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Capital stocks and their user costs for West German Agriculture: A documentation

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Contents

1 Introduction

Most empirical work in agricultural economics tries to explain or to project the use of agricultural inputs and outputs. The measurement of any of these is fraught with numerous difficulties. For capital, they tend to be the worst.

- For all other input or output quantities some proxies are observed and collected in official statistics on an annual basis like purchases of fertilizer, production of grains, use of land and labour. For capital we can also observe yearly purchases of investment goods. Because these are used over several years, however, we cannot identify capital input with investments. Only some part of this year's investment purchases can be imputed to represent present capital use and there is a myriad of ways to do so.

- If this year's investments do not represent this year's capital input then the price of the capital input will not coincide with the price of investment goods. We can observe the (rental) price of using the capital stock in this year only when it is rented, but this is the exeption rather than the rule. Again, there are many ways to allocate the cost of investment goods to the years of their service live.

For partial analysis, we might be tempted to simply neglect capital. However, in a comprehensive analysis, this easy solution is no way out. For productivity analysis, production function estimation and supply side analysis in general we need at least some measure of the *quantity* of capital. If we want to explain it's movement or if we want to aggregate (types of) capital, it's price is needed as well.

The following sections decribe in detail how to compute time series for the aggregate capital stock in German agriculture and it's user cost. The resulting series are given in section 5 and for these results the paper serves as a technical documentation. For readers less interested in the German example the paper should provide as a step by step introduction into the methods involved in the computation of capital stocks in general.

2 Theoretical background

2.1 Perpetual inventory method for capital stocks

The general principles of the perpetual inventory method are well known (e.g. Kirner 1968, Behrens 1980, Ball, Matson, Somwaru 1992) and may be summarized as follows. The capital stock is composed of the assets purchased in the past. Only those assets that have not yet reached the end of their useful service life can enter the present capital stock, however. Older assets have been scrapped before (*discards*). Old assets that have not been scrapped yet probably have lower productive capacities than younger assets. Consequently, there is physical depreciation (*decay*) of each single item which must be taken into account in any measure of capital stock. Both discards and decay cause the productive capacity of older investment cohorts to be lower than that of the most recent investment cohort. The productive capacity of an investment cohort relative to the youngest one is called *cohort efficiency* in this study. We will assume that it *only* depends on age. This neglects any dependency of discards and decay on economic incentives but simplifies things considerably.

If there is an initial capital stock, the following equation summarizes the perpetual inventory method to calculate the capital stock K_t in year t:

$$K_t = \Sigma_{i=0}^{t-1} e(i)\, I_{t-i} + e(t)\, K_0 \tag{1}$$

i = age of the investment cohort

e(i) = cohort efficiency at age i [= cohe(age)][1]

I_{t-i} = investments in year t-i

For the youngest cohort, e(0) = 1. Conversely, if T is the maximum useful service live of assets of our type, e(i) = 0 for any i > T, i.e. for all cohorts completely scrapped. An example of a cohort efficiency function is shown in the following figure 1. The procedure to construct this cohort efficiency function from underlying assumptions on discards and decay will be explained later.

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1 All expressions in [special] typing refer to names used in the acompanying file [capsg.xls] which contains the initial data, the „visual basic“ program to do the calculations and the detailed results.

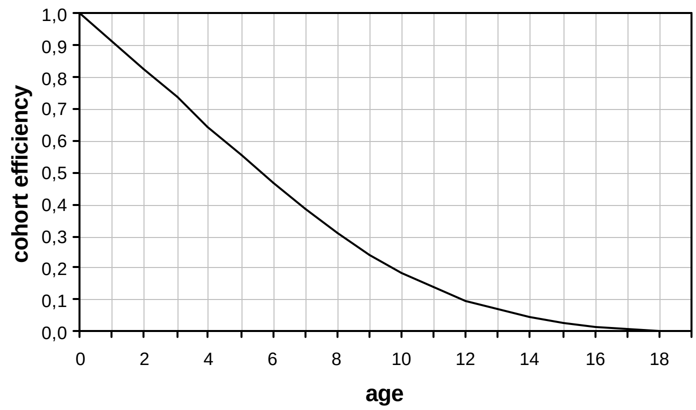

Figure 1: Cohort efficiency of machinery as a function of age (in years)

Equation (1) requires additional comments on the exact date in year t that we have in mind for the capital stock K_t. Figure 1 essentially depicts the cohort efficiency for *end year stocks*, because only then will this year's investments (i = 0) enter this year's capital stock (at the end of the year) with a weight of 1.

On the other hand, if we want the stock that is determining this year's production there are essentially 2 possibilities, beginning year stocks and mid year stocks. For *beginning year stocks* $e_b(0) = 0$, because this year's investments only become part of next year's stock (at the beginning of the year), e.g. as in Ball et al. 1993 and here in most runs. Assuming that investments usually occur on December 31, $e_b(1) = 1$, because last year's investment goods would have an age of 0 at the beginning of the year (runs[2] 1 and 9). Alternatively and more realistically, we may assume a uniform distribution of investments over the year or that investments usually occur on June 30. In this case, the average age of investment goods purchased last year will be 0,5 and $e_b(1)$ will be slightly smaller than 1 (runs 2-4, 6-8).

In the case of *mid year stocks*, this year's investments enter this year's stock of capital, but only if they are made in the first half of the year. For simplicity we will assume that investments occur on June 30. This results in an average cohort efficiency for this year's investments of $e_m(0) = 0{,}5$, because it is 0 for all investments in the second half and 1 for the first half of the year (run 5).

2 The different „runs“ of capital stock calculations conducted for this study are explained in more detail in section 5.

2.2 User cost of capital

For most purposes like productivity calculations and econometric analysis, we do not only need the stock of capital but also its price, i.e. the cost per period of using the capital stock. This user cost of capital summarizes nominal interest, asset revaluation and depreciation for the decline of physical capacity over time as it is expressed by the cohort efficiency function. In a static framework, this price per period, i.e. the rental price, will be equated to the marginal value product of capital. Given that rental prices are usually not available, the user cost of capital has to be derived from asset prices.

This derivation starts from the intertemporal profit maximization problem of the firm:

$$\max_{I_t} \sum_{t=1}^{\infty} \left\{ (1+r)^{-t} \left[\pi_t(K_t) - P_t I_t \right] \right\} \tag{2}$$

s.t. $$K_t = \sum_{i=0}^{t-1} e(i) I_{t-i} + e(t) K_0 = \sum_{i=1}^{t} e(t-i) I_i + e(t) K_0$$

where

I_t = Investment in year t (t = 1, 2, 3, ...)
e(i) = relative efficiency of a cohort of assets of age i [cohe(age)]
r = nominal interest rate (assumed constant)
P_t = asset price in year t
K_t = capital stock in year t
π_t = Momentary, restricted profit function in year t. Depends on the capital stock, other fixed factors and prices (indicated by subscript t).

The Lagrangean associated with problem (2) looks as follows:

$$L(.) = \sum_{t=1}^{\infty} \left\{ (1+r)^{-t} \left[\pi_t(K_t) - P_t I_t \right] \right\} + \sum_{t=1}^{\infty} \left\{ \lambda_t \left[\sum_{i=1}^{t} \left(e(t-i) I_i \right) + e(t) K_0 - K_t \right] \right\} \tag{3}$$

First order conditions include:

$$\frac{\partial L}{\partial K_s} = (1+r)^{-s} \frac{\partial \pi_s}{\partial K_s} - \lambda_s = 0 \quad , s = 1, 2, 3, \ldots \tag{4}$$

$$\frac{\partial L}{\partial I_\tau} = -(1+r)^{-\tau} P_\tau + \sum_{t=\tau}^{\infty} \lambda_t \, e(t-\tau) = 0 \quad , \tau = 1, 2, 3 \ldots \tag{5}$$

Using (4) in (5) yields:

$$\sum_{t=\tau}^{\infty} \left((1+r)^{\tau-t} e(t-\tau) \frac{\partial \pi_t}{\partial K_t} \right) = P_\tau \quad , \tau = 1, 2, 3, \ldots \tag{6}$$

The present value of all future increases of profits due to a unit increase of today's investment will be equated to today's price of the asset. With non-static expectations and $e(t-\tau)$ not geometric ($e(t-\tau) \neq (1-\delta)^{t-\tau}$), there is no explicit expression for $\partial\pi_1/\partial K_1$ which equals the user cost of capital in the next year. Simplification is only possible with *static expectations*. In this case P_τ and the prices implicit in $\partial\pi_\tau/\partial K_\tau$ will be constant and the optimal capital stock will also be constant, as the FOC assume the same form for all τ. Therefore, we can take the marginal profit out of the infinite sum to obtain a formula for the user cost of capital:

$$\frac{\partial\pi_\tau}{\partial K_\tau} = uc_\tau = P_\tau \Big/ \sum_{i=0}^{\infty} \left\{(1+r_\tau)^{-i} \cdot e(i)\right\}, \quad \tau = 1, 2, 3, \ldots \tag{7}$$

A time subscript has been added for the interest rate also, because it will vary from year to year in the calculations. However, as the decision makers are assumed to form static expectations, they will apply the present interest rate to all future periods. Essentially therefore, the user cost follows from multiplying the asset prices P_τ with a time varying factor $1/\Sigma\{.\}$ [= ucfac]. The development of the resulting user cost series will be dominated by the movement of the asset prices, because the variation of the interest rate will be much smaller than the variation of the strongly upward trending asset prices.

With e(i) properly chosen, equation (7) applies to beginning year (subscript b), mid year and end year stocks (subscript e). However, because $e_b(0) = e_e(-1) = 0$, $e_b(1) = e_e(0) = 1$, $e_b(2) = e_e(1)$ etc., we can rewrite the user cost of beginning year stocks as

$$uc_\tau = P_\tau \Big/ \sum_{i=1}^{\infty} \left\{(1+r_\tau)^{-i} \cdot \hat{e}(i-1)\right\} = P_\tau(1+r_\tau) \Big/ \sum_{i=0}^{\infty} \left\{(1+r_\tau)^{-i} \cdot \hat{e}(i)\right\} \tag{8}$$

if we „borrow“ the cohort efficiencies $\hat{e}(i) = e_e(i)$ applying to end year stocks from above[3]. Checking this formula for the special case of a geometric cohort efficiency, $e(t-\tau) = (1-\delta)^{t-\tau}$, the familiar result ($uc_\tau = P_\tau\,(r+\delta)$) emerges.

Equation (8) looks different from the one presented in Ball, Matson, Somwaru 1992. They start with an optimality condition stated in Coen 1975 to end up with the following expression for the user cost of capital with static expectations:

[3] Thus ucfac has been calculated slightly differently for beginning year and mid year stocks.

$$uc_\tau = r_\tau \; P_\tau \Big/ \left[1 - \sum_{i=1}^{\infty} \left\{ (1 + r_\tau)^{-i} \cdot m(i) \right\} \right] \tag{9}$$

where m(i) = e(i-1) - e(i) , the mortality at age i, r_τ is the interest rate and P_τ the asset price. The equivalence of (8) and (9) is not evident, but for a geometric cohort efficiency, we may easily check that we get the same result[4] ($uc_\tau = P_\tau\,(r+\delta)$).

3 A necessary distinction: decay, discards and depreciation

As mentioned above we are carefully distinguishing between *decay*, *discards* and *depreciation*.

Decay is the loss of productive capacity that affects each individual machine while in use. A one year old tractor will require less repair and perform better than the same tractor 5 years later. In special cases („one-hoss-shay“) is possible to imagine that there is no decay, say for a light bulb that burns for 5 years and then suddenly breaks down. In general we will observe a loss of efficiency that is related to the asset's age and service live by an *item efficiency function* ε(i, L).

Discards are the items from the capital stocks that are being scrapped, because they are out-dated, broken or require costly repairs. The end of the asset's useful service live will depend on it's type, quality, utilization, maintenance etc. Because these factors are not observed statistically, they will be modelled as causing the actual service lives of all buildings and machinery items to be dispersed around some mean service live. The aggregate efficiency of a cohort of assets then declines because an increasing proportion of the initial assets are being *scrapped* and in addition

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4 In Ball, Matson, Somwaru 1992 the user cost of capital in a certain year t has been calculated aggregating *vintage specific* user costs, where these were based on the historical interest rates when the vintage had been installed. This procedure is inconsistent with the decision problem (2) and returns to some degree to a valuation at purchase costs instead of replacement costs. The FOC (4) implies that there is a common marginal product of the total capital stock, i.e. for the contribution of each vintage $e(i)I_{t-i}$. Historical interest rates explain the value of the initial condition K_0, but they are irrelevant for today's decisions. Investment (or disinvestment) in the current year will be made to equalize $\partial\pi_1/\partial K_1$ with the current relevant user cost uc_1 which depends only on current prices (with static expectations). The initial conditions and therefore historical incentives only determine the size (and sign) of current investment to obtain the equality $\partial\pi_1/\partial K_1 = uc_1$.

because each item *decays* even before being scrapped. This decline due to the combined effects of discards and decay is expressed by the cohort efficiency function.

Depreciation is the decline of economic value of an asset that corresponds to the loss of productive capacity and which may be observed in second hand markets. Depreciation is more accelerated than the loss of productive capacity. This is most clearly seen at our extreme example of the light bulb. After 4 years, it's value has dropped to 20% of the original price, but the productive capacity is still 100% (for one more year). Because the data in this study will be used to characterize the *physical quantities* of capital to be used in production, only decay and discards are directly relevant here. On depreciation and replacement values see Ball et al. 1993.

4 Decay and item efficiency

The item efficiency function $\varepsilon(i, L)$ [eff(age,l)] relating the efficiency of each asset item to it's age i and service live L is approximated by a rectangular hyperbola:

$$\varepsilon(i,L) = (L-i) / (L-b\,i), \qquad 0 \leq i \leq L$$

$$\varepsilon(i,L) = 0, \; i > L \tag{10}$$

where b is a curvature parameter. This function incorporates many of the commonly used forms of decay as special cases. The upper limit of b is 1. This corresponds to the 'one-hoss shay' form of decay where an asset (the light bulb above) is fully productive until it reaches the end of its service life, at which point its productivity falls to zero. For $0 < b < 1$, decay occurs at an increasing rate over time. If b is zero, the function corresponds to linear physical decay, i.e. in even increments over the life of the asset. Finally, if b is negative, decay occurs most rapidly in the early years of service live corresponding to accelerated forms of decay such as geometric decay. Some possible values of b and their corresponding item efficiency functions are depicted in figure 2.

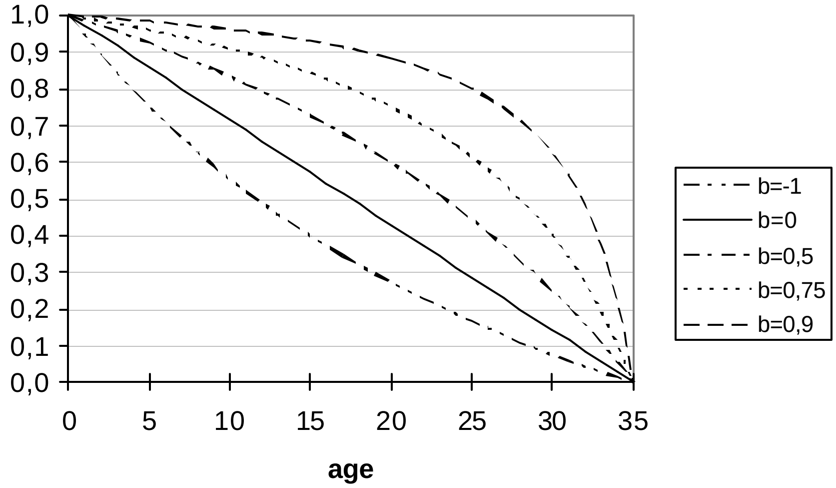

Figure 2: **Efficiency of an asset item with a 35-year service life under various forms of decay**

Anecdotal evidence suggests that decay occurs at an increasing rate over time rather than being concentrated in the first years. Especially for buildings, there will be very little decay in the first years. Therefore, the value of b has been set to 0,5 for machinery and 0,75 for buildings, following the arguments in Ball et al 1993.

In addition the relative efficiency of an asset depends on it's service life, as may be seen from figure 3.

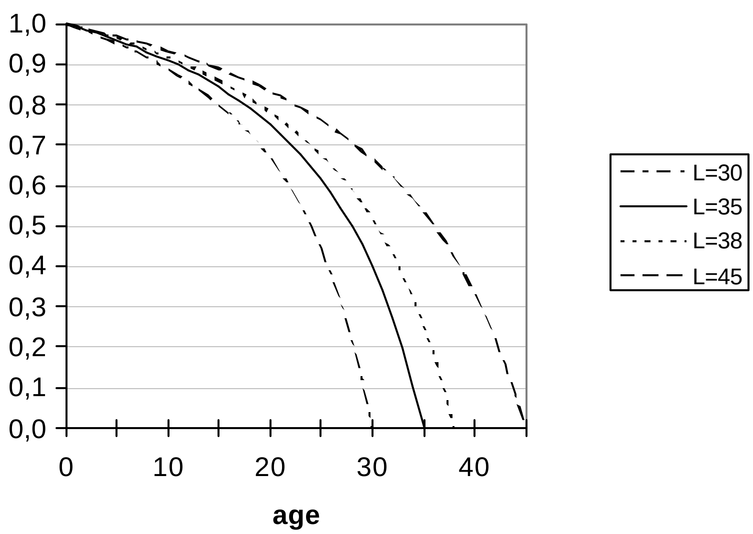

Figure 3: **Efficiency functions for different service lives L (in years, parameter b = 0,75)**

5 Discards and cohort efficiency

Although each single asset item has a single service live, for a whole cohort of machinery or building items there will exist a distribution of service lives around some mean due to differences of type, quality, utilization, maintenance etc. within a cohort. We will work with the normal distribution and, to compare with a highly skewed distribution, with the log-normal. The parameters of both are determined if the mean and the variance or standard deviation is known.

The mean service live $\bar{L}$ for machinery [lmeanm] will be taken to be 10 years in most calculations as in Behrens 1981. This value might appear low compared to older capital stock calculations in Hrubesch 1967 (13,6 years), Kirner 1968 (15 years). Recent years might have seen a shortening of service lives, however, due to a higher utilization per year and increased structural change. On the other hand, even in 1991, the average age of all tractors registered at the federal motor vehicle office (Kraftfahrtbundesamt) was still approximately 19 years (Beck 1994, p. 94). While a large part of these will be in use only occasionally and other machinery items might have shorter service lifes, the value of $\bar{L} = 9$ used in Ball et. al. 1993 appears to be somewhat small and has been increased slightly.

For buildings we assume a mean service live [lmeanb] of 35 years. Again, this is somewhat lower than in older studies, i.e. in Hrubesch 1967 (50 years), Kirner 1968 (70 years) or Behrens 1981 (50 years). More recent studies, on the other hand, also preferred shorter mean service lives. Hockmann 1988 assumed 20 years, Folmer 1989 35 years and Ball et al. 38 years for buildings. Again it is structural change that suggests to set the mean service considerably lower than a mean physical life. If some 3% of all farms close down each year, a certain number of (older) buildings will be removed from productive use even if they could be used technically and have not been scrapped. However, because the appropriate values are not known precisely, the sensitivity of results with respect to service lives has been checked (see below, run 3).

In addition to the mean, we need the variance or the standard deviation of the distribution [stab]. Here we assume that the standard deviation is 50% of the mean, as in Ball et al. 1993. In the case of the normal, for example, this implies that the service lives of 95% of all assets fall in the range $\bar{L} \pm 1{,}96*(0{,}5*\bar{L}) = [0{,}02\ \bar{L};\ 1{,}98\ \bar{L}]$ = [lmin; lmax]. The variances are even less known than the means and have been subjected to sensitivity analysis as well (run 7).

From both the normal and the log-normal distribution the lower and upper tails will be truncated such that only the range from 0,025 to 0,975 of the original cdf is retained. For the technical details in the case of the log-normal see the appendix 1. The truncation is necessary, because the data on investments do not stretch infinitely far in the past (see below) and because the normal would yield a positive probability of negative ages. The probabilities in the remaining admissible range are scaled up with a factor 1/0,95 to obtain densities that integrate to one again[5].

With the pdf at hand, the cohort efficiency function e(i) [cohe(age)] can be constructed as a weighted sum of the item efficiency functions $\varepsilon(i, L)$ for each possible service live L using the density at each service live, pdf(L), as weights. The cohort efficiency function thus reflects the decline of the average efficiency of a cohort due to decay and due to discards:

$$e(i) = \int_{L_{min}}^{L_{max}} \varepsilon(i, L)\, pdf(L)\, dL \tag{11}$$

where pdf(L) comes from a truncated normal or log-normal distribution. This is a problem of numerical integration for which different procedures are available. One of the more precise is Simpson's approach (e.g. Berck, Sydsaeter 1991, p. 39)

$$\begin{aligned} e(i) \approx \frac{h}{3}\Big\{ & \varepsilon(i, L_{min})\, pdf(L_{min}) \\ & + \sum_{s=1}^{n-1}\Big[\left(3-(-1)^s\right)\varepsilon(i, L_{min} + s\,h)\, pdf(L_{min} + s\,h)\Big] \\ & + \varepsilon(i, L_{max})\, pdf(L_{max})\Big\} \end{aligned} \tag{12}$$

where n is the (even) number of steps [steps], and h = (L_{min} - L_{max})/n is the step size [step] in the numerical integration.

The calculation of cohort efficiencies as a weighted aggregation of item efficiencies is illustrated in figure 4. For example, the top cohort efficiency for a 1 year old machine, approximately 0,90 (if $\bar{L}$ = 9, b = 0,5), is the weighted average of the top item efficiencies weighted by the pdf at the bottom of the figure. The cohort efficiencies for ages of 7 and 13 years follow analogously.

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5 In the case of the log-normal, truncation of the 2,5% lower and upper tails reduces the mean of the truncated distribution, because the log-normal is heavily skewed to the right. This „downward bias" of the mean proved to be approximately 2,5% in preliminary calculations and has been corrected by an upward scaling of the mean to be used in the log-normal.

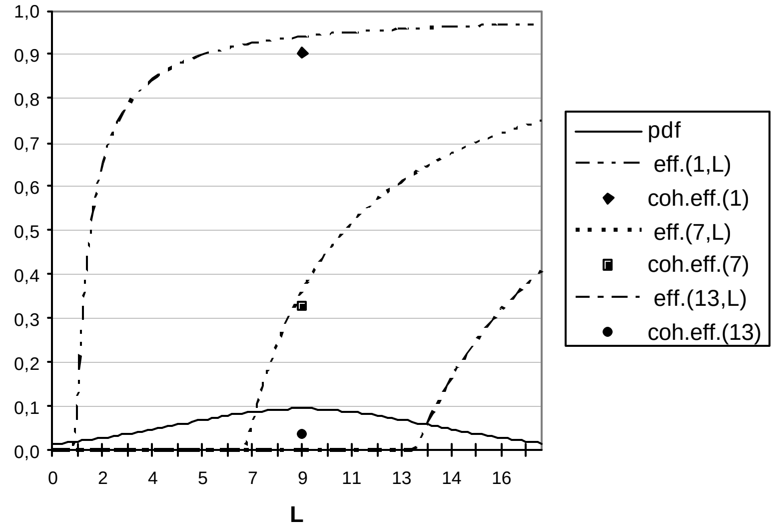


Figure 4: **Normal pdf, item efficiency and cohort efficiency as a function of service life and age for machinery (with $\bar{L}$ = 9, b = 0,5)**

The following figure 5 illustrates the effects of choosing the log-normal instead of the normal probability distribution in this kind of calculation.

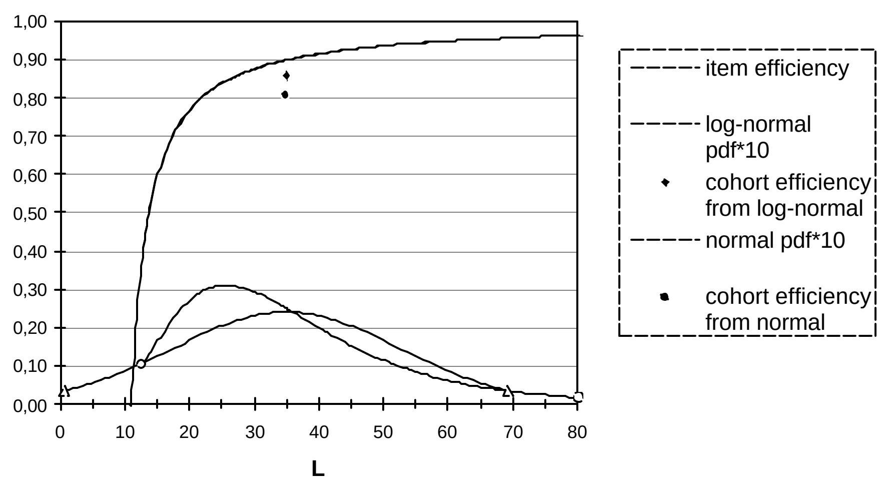


Figure 5: **Cohort efficiency for an 11 year old building ($\bar{L}$ = 35, b = 0,75) from a normal and log-normal probability distribution**

The figure shows the different shape and 2,5% cut off points for the two distributions. The log-normal yields a somewhat higher cohort efficiency for an 11 year old building than the normal (0,85 vs. 0,80) because it assigns very low (= 0 for the *truncated* log-normal) probabilities to service lives under 12,7 years. However, switching from the normal to the log-normal does not increase the cohort efficiency at all ages. The complete cohort efficiency functions for the two distributions are depicted in the following figure 6.

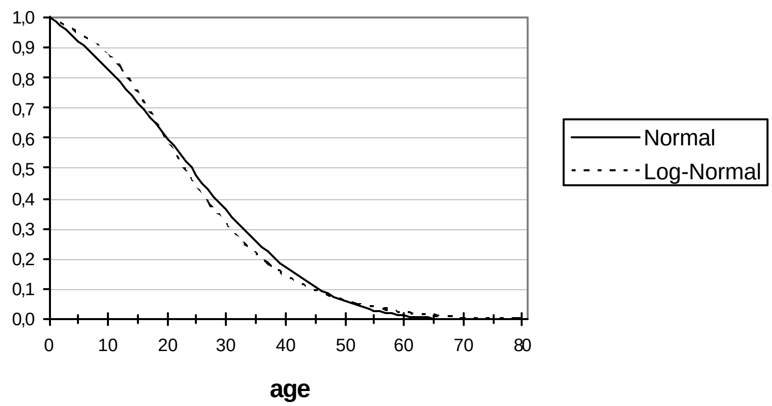


Figure 6: Cohort efficiencies for buildings $\bar{L}$ = 35, b = 0,75) based on a normal and log-normal probability distribution

As is evident from figure 6, the differences are very small. The log-normal yields smaller cohort efficiencies at ages around the mean service life and higher cohort efficiencies at high ages. From this, it is to be expected that the capital stocks and capital costs are not very sensitive to the choice of the distribution (see run 8 in section 5). Therefore the normal distribution has been retained as the standard assumption due to it's ease of interpretation.

The following figure 7 illustrates the effect of assuming a smaller dispersion of service lifes on the cohort efficiency, maintaining the normal pdf. More specifically we will assume that the standard deviation is only 39% of the mean or equivalently that 80% of all assets have service lives in the range $\bar{L} \pm 50\%*\bar{L}$. A standard deviation of 50% of the mean implies, on the contrary, that only 68% of all assets have service lives within this range.

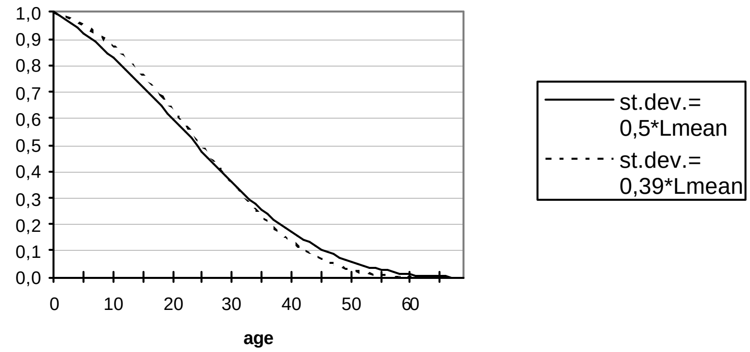


Figure 7: Cohort efficiencies for buildings ($\bar{L}$ = 35, b = 0,75) for normal distributions of L with different standard deviations

A smaller standard deviation of service lives raises the efficiency of young cohorts and lowers it for old ones because a smaller percentage of young cohorts and a higher percentage of old cohorts will be discarded, if the service lives are more concentrated around the mean. Given that these differences are small and go in opposite directions, large differences in the resulting capital stocks and capital costs would be surprising (see run 7). In view of the heterogeneity of our categories „machinery“ and „buildings“, however, the higher dispersion seems to be more appropriate and will be maintained.

Whereas these variations of the variance and skewness of the distribution do not have tremendous effects on the cohort efficiencies, any increase in the mean service life or in the parameter b raises the item efficiencies associated with each service live and consequently the cohort efficiency at *all* ages (see figures 1 and 2 above). This is illustrated for a variation of the curvature parameter in figure 8

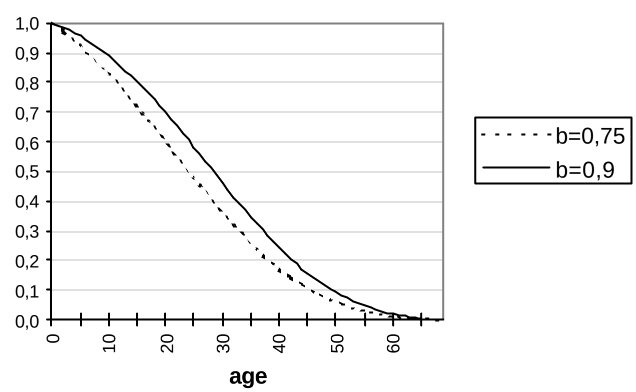


Figure 8: Cohort efficiencies for buildings with different curvature parameters ($\bar{L}$ = 35, st.dev. = 0,5$\bar{L}$, normal pdf)

Variations that raise or lower the cohort efficiencies of *all* ages will have a marked effect on the resulting capital stocks, see eq. (1). The user cost of capital, on the other hand, will vary in the opposite direction, because the cohort efficiencies enter the denominator of (7). Therefore the effect on total capital cost, i.e. the product of capital stock and user cost, is ambiguous and likely to be small (see runs 3 and 6).

6 Data and results

Investment series

Estimation of capital stocks requires annual data on gross expenditures for capital goods. To conform with the notion of quantities, the data have to be expressed in constant prices. For the calculation of user costs of capital corresponding price indices are required as well.

The principal source for these data is the Economic Accounts of Agriculture as published by Eurostat or the German ministry of agriculture. They date back until 1949. To calculate the capital stock of buildings in 1965, however, data on investments from 1896 onwards are needed, if the cohort efficiency function drops to zero only after an age of 70 years (see the previous figure 8). Of course, these data are not directly available from published statistical yearbooks. Fortunately Kirner (1968) has estimated investments in machinery and buildings for 18 sectors including agriculture in constant 1954 prices, taking into account damages in wartimes and changes in the German territory.

Because data corresponding to Kirner's were not available for all EU-9 members, Ball et al. 1993 relied on a heroic assumption used already in Behrens 1979, i.e. that investments grew linearly from a level of 0 in 1850 to the observed value in 1950. Checking this assumption with a long French series on investments in buildings, where investments were actually *declining* instead of rising, Ball et al. found that growth rates and even the level of the French capital stock series was little affected by the Behrens assumption (1993, p. 445). The reason is the decline of the cohort efficiency below 0,5 after about 25 years. Old investment cohorts do matter, but they did not seem to matter much.

For West Germany, the following table 1 shows the capital stocks resulting from the assumptions of Ball et al.[6] and the two approaches to meet the data requirements. Apart from the levels in the years 1965 and 1992 and the usual average yearly growth rates between these years, the table gives the root mean square yearly growth rates (RMS(%change)[7]) from 1965 to 1992. This indicates the average yearly growth rates where positive and negative changes do not cancel and large changes are weighted more heavily due to the squares. Summary indicators for the average yearly *deviations* of the resulting series are given at the bottom of the table. First, the root mean square deviation of the yearly *growth rates* (RMSD(%changes)[8]) indicates the average (absolute) deviation of the yearly growth rates of each approach. Second, to compare the average deviation in the *levels*, the table shows the percentage root mean square deviation of the levels (%RMSD(levels)[9]).

Table 1: **Capital stocks with the assumptions in Ball et al. 1993 using Behrens' assumption or the Kirner data**

	buildings	machinery
Kirner data:		
1965	46109	38884
mean % growth per year	*0,71%*	*0,06%*
1992	55432	39450
RMS(%change)	*1,91%*	*2,21%*
Behrens' assumption:		
1965	39939	38876
mean % growth per year	*1,23%*	*0,06%*
1992	54895	39450
RMS(%change)	*2,55%*	*2,21%*
Deviations:		
RMSD(%changes)	*0,70%*	*0,00%*
%RMSD(levels)	5,85%	0,00%

6 End of year stocks, investment at December 31, $\bar{L}$ = 38 (buildings) / 9 (machinery), standard deviation = 0,5 $\bar{L}$, normal pdf, b = 0,75 (buildings) / 0,5 (machinery), contained as "run 1" in the file [capsg.xls]. The values in table 1 differ slightly from those given in Ball et al. due to different precision in the computations and minor revisions of the underlying investment series.

7 $\text{RMS}(\%\text{change}) = \sqrt{\frac{1}{T-1}\sum_{t=2}^{T}\left(\frac{X_t - X_{t-1}}{X_{t-1}}\right)^2}$, X_t = capital stock.

8 $\text{RMSD}(\%\text{changes}) = \sqrt{\frac{1}{T-1}\sum_{t=2}^{T}\left[\left(\frac{X_t - X_{t-1}}{X_{t-1}}\right) - \left(\frac{B_t - B_{t-1}}{B_{t-1}}\right)\right]^2}$, B_t from benchmark, X_t from alternative run.

9 $\%\text{RMSD}(\text{levels}) = \sqrt{\frac{1}{T}\sum_{t=1}^{T}\left[\frac{X_t - B_t}{B_t}\right]^2}$, B_t from benchmark, X_t from alternative run.

Evidently old investment data prior to 1952 (where they differ) do not matter for machinery stocks. On the contrary, the difference in the 1965 building stock levels is sizable although the stocks do converge later. Because the building stock is more or less stagnating after the 60s (see figure 9), the resulting differences in the growth are relatively more important than in the French example reported above.

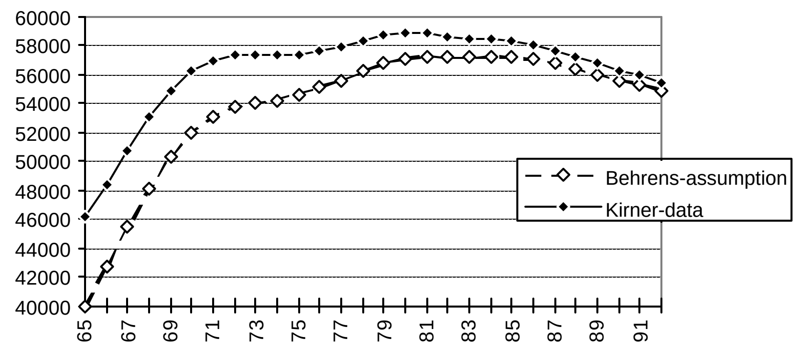


Figure 9: **Capital stocks for buildings with the parameters in Ball et al. 1993 using Behrens' assumption or the Kirner data**

When it comes to computations for EU countries with missing data the results above recommend additional effort to compile the neccessary data or to obtain more reliable assumptions than Behrens' solution.

Interest rates

The first question is whether nominal or real interest rates should be used. In Ball, Matson, Somwaru 1992 nominal interest rates have been deflated for general inflation and expected (ex ante) real interest rates have been calculated from ARIMA-forecasts. This example will not be followed here, because the explicit derivation above presupposes static expectations for prices and a *constant* interest rate, see eq. (6). In addition, it seems to be inconsistent, to use ARIMA-forecasts for one variable and static expectations for all others. Finally, correcting the interest rate for inflation is only necessary when modelling the savings decision with imperfect capital markets. The explicit model above, on the contrary, is a simple intertemporal profit maximization with a given interest rate from a perfect capital market.

The interest rates were obtained from data of the German farm accountancy network („TBS“) as published in the annual German report on agriculture (Agrarbericht, „Haupterwerbsbetriebe“).

Therefore some interest rate subsidies have been taken into account implicitly, because the interest rates were calculated as the ratio of interest actually paid to debt. A problem with this series is that these interest rates are an average for old and new debt. If interest rates for old debt are not variable, then these average interest rates will develop considerably smoother than interest rates only for new credit which conform better to the theory of static expectations. Because the advantage of using an interest series specifically relevant for agriculture was deemed more important, we retained the interest rates from the German farm accountancy network.

As an alternative source, there is an interest rate series published in the framework of the economic accounts of agriculture („LGR“). Conceptually, this is also an average rate which is consistent with series on interest paid and total debt (e.g. BML, AB 1995, MB p. 35). Because the variation of these interest rates is even lower, they have been used only in exploratory calculations.

Results

Several runs of capital stock and user cost calculations have been carried out for machinery and buildings in West German agriculture from 1965-1992. They may be found in detail in the accompanying file [capsg.xls, table kirner!]:

1 End of year stocks, *investment at December 31, $\bar{L}$ = 38 (buildings)* / 9 (machinery), standard deviation = 0,5 $\bar{L}$, normal pdf, b = 0,75 (buildings) / 0,5 (machinery),

2 End of year stocks, investment at June 30 (or uniform over the year), *$\bar{L}$ = 38 (buildings)* / 9 (machinery), standard deviation = 0,5 $\bar{L}$, normal pdf, b = 0,75 (buildings) / 0,5 (machinery),

3 End of year stocks, investment at June 30, *$\bar{L}$ = 45 (buildings) / 12 (machinery)*, standard deviation = 0,5 $\bar{L}$, normal pdf, b = 0,75 (buildings) / 0,5 (machinery),

4 End of year stocks, investment at June 30, $\bar{L}$ = 35 (buildings) / 10 (machinery), standard deviation = 0,5 $\bar{L}$, normal pdf, b = 0,75 (buildings) / 0,5 (machinery),

5 *Mid year stocks*, investment at June 30, $\bar{L}$ = 35 (buildings) / 10 (machinery), standard deviation = 0,5 $\bar{L}$, normal pdf, b = 0,75 (buildings) / 0,5 (machinery),

6 End of year stocks, investment at June 30, $\bar{L}$ = 35 (buildings) / 10 (machinery), standard deviation = 0,5 $\bar{L}$, normal pdf, *b = 0,9 (buildings) / 0,75 (machinery),*

7 End of year stocks, investment at June 30, $\bar{L}$ = 35 (buildings) / 10 (machinery), *standard deviation = 0,39 $\bar{L}$*, normal pdf, b = 0,75 (buildings) / 0,5 (machinery),

8 End of year stocks, investment at June 30, $\bar{L}$ = 35 (buildings) / 10 (machinery), standard deviation = 0,5 $\bar{L}$, *log-normal pdf,* b = 0,75 (buildings) / 0,5 (machinery),

9 End of year stocks, *investment at December 31*, $\bar{L}$ = 35 (buildings) / 10 (machinery), standard deviation = 0,5 $\bar{L}$, normal pdf, b = 0,75 (buildings) / 0,5 (machinery).

The results will be presented in comparison to a base run to show the sensitivity with respect to the assumptions. Table 2 gives the complete results for run #4 which shall serve as the benchmark. The underlying cohort efficiencies for machinery and buildings are depicted in figures 1 and (e.g.) 6 above.

Table 2: **Stocks [capi4], user costs [uci4] and total capital costs [icost4] for buildings [i=b], machinery [i=m] and the (Törnquist) aggregate capital stock in the benchmark run 4**

year	capb4	ucb4	bcost4	capm4	ucm4	mcost4	cap4	uc4	ccost4
1965	42871	0,023	974	38919	0,090	3522	10503	0,428	4495
1966	45135	0,024	1066	40978	0,093	3820	11059	0,442	4887
1967	47500	0,024	1145	42097	0,096	4029	11421	0,453	5174
1968	49746	0,023	1159	41332	0,094	3899	11377	0,445	5058
1969	51532	0,027	1389	39891	0,106	4212	11168	0,502	5601
1970	52872	0,029	1530	39890	0,108	4305	11241	0,519	5835
1971	53598	0,035	1886	39966	0,115	4613	11299	0,575	6499
1972	53891	0,039	2098	39122	0,122	4785	11149	0,617	6883
1973	53829	0,041	2213	38453	0,127	4886	11013	0,645	7099
1974	53724	0,047	2546	38970	0,138	5369	11107	0,713	7915
1975	53812	0,053	2834	38733	0,151	5859	11067	0,785	8693
1976	53970	0,052	2793	38606	0,163	6299	11052	0,823	9091
1977	54198	0,051	2775	39079	0,168	6572	11161	0,837	9347
1978	54535	0,054	2921	40220	0,175	7054	11411	0,874	9975
1979	54789	0,058	3196	41547	0,182	7580	11691	0,922	10777
1980	54862	0,068	3710	42826	0,192	8235	11945	1,000	11945
1981	54777	0,082	4469	43055	0,207	8904	11982	1,116	13373
1982	54488	0,092	5024	42419	0,226	9593	11844	1,234	14617
1983	54236	0,094	5082	41702	0,238	9905	11693	1,282	14987
1984	54091	0,088	4759	41725	0,239	9959	11687	1,259	14718
1985	53835	0,087	4709	41140	0,244	10037	11558	1,276	14746
1986	53519	0,088	4728	40546	0,250	10123	11423	1,300	14851
1987	53038	0,088	4693	39868	0,251	10012	11260	1,306	14705
1988	52494	0,087	4558	39106	0,250	9787	11076	1,295	14345
1989	51922	0,088	4563	38674	0,253	9800	10954	1,311	14363
1990	51357	0,092	4702	38872	0,260	10113	10954	1,352	14815
1991	50954	0,101	5160	39360	0,271	10664	11019	1,436	15824
1992	50363	0,111	5571	40148	0,284	11416	11125	1,527	16987

The results may be followed in detail also in the following figures 10 and 11 where the results of alternative runs are included as well.

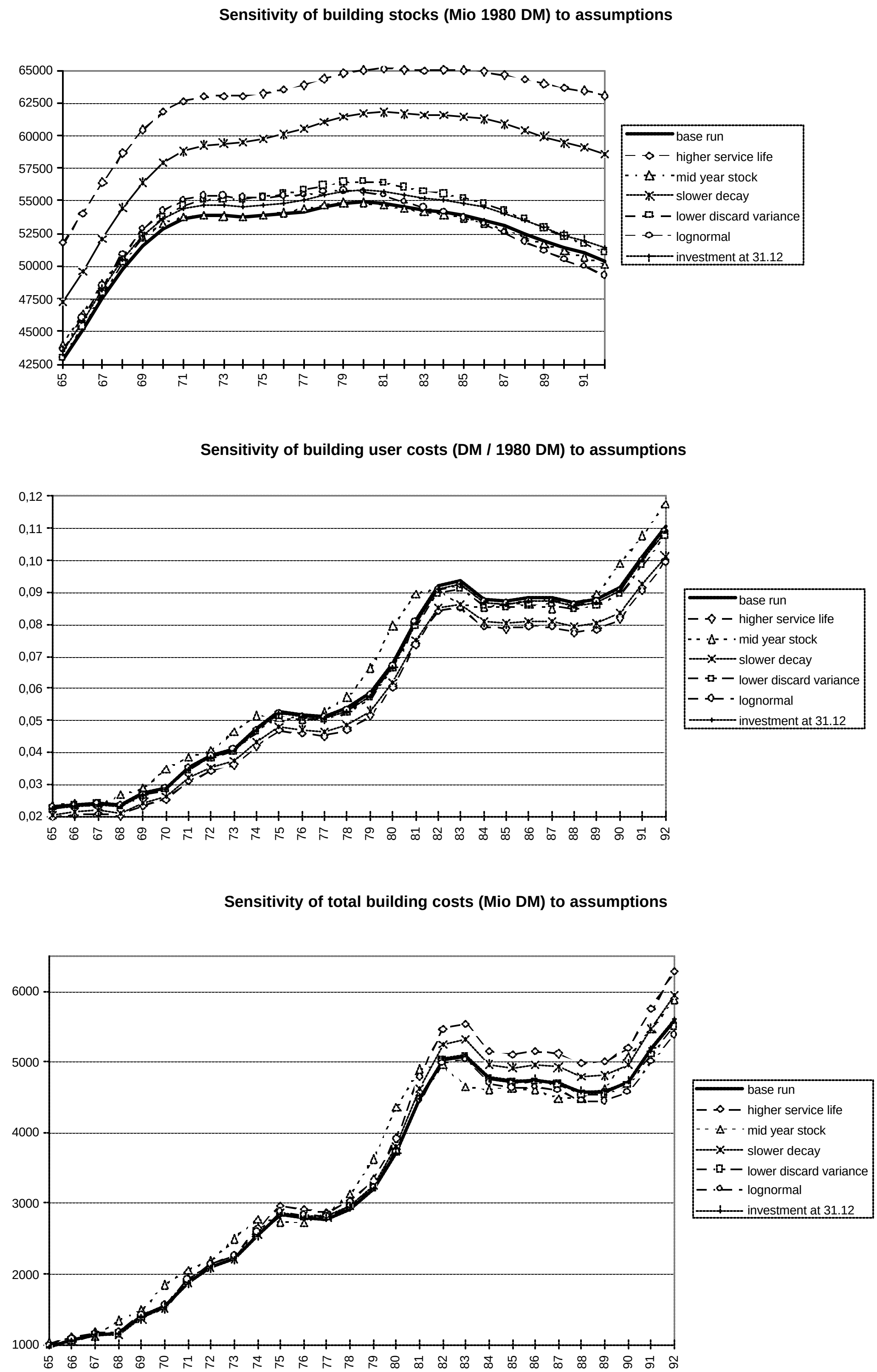
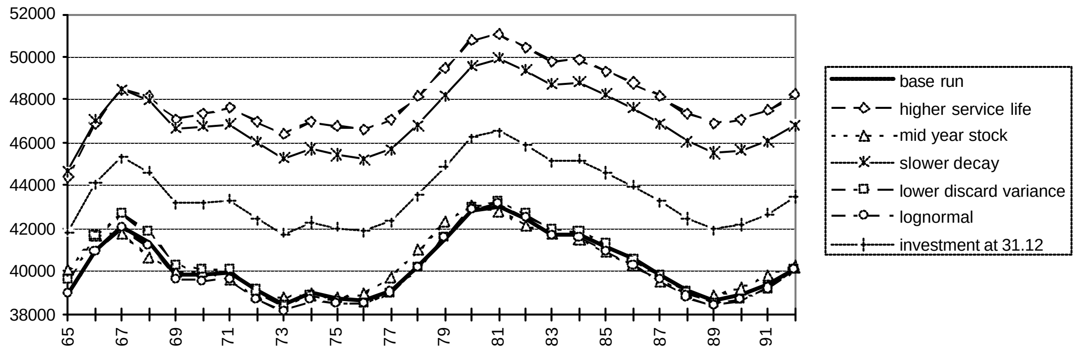
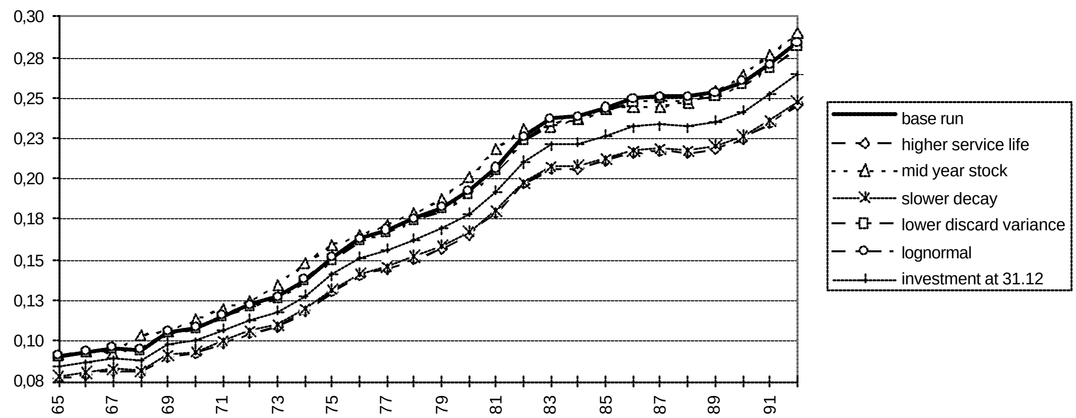


Figure 10: Stocks, user costs and total costs for buildings in the base run 4 and alternative runs

Sensitivity of machinery stocks (Mio 1980 DM) to assumptions



Sensitivity of machinery user costs (DM / 1980 DM) to assumptions



Sensitivity of total machinery costs (Mio DM) to assumptions

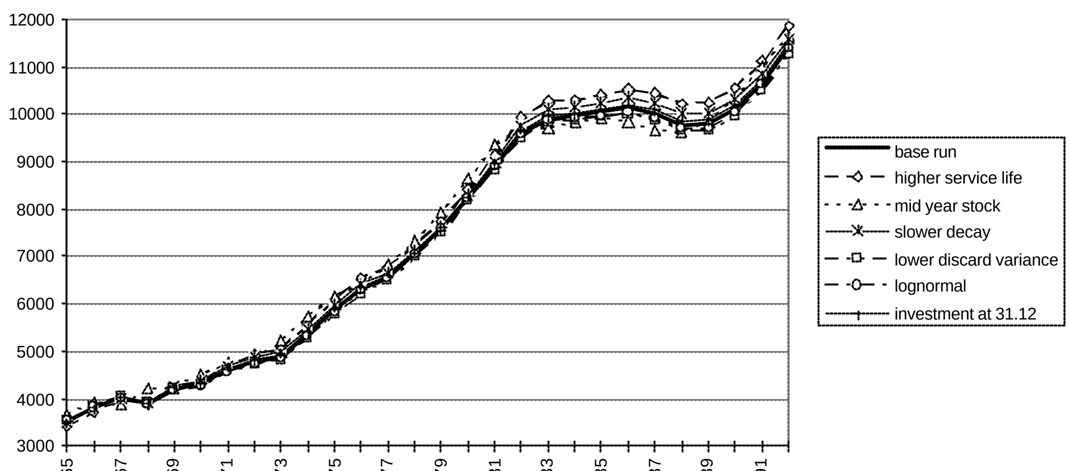


Figure 11: **Stocks, user costs and total costs for machinery in the base run 4 and alternative runs**

Both the buildings and the machinery stock show a peak around 1980 with the 1992 levels approximately equal to those at the end of the 60s, i.e. the levels are stagnating to a large extent. The machinery stocks are fluctuating more because of their considerably shorter service lifes.

The user costs are rising quickly, both for buildings and for machinery, mainly due to the rising asset prices which follow general inflation more or less. With increasing prices and limited fluctuations in quantities their product, i.e. capital costs are also increasing considerably in the period observed.

These comments apply to the results of the base run as well as for the alternative runs. Only the levels are fairly sensitive to some changes in the underlying assumptions. The differences are frequently too small to be traced in the graphical displays which illustrate more of the similarities than of the differences. Therefore the following table 3 presents the numerical results for selected years together with summary indicators of the deviations of the alternative runs from the benchmark run 4.

Table 2: **Stocks, user costs and total capital costs for buildings, machinery and aggregate capital in the base run 4 and alternative runs**

	buildings			machinery		
	stocks	user costs	total costs	stocks	user costs	total costs
Run 4: base run						
1965	42871	0,023	974	38919	0,090	3522
mean % growth per year	*0,62%*	*6,28%*	*6,94%*	*0,12%*	*4,50%*	*4,63%*
1992	50363	0,111	5571	40148	0,284	11416
RMS(% per year)	*2,04%*	*9,83%*	*10,49%*	*2,09%*	*5,54%*	*5,97%*
Run 3: higher service live						
1965	51779	0,019	1009	44398	0,077	3422
mean % growth per year	*0,76%*	*6,47%*	*7,28%*	*0,32%*	*4,57%*	*4,90%*
1992	63048	0,099	6272	48250	0,246	11870
RMSD(%changes)	*0,45%*	*0,66%*	*0,84%*	*0,50%*	*0,24%*	*0,58%*
%RMSD(levels)	20,09%	11,84%	6,82%	19,70%	14,24%	3,36%
Run 5: mid year stocks						
1965	44013	0,023	1026	40077	0,092	3676
mean % growth per year	*0,50%*	*6,42%*	*6,95%*	*0,01%*	*4,52%*	*4,53%*
1992	50083	0,117	5880	40218	0,289	11638
RMSD(%changes)	*0,25%*	*7,66%*	*7,73%*	*0,98%*	*4,01%*	*3,87%*
%RMSD(levels)	1,01%	8,32%	8,58%	1,10%	3,70%	3,96%
Run 6: slower decay						
1965	47247	0,020	963	44635	0,078	3490
mean % growth per year	*0,83%*	*6,36%*	*7,24%*	*0,18%*	*4,53%*	*4,73%*
1992	58549	0,101	5933	46818	0,248	11595
RMSD(%changes)	*0,26%*	*0,31%*	*0,48%*	*0,33%*	*0,12%*	*0,37%*
%RMSD(levels)	12,72%	9,26%	3,47%	17,09%	13,40%	1,69%
Run 7: lower discard variance						
1965	42937	0,022	956	39608	0,090	3559
mean % growth per year	*0,66%*	*6,25%*	*6,95%*	*0,04%*	*4,48%*	*4,53%*
1992	50978	0,108	5491	40038	0,281	11255
RMSD(%changes)	*0,20%*	*0,09%*	*0,20%*	*0,18%*	*0,07%*	*0,20%*
%RMSD(levels)	2,27%	2,45%	0,74%	0,67%	1,01%	1,06%
Run 8: lognormal						
1965	43614	0,023	985	38946	0,091	3526
mean % growth per year	*0,47%*	*6,25%*	*6,75%*	*0,11%*	*4,49%*	*4,61%*
1992	49283	0,109	5379	40067	0,284	11377
RMSD(%changes)	*0,25%*	*0,11%*	*0,30%*	*0,17%*	*0,03%*	*0,17%*
%RMSD(levels)	2,00%	1,11%	1,84%	0,57%	0,10%	0,62%
Run 9: investment at 31.12						
1965	43542	0,022	973	41845	0,084	3508
mean % growth per year	*0,64%*	*6,29%*	*6,97%*	*0,15%*	*4,51%*	*4,67%*
1992	51392	0,109	5605	43477	0,264	11485
RMSD(%changes)	*0,02%*	*0,03%*	*0,04%*	*0,10%*	*0,04%*	*0,11%*
%RMSD(levels)	1,75%	1,50%	0,32%	8,41%	7,33%	0,57%

Some comments may be helpful in the interpretation of the results presented above.

Run 3 with 20-30% longer service lives shows the highest differences to the benchmark run. The levels of the *building stocks* are on average 20% higher than in run 4 (there are no negative deviations, see figure 10). Even the yearly changes deviate on average (in root mean square) by 0,5 percentage points which appears to be serious compared with the mean yearly growth rate of 0,6% in the base run. However, figure 10 showed that run 3 and the base run develop more or less parallel which is more intuitive when the 0,5 percentage point deviation is measured against the root mean square growth rate of 2,0%. The *user costs* for buildings are 12% lower than in the benchmark run. Here, the yearly changes deviate by 0,7 percentage points which is absolutely higher but relative to the average yearly growth rate much lower than for the stocks. In the *total cost* for buildings the differences in the levels cancel to a large extent with an average difference of 7% remaining. The yearly changes of the total building costs, however, deviate on average slightly more than the user costs, i.e. by 0,8 percentage points. On the other hand, total buildings cost is the variable with the highest root mean square changes (above 10% per year) such that a series deviating from the benchmark by 0,8 percentage points still follows it closely. Looking at the consequences of longer service lives for machinery, we see a similar pattern with the differences in the stock and user cost levels cancelling to a large extent and the deviations in the stocks being higher than for the user costs but still appearing moderate when measured against the root mean square growth rate of 2,1%. Overall, the results show that capital stock calculations are only moderately sensitive to the choice of the service lives when the focus is on the yearly *changes* but that the *levels* depend strongly on these choices.

In **run 6** an increased parameter b reflects a slower decay, i.e. a slower loss of efficiency for each item in the first years of it's service live. Consequently the capital stocks rise and the user costs decline markedly such that the effects in run 6 are very similar to those of longer service lifes in run 3 (see already Kirner 1968, p. 21). With the simulated changes in the parameter b, the effects are smaller than for the changes of the service lives, but this depends of course also on the magnitude of the changes that have been simulated.

Run 9 shows the consequences of assumptions as to the timing of *investment* during the year. To impute an age of 0 instead of 0,5 to this year's investments for end year stocks is a convenient simplification. It amounts to a half year shift of the cohort efficiency function of the base run to the right[10], because decay and discards are postponed correspondingly. The result is, as in runs 3 and 6,

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10 The cohort efficiency for machinery shown in figure 1 corresponds already to this run 9 with investments at 31.12. The function for the base run would be located slightly to the left, therefore.

a higher stock and a lower user cost compared to the base run. Because a half year shift does not matter much for buildings with a service live of 35 years, the effects are sizable only for machinery and here only in the levels. When the percentage changes matter, even machinery is hardly affected from the simplifying assumption that investments take place on December 31.

Run 5, on the other hand, shows the consequences as to the point in time when the *capital stock* is observed. Switching to mid year stocks does not only change the weights for a given set of investments and prices that enter the calculation, but it causes this year's investments and prices to enter whereas they did not in the base run. Any change in the series will be felt about a year earlier with mid year stocks, what may be seen most clearly in the display of the user costs, as they depend *only* on the most recent data (investment good prices, see in particular figure 10). The difference of the base run and mid year stocks is thus essentially a lag in the series and this results in considerable differences in levels and root mean square growth rates, even when measured against the root mean square growth rates of the base run. For the stocks the effects of the lag are strongly dampened due to the inclusion of past investment cohorts. Overall the differences suggest that the choice between end year and mid year stocks might be crucial for econometric analysis relying on this type of data.

Reducing the variance of the service live distribution in **run 7** from 0,5 $\bar{L}$ to 0,39 $\bar{L}$ has only little consequences. This might be due to the change in the assumption being only small. On the other hand, as noted above, big changes would be surprising because a reduction in the variance raises the efficiency for young cohorts but lowers it for older cohorts. This is quite reassuring, because the dispersion parameter is one of the least well known.

The same argument applies to switching to the log-normal distribution in **run 8**. The largest difference to the base run is in the building stock levels which is still small with a root mean square average of 2%. The yearly changes of the building stocks deviate on average by only 0,25 percentage points from the benchmark run. Nevertheless, in figure 10 we recognize that the decline in the building stock from the peak in 1980 is noticeably stronger according to the lognormal assumption, i.e. even small deviations may make a difference when they do not cancel over several years.

Summing up the sensitivity analysis, we may note that moderate changes in the assumptions have only little effects on the results. Exceptions from this rule are the service lifes and decay parameters, when the levels of stocks and user costs are important and the choice between mid year and end year stocks, when the yearly changes of the costs matter.

The latter is certainly the case in any attempt to *explain* the development of the capital stock. However, this is beyond the scope of the present paper.

Summary

A step by step exposition of the perpetual inventory method for capital stock calculations and associated user costs is given. Assumptions are made for mean service lives and their distributions, the form of decay and the exact timing of variables. Using data on buildings and machinery for West German agriculture, the sensitivity of stocks, user costs and total capital costs with respect to these assumptions is checked. In this way, a technical documentation together with a full presentation of benchmark results for capital and its cost in West German agriculture is provided. At least medium term movements of the benchmark are shown to be fairly robust to assumptions.

Zusammenfassung

Die Methoden der Berechnung von Kapitalstöcken und zugehörigen Kapitalkosten werden Schritt für Schritt erläutert. Dazu gehören Annahmen über die durchschnittliche Lebensdauer, ihre Verteilung, den Verschleiß und die genauen Zeitpunkte für Investitionen und Kapitalstockmessungen. Am Beispiel der Gebäude und Maschinen in der westdeutschen Landwirtschaft werden Sensitivitätsanalysen durchgeführt. Hierdurch werden die detailliert wiedergegebenen Ergebnisse einer Referenzspezifikation zum Kapital und seinen Kosten in der westdeutschen Landwirtschaft technisch dokumentiert. Zumindest bezüglich seiner mittelfristigen Entwicklung ist der Referenzlauf weitgehend robust bezüglich der Anahmen.

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Appendix

The log-normal pdf, mean and variance are (e.g. Hawkins, Weber 1980, p. 141)

$$f(L) = \frac{1}{L\sqrt{2\pi}\sigma} e^{\left(-\frac{(\ln L-\mu)^2}{2\sigma^2}\right)} \tag{1}$$

$$\overline{L} = e^{\mu+0,5\sigma^2} \tag{2}$$

$$Var(L) = e^{2\mu+\sigma^2}(e^{\sigma^2} - 1) \tag{3}$$

If we express the standard deviation in terms of the mean (st.dev. = k $\bar{L}$, usually k=0,5) the parameters result as follows:

$$k\,\overline{L} = \sqrt{Var(L)} = e^{\mu+\sigma^2/2}(e^{\sigma^2} - 1)^{1/2} = \overline{L}\,(e^{\sigma^2} - 1)^{1/2}$$

$$\Leftrightarrow k^2 = e^{\sigma^2} - 1 \Leftrightarrow \ln(1+k^2) = \sigma^2 \tag{4}$$

$$\overline{L} = e^{\mu+0,5\sigma^2} = e^{\mu+0,5\ln(1+k^2)} = e^{\mu}(1+k^2)^{0,5}$$

$$\Leftrightarrow \overline{L}\,(1+k^2)^{-0,5} = e^{\mu} \Leftrightarrow \ln\overline{L} - 0,5\ln(1+k^2) = \mu \tag{5}$$

For the 95% cut off points we may use the fact that lnL is distributed normally, if L is distributed log-normally, i.e.

$$prob(L_{min} < L < L_{max}) = \Phi\left(\frac{\ln L_{min} - \mu}{\sigma}\right) - \Phi\left(\frac{\ln L_{max} - \mu}{\sigma}\right) = 95\%$$

where $\Phi(.)$ is the standard normal cdf, from which it follows that:

$$\ln L_{min} = \mu - 1{,}96\sigma \Leftrightarrow L_{min} = e^{\mu-1,96\sigma},\ \ \ln L_{max} = \mu + 1{,}96\sigma \Leftrightarrow L_{max} = e^{\mu+1,96\sigma} \tag{6}$$

These results where used in the computations of run 8.