with respect to the 2SLS and R3SLS mappings (without any inequality constraints imposed) were calculated for all meats jointly, for all meats but fish, all meats but fish and chicken, and between all possible pairs of meats (see table 5). Again, these probabilities represent the proportion of the bootstrap sample outcomes that satisfy the relevant inequality restrictions on the elements of the Hicksian elasticity matrix. Since the matrix of substitution effects is not symmetric in the case of the 2SLS mappings, the signs of two Hicksian elasticities need to be checked for each pair of meats in this case. Without exception, the model with neoclassical equality restrictions imposed provides greater posterior support for net substitutability than the unrestricted one. However, for both mappings, the posterior probabilities that all meats, or all meats but fish, are simultaneously net substitutes is .006 or less, indicating a strong rejection of net substitutability by the model. Overall, high posterior probabilities for some pairs of goods suggest that there are subsets of meat and fish commodities that might reasonably be considered substitute goods, although complementarity among some subsets of goods is also strongly supported.

It would be possible to generate posterior means for coefficients and elasticities that satisfy net substitutability for all meats, since the posterior probability of net substitutability is still positive for the R3SLS mappings. For this purpose one would need to increase considerably the bootstrap sample size until enough outcomes satisfied the inequality restrictions to yield stable estimates of posterior expectations. Given the overwhelming rejection of the net substitutability proposition, we feel that the usefulness of such results would be quite limited and so we refrain from this exercise.

Table 5. Posterior Probabilities of Concavity, Consistent Budget Shares, and Net Substitutability: 2SLS and R3SLS Mapping with Ignorance Prior

Prior Restrictions	2SLS Mapping	R3SLS Mapping
Concavity	0.0036	0.2802
Consistent Budget Shares	1.0000	1.0000
Net Substitutability		
All Meats	0	0.001
All Meats but Fish	0.0002	0.006
Wagyu Beef, IQ Beef, Pork	0.0546	0.46
Wagyu Beef, IQ Beef	0.6928	0.8008
Wagyu Beef, Pork	0.5924	0.9762
Wagyu Beef, Chicken	0.804	0.948
Wagyu Beef, Fish	0.1286	0.7068
IQ Beef, Pork	0.1844	0.5984
IQ Beef, Chicken	0.0588	0.1134
IQ Beef, Fish	0.7832	0.9144
Pork, Chicken	0.093	0.275
Pork, Fish	0.7208	0.993
Chicken, Fish	0.7596	0.9304

5.6 South Korean and Taiwanese Prior Elasticities

We now evaluate the prior information on South Korean and Taiwanese Marshallian own-price elasticities for pork and chicken to assess their informational content relative to the likelihood function for the Japanese demand model. Posterior probabilities defined in terms of the proportion of bootstrap outcomes that satisfy inequality constraints cannot be applied here since the prior information on elasticities is formulated in terms of bivariate normal distributions. Instead, we use the previously described prior influence measure m_p for evaluation purposes. Relative to the Japanese likelihood defined by the R3SLS mapping the Taiwanese prior has a greater impact on the posterior (m_p = 0.49) than the South Korean prior (m_p = 0.73). Since both priors had the same levels of dispersion by definition, and since in hindsight the Taiwanese prior means are more distant from the means of the Japanese likelihood (compare the prior means to the posterior means of the R3SLS mapping based on the ignorance prior, table 2), this result was not surprising. Thus the South Korean price response would appear to be more in accord with Japanese demand than is price response in Taiwan.

One should not jump to the conclusion that higher values of m_p necessarily justify the use of certain priors for estimation purposes and lower values do the opposite because of data/prior information compatibility considerations. Adopting such a rule leads to a situation where only priors that do not add much information to the posterior are used. Estimates that represent improvements over purely data-based results can only be achieved if effective prior information is included in the analysis. The question of proper use of priors can be answered only by assessing the validity of the prior. In the case at hand, if the analyst felt strongly that preferences in the Pacific Rim were not much different among countries (Capps et al., p. 223 did not), it would be justified to combine the different sources of information via the Bayesian approach and obtain estimates that were founded on a broader information base and consequently were more precise (posterior variances were notably smaller for a large majority of the coefficient when either of the prior elasticity distributions was imposed--results available upon request). The smaller the measure m_p the more the prior influences the posterior distribution and inferences derived from it, and the more important is the issue of prior validity. Given current conventions in economic analysis, one most likely would have an easier time justifying a concavity prior with a lower value of m_p (0.28 in the current application) than the Taiwanese prior with $m_p = 0.49$ (at least among a group of neoclassically trained economists), despite the apparently larger conflict that the concavity prior has with the data-based information.

6 Summary and Conclusions

The application of robust Bayesian bootstrap analysis to the Japanese AIDS meat demand system demonstrated that the method is relatively straightforward to implement in a typical econometric model. The technique is a useful tool for the evaluation and/or incorporation of different types of prior information on model parameters. In particular, the technique does not require an explicit likelihood function specification, and combinations of equality restrictions and other types of prior information can be straightforwardly incorporated into the R3SLS mapping to obtain posterior distribution of parameters via bootstrap simulation. Furthermore, unrestricted 2SLS mappings allow an evaluation of the support from the data for the various types of prior information.

The Japanese meat demand model was largely consistent with symmetry, homogeneity and additivity constraints on model parameters. The model was also completely consistent with restricting budget shares to the unit simplex. Net substitutability between certain pairs of meats was supported more by the R3SLS mapping than by the unrestricted mapping, but very low posterior probabilities regarding net substitutability between all meats suggests that not all meats are net substitutes in the diet of Japanese consumers. It was also found that Japanese demand for pork and chicken was more in accord with South Korean than Taiwanese demand in terms of price responsiveness. However, all substantive model results need to be tempered by the fact that posterior probabilities for concavity of the cost function were low, especially for the 2SLS mapping, suggesting possible deficiencies in the model specification or data. Further analysis would be advisable before conclusions generated from model results were considered definitive.

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APPENDIX

TABLES OF DATA AND INSTRUMENTS

Table A1. Expenditure Shares, Total Expenditure, Stone's Price Index, and Prices of Wagyu Beef, Import Quality Beef, Pork, Chicken, and Fish Used in the Japanese AIDS Demand Model

	Expenditure Shares							Prices				
Year	Wagyu Beef	IQ Beef	Pork	Chicken	Fish	Total Expenditure	Stone's Index	Wagyu Beef	IQ Beef	Pork	Chicken	Fish
65	0.1010	0.0347	0.1910	0.0892	0.5840	12.931	0.3619	1079.0	867.4	745.0	718.0	265.3
66	0.0743	0.0340	0.2270	0.0975	0.5670	13.922	0.3859	1279.9	1085.3	694.0	724.0	279.2
67	0.0744	0.0388	0.2110	0.1010	0.5750	16.269	0.4226	1612.1	1169.9	714.0	728.0	314.4
68	0.0669	0.0389	0.2100	0.0990	0.5860	19.152	0.4704	1745.4	1102.2	849.0	744.0	354.8
69	0.0748	0.0448	0.2190	0.1060	0.5550	21.618	0.5230	1720.0	1034.5	960.0	748.0	400.0
70	0.0721	0.0502	0.1780	0.1100	0.5910	25.299	0.5689	1783.5	1246.1	909.0	767.0	474.6
71	0.0664	0.0548	0.1810	0.0990	0.5990	29.102	0.6202	1819.4	1343.4	930.0	712.0	549.5
72	0.0620	0.0625	0.1820	0.1000	0.5930	33.168	0.6712	1967.5	1466.1	992.0	724.0	601.2
73	0.0514	0.0932	0.2010	0.1020	0.5520	38.524	0.8001	3002.0	2113.5	1120.0	801.0	689.9
74	0.0452	0.0598	0.1870	0.1040	0.6040	47.082	0.9058	3057.0	1631.1	1240.0	960.0	837.6
75	0.0504	0.0713	0.2010	0.0927	0.5850	55.329	1.0608	3469.6	2466.8	1550.0	1000.0	951.1
76	0.0527	0.0705	0.1960	0.1020	0.5790	62.773	1.1847	4201.6	2695.3	1680.0	1110.0	1069.9
77	0.0569	0.0665	0.1830	0.0980	0.5950	66.696	1.2422	4246.0	2513.3	1590.0	1040.0	1203.9
78	0.0571	0.0714	0.1790	0.0990	0.5930	72.636	1.2680	4100.0	2581.0	1570.0	1030.0	1253.9
79	0.0511	0.0882	0.1830	0.0992	0.5780	75.661	1.3066	4349.7	2966.1	1500.0	993.0	1314.4
80	0.0458	0.0801	0.1720	0.1120	0.5900	78.604	1.3478	4571.8	2691.0	1450.0	1140.0	1380.7
81	0.0430	0.0807	0.1710	0.1140	0.5910	82.577	1.3975	4531.6	2526.0	1530.0	1200.0	1444.9
82	0.0434	0.0850	0.1680	0.1140	0.5890	84.840	1.4580	4559.1	2672.0	1570.0	1180.0	1533.8
83	0.0488	0.0855	0.1750	0.1180	0.5720	84.652	1.4398	4582.4	2638.2	1630.0	1190.0	1492.7
84	0.0565	0.0806	0.1720	0.1180	0.5730	87.798	1.4317	4533.7	2557.8	1640.0	1170.0	1486.2
85	0.0575	0.0832	0.1660	0.1160	0.5780	90.544	1.4445	4565.5	2642.4	1540.0	1150.0	1531.0
86	0.0528	0.0877	0.1570	0.1150	0.5880	94.657	1.4785	4612.0	2733.4	1500.0	1110.0	1599.5

Source: Wahl and Hayes (table A-1).

Table A2. Principal Components Used as Instruments in the Japanese AIDS Demand Model

Year	Principal Components 1	2	3	4	5	6	7	8	9	10
65	-13.2635	-2.6303	-0.0802	-2.2503	-2.1223	-2.9645	-1.3349	-1.1365	-0.0536	0.5837
66	-12.1535	-1.8245	-0.5728	-2.0269	-1.6034	-0.7514	0.2543	0.2085	-0.5307	-1.0308
67	-11.8614	-1.7275	-0.7103	-1.6891	-0.2023	0.5807	1.8857	0.9330	0.3583	-0.5214
68	-10.9681	-1.3701	-0.3346	-0.8869	0.7136	0.9470	1.4353	0.6939	0.1703	-0.2322
69	-9.8652	-0.9649	-0.4336	-0.3525	1.5796	0.9361	1.3632	0.3398	0.0460	0.6291
70	-8.5419	-2.1155	-0.5644	0.7534	1.7702	1.2576	-2.2821	0.3328	-0.8785	0.8049
71	-7.0257	-3.0196	-0.3883	0.7048	1.1139	0.7351	-2.2892	0.3538	0.0187	0.7234
72	-5.4682	-0.9750	4.6931	5.1201	0.4407	-2.5225	0.5797	1.1682	-0.1140	-0.5250
73	-4.5689	2.8661	4.0864	0.4634	-0.6821	1.9403	0.1956	-2.0011	0.0758	0.4215
74	-3.0591	3.2400	0.8372	0.4892	0.0910	1.4005	-0.3865	-1.1184	0.9098	-1.4963
75	-1.8279	3.0937	-0.7379	1.0088	0.3136	0.0732	0.3579	-1.0778	1.3125	0.8413
76	1.5073	1.6144	-4.0178	1.9761	0.0341	0.0812	0.3956	-0.6853	0.1982	-0.7248
77	4.1596	-0.1022	-5.0094	3.0636	-0.7238	-0.3236	-0.3048	-0.1082	-0.1932	-0.3716
78	4.2198	4.5371	-1.4798	0.7726	-1.1584	-0.8556	1.1992	0.6644	0.0749	1.5898
79	5.7491	4.6236	1.0874	0.1951	-2.4125	0.8912	0.0235	-0.0383	-1.8404	0.2071
80	6.7648	3.6417	0.4955	-1.0081	-0.9006	1.3984	-1.2099	1.9800	-0.6236	-0.4568
81	7.0179	3.9438	0.6274	-1.5886	0.5410	-0.7331	-1.4631	0.4824	0.8815	-0.8012
82	8.8061	2.5449	0.1688	-1.3794	0.8642	-1.6610	-0.1758	0.1086	0.7939	0.3333
83	9.5938	1.7673	0.6127	-2.0749	2.1465	-1.1807	-0.4410	-0.0176	0.1216	-0.2603
84	11.1651	-0.6528	0.6325	-1.5462	1.1951	-0.1285	1.8618	-0.0900	-0.7552	0.6404
85	13.0217	-6.5786	-0.0673	-0.0168	1.2588	-0.4571	0.4873	-1.5325	-1.5543	-0.6717
86	16.5981	-9.9116	1.1555	0.2725	-2.2568	1.3368	-0.1520	0.5402	1.5820	0.3177

Source: The instruments were provided by T. I. Wahl, Department of Agricultural Economics, Washington State University, Pullman.

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