Bayesian Estimation of Non-Stationary Markov Models Combining Micro and Macro Data

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Abstract: We develop a Bayesian framework for estimating non-stationary Markov models in situations where macro population data is available only on the proportion of individuals residing in each state, but micro-level sample data is available on observed transitions between states. Posterior distributions on non-stationary transition probabilities are derived from a micro-based prior and a macro-based likelihood using potentially asynchronous data observations, providing a new method for inferring transition probabilities that merges previously disparate approaches. Monte Carlo simulations demonstrate how observed micro transitions can improve the precision of posterior information. We provide an empirical application in the context of farm structural change.

Keywords: data combination; Markov process; micro and macro data; transition probabilities

JEL classification codes: C11, C81

1. Introduction

A new Bayesian framework for inferring the transition probabilities of non-stationary Markov models is developed in this paper. Non-stationary Markov models facilitate analysis of factors influencing the probability that an individual will transition between predefined states. Data used for estimating Markov models can either be panel data, where the specific movement of an individual between states is observed over time, or aggregated data, providing only the number of individuals residing in each state over time. Following Markov terminology, we refer to such panel data and aggregated data as *micro* and *macro* data, respectively. The overall objective of our approach is to combine micro and macro information into a unified and consistent methodology for estimating transition probabilities.

In recent years Markov models have been popular in the context of farm structural change analysis (Karantininis 2002, Huettel and Jongeneel 2011, Zimmermann and Heckelei 2012a, Zimmermann and Heckelei 2012b, Arfa et al. 2014). For estimation, they rely on the generalized cross-entropy (GCE) approach, proposed by Golan and Vogel (2000) and first applied in a Markov context by Karantininis (2002). The GCE approach allows including prior information in the estimation. In the Markov context, prior information is typically specified for the transition probabilities and can be based on previous studies, theory or expert knowledge. Considering prior information allows estimating ill-defined systems which is the major strength of the GCE approach. On the other hand the use of prior information is often criticized for introducing subjective prior beliefs. This criticism is addressed by Zimmermann and Heckelei (2012a) using macro data from the farm structural survey (FSS; Council Regulation (EC) No 1166/2008) in combination with micro data from the Farm accountancy network (FADN; Council Regulation (EC) No 1217/2009). The micro data is used to specify prior information on the transition probabilities in the GCE approach avoiding an otherwise rather ad hoc prior specification. Despite these contribution general shortcomings of the GCE approach persists. Particularly, the rather non-transparent way prior information is specified and used in estimation makes it difficult for the researcher and the research community to assess its influence. Further, it is not possible to specify an ignorance (or non-informative) prior for cases where no prior information is available (Heckelei et al., 2008). An additional shortcoming of the approach proposed by Zimmermann and Heckelei (2012a) is that it ignores the precision of prior information in the micro data. Thus micro and macro data is not weighted in estimation and the final results are independent of the size of the micro sample. Further, the approach requires the specification of reference distributions for residuals, including the specification of support points, which determine the signal-to-noise ratios in the Markov transition equations *a priori*. Lastly, since FSS data is only available every two to three years, the approach requires interpolating FSS macro data on a yearly basis.

Apart from the analysis of farm structural change the idea of combining micro and macro data was considered previously in the context of a medical application by Hawkins and Han (2000). They analyzed macro data obtained in repeated independent cross sectional surveys within a city district together with limited micro data obtained from respondents who were 'coincidently' interviewed in two consecutive cross sectional surveys. The behaviour under study was the benefits of an intervention program attempting to modify drug use-related behaviour, and their Markov model was a two-state process relating to awareness, or not, of the health consequences of not bleaching shared drug needles. They defined a linear model, within the Classical statistical framework, that explained the binary marginal probabilities of being in one of the two awareness states in a certain time period (based on 'standard observed proportion estimates' from aggregate data) as well as transition probabilities relating to transitions between the two states (from the micro data). Generalizations of Hawkins and Han's binary state model to multinomial transitions are conceptually possible, but the parameter dimensionality, as well as the complexity of the covariance structure and constraint set imposed by the sampling design, quickly renders their general linear model approach intractable as the number of states increase beyond two.

In contrast to the previous two Classical approaches, our Bayesian framework provides a flexible and tractable method of combining micro and macro data generating processes that is logically consistent and coherent within the tenets of the probability calculus while accommodating a relatively large number of Markov states. The rather complicated linkages between transition probabilities and observed Markov state outcomes, and the complex parametric constraints and covariance matrix structure of the combination of micro and macro data generating processes, are specified consistently as a matter of course in specifying the posterior probability distribution for the parameters of the transition equation. Moreover, the Bayesian framework allows prior information to be incorporated into the estimation of non-stationary Markov models within an established coherent probabilistic framework. In addition, the Bayesian methodology provides a natural and relatively straightforward way of combining data observations at either the macro or micro level that are asynchronous[1], which is in contrast to the methods offered heretofore. Also we devise different specifications for both ordered and unordered Markov states, which is yet another flexible feature of the method. Overall, the Bayesian approach that we present offers a tractable full posterior information approach for combining micro and macro data-based information on non-stationary transition probabilities that allows the estimation of functional relationships linking transition probabilities with their determinants.[2]

Even though our focus is on combining micro and macro data it should be pointed out that the approach is also relevant for cases when no micro data is available. In these cases the approach allows specifying an ignorance prior such that a consistent estimation of non-stationary Markov model with only macro data is possible. This is an important feature compared to the GCE approach in which each specification (even a uniform distribution) implies some form of prior information (Heckelei et al., 2008).

Despite the analysis of farm structural change which is the primary focus here the proposed approach is equally relevant for other applications for which both macro and micro data is available. One example is an analysis of voter transitions in political science. Here, macro data on the vote shares of candidates is available from official statistics, whereas micro data can be obtained from voter (transition) surveys (McCarthy and Tyan 1977, Upton 1978). Additional examples of similar data situations can be found in the context of Ecological inference problems, which are closely related to Markov processes (Wakefield 2004, Lancaster *et al.* 2006). In general the proposed approach is relevant for all situations in which the micro sample is relatively small compared to the macro data. If the micro sample is relatively large the macro sample does not contribute additional information such that an approach relaying exclusively on the micro data is sufficient.

The paper is organized as follows: First, the Bayesian framework for non-stationary Markov models is developed in section 2. Two different specifications of the transition probabilities, that of ordered and unordered Markov states, are discussed, appropriate likelihood functions and prior densities are defined, and issues relating to computational implementation are identified. Then the design and results of a Monte Carlo simulation experiment are presented in section 3 and used to assess how the inclusion of prior information affects the posterior as well as the numerical stability of the sampling algorithm, and the degree to which estimator performance is improved under different micro sample sizes for both specifications. In section 4 the methodological framework is applied empirically in the context of an analysis of farm structural change in Germany. The application demonstrates how the framework can facilitate estimation in a situation where estimation with either micro or macro data alone would suffer from several limitations. Section 5 provides conclusions and a discussion of areas for further research.

[1] By "synchronous" we mean both that observations over time occur in sequence without gaps (follow a tact) and that the micro and macro data are observed for the same time units.
[2] In their pedagogical contribution to the use of MCMC computational methodology Pelzer and Eisinga (2002) include an example of a Bayesian approach specifically designed for a two state Markov model which depends crucially on the characteristics of a Bernoulli process. The specification of prior information in their example is effectively ad-hoc, whereas our specification is fully consistent with the structure of the data generating process. Moreover, their example does not generalize to either stationary or non-stationary multinomial Markov processes.

2. Bayesian Approach for Non-Stationary Markov Models

Markov processes provide a conceptual model for the movement of individuals between a finite number of predefined states, $i = 1,...,k$, within the context of a stochastic process. The k states are mutually exclusive and exhaustive. A Markov process is characterized by a $(k \times k)$ transition probability (TP) matrix[3] $\mathbf{P}_t$. The elements P_{ijt} of $\mathbf{P}_t$ represent the probability that an individual moves from state i in time $t-1$ to j in time t. The $(k \times 1)$-vector $\mathbf{n}_t$ denotes the number of individuals in each state i at time t and evolves over time according to a (first order) Markov process

$$\mathbf{n}_t = \mathbf{P}_t' \mathbf{n}_{t-1}. \tag{1}$$

In a non-stationary Markov process, the TPs change over time periods[4] $t = 0,1,...,T$. Data used for estimating a non-stationary Markov process can either be macro or micro level. In the case of macro data, only the aggregate numbers of individuals in the states, $\mathbf{n}_t$, is observed at each time period. For micro data, the movement of each individual between states is also observed over time. Thus, the $(k \times k)$-matrix $\mathbf{N}_t$ with elements n_{ijt} representing the number of individuals that transition from state i at $t-1$ to j in t, is directly observed.

In this section we assume data observations are synchronous, as defined in footnote 1, both for ease of exposition and to be consistent with precedence in the literature. However, the proposed approach is considerably more flexible in that asynchronous data can be analyzed in a straightforward way, and in the empirical application in section 4, macro data available only every two to three years will be combined with yearly micro data. Similarly, the reverse case, where macro data has a higher temporal resolution than the micro data, can be considered as well.

The structural specification of the TP matrix $\mathbf{P}_t$ depends on the underlying behavioural model. In the following subsection we review TP matrix specifications corresponding to ordered as well as unordered Markov states to define notation and establish the foundation for the definition of the posterior. Then the data likelihood function, representing the macro data, and a prior density, representing the micro data are defined and combined into the posterior distribution for the TPs.[5] The last subsection presents computational methodology relating to the use of the posterior distribution for inference purposes.

Specification of the Transition Probability Matrix

For appropriate specification of the TPs, the nature of the relationship between Markov states need to be considered, and we discuss two different behavioural models that differentiate between ordered and unordered Markov states. We argue that for ordered

[3]Bold letters are used for vectors or matrices.
[4]Depending on the problem context, one could also consider only two time periods observed over various regions, or a combination of multiple time and regional observations.
[5] In his dissertation, Rosenqvist (1986) introduces the conceptual rudiments of combining micro and macro data in a prior-likelihood framework. However, the analysis was restricted to stationary processes with synchronous observations and the micro and macro data observations were assumed to be disjoint. Our Bayesian framework is not constraint by any of these assumptions and moreover, we provide a tractable empirical method of implementation.

Markov states the ordered logit model is superior to the more common multinomial logit model with respect to both model assumptions and from a computational point of view.

In cases where the states of the Markov process are unordered, the multinomial logit model is a suitable specification for the TPs[6]. The specification based on the multinomial logit model assumes that the transition of individuals between different states can be represented by a random utility model. The utility that would accrue to individual l upon moving from state i in $t-1$ to j in t is denoted as $U_{ijtl}=V_{ijt}+\varepsilon_{ijtl}$, where the deterministic component of utility is specified as $V_{ijt}=\mathbf{z}'_{t-1}\mathbf{b}_{ij}$, with $\mathbf{z}_{t-1}$ being a vector of lagged exogenous variables. The deterministic part varies only over time and not over individuals because aggregated (macro) data is considered. Consequently, the deterministic component of utility reflects exogenous variables that affect the utility of all individuals alike. The random error ε_{ijtl} varies over time and individuals. It is assumed that an individual chooses a transition that maximizes her utility U_{ijtl}. The assumption that ε_{ijtl} are *iid* random draws from a Gumbel distribution result in a multinomial logit specification for each row of $\mathbf{P}_t$.

If the Markov states are ordered, an ordered choice model is an appropriate specification for the underlying behavioral model. In this case it is assumed that there exists an unobserved continuous latent variable Y^*_{itl} for each individual l that determines the outcome of the observed variable Y_{itl} according to

$$Y_{itl}=j \quad \text{if} \quad c_{j-1}<Y^*_{itl}\leq c_j \quad \forall\, i,j=1,\ldots,k \tag{2}$$

where the c_j's are the thresholds for each Markov state, with $c_o\equiv-\infty$ *and* $c_k\equiv\infty$. The index i indicates that an individual was in state i at $t-1$. The unobserved latent variable Y^*_{itl} consists of a deterministic part $\mathbf{z}'_{t-1}\boldsymbol{\beta}_i$ plus a random part ε^*_{itl}. The vector of unknown parameters $\boldsymbol{\beta}_i$ are allowed to differ between the k different states in $t-1$. As in the preceding multinomial logit model, the deterministic part varies over time but not over individuals. Assuming that ε^*_{it} are *iid* random draws from a logistic distribution[7] results in an ordered logit model for each row of $\mathbf{P}_t$.

One important difference between the ordered logit and the multinomial logit model is that only one error term, instead of one error term for each alternative, is considered for each individual. This implies that the assumption of 'Independence of Irrelevant Alternatives' (IIA) does not apply to the ordered logit model. This is more appropriate whenever the alternatives are ordered since in this case it can be expected that the error associated with one state is more similar to the error of an alternative close to it than to an alternative further away (Train 2009). Also from a computational point of view, the ordered logit specification is often

[6] A multinomial probit model could be an appropriate alternative for the error structure specification, but is left for future work.

[7] Assuming that the ε^*_{it} are random draws from a normal distribution would result in a probit (see footnote 6).

preferable since only $k n_z + (k-2)k$ parameters[8] need to be estimated, as compared to $k(k-1)n_z$ parameters for the multinomial logit model.

A further advantage of the ordered choice model is that the interpretation of the latent variable is often straightforward. For example, in the case of farm structural change noted in the introduction, where Markov states refer to firm size classes, the latent variable can be interpreted directly as firm size (see section 4). In the medical context where classes refer to different stages of illness, the latent variable can be interpreted as the degree of illness. However, the decision between an ordered and unordered choice model is not always straightforward and can depend on the problem context as well as decision makers' behavioural characteristics. In the voter transition example, one could regard the candidates as unordered choices, but alternatively one could also argue that they are ordered according to a one-dimensional political spectrum ('right' to 'left'), in which case both models have justification and the choice between the two must be guided by additional theoretical and/or substantive behavioural arguments.

Posterior

The posterior is defined as the joint density of a micro data prior and a macro data likelihood. Since micro and macro data are interdependent, the likelihood is the conditional density of the macro data given the micro data. The prior density represents information derived from a sample of micro observations on state transitions. It should be pointed out that the distinction between prior and likelihood is somehow artificial. Both are likelihood specification representing two different data sets. Also they are sampled at the same time which usually distinguishes prior and likelihood information. The labeling is thus more a convention and is motivated from the works in the context of the entropy estimation by Zimmermann and Heckelei (2012a), mentioned above, using micro data to specify the support and prior densities in an entropy estimation based on macro data.

The foundation for the likelihood function is provided by the first-order non-stationary Markov process proposed by MacRae (1977). For the specification of a macro data based likelihood function MacRae (1977) points out that the nature of the likelihood specification depends critically on whether the state proportions, $\mathbf{x}_t$, are observed over time for the entire population of size N, which she refers to as *perfect observations,* or whether the state proportions, $\mathbf{y}_t$, are only a random sample of size $M_t < N$ drawn and observed at each time period, referred to as *imperfect observations*. In the case of perfect observations the distribution of $\mathbf{x}_t$ is fully characterized by $\mathbf{x}_{t-1}$. However, for imperfect observations the distribution of $\mathbf{y}_t$ also depends on earlier observations, $y_{t-2},...,y_0$, which provide additional information on y_t. For the latter case MacRae (1977) proposed a limited information likelihood approach which is appropriate whenever macro data is available for only a sample of the population. In the following, we focus on the case of perfect observations, i.e., a census type of macro data set, which characterizes the type of data available in our empirical application provided in section 4.

[8] If a constant is included and c_1 is normalized to zero $(k-2)k$ cut points need to be estimated in addition to one parameter for each explanatory variable and state ($k n_z$) .

MacRae (1977) shows that in the case of perfect observations, the state proportions are distributed as a weighted sum of independent multinomial random variables with probabilities equal to the corresponding rows in $\mathbf{P}_t$ and weights equal to the state proportions in $t-1$. The resulting likelihood function is given by

$$L\left(\boldsymbol{\beta}\middle|\mathbf{n}_0,\mathbf{n}_1,\ldots,\mathbf{n}_T\right)=\prod_{t=1}^{T}\sum_{\mathbf{H}_t\in\mathbb{H}_t}\prod_{i=1}^{k}\left(n_{i,t-1}!\right)\left(\prod_{j=1}^{k}P_{ijt}^{\eta_{ijt}}/\eta_{ijt}!\right). \tag{3}$$

The n_{it}'s are the elements of the data vector $\mathbf{n}_t$. The matrix $\mathbf{H}_t$ is of dimension $(k\times k)$ and has entries η_{ijt} denoting the (unobserved) number of individuals transitioning from state i at time $t-1$ to state j at time t. The summation involving $\mathbf{H}_t$ in likelihood expression (3) is over the set $\mathbb{H}_t$ of all matrices $\mathbf{H}_t$ having rows that sum to corresponding elements in $\mathbf{n}_{t-1}$ and columns that sum to the corresponding entries in $\mathbf{n}_t$, so that

$$\mathbb{H}_t=\left\{\mathbf{H}_t\middle|\mathbf{1}_k'\mathbf{H}_t=\mathbf{n}_t',\ \mathbf{H}_t\mathbf{1}_k=\mathbf{n}_{t-1}\right\}, \tag{4}$$

with $\mathbf{1}_k$ being a $(k\times 1)$ vector of ones. The set of matrices represented by $\mathbb{H}_t$ is the collection of all conceptually possible outcomes of between-states transition numbers when moving from observed state distribution $\mathbf{n}_{t-1}$ in time $t-1$ to the observed state distribution $\mathbf{n}_t$ in time t. With micro data available we observe that some transitions have occurred at the micro level. Let $\mathbf{N}_t^*$ denote the micro data i.e. a matrix of observed transitions with n_{ijt}^* being the number of state *i*-type units in time $t-1$ that we observed to be state *j*-type unites in time t. The likelihood of the event of moving from $\mathbf{n}_{t-1}$ to $\mathbf{n}_t$ changes given that certain ways of transitioning to achieve $\mathbf{n}_t$ are ruled_out by the $\mathbf{N}_t^*$ observations. Particularly, the set of all possible combination is now defined as

$$\mathbb{H}_t^*=\left\{\mathbf{H}_t:\mathbf{1}_s'\mathbf{H}_t=\mathbf{n}_t',\mathbf{H}_t\mathbf{1}_s=\mathbf{n}_{t-1}\text{ and }\mathbf{H}_t\geq\mathbf{N}_t^*\right\} \tag{5}$$

such that the likelihood becomes

$$L\left(\boldsymbol{\beta}\middle|\mathbf{n}_0,\mathbf{n}_1,\ldots,\mathbf{n}_T;\mathbf{N}_1^*,\ldots,\mathbf{N}_T^*\right)=\prod_{t=1}^{T}\sum_{\mathbf{H}_t\in\mathbb{H}_t^*}\prod_{i=1}^{k}\left(n_{i,t-1}!\right)\left(\prod_{j=1}^{k}P_{ijt}^{\eta_{ijt}}/\eta_{ijt}!\right) \tag{6}$$

The number of elements in set $\mathbb{H}_t$ or $\mathbb{H}_t^*$ increases exponentially with the number of states, making the implementation of expression (3) or (6) for larger samples challenging (or intractable) from a computational point of view. For example, in the case of only 3 states and 200 observations, there are over 2.5 million combinations of (3×3)-matrices possible if approximately the same number of individuals reside in each of the three states. For the unconditional likelihood (3) this dimensionality problem can be approached using a large sample approximation that avoids the computation of the set $\mathbb{H}_t$ (see Hawkes 1969 and

Brown and Payne 1986). The large sample approximation used the property that the multinomial distribution can be approximated with a multivariate normal distribution in large samples. In our case each i-th row $\mathbf{H}_{it}$ of $\mathbf{H}_t$ is multinomial with size $n_{i,t-1}$ over $1,...,k$ categories. If $n_{i,t-1}$ is large $\mathbf{H}'_{it}$ is approximately multivariate normal with mean $\boldsymbol{\mu}_i = \mathbf{P}^*_{it} n_{i,t-1}$, where $\mathbf{P}^*_{it}$ denotes the i row of $\mathbf{P}_t$ without the element of the last column, and covariance matrix $\mathbf{V}_i = n_{i,t-1}\left[diag\left(\mathbf{P}'^*_{it}\right) - \mathbf{P}'^*_{it}\mathbf{P}^*_{it}\right]$, where $diag(\cdot)$ denotes a square matrix with the argument vector as the main diagonal and zero off-diagonal elements. Since transitions between observations are independent, each row of $\mathbf{H}_t$ is independent and the probability of $\mathbf{H}_t$ is approximately equal to a multivariate normal random $k(k-1)\times 1$ vector $M_t = \left[\mathbf{H}^*_{1t} .... \mathbf{H}^*_{kt}\right]'$ with mean $\boldsymbol{\mu} = [\boldsymbol{\mu}'_1 ... \boldsymbol{\mu}'_k]'$ and variance

$$\mathbf{V} = \begin{bmatrix} \mathbf{V}_1 & 0 & \dots & 0 \\ 0 & \mathbf{V}_2 & \dots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \dots & \mathbf{V}_k \end{bmatrix}. \tag{7}$$

Defining $B = \left[\mathbf{I}^*_1 ... \mathbf{I}^*_k\right]$, with $\mathbf{I}^*_i$ being an identity matrix of size $k-1$, we have $\mathbf{BM}_t = \mathbf{n}_t$. Using that each linear transformation of a multivariate normal random variable is also multivariate normal it follows that $\mathbf{n}_t$ is multivariate normal with mean $\mathbf{B}\boldsymbol{\mu} = \mathbf{P}^*_t \mathbf{n}^*_{t-1}$ and variance

$$\mathbf{BVB}' = diag\left(\mathbf{P}'^*_t \mathbf{n}^*_{t-1}\right) - \mathbf{P}'^*_t \mathbf{n}^*_{t-1} \mathbf{P}^*_t = \boldsymbol{\Gamma}, \tag{7}$$

where $\mathbf{P}^*_t$ and $\mathbf{n}^*_t$ is equal to $\mathbf{P}_t$ and $\mathbf{n}_t$ without the last column and row, respectively. Therefore, the probability of $\mathbf{n}_t$ given $\mathbf{n}_{t-1}$ can be approximated by a normal density such that $P\left(\mathbf{n}_t \middle| \mathbf{n}_{t-1}\right) \approx \phi\left(\mathbf{n}_t; \mathbf{P}^*_t \mathbf{n}_{t-1}, \boldsymbol{\Gamma}\right)$. From this it follows that (3) can be approximated by a large sample log-likelihood, L_{la}, given by

$$\begin{aligned} & L_a\left(\boldsymbol{\beta} \middle| \mathbf{n}_0, \mathbf{n}_1, ..., \mathbf{n}_T\right) = \\ & \sum_{t=1}^{T} -0.5\left(\log\left|\boldsymbol{\Gamma}_t\right| + \left(\mathbf{n}^*_t - \mathbf{P}^{*\prime}_t \mathbf{n}_{t-1}\right)' \left(\boldsymbol{\Gamma}_t\right)^{-1} \left(\mathbf{n}^*_t - \mathbf{P}^{*\prime}_t \mathbf{n}_{t-1}\right)\right). \end{aligned} \tag{8}$$

When considering the micro observations, $\mathbf{H}_{it}$ is still multinomial with size $n_{i,t-1}$ over $1,...,k$ categories except that the constraint $\mathbf{H}_{it} \geq \mathbf{N}^*_{it}$ need to be considered. As argued above the approach is intended for situation in which the micro data is only available for a fraction of the observation in the macro data. In these situations the limits imposed by $\mathbf{H}_{it} \geq \mathbf{N}^*_{it}$ are hardly binding such that the approximation of $\mathbf{H}_t$ by a multivariate normal remains valid. The validity of this large sample approximation is assessed in the Monte Carlo simulations considering different sizes of the micro sample.

The specification of the prior density $p(\boldsymbol{\beta})$, considers the underlying sampling distribution of the micro observations. Recall that n_{it} is the number of individuals that were in state i at time t, let $\mathbf{X}_t^i$ be the vector of shares across states in t for individuals who were in state i in $t-1$, and let $\mathbf{P}_{it}$ be the i-th row of $\mathbf{P}_t$. The propensity of each individual in the micro sample to transit between states is in accordance with the appropriate elements of $\mathbf{P}_t$. Analogous to the case of macro data, the distribution across states in t of individuals who were in state i in $t-1$ is multinomial around mean $\mathbf{P}_{it}$ with size n_{it}. The observed number of individuals in each of the k states in t, n_{it}, $i=1,...,k$, is then the corresponding weighted sum of vectors $\mathbf{X}_t^i$, $i=1,...,k$. Therefore, the prior density can be represented as a likelihood similar to (3), except that now information about the individual transitions n_{ijt} is available, making the summation over the set $\mathbb{H}_t$ unnecessary because the actual transitions are observed. Hence the likelihood simplifies to

$$p(\boldsymbol{\beta}) = L(\boldsymbol{\beta}|\mathbf{N}_1,...,\mathbf{N}_T) = \prod_{t=1}^{T}\prod_{i=1}^{k}(n_{i,t-1}!)\left(\prod_{j=1}^{k}\mathbf{P}_{ijt}^{\;n_{ijt}} / n_{ijt}!\right), \tag{9}$$

where the $(k \times k)$-matrix $\mathbf{N}_t$ has elements n_{ijt} representing the number of individuals that transition from state i at $t-1$ to j in t. We emphasize that for the case of aggregated data discussed above, the distribution of $\mathbf{n}_t$ differs between imperfect and perfect observations, while for micro observations, this distinction does not apply. In the latter case, the distribution of $\mathbf{x}_t$ is fully characterized by $\mathbf{x}_{t-1}$ regardless of whether a sample or the entire population is observed. The fundamental difference is that in the case of micro observations, individuals in the sample in time period t are all the same as in $t-1$ which is usually not the case for macro data. Consequently, information earlier than $\mathbf{x}_{t-1}$ contains no additional information about $\mathbf{x}_t$.

Computational Implementation

In order to conduct inference in the model depicted above, integrating and/or taking expectations based on the posterior density $h(\boldsymbol{\beta}|\mathbf{d}) \propto L(\boldsymbol{\beta}|\mathbf{n}_0,\mathbf{n}_1,...,\mathbf{n}_T;\mathbf{N}_1^*,...,\mathbf{N}_T^*)\,p(\boldsymbol{\beta})$ or on its approximation $h_a(\boldsymbol{\beta}|\mathbf{d}) \propto L_a(\boldsymbol{\beta}|\mathbf{n}_0,\mathbf{n}_1,...,\mathbf{n}_T)\,p(\boldsymbol{\beta})$ is required. An analytical approach to such computations is generally intractable. Instead a Monte Carlo integration approach is implemented based on sampling from the posterior density via a Metropolis Hastings (MH) algorithm.[9] For our purposes, we evaluate the optimal Bayesian estimator under quadratic

[9] An interesting alterative to the simple random walk MH sample would be the development of a data augmentation sample algorithm, in the spirit of Albert and Chib (1993), for a non-stationary Markov model using aggregated data. Our first implementation of such an algorithm, building on Musalem et al. (2009) who proposed a concept to consider aggregated data in an simple ordered logit model, suffered, however, form slow convergence problems. Convergence problems are known for the Albert and Chib (1993) algorithm and could be overcome using alternatives such as those proposed by Frühwirth-Schnatter and Frühwirth (2007) or Scott (2011). These algorithms, however, focus on simple multinomial logit models and are not directly transferable to the Markov case using aggregated data.

loss, the posterior mean, by calculating the mean of an *iid* sample from $h(\boldsymbol{\beta}|\mathbf{d})$ for sufficiently large sample sizes.

Specifically, a random walk MH algorithm with a multivariate normal generating density is employed.[10] The variance of the proposal density is adjusted such that an acceptance rate in the interval $[.2, .3]$ is obtained. In cases where the number of parameters to be estimated is large, a 'Block-at-a-Time' algorithm proposed by Chib and Greenberg (1995) is employed in which the parameters to be estimated are divided into blocks.

3. Monte Carlo Simulation of Prior Information Effects

In this section we analyze the influence of prior information, in the form of a sample of micro observations, on the posterior distribution and associated estimators' performance as well as on the behaviour of the sampling algorithm. Based on an underlying population of $n_{ind} = 10,000$ individuals, four different scenarios are considered regarding the availability of prior information, including a case of no micro observations, and micro samples of n = 100, 500, and 1000. The scenarios are further distinguished by the number of Markov states ($k = 3,4,5$). Data is generated for $T = 100$ time periods and $n_z = 6$ explanatory variables including a constant. All simulations are undertaken for a Markov model based on either the multinomial logit specification or the ordered logit specification discussed above, and are performed using Aptech's GAUSS™ 11.

Data Generating Process

The data generating process distinguishes between the two different behavioural models, based on the multinomial logit and ordered logit specification discussed in section 2. In both cases the parameterization is chosen so that the deterministic part constitutes roughly one third of the model's total variation. Furthermore, in both cases n_{ind} individuals are considered that transition over time between the k states in accordance with the underlying behavioural model. The initial state of each individual in $t = 1$ is randomly chosen with probability equal to $u_i \ \forall i = 1,...,k$, where the probability is the same for all individuals and given by $u_i = \tilde{u}_i \Big/ \sum_{h=1}^{k} \tilde{u}_h$ with $\tilde{u}_i \sim iid\ \mathcal{U}(0,1)$, where $\mathcal{U}(a,b)$ denotes the continuous uniform distribution on the interval *a* to *b*.

In the multinomial logit model each individual l chooses the state of the next period based on the utility, U_{ijtl}, associated with a specific transition from state i in $t-1$ to j in t. The utility $U_{ijtl} = V_{ijt} + \varepsilon_{ijtl}$ consists of a deterministic part $V_{ijt} = \mathbf{z}'_{t-1}\mathbf{b}_{ij}$ and an individual random part ε_{ijtl} and is generated by drawing the elements of the (lagged) exogenous variables $\mathbf{z}_{t-1}$ from $\mathcal{N}(1,4)$ and the elements of the $(n_z \times 1)$ 'true' parameter vectors $\mathbf{b}_{ij}$ from $\mathcal{U}(-1,1)$. Since only differences in utilities are relevant, the parameters of the last

[10] To mitigate computer overflow problems the Metropolis acceptance ration is calculated as
$\alpha\left(\boldsymbol{\beta}^{(r)}, \boldsymbol{\beta}^{can}\right) = \min\left[\exp\left(\ln h\left(\boldsymbol{\beta}^{can}\middle|\mathbf{d}\right) - \ln h\left(\boldsymbol{\beta}^{(r)}\middle|\mathbf{d}\right)\right), 1\right]$.

alternative are set to zero, $\mathbf{b}_{ik} = \mathbf{0} \; \forall \; i = 1,...,k$, in order to identify the model. To obtain a logit model, the ε_{ijtl} are drawn from a Gumbel (type I extreme value) distribution, specified by $F_g\left(\varepsilon_{ijtl};0,3\right) = \exp\left(-e^{-\varepsilon_{ijtl}/3}\right)$. In each time period an individual chooses the transition that maximizes utility, moving from state i in $t-1$ to state j in t if $U_{ijtl} = Max\left(U_{i1tl}, U_{i2tl}, ..., U_{iktl}\right)$.

For the ordered logit model, the transition between states is based on a latent index value $Y_{itl}^{*} = \mathbf{z}_{t-1}^{\prime}\boldsymbol{\beta}_i + \varepsilon_{itl}^{*}$ consisting of a deterministic part $\mathbf{z}_{t-1}^{\prime}\boldsymbol{\beta}_i$ and a random part ε_{itl}^{*}. The index value is generated by drawing the elements of the (lagged) exogenous variables $\mathbf{z}_{t-1}$ from $\mathcal{N}\left(1,4\right)$ and the elements of the $\left(n_z \times 1\right)$ *true* parameter vectors $\boldsymbol{\beta}_i$ from $\mathcal{U}\left(-1,1\right)$. The random errors ε_{itl}^{*} are *iid* random draws from a logistic distribution, specified by $F_l\left(\varepsilon_{itl}^{*};0,2.3\right) = \left(1+\exp\left(-\varepsilon_{itl}^{*}/2.3\right)\right)^{-1}$. The latent index value determines the outcome of Y_{itl} for each individual in each time period according to (2).

Using the above sampling design a micro dataset for n_{ind} individuals and T time periods is obtained for both the multinomial logit and the ordered logit specification, and represents the full population of individuals under study. For the specification of the prior density, random samples of size 100, 500, and 1000 are drawn without replacement from these micro datasets. The population is transformed into macro datasets by summing up the number of individuals in each state in each time period.

In order to avoid dependency of the results on a specific set of parameters, $n_{true} = 10$ *true* models are generated using the data generating process. For each of the n_{true} true models the process is repeated $n_{rep} = 20$ times with the same parameters, but with new draws of the random errors ε_{ijtl} or ε_{itl}^{*} in each repetition.

Performance Measures

The influence of prior information is assessed by a comparison of measures characterizing features of the posterior density, including performance of the posterior mean of the density, representing the minimum quadratic risk estimate of $\boldsymbol{\beta}$. The effect of prior information on the numerical stability of the sampling algorithm is also analyzed. For the Monte Carlo simulation a fixed burn-in period and a fixed sample size is employed for the MH sampler. Even though appropriate burn-in periods and sample sizes are found using graphical measures in trial runs for each scenario and resulted in substantially large burn-in periods, it still cannot be guaranteed that the MH sample will converge correctly for every simulation run. Therefore, Box-Whisker-Plots are employed to detect outliers among the sum of squared errors of the $n_{true}n_{rep}$ simulations as an indication that the MH sample had not converged appropriately. Measures characterizing the posterior density and performance measures relating to the estimator are then calculated based on only those runs that were not designated as outliers.

The effect of prior information on the spread of the posterior is assessed based on posterior variances, and is calculated on the basis of the posterior sample outcomes. The total variance of the posterior density is calculated by summing over the posterior variances of all n_z parameters for each run, and then the mean over all $n_{true}n_{rep}$ simulation runs (outliers excluded) is calculated to obtain one scalar value measure of the total variance.

The analysis of the influence of prior information on the Bayes estimator (posterior mean) is based on the mean square error (MSE) criterion, calculated as the mean sum of squared errors between estimated and true parameter values, where the mean is calculated over all of the $n_{true}n_{rep}$ simulation runs not detected as outliers. The MSE is further decomposed into variance and bias components, where the squared bias is again summed over all parameters. The distribution of the sum of squared errors together with the number of outliers detected for each scenario provides an assessment of the numerical stability of the MH sampler, and the effects of prior information on that numerical stability.

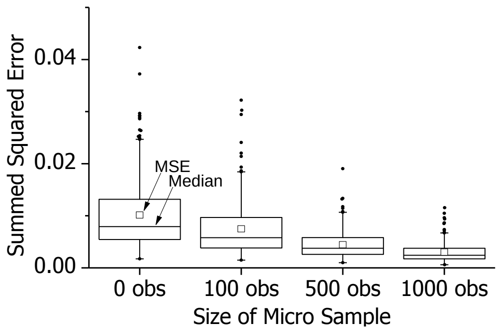
Results of the Monte Carlo Simulation

The results of the Monte Carlo Simulations for the multinomial logit model are presented in Fig. 1. Results show that incorporating prior information in the form of a micro sample decreases the total variance of the posterior density, and more so the larger the micro sample. The variance reduction effect of prior information becomes even more pronounced the greater the number of Markov states being considered. Similarly, prior information decreases the MSE of the estimator, and a greater number of Markov states accentuate this effect. Decomposing the MSE into bias and variance suggests that the MSE is primarily determined by the variance of the estimator. In all scenarios the share of the squared bias is only 4 to 9 % of total MSE.

The distribution of the sum of squared errors, as depicted in the Box-Whisker-Plots in Fig. 1, provides information about the numerical performance of the MH sampling algorithm. Results show that more simulation runs are detected as outliers in the no prior information scenario (i.e. micro sample with 0 obs.), especially when considering $k=4$ or $k=5$ Markov states. This observation indicates problems relating to the numerical stability of the MH sampler, in the sense that the algorithm does not converge correctly for some simulation runs. When considering a micro sample as prior information, substantially fewer simulation runs are detected as outliers, indicating that the use of prior information improves the numerical stability of MH sampler.

Comparable results are obtained for the ordered logit model as depicted in Fig. 2. Similar to the multinomial logit simulation, results indicate that prior information reduces the variance of the posterior density, and more so the larger the micro sample considered. The same can be observed for the MSE, which decreases with increasing micro sample size. If prior information is considered the MSE is mainly determined by the variance of the estimator such that the share of the squared bias is only 4 to 6 % of total MSE in all scenarios. For the no prior information scenarios, however, the bias share is substantially larger, being between 23 and 28 %.

Number of Markov states: k=3

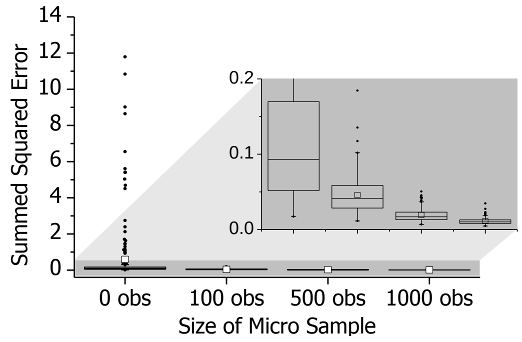


	Size of micro sample			
Measures	0	10 0	50 0	10 00
Variance[a], Posterior	0. 00592	0. 00496	0. 00312	0. 00219
MSE[a], Estimator	0. 00892	0. 00672	0. 00414	0. 00275
Sq. Bias[a], Estimator	0. 00042	0. 00041	0. 00024	0. 00014
Outlier[b]	12	9	7	9

Sample: 50,000; Burn-In: 100,000; Blocks: 1;

σ: 1/800; num. o coef.: 36

k=4

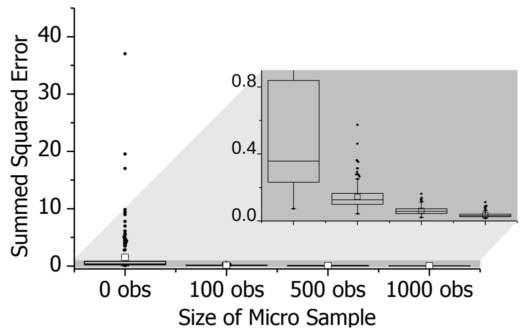


	Sizeof micro sample			
Measures	0	100	500	1000
Variance[a], Posterior	0.03585	0.02433	0.01340	0.00891
MSE[a], Estimator	0.09394	0.04425	0.01802	0.0106
Sq. Bias[a], Estimator	0.0080	0.021	0.00108	0.0045
Outlier[b]	34	0	9	11

Sample: 100,000; Burn-In: 200,000; Blocks: 1;

σ: 1/870; num. of oef.: 72

k=5



	Size of micro sample			
Measures	0	10 0	50 0	10 00
Variance[a], Posterior	0. 18702	0. 10570	0. 04839	0. 02992
MSE[a], Estimator	.3 92	0. 13130	0. 0586	0. 03296
Sq. Bias[a], Estimator	0. 03678	0. 00758	0. 00336	0. 00179
Outlier[b]	3	1	7	9

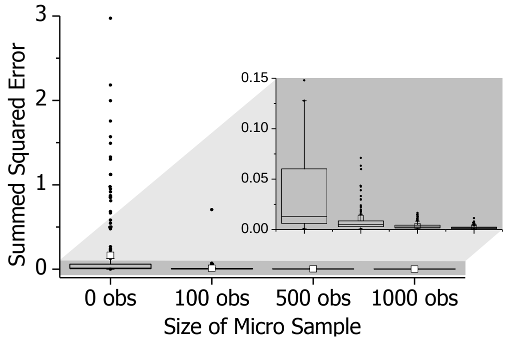
Sample: 250,000; Burn-In: 500,000; Blocks: 2;

σ: 1/580; num. of coef.: 120

[a] Calculated without simulation runs detected as outliers. [b] Note that due to the illustration the number of outliers cannot be derived from the figures directly. Source: estimated

Figure 1. Results for the multinomial logit model of a Monte Carlo simulation to analyze the influence of prior information, in the form of a micro sample, on the posterior and the posterior mean estimator.

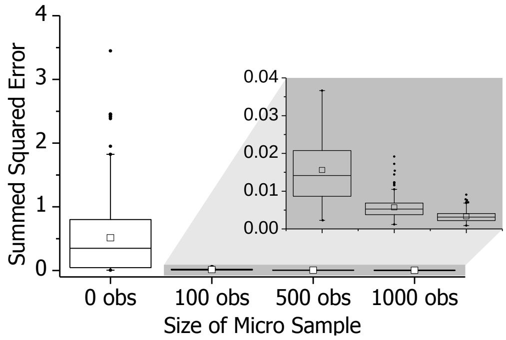
Number of Markov states: k=3



	Size of micro sample			
Measures	0	100	500	1000
Variance[a], Posterior	0.00557	0.00352	0.00197	0.00139
MSE[a], Estimator	0.02149	0.00530	0.00290	0.00168
Sq. Bias[a], Estimator	0.00532	0.00022	0.00018	0.00008
Outlier[b]	33	18	14	15

Sample: 20,000; Burn-In: 50,000; Blocks: 1; σ: 1/730; num. of coef.: 21

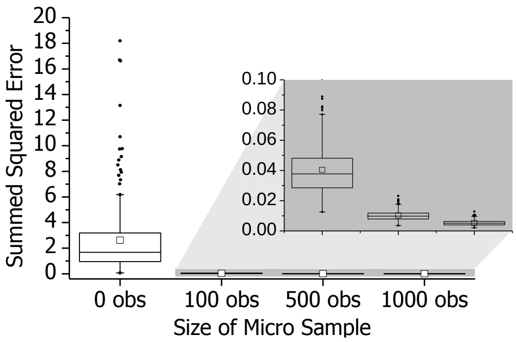
k=4



	Size of micro sample			
Meures	0	10	500	1000
Variance[a] Posterior	0.02031	0.00906	0.00435	0.00274
MSE[a], Estimator	0.43143	0.01457	0.00526	0.00320
Sq. Bias[a], Estimator	0.11923	0.00069	0.00023	0.00020
Outlier[b]	8	5	11	5

Sample: 30,000; Burn-In: 70,000; Blocks: 1; σ: 1/700; num. of coef.: 32

k=5



	Size of micro sample			
Measures	0	10	500	1000
Variance[a], Posterior	0.05809	0.02361	0.00851	0.00483
MSE[a], Estimator	1.97938	0.03864	0.00979	0.00516
Sq. Bias[a], Estimator	0.45399	0.00196	0.00047	0.00029
Outlier[b]	15	7	9	7

Sample: 50,000; Burn-In: 100,000; Blocks: 1; σ: 1/750; num. of coef.: 45

[a] Calculated without simulation runs detected as outliers. [b] Note that due to the illustration the number of outliers cannot be derived from the figures directly. Source: estimated.

Figure. 2. Results for the ordered logit model of a Monte Carlo simulation to analyze the influence of prior information, in the form of a micro sample, on the posterior and the posterior mean estimator.

The number of outliers detected by the Box-Whisker-Plots is used again to assess the numerical stability of the MH sampler. The results are consistent with the findings in the multinomial logit case, where performance of the MH sampler improves the larger the micro sample size considered as prior information. It is worth noting that the numerical problems in cases without prior information persist in the ordered logit model compared to the multinomial logit model even though substantially fewer coefficients need to be estimated (e.g. 25 compared to 120 for $k = 5$).

Overall the results suggest that without prior information, alternative individualized sampling strategies or extensions of the simple MH sampler (e.g. Parallel Tempering (Liu 2008) or Multiple Try Method (Liu *et al.* 2000)) should be considered for successful sampling from the posterior, which could not be automated for the Monte Carlo simulations. This suggests that through prior information, the computational demands with respect to the sampling algorithm are reduced and that more precise estimation can be achieved with the simple MH sampler in both the multinomial and the ordered logit model with a moderately sized micro sample. Given the fact that micro data leads to an improvement of the performance of the estimator the Monte Carlo results also indicate that for cases where the size of the micro data is relatively small compared to the macro data the large sample approximation (8) can also be used for the conditional macro data based likelihood (6).

4. Empirical Application: Structural Change in German Farming

The Bayesian estimation framework developed in section 2 is used to combine micro and macro data from two different data sources in an empirical analysis of structural change in German farming. The application demonstrates how the approach facilitates estimation of non-stationary TPs in a situation in which estimation with either macro or micro data alone would be substantially debilitated. Further, it illustrates how asynchronous data, in this case consisting of yearly micro data and macro data available only every two to three years, can be consistently combined in estimation. The application provides an alternative inferential approach to Zimmermann and Heckelei (2012a) mentioned in section 1, who were the first to consider using the same data sources to analyze farm structural change, using a generalized cross entropy approach to estimation.

Both the multinomial logit and the ordered logit model of the TPs are applied to provide two different perspectives on the evolution of structural change. The multinomial logit model is applied in an analysis of changes in farm specialization (for example, the transition from a crop producing to a milk producing farm). In this case the states constitute five different farm types as well as an entry/exit class (see Table 1). The entry/exit class is used to represent farms that enter or quit farming. The six states are mutually exclusive, and with the entry/exit class included, are also exhaustive. Since no clear order can be assumed for the farm types, the multinomial logit model is the appropriate specification. The second analysis perspective concerns the transition of farms between an entry/exit class and three classes representing different sizes of operation. Here an ordering (entry/exit, small, medium, large) of the states can be assumed such that the ordered logit model can be applied. The four states are again mutually exclusive and exhaustive.

Table 1. Definition of farm types and size classes*

	State	Description
Farm types considered in the multinomial logit model	E/E	Entry/Exit class
	COP crops	Specialist Cereals, Oilseed And Protein Crops; Specialist Granivores
	Other crops	Specialist other field crops; Mixed crops
	Milk	Specialist milk
	Other livestock	Specialist sheep and goats; Specialist cattle
	Mix	Mixed livestock; Mixed crops and livestock
Size classes considered in the ordered logit model	E/E	Entry/Exit class
	Small	16 -< 40 Economic Size Units (ESU)
	Medium	40 -< 100 Economic Size Units (ESU)
	Large	>100 Economic Size Units (ESU)

* In the FSS and the FADN farm are classified by type of farming and size classes based on the concept of Standard Gross Margin and Economic Size Units (ESU) (Commission Decision 85/377/ECC and following amendments)

Sources for Micro and Macro Data

Two different data sources, namely the Farm Structural Survey (FSS) and the Farm Accountancy Data Network (FADN), provide the macro and micro data, respectively. The FSS is a census of all agricultural holdings (above a specific size limit) conducted every two to three years. The available FSS data do not allow tracking an individual farm over time so that only macro data can be derived from the survey. The FADN provides detailed farm level information from a sample of farm holdings on a yearly basis. Using information associated with farms that remained in the sample over several years, micro data on transitions between predefined states can be derived. The advantage of FADN is that it provides more detailed information with a higher temporal resolution compared to the FSS

Table 2. Available FADN and FSS years

Year	FADN years (t)	FSS years (λ)	Year	FADN years (t)	FSS years (λ)
1989	0		1999	10	
1990	1	0	2000	11	4
1991	2		2001	12	
1992	3		2002	13	
1993	4	1	2003	14	5
1994	5		2004	15	
1995	6	2	2005	16	6
1996	7		2006	17	
1997	8	3	2007	18	7
1998	9		2008	19	

The stratified sampling plan applied in FADN aims to obtain a sample of farms that encompass different farm types and size classes. However, the sample is not necessarily fully representative of the transitions between these farm types and classes. While the macro data derived from the FSS is less detailed and available only every two to three years, the information that it contains is representative of the entire population. An additional limitation of the micro data derived from the FADN is that no information about entry or exit of farms to or from the sector can be derived. The reason is that no distinction is made between farms that quit farming and farms that are simply not selected by the sampling scheme (the same applies for entry). In contrast, in the FSS data, because the total number of farms in the population is assessed, information about entry and exit can be derived. This is commonly accounted for in Markov-type models by adding a catch-all entry/exit category. The number of farms in this entry/exit class[11], which is unobservable, is defined as a residual between an assumed maximum number of farms (e.g. 20% more than the maximum number of farms observed in any year during the estimation period[12]) and the observed number of farms in the particular year.

Both datasets are available at a regional level for the entire EU 27. However, the specific example is restricted to 7 West German *Laender*[13] for which a relatively long time period is available. Here, FADN data is available from 1989 to 2008 on a yearly base while the FSS data is available from 1990 to 2007 for every two or three years (see Table 2).

Implementation

Estimation of TPs would in principle be possible with either micro or macro data alone. However, each approach would have substantial limitations. If only macro data were used one would need to address the problem that FSS data is only available every two or three years. If only FADN micro data were used no information about entry and exit of farms can be obtained. Only information about transitions between states, conditional on the farm being active and remaining active, can be derived. This is particularly problematic given that the rapid decline of farm numbers is the most obvious pattern of structural change observed in the last decades and hence of central interest. The combination of micro and macro data allows exploiting the advantages of each data source while mitigating their disadvantages. Using the framework delineated in section 2, it is straightforward to analyze both macro data available only every two or three years and yearly micro data in a consistent way. Moreover, it is possible to exploit the information in the macro data concerning entry and exit while using a non-informative prior for the entry/exit transitions.

In consideration of macro data being available only every two to three years, the large sample likelihood function (8) can be adjusted to apply to the available data as

$$L_a\left(\boldsymbol{\beta}\middle|\mathbf{n}_\lambda, \forall\lambda\in\Lambda\right) = \sum_{\lambda\in\{\Lambda-0\}} -0.5\left(\log\left|\boldsymbol{\Gamma}_\lambda\right| + \left(\mathbf{n}_\lambda^* - \boldsymbol{\Pi}_\lambda^{*\prime}\mathbf{n}_{\lambda-1}\right)'\left(\boldsymbol{\Gamma}_\lambda\right)^{-1}\left(\mathbf{n}_\lambda^* - \boldsymbol{\Pi}_\lambda^{*\prime}\mathbf{n}_{\lambda-1}\right)\right), \tag{10}$$

[11] One might also categorize this class as the number of farms that are inactive or that are idle.

[12] The assumed maximum number of farms was chosen ad hoc. Note that this value can be chosen arbitrarily without its value impacting the main results of principal interest. It only influences the absolute size of the TPs in the row of the entry/exit state that are defined in combination with the number of farms in the entry/exit state. The choice of the "20% more than the maximum observed number of farms" could be motivated from a Bayesian perspective by viewing the choice of the maximum number of farms in a hierarchical Bayesian formulation. A uniform prior density between 0 and 40% could be defined to represent prior beliefs about the number of individuals thought to be idle or potential farming entrants. In this instance, since no information about the true maximum number of farms is available in the data the optimal Bayesian estimation under squared error loss would be 20%, equivalent to the mean of the posterior density.

[13] Baden-Württemberg, Bavaria, Hesse, Lower Saxony, North Rhine-Westphalia, Rhineland-Palatinate, Schleswig-Holstein

where $\mathbf{n}_\lambda$ denotes the observed macro data in the FSS years $\lambda \in \Lambda$ with Λ being a set of all FSS years for which a pair of sequential observations are available such that $\mathbf{n}_\lambda$ and $\mathbf{n}_{\lambda-1}$ are both observed, $\lambda = 0$ begins the first of the FSS years, and $\lambda - 1$ refers to the FSS year previous to λ (see Table 2). Further, $\mathbf{\Pi}_\lambda$ represents the TPs between FSS years which are calculated by multiplying the yearly TPs, represented by $\mathbf{P}_t$, accordingly. For example the first TP matrix between FSS years (1990 to 1993) is calculated as $\mathbf{\Pi}_1 = \mathbf{P}_2\mathbf{P}_3\mathbf{P}_4$ and the second (1993 to 1995) is defined by $\mathbf{\Pi}_2 = \mathbf{P}_5\mathbf{P}_6$. The remaining years follow accordingly based on the mapping of FSS and FADN years given in Table 2. As we had done previously, $\mathbf{n}_\lambda^*$ represent $\mathbf{n}_\lambda$ without the last row and $\mathbf{\Pi}_\lambda^*$ represent $\mathbf{\Pi}_\lambda$ without the last column. The definition of $\mathbf{\Gamma}_\lambda$ follows from (7) where FADN years (t) are replaced by FSS years (λ) and $\mathbf{P}_t^*$ by $\mathbf{\Pi}_\lambda^*$. A non-informative prior distribution with respect to the entry/exit class, defined as the first state ($k = 1$), is obtained by adjusting (9) to (note the difference for the index i, j)

$$p(\boldsymbol{\beta}) = \prod_{t=1}^{T} \prod_{i=2}^{k} \left(n_{i,t-1}!\right) \left(\prod_{j=2}^{k} \mathbf{P}_{ijt}^{\ n_{ijt}} / n_{ijt}! \right). \tag{11}$$

For the multinomial and the ordered logit model two different model specifications are chosen. For the multinomial logit model the observations are pooled across different regions. For the ordered logit model, which requires fewer parameters, a fixed effects panel model is estimated by including regional indicator variables for all (except one) regions. Policy indicator variables are used as explanatory variables in both cases to model the effects of major shifts in EU agricultural policy on structural change. Specifically, these variables include an indicator for the *Mac Sherry Reform* in 1993 (zero before 1993, one otherwise), an indicator for the *Agenda 2000* in 2000 and an indicator for the *Mid Term Review* in 2003[14] in addition to a constant and, in the ordered logit model, the regional indicator variables.[15]

Results

Table 3 provides the estimated TP matrix (averaged over all regions and time periods) between the five farm types and the E/E class obtained from the multinomial logit model. The TP matrix displays a reasonable pattern of magnitudes. As expected we obtain relatively high diagonal elements for the TP matrix, indicating that most farms remain in their current farm type. TPs between the substantially different farm types of crop (*COP crop* and *Other crop*) and livestock (*Milk* and *Other livestock*) enterprises are near zero while higher TPs are obtained for transitions between the two relatively similar crop farm types and the two livestock farm types. Further we observer relatively high TPs between all farm types and the *Mix* farm type which represents farms without one major specialization such that movement to or from any other class is likely if one branch of a farm gains importance. Comparisons with TP matrices calculated from the FADN micro data illustrates how prior information is

[14] We chose 2003 from the *Mid Term Review* where it was agreed; even so the final implementation was in 2005/06. The decision is based on the reasoning that farmers might already start to adapt to the agreed changes as some as they are decided.

[15] The mean posterior estimator is calculated based on a sample of 100,000 draws from the posterior, after a burn-in-period of 200,000 iterations. The variance of the multivariate normal proposal density is $\left(1/350 \times \mathbf{I}\right)$ and $\left(1/400 \times \mathbf{I}\right)$ which resulted in an acceptance rate of 0.26 and 0.24 for the multinomial logit model and the ordered logit model, respectively.

updated using the macro data information (upper part of Table 3). Although the two TP matrices are not directly comparable[16], the general pattern described above is already contained in the calculated TP matrix, which is then updated by the information in the FSS macro data. In addition to the results on the TPs, a comparison between the observed numbers of farms in the FSS years with the yearly fitted values suggests that the combination of FSS data with yearly FADN data is well suited to recover the observed farm numbers and to provide yearly estimates for the number of farms between FSS years (see Fig. A1 in the supplementary data).

Table 3. Comparison of transition probabilities (TPs) between farm types and between size classes calculated from FADN micro data and estimated TPs using FADN micro and FSS macro data (averaged over all regions and time periods).

	Calculated TP from the FADN micro data							**Estimated TP using FADN micro and FSS macro data**					
Transition probabilities for transition between farm types													
	E/E	COP Crop	Other Crop	Milk	Other Livest.	Mix		E/E	COP Crop	Other Crop	Milk	Other Livest.	Mix
E/E	---	---	---	---	---	---	E/E	91	2	2	2	2	1
COP Crop	---	84	5	0	0	11	COP Crop	13	74	4	0	0	9
Other Crop	---	6	87	0	0	7	Other Crop	5	3	85	0	0	6
Milk	---	0	0	96	2	2	Milk	4	1	0	92	2	2
Other Livest.	---	0	0	14	72	14	Other Livest.	9	0	0	14	60	17
Mix	---	6	4	3	2	85	Mix	4	4	4	3	3	83
Transition probabilities for transition between size classes													
	E/E	Small	Medium	Large				E/E	Small	Medium	Large		
E/E	---	---	---	---			E/E	90	4	6	0		
Small	---	90	10	0			Small	11	85	5	0		
Medium	---	5	91	4			Medium	0	7	86	7		
Large	---	0	9	91			Large	15	0	5	79		

Table 3 (lower part) provides a TP matrix for the three size classes and the entry/exit class estimated using the ordered logit Markov approach in comparison to a TP matrix for the three size classes calculated from the FADN micro data (both averaged over all regions and time periods). Again the estimated TPs depict reasonable patterns and indicate how prior information is updated using FSS macro data. As expected, farms are most likely to remain in their current size class or transit to the immediate neighboring one. Farm entry is most likely to happen in the small or medium class and only very rarely in the large size class. Only with respect to farm exit results do not match the intuitive expectation. Naturally one would expect that farm exit rates are highest for small farms and decline for the medium and large class. Estimated exit TP, however, are largest for the large size class followed by the small and the medium size class. This might indicate that results overestimated the true exit rate from the large class while the exit rate from the medium class is underestimated. Nevertheless, the comparison between observed number of farms in the FSS years and the fitted values based on the estimated TPs shows that total exits rates are well matched (see Fig. A2 in the supplementary data).

[16] As noted above no information about entry and exit is provided in FADN such that the calculated TP matrix gives the probability that a farm moves to another state conditional on the farm being active before and remaining active.

5. Conclusion

We propose a Bayesian framework for analyzing non-stationary Markov models that allows micro and macro data to be combined in estimation. In contrast to earlier approaches for combining micro and macro data offered in the literature, the Bayesian framework offers a general full posterior information approach for combining micro and macro data-based information on TPs and allows the estimation of functional relationships that link TPs with their determinants. Our Monte Carlo simulations show how prior information, in the form of a micro sample of data, can improve the accuracy of posterior information on the parameters of interest as well as the numerical stability of the estimation approach.

An application of the approach in the context of farm structural change underscored the advantages of the approach in an empirical setting. The combination of micro and macro data based on the proposed framework allows one to take advantage of information in each data set while mitigating the respective disadvantages of using either data set in isolation. Moreover, it was shown that the approach allows combining two dataset with different temporal resolution (yearly FADN micro data in combination with FSS macro data available only every two or three years). In this respect the proposed framework could also be useful for deriving TPs for shorter time intervals (e.g., months) from TPs for longer intervals (e.g., years). Such problems arise in several areas of inquiry such as network theory (Estrada 2009), land use change (Takada *et al.* 2010), chronic disease analysis (Charitos *et al.* 2008) or the analysis of credit risk (Jarrow 1997) (see Higham and Lin (2011) for a general discussion of the problem).

The general findings and the proposed approach are subject to some limitations. First, the likelihood specification presented here is applicable for aggregated data observed for the entire population. For other situations alternative likelihood specifications, such as MacRae's (1977) limited information likelihood specification, need to be considered for use in the proposed Bayesian framework. Secondly, the number of model parameters increases with the number of Markov states, often limiting the number of states that can be feasibly considered in empirical applications. The proposed ordered logit approach moderated this problem significantly, but other model specifications based on continuous Markov chains, such Piet (2010), could provide further improvement in this respect.

Overall, this paper contributes to the existing literature by providing an analysis framework that allows for combining micro and macro data information relating to non-stationary Markov models in a way that is consistent with the established tenets of the probability calculus and leads to a minimum loss estimator that is based on full posterior information. The approach is relevant for a broad range of empirical applications in which macro data is available at the population level while micro data is only available for a subsample and one is interested in quantifying the effect of factors that cause individuals to switch between predefined states.

6. References

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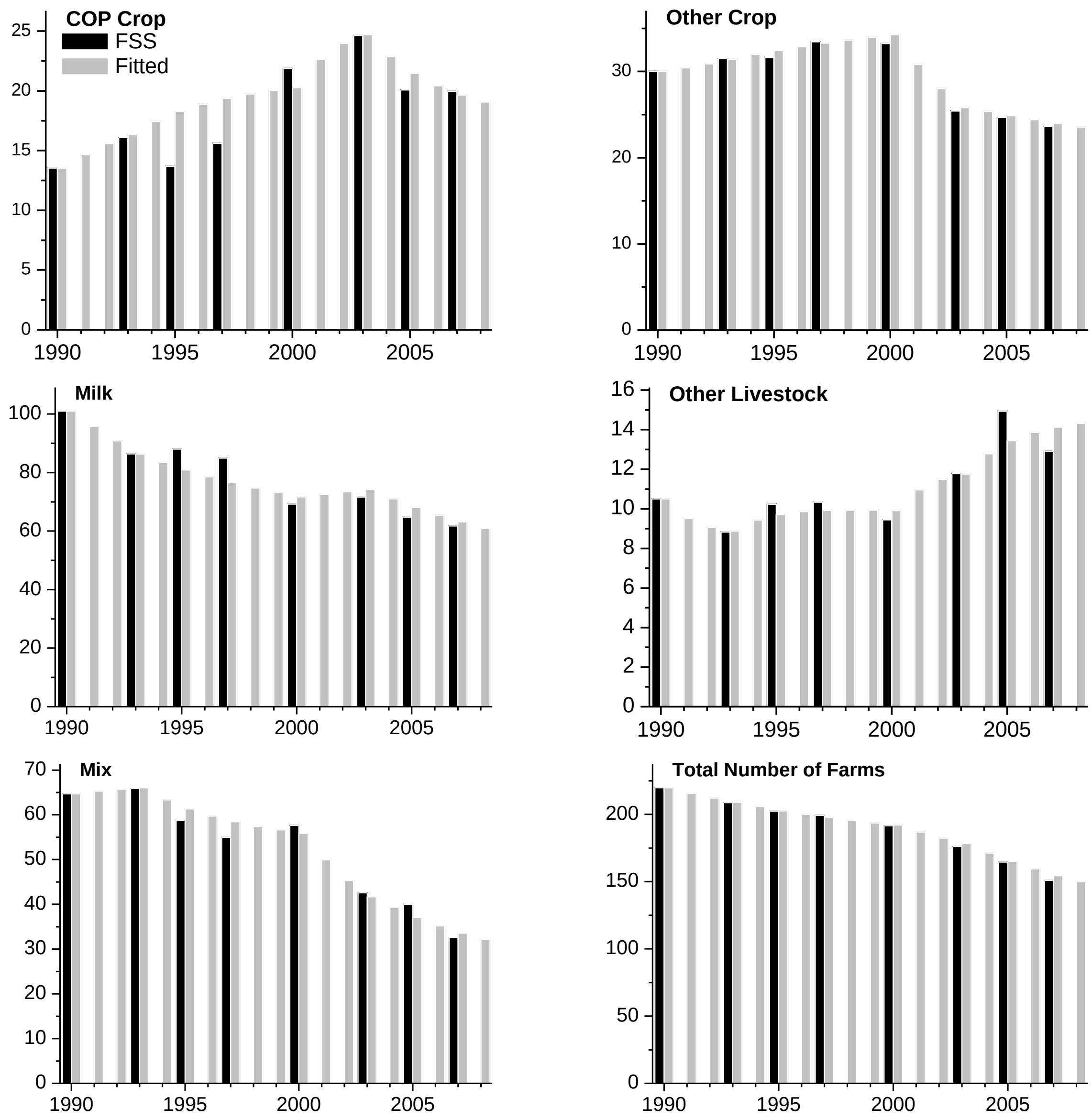
7. Appendix

Appendix A1: Number of farms (in 1000) observed in the FSS dataset and fitted values of the Markov multinomial logit model. Results aggregated over all considered regions and differentiated between the five different farm types and the total number of farms.



Appendix A2: Number of farms (in 1000) observed in the FSS dataset and fitted values of the Markov ordered logit model. Results aggregated over all considered regions and differentiated between the three different size classes and the total number of farms. The figures illustrate that fitted values follow closely the observed FSS macro data available every 2-3 years. Additionally to approach provides a direct prediction of macro data for years between FSS years.