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Bruce A. Babcock, Alicia L. Carriquiry, and Hal S. Stern

*Working Paper 96-WP 147*  
January 1996

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## ABSTRACT

The value of soil-test information in planning fertilizer application levels is determined by using agricultural field-plot data to estimate the posterior distribution of mean soil-nitrate concentrations at a given location. Optimal decisions concerning fertilizer application levels are made with respect to this posterior distribution. Average reductions in fertilizer application rates range from 15 to 41 percent, depending on the form of prior information that is available. These reductions are achieved by increasing the variability of application rates over time. Disregarding the uncertainty that remains after soil testing significantly overstates the expected benefits of soil testing.

**Keywords:** Bayesian methods, fertilizer rates, posterior distributions, soil tests.

## 1. Introduction

Growing concern over agriculture's impact on the environment has increased the call for methods that reduce application rates of chemical inputs. For example, the Committee on Long-Range Soil and Water Conservation (1993, p.57), recently concluded that "Increasing the efficiency with which nutrients, pesticides, and irrigation water are used in farming systems should be a fundamental objective of policies to improve water quality." Increasing efficiency often requires acquisition of information that decreases uncertainty about the productivity of individual input applications. Such knowledge allows farmers to better match an application's value with its cost. For example, Integrated Pest Management technologies typically include a scouting program that determines when pest numbers are large enough to justify a pesticide application (Carlson and Wetzstein, 1993). Soil tests are used to determine when fertilizer should be applied and at what rate (Blackmer et al., 1992). Environmental benefits can occur if resolution of uncertainty about an input's productivity decreases the incentive for producers to apply "insurance" amounts of inputs, thereby decreasing average application rates. Such a reduction can however be obtained only by increasing the variability of application rates. That is, increased information leads to increased variability in optimal application rates. The resulting application plans are variable-rate plans.

The evaluation of a test or measurement that provides the information making variable-rate plans possible is difficult because there can remain appreciable uncertainty about the relevant state of nature after the information is obtained. The residual uncertainty may be due to measurement error, or to the fact that information is obtained from a random sample of the relevant population (sampling error). In either case, if producer utility depends nonlinearly on the true state of nature, then decisions that account for the residual uncertainty will differ from decisions that treat the information as absolute truth (DeGroot, 1969). Thus, to conduct an evaluation of information requires explicit consideration of how uncertainty affects utility, the extent to which the uncertainty is resolved by a test or measurement, and the amount and form of prior information.

Babcock and Blackmer (1992) determined that the potential benefits from

adoption by producers of a late-spring soil nitrate test in dry-land corn production are large. Public (environmental) benefits would accrue from a potential reduction in nitrogen fertilizer rates of up to 40 percent. Private benefits consist of economically significant cost-savings and smaller yield increases. Babcock and Blackmer (1992) presented their results as potential benefits because they treated the soil test as providing perfect information about the true nutrient concentration in the soil. This paper relaxes the assumption of perfect information and uses Bayesian methods to demonstrate the extent to which residual risk after soil is tested reduces the benefits of the test.

We examine the value of the information provided by a soil nitrate test used in non-irrigated corn grown in a continuous corn rotation. In Section 2, we discuss the problem of estimating the production function that relates crop yield to available nitrogen. The optimal level of available nitrogen and the corresponding crop yield are then used to determine the maximum profit that a producer should expect, both ignoring and considering the information provided by the soil test (Subsections 2.1 and 2.2, respectively). It is shown that, to maximize expected profit, either the unconditional (when soil tests are ignored) or the conditional (given the soil test) distribution of available nitrogen must be determined. Two data sets obtained from experimental plots are described in Section 3. These data are used to estimate the prior distribution of soil nitrate levels and the sampling distribution (likelihood function) of the soil-test measurement. The resulting estimated distributions of nitrate levels (either with or without the soil test) are used to calculate the distribution of expected profit-maximizing nitrogen fertilizer application levels; the distribution is then used to determine the value of soil testing. To demonstrate the importance of accounting for residual uncertainty, results using the Bayesian decision rules are compared in Section 4 to results obtained by assuming that the soil-test information is perfect. Section 5 contains concluding remarks.

## 2. The Decision Model

Kanwar and Baker (1992) reported that, in Iowa alone, more than \$300 million worth of nitrogen fertilizer (approximately one million tons avoirdupois of nitrogen) was applied to corn in 1990. The discussion that follows refers

to a single site, i.e., we consider an individual farmer and estimate the yield function at a single location. The production decision analyzed is the per-acre amount of nitrogen fertilizer to apply in the late spring. All other input decisions are assumed to have been made before fertilizer is applied. Following Babcock and Blackmer (1992), mean yield  $y$  is assumed here to be a function solely of the mean concentration of available nitrates  $\mu$  (measured in ppm) in the top 12-inch layer of soil at the time of rapid plant uptake. The resulting yield response function represents expected yield at this site at the time the fertilizer decision is made, conditional on other factors being fixed (by the design of the experiment) at levels that are not limiting to yield.

A producer can alter  $\mu$  by applying an amount of nitrogen fertilizer,  $A$ , measured in lbs/ac. A linear relationship between  $\mu$  and  $A$  is assumed, so that  $\mu + Ak$  is the nitrogen concentration after applying the fertilizer where  $k$  is a multiplicative constant that transforms lbs/ac to ppm. [ $k = 1/7.62$  for the calculations in the paper.]

If initial nitrate levels and the yield response function  $f(\mu)$  were known, then the decision problem would have a straightforward solution: fertilizer should be applied to bring nitrate levels from the initial level to the level that maximizes profits. In any given year, however, the starting level of nitrate in the soil is unknown. Yearly changes in the level are expected because of weather-dependent losses from leaching and denitrification, and gains from the fixation of atmospheric and organic nitrogen sources (Hanley, 1990). This paper assesses the value of a soil test for reducing uncertainty about initial nitrate levels. Field conditions are assumed to be such that a producer can always apply nitrogen fertilizer after the soil test is taken. Feinerman et al. (1990) and Babcock and Blackmer (1992) examined the effects on optimal decisions of alternate assumptions concerning the probability that late fertilizer cannot be applied. Their results indicate that increasing the probability that late spring fertilizer cannot be applied increases preplanting fertilizer rates and decreases expected late applied rates.

Let  $E(y|\mu) = f(\mu)$  represent the production relation, the mean yield for a given soil nitrate concentration. Previous analyses of yield response data of the type used in this study support the existence of a yield plateau and an approximately linear response to soil nitrates when nitrates are limiting.

(See, for example, Figures 2 and 3 in Binford et al., 1992). This linear response and plateau (LRP) model can be written as a change point linear regression model (e.g., Carlin et al., 1992),

$$E(y|\mu) = f(\mu) = y_p - \beta(\mu^* - \mu)I_{\{\mu < \mu^*\}} \quad (1)$$

where  $y_p$  is the plateau yield,  $\beta$  is an unknown fixed regression coefficient,  $\mu^*$  is the unknown value of  $\mu$  at which the plateau begins, and  $I$  is an indicator variable that takes the value 1 if  $\mu < \mu^*$  and takes the value 0 otherwise. We further assume that the distribution of yield given the LRP parameters is Gaussian with mean  $f(\mu)$  and variance  $\sigma^2$ .

### 2.1 Optimal fertilizer level with no soil-test information

Let the density function  $g_0(\mu)$  represent a producer's prior information about the nitrate level,  $\mu$ , at a particular location before fertilizer application and let  $g_A(\mu)$  represent the deterministic rightward shift of  $g_0(\mu)$  that results from applying fertilizer at level  $A$ ,

$$g_A(\mu) = g_0(\mu - Ak), \mu \geq Ak.$$

Figure 1 illustrates the LRP production function of a representative producer with the prior distribution  $g_0(\mu)$  illustrated at bottom left. In this illustration, when  $A = 0$  (i.e., at  $g_0(\mu)$ ) the probability that  $\mu > \mu^*$ , i.e., that nitrogen levels are above the plateau threshold, is approximately zero. That is, nitrogen is certainly a limiting input. At  $A = 100$ , the probability that nitrogen is limiting decreases to approximately 0.5 under the prior shifted to account for  $A$ , and at  $A = 200$  the probability that  $\mu > \mu^*$  is essentially 1 and nitrogen is definitely not a limiting input. The level of  $A$  that maximizes expected profits equalizes the probability-weighted cost of being caught short of nitrates with the unit cost  $P$  of  $A$  (Babcock, 1992). When  $P$  is inexpensive relative to its value in production, then the optimal  $A$  will lie between 100 and 200 lbs/ac. That is, in this example, the optimal probability of being caught short of nitrogen is between one-half and zero.

More formally, for known  $y_p$ ,  $\beta$ , and  $\mu^*$ , the optimal level of nitrogen  $A$  is defined as the level that maximizes expected profit  $E(\pi)$ , which is given by

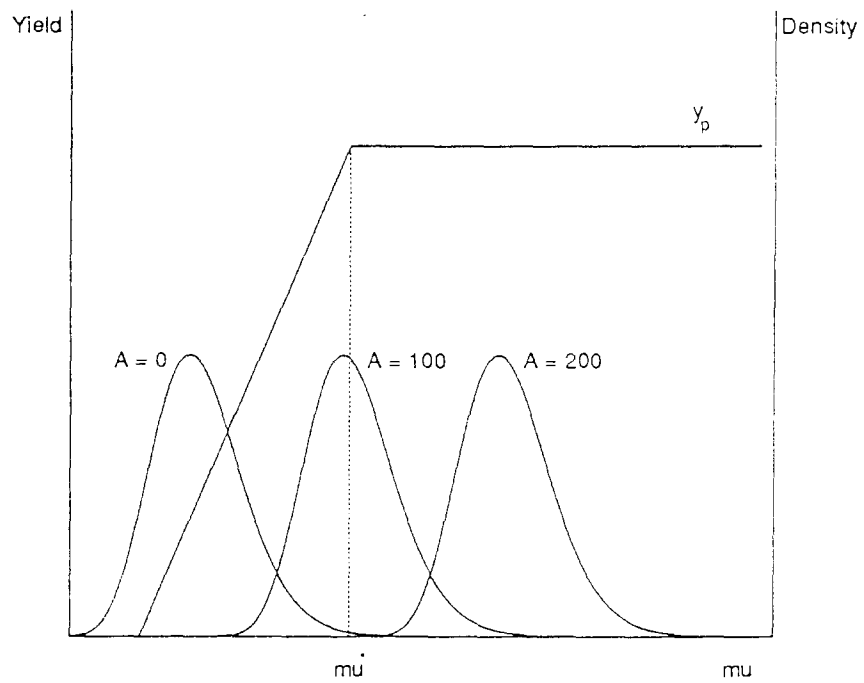


Figure 1: The linear response and plateau yield function (left vertical axis) and distribution of nitrate level under three assumptions concerning fertilizer application ( $A = 0$ ,  $A = 100$ ,  $A = 200$ ).

$$\begin{aligned}
E(\pi) &= \int_{Ak}^{\infty} [P_o f(\mu) - PA] g_A(\mu) d\mu \\
&= P_o \int_{Ak}^{\mu^*} (\alpha + \beta\mu) g_A(\mu) d\mu + P_o \int_{\mu^*}^{\infty} y_p g_A(\mu) d\mu - PA, \quad (2)
\end{aligned}$$

where  $\pi$  is profits per acre,  $P_o$  is the price of output, and  $\alpha = y_p - \beta\mu^*$ . The necessary condition (assumed here to be sufficient) for maximizing profits is

$$\int_{Ak}^{\mu^*} (\alpha + \beta\mu) \frac{\partial g_A(\mu)}{\partial A} d\mu - (\alpha + \beta Ak) k g_A(Ak) + y_p \int_{\mu^*}^{\infty} \frac{\partial g_A(\mu)}{\partial A} d\mu = \frac{P}{P_o}. \quad (3)$$

The solution  $A^*$  to (3) is the level of nitrogen fertilizer that maximizes expected profit without use of the soil test. Maximum expected profit is found by substituting  $A^*$  into (2) and then subtracting application costs.

The maximization problem does not account for fertilizer carryover from year to year. Experiments that tracked  $^{15}\text{N}$ -labeled fertilizer one and two years after application found that an average of two percent of nitrogen used by corn in Iowa was recovered from previous years' application (Binford and Blackmer, 1991). This indicates that errors made from maximizing current expected profits rather than a discounted stream of expected profits are small.

## 2.2 Optimal fertilizer level with a soil test

A soil test yields an estimate  $N$  of the current level of  $\mu$ . Let  $p(\mu|N)$  be the posterior distribution of  $\mu$  given the soil test result. Formally,

$$p(\mu|N) \propto g_0(\mu) h(N|\mu), \quad (4)$$

where  $h(N|\mu)$  is the sampling distribution of soil test results. After observing  $N$ , the farmer determines the level  $A$  of fertilizer that maximizes profits conditional on  $N$ . The expected profit given  $N$  is given by an expression like (3) with  $g_A(\mu)$  replaced by  $p_A(\mu|N)$ , where  $p_A(\mu|N) = p(\mu - Ak|N)$ . Denote the fertilizer application level that maximizes expected profits for a given  $N$  as  $A^*(N)$ . If  $N$  is sufficiently large, then  $A^*(N)$  may be zero.

Conditional maximum expected profits (neglecting application costs and the cost of testing) are

$$E(\pi|N) = \int_{A^*(N)_k}^{\infty} [P_o f(\mu) - PA^*(N)] p_{A^*(N)}(\mu|N) d\mu \quad (5)$$

which can be expanded to show separately the effect of the linear portion of the production relation and the plateau as in ( 3).

Unconditional or *ex ante* expected profits are found by integrating ( 5) over the range of  $N$ :

$$E(\pi) = \int \int_{A^*(N)_k}^{\infty} [P_o f(\mu) - PA^*(N)] p_{A^*(N)}(\mu|N) h(N) d\mu dN, \quad (6)$$

where  $h(N)$  is the marginal distribution of soil test results,

$$h(N) = \int h(N|\mu) g_0(\mu) d\mu, \quad (7)$$

### 3. Data and Estimation Results

The three functions needed to implement the preceding model and estimate the value of soil testing are 1) the crop production function  $f(\mu)$ , 2) the prior density function of nitrates  $g_0(\mu)$ , and 3) the sampling distribution  $h(N|\mu)$  of soil test results. The posterior distribution of soil nitrate level is obtained in the usual manner (see expression ( 4)) by combining the prior distribution and the likelihood function. The production function and the prior density of nitrates are estimated from data obtained in two sets of experiments conducted in Iowa between 1985 and 1991. Recall that we take the position of a single producer and therefore do not address site-to-site variability in estimating the production function.

#### 3.1 Estimating the Production Function

Data collected from a set of experiments designed to determine the relationships between corn yields and fertilizer applications were divided into two subsets. One subset was used to estimate the production function ( 1) and the other was used to estimate the parameters of one of the two prior distributions used in this study.

Data collected from a single site over a six year period (1986-1991) were used to estimate the unknown parameters in the production function  $f(\mu)$ .

The experiments involved three replications of 10 rates of preplant nitrogen fertilizer each year. The experimental site containing the 30 plots was selected for uniform growing conditions. Each year, all other inputs were applied at constant levels thought to be non limiting to crop yields at all ten fertilizer rates. Data consisted of  $T = 180$  nitrate test results obtained from the 30 experimental plots on each of six years. The estimation of the production function is complicated by a measurement error problem. The LRP production function is assumed to relate yield to actual soil nitrate concentrations. However, only soil-test-based estimates of nitrate concentrations are available. To minimize this problem in the current study we average the three test results for each rate of fertilizer use, which yields a data set for analysis consisting of  $n = 60$  observations. The LRP function was estimated using LSQ, a nonlinear least squares procedure in the software package TSP (see references). This procedure estimates the parameters in a nonlinear function with a finite number of non differentiable points. The resulting estimated regression equation was

$$\hat{y} = 139.62 - 2.98(25.52 - \mu)I_{\{\mu < 25.52\}}. \quad (8)$$

Estimated standard errors for  $\hat{y}_p$ ,  $\hat{\beta}$ , and  $\hat{\mu}^*$  were, respectively, 4.17, 0.39, and 2.06, (on 45 error degrees of freedom) .

### 3.2 *Choice of a prior density*

The prior distribution represents the farmer's prior information about the amount of nitrogen present in the soil before obtaining any soil-test information. To demonstrate the effects of prior information on the evaluation of soil testing, the analysis is conducted for two different prior distributions, a non informative uniform prior and a three-parameter gamma distribution.

The uniform prior distribution,  $\mu \sim U(a, b)$ , specifies an interval  $(a, b)$  that is believed to contain  $\mu$  and further specifies that a priori,  $\mu$  is equally likely to take any value in the interval. The random variable  $\mu$  has a distribution with density

$$g_0(\mu|a, b) = 1/(b - a),$$

and with first two moments given by  $E(\mu|a, b) = (a + b)/2$  and  $Var(\mu|a, b) = (b - a)^2/12$ . A uniform prior distribution is appropriate when the only infor-

mation the producer has is the range of likely nitrate concentrations. In this study, information about the values of  $a$  and  $b$  was obtained from the data set described in the next paragraph. Based on this empirical evidence, it was established that  $a = 3$  ppm and  $b = 30$  ppm, so that under the uniform prior distribution  $\mu$  has mean 16.5 ppm and standard deviation 7.8 ppm. Nitrate concentrations less than 3 ppm are possible but do not seem realistic for the site of interest – in general, a prior distribution with some small probability mass allocated to values near zero would be appropriate.

The informative prior distribution that we use is based on an analysis of nitrate concentrations at four sites (not including the site used to estimate the production function). The data actually represent soil-test results rather than actual ground concentrations but we ignore the measurement error issue in constructing the prior distribution. Data from four sites collected over a number of years (five in all, 1987-91) were selected to represent the type of variation to be expected in nitrate levels on a homogeneous field. The relevant observations for gathering prior information are those corresponding to zero fertilizer application (other observations corresponding to nonzero fertilizer applications are ignored here). Pooling the data from the four sites to estimate a representative prior distribution of nitrate levels across the field is appropriate only if the sites are homogeneous with respect to the forces that generate nitrate levels. If the four sites are not homogeneous, then the pooled data might overstate the amount of variability relative to what would be expected on a single producer's field. For this analysis we used classical methods to determine if pooling of observations from different sites (and different years) can be justified. We fitted a linear model with site, year, site  $\times$  year, and replication as fixed effects, and then tested the null hypotheses of a constant mean nitrate over sites. The hypothesis of equal means could not be rejected at the 0.05 level so the full set of 60 observations was pooled to gather information about  $\mu$ .

Kernel estimates (Silverman, 1986, Chapter 3) of the distribution of the 60 test results from the zero application rate plots indicated that a skewed (to the right) distribution might be a reasonable choice for an informative prior for  $\mu$ . Therefore, a three-parameter gamma distribution was chosen to represent the prior information about the value of  $\mu$ . The density of the three-parameter gamma is

$$g_0(\mu|\theta, \lambda, \xi) = \frac{(\mu - \xi)^{\theta-1} \exp[-(\mu - \xi)/\lambda]}{\lambda^\theta \Gamma(\theta)}, \quad \theta, \lambda, \xi > 0; \mu > \xi. \quad (9)$$

The first two moments of a three-parameter gamma random variable are given by  $E(\mu|\theta, \lambda, \xi) = \xi + \lambda\theta$  and  $\text{var}(\mu|\theta, \lambda, \xi) = \lambda^2\theta$ . The location parameter  $\xi$  was fixed, in this study, to be equal to 3 for reasons described in discussing the uniform prior. The remaining prior parameters  $\theta$  and  $\lambda$  were estimated from the set of 60 observations using a maximum likelihood procedure in TSP. Parameter estimates were  $\hat{\theta} = 2.094$  and  $\hat{\lambda} = 3.191$ , implying that under the three-parameter gamma distribution the variable  $\mu$  has mean 9.7 ppm and standard deviation 4.6 ppm. Under the gamma distribution the probability assigned to values greater than 30 ppm is  $< 0.003$ . The range of values of  $\mu$  under both the uniform and gamma priors is approximately the same, but the mean and variance of the uniform prior density are noticeably larger.

### 3.3 Estimation of the Sampling Distribution

The experimental dataset described in Section 3.2 (four sites for five years) includes three replications of each of ten fertilizer application levels. Each of the 600 observations in the experiment is the soil-test result for a single replication of a single fertilizer level at given site in a given year. Sixty of the observations are used to construct a plausible prior distribution  $g_0(\mu)$ . The remaining 540 observations (corresponding to nonzero fertilizer application levels) are used to explore the shape of the soil-test sampling distribution  $h(N|\mu)$ .

Means and standard deviations over the three replicates at a given level of fertilizer are plotted in Figure 2 (180 points in all). The standard deviation of test results seems to be approximately proportional to the mean nitrate concentration, and, conditional on mean concentration, the data do not appear skewed. A normal distribution with mean equal to  $\mu$  and variance equal to  $\gamma^2\mu^2$  (for unknown  $\gamma$ ) was chosen to represent the sampling distribution of soil test results for a given true concentration  $\mu$ ,  $h(N|\mu)$ . This distribution will be a reasonable approximation as long as  $\gamma$  is small, for otherwise some probability would be assigned to negative soil test results. In this discus-

sion we implicitly assume that the nitrogen level of the soil prior to fertilizer application is the same for all three replicates at a given fertilizer level.

Although this dataset suggests the functional form of the sampling distribution, it may not be appropriate for estimating  $\gamma$ , which defines the distribution of soil tests on a single homogeneous plot, because the 540 observations reflect the results on four different plots over five years. Instead, a third set of data in which multiple soil tests were conducted on the same plots each year for three years was used to estimate  $\gamma$ . Data for this study were obtained without applications of nitrogen fertilizer. For each plot, sample means and variances were calculated. Following Cochran (1977, Chapter 6) the parameter  $\gamma$  was estimated using the ratio estimator

$$\hat{\gamma}^2 = \frac{\sum_{i=1}^n S_i^2}{\sum_{i=1}^n \bar{N}_i^2 - \frac{1}{r_i} S_i^2}, \quad (10)$$

where  $n$  is the number of plots,  $r_i$  is the number of soil tests conducted on the  $i$ -th plot, and  $\bar{N}_i$  and  $S_i^2$  are the mean and variance of soil test results in the  $i$ -th plot. With  $\sum_{i=1}^n r_i = 105$ , the estimate of the scale parameter was  $\hat{\gamma}^2 = 0.01507$ , with a standard error of 0.005.

Given  $\hat{\gamma}^2$  (ignoring uncertainty in this estimate), the posterior distribution of  $\mu$  can now be obtained. For the uniform prior density,

$$\begin{aligned} p^u(\mu|N) &= [h^u(N)]^{-1} (2\pi)^{-\frac{1}{2}} (\gamma\mu)^{-1} \exp\{-0.5(\gamma\mu)^{-2}(N - \mu)^2\} (27)^{-1}, \\ &3 \leq \mu \leq 30, \end{aligned} \quad (11)$$

where

$$\begin{aligned} h^u(N) &= \int_3^{30} \frac{1}{27} (2\pi)^{-\frac{1}{2}} (\gamma\mu)^{-1} \exp\{-0.5(\gamma\mu)^{-2}(N - \mu)^2\} d\mu, \\ &0 < N < \infty \end{aligned} \quad (12)$$

is the normalizing constant and the superscript  $u$  denotes the use of the uniform prior distribution.

When the prior distribution is the three parameter gamma distribution, then the posterior distribution of  $\mu$  has the form

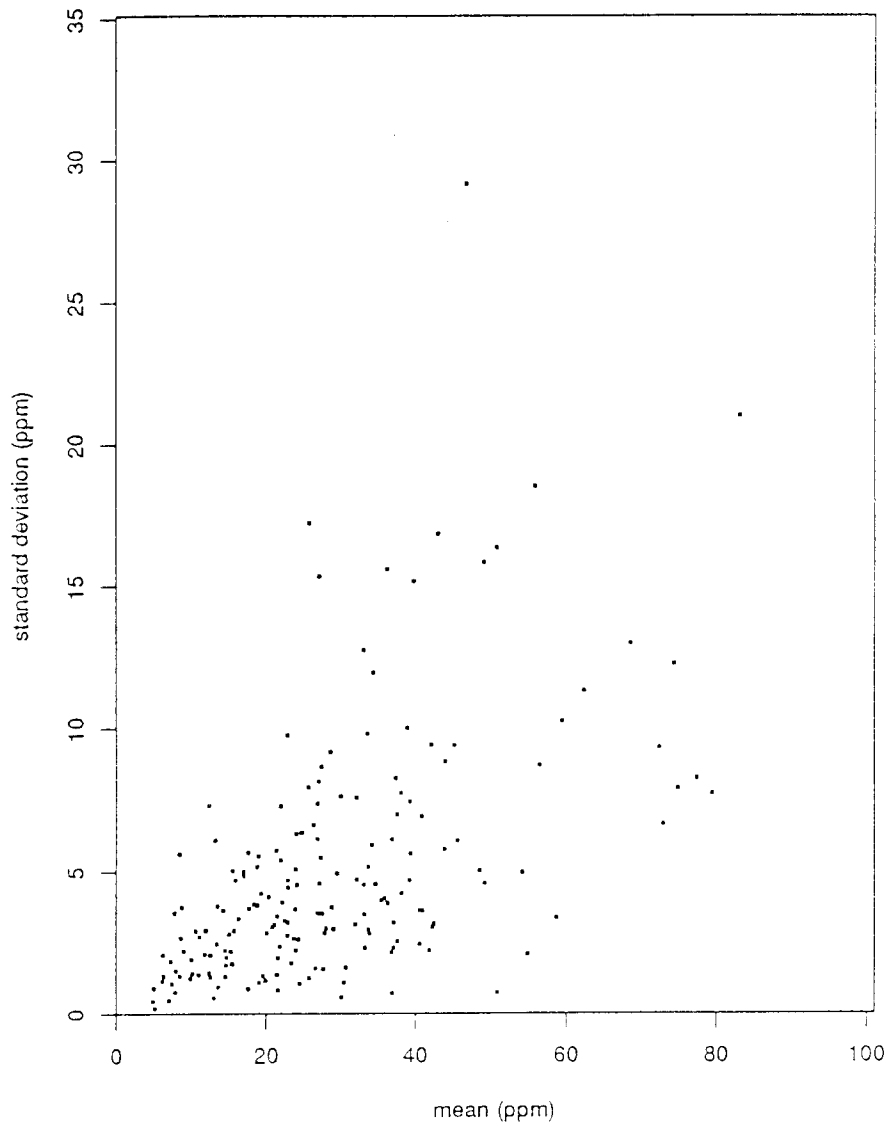


Figure 2: Soil test variability and mean nitrate concentration for 180 sample plots with varying fertilizer application rates. Each point is the mean of three replicates.

$$p^g(\mu|N) = \frac{\exp\{-0.5(\gamma\mu)^{-2}(N - \mu)^2\}}{[h^g(N)](2\pi)^{\frac{1}{2}}(\gamma\mu)} \times \frac{\exp\{-\lambda^{-1}(\mu - 3)\}}{(\mu - 3)^{-(\theta-1)}\lambda^\theta\Gamma(\theta)}, 3 \leq \mu \leq \infty \quad (13)$$

with

$$h^g(N) = \int_3^\infty (2\pi)^{-\frac{1}{2}}(\gamma\mu)^{-1} \exp\{-0.5(\gamma\mu)^{-2}(N - \mu)^2\} \times \frac{(\mu - 3)^{\theta-1} \exp\{-\lambda^{-1}(\mu - 3)\}}{\lambda^\theta\Gamma(\theta)} d\mu. \quad (14)$$

In some Bayesian analyses we can summarize the posterior distribution using simulations from the posterior distribution and do not need to calculate the normalizing constant. In this application, however, the posterior distribution (either ( 11) or ( 13)) will be integrated with respect to the marginal densities of  $N$  (either ( 12) or ( 14)) to obtain the expected profit, the expected nitrogen application, and expected yield. Graphs of  $h^u(N)$  and  $h^g(N)$  are shown in Figure 3. These densities were calculated using 20-point Gauss-Legendre quadrature. The errors that were made in this study using Gauss-Legendre quadrature were approximately equal to 0.001. With the gamma prior, the marginal density of  $N$  takes on the gamma shape. When the prior is the uniform density, the marginal distribution of  $N$  is flat for the most part except near the tails where some values of  $N$  are less likely to occur because the most relevant values of  $\mu$  have zero probability under the prior. The upper tail contains more probability than the lower tail, because the variance of test results are proportional to  $\mu$ .

Graphs of the two posterior density functions  $p^u(\mu|N)$  and  $p^g(\mu|N)$ , for three outcomes of the soil test  $N$  are shown in Figure 4. Because the variance of soil test results increases as  $\mu$  increases, both posterior densities are positively skewed for all  $N$ . The mode of  $p^u(\mu|N)$  equals  $N$ , as long as  $N$  is in the range supported by the prior distribution, because the sampling distribution of test results dominates the posterior density when the prior distribution is non informative. When the prior is the three parameter gamma distribution, the posterior density gets shifted to the left reflecting the prior information favoring small values of  $\mu$ . This effect is most extreme for large values of  $N$ .

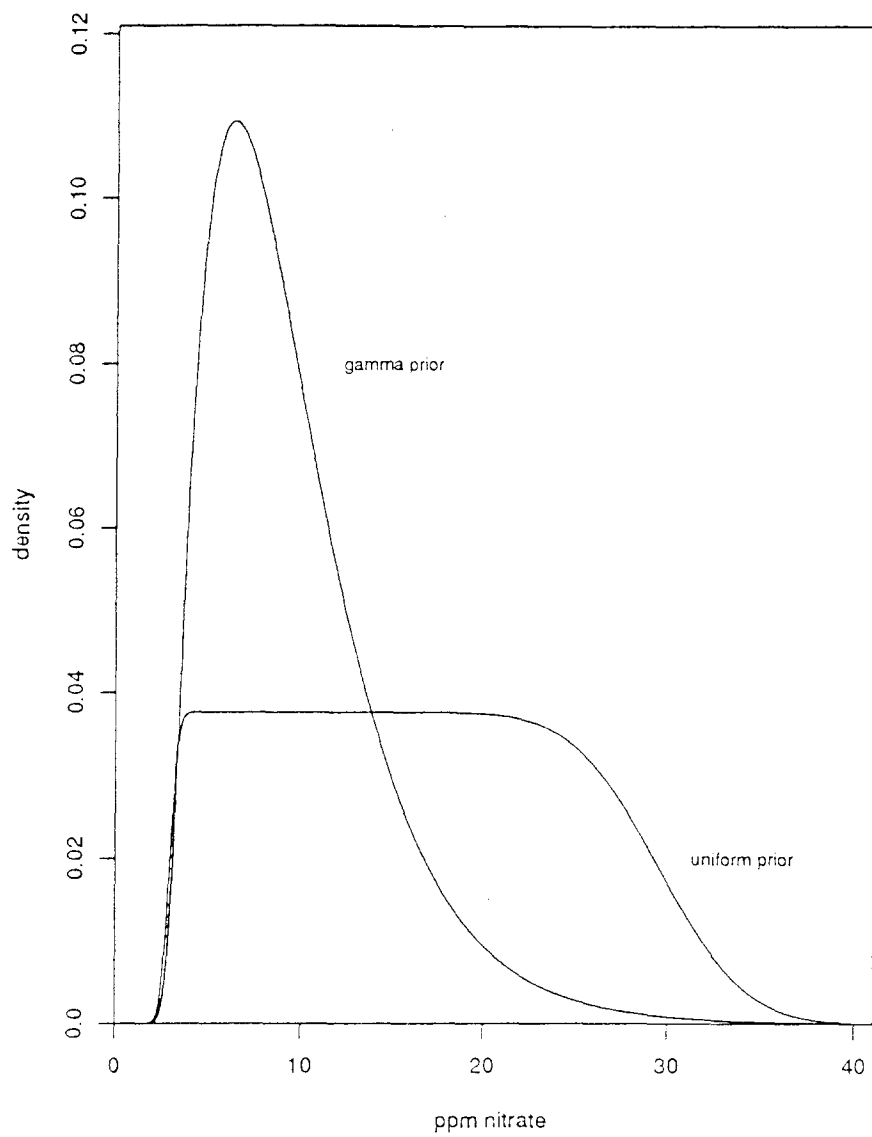


Figure 3: *Derived marginal distributions for the soil test result  $N$  under two prior densities for the true soil nitrate level  $\mu$ .*

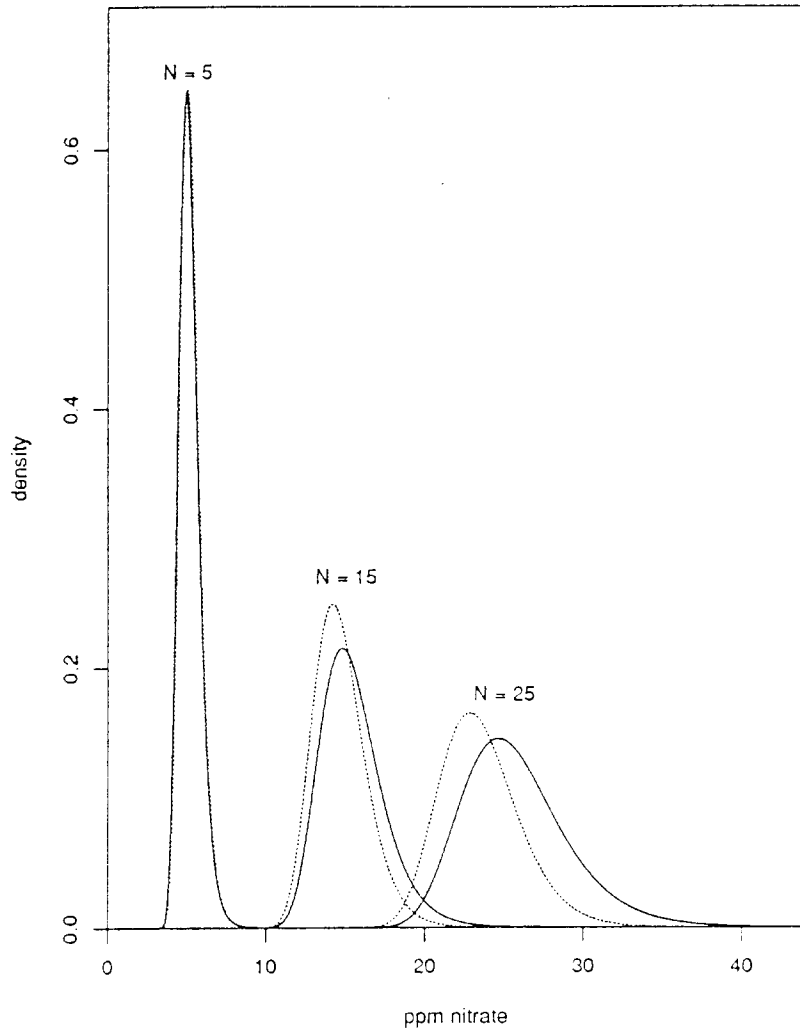


Figure 4: Posterior densities for the true soil nitrate level given three possible soil test values ( $N = 5$ ,  $N = 15$ ,  $N = 25$ ). For each  $N$ , the dashed posterior density is derived from the gamma prior distribution and the solid posterior density is derived from the uniform prior distribution.

<i>Uniform Prior</i>    <i>Gamma Prior</i>			
Optimal Solution without Soil Test			
$A$	140.2 lbs	$A$	152.6 lbs
$E\{y\}$	138.7 bu	$E\{y\}$	139.5 bu
$E\{\pi\}$	\$324.2	$E\{\pi\}$	\$324.3
Optimal Solution with Soil Test			
$E\{A\}$	82.7 lbs	$E\{A\}$	127.7 lbs
$E\{y\}$	139.2 bu	$E\{y\}$	139.3 bu
$E\{\pi\}$	\$334.2	$E\{\pi\}$	\$327.2

Table 1: Optimal yields, quantities, and profits for two prior distributions for  $\mu$ . We assume \$0.15/lb for fertilizer, \$2.50/bu for corn, and \$1.50/ac for application costs.

#### 4. Results

The value of soil testing is estimated for a corn price of \$2.50/bu and a nitrogen fertilizer price of \$0.15/lb. Application costs are \$1.50/ac. The producer who does not use the soil test maximizes expected profits with respect to the unconditional (or prior) distribution of  $\mu$ , as in expression ( 6). The upper part of Table 1 presents the results obtained with no soil test.

Optimal levels of fertilizer, expected yields, and expected returns over fertilizer and application costs are provided for each of the two prior densities considered in this paper. Expected yields and profits under the two priors are similar, even though optimal fertilizer use is about eight percent smaller with the uniform prior. The reason for the smaller average optimal fertilizer application rate with the uniform prior is the greater prior probability that soil nutrient concentrations are above the critical level of 25.52 ppm.

The bottom part of Table 1 presents expected nitrogen applications, yields, and profits for the soil-test user with the two priors. The reduction in expected optimal fertilizer applications from use of the soil test is much greater with the uniform prior than with the gamma prior. This difference arises because the uniform prior has greater variance than does the gamma prior. In addition, the probability of receiving low test values (less than 10 ppm) is much greater for the gamma prior than for the uniform prior. Thus, fertilizer applications with the gamma prior remain large in most years, and the soil test does not have as great an impact as it does with the uniform prior. This differential impact is reflected in the change in expected returns due to adoption of the soil test. Without considering the per-acre cost of the soil test, expected profits increase by \$10.03 with the uniform prior and by \$2.93 with the gamma prior. This difference illustrates the sensitivity of the increase in expected profits to the form and amount of prior information about nitrate concentrations. Producers with less specific prior information who consequently face considerable uncertainty about soil nitrate levels will be more likely to adopt the soil test than those with strong prior information.

It is important to emphasize that although average behavior varies under the two priors, individual decisions concerning fertilizer application do not depend on the prior very much. Figure 5 shows the optimal application level for different soil test results. Notice that the optimal application level is not very sensitive to the form of the prior distribution used. There is little difference over the range for which we expect to see  $N$  most often. The largest differences occur for large soil test results. In that case, the information from the soil test does not agree with the gamma prior. Consequently, under that prior, we discount the test and apply about 23 lbs/ac., more than would be needed under the uniform prior. Over the range for which we expect to see  $N$  most often, there is little difference.

The results in Table 1 indicate that adoption of the soil test can greatly reduce fertilizer applications, particularly when producers are uncertain about soil nitrate levels. The soil test reduces optimal fertilizer applications by 41 percent with the uniform prior, and by 15 percent with the gamma prior. These reductions are obtained with little change in expected yields. That is, adoption of the late spring test can lead to increases in the efficiency with

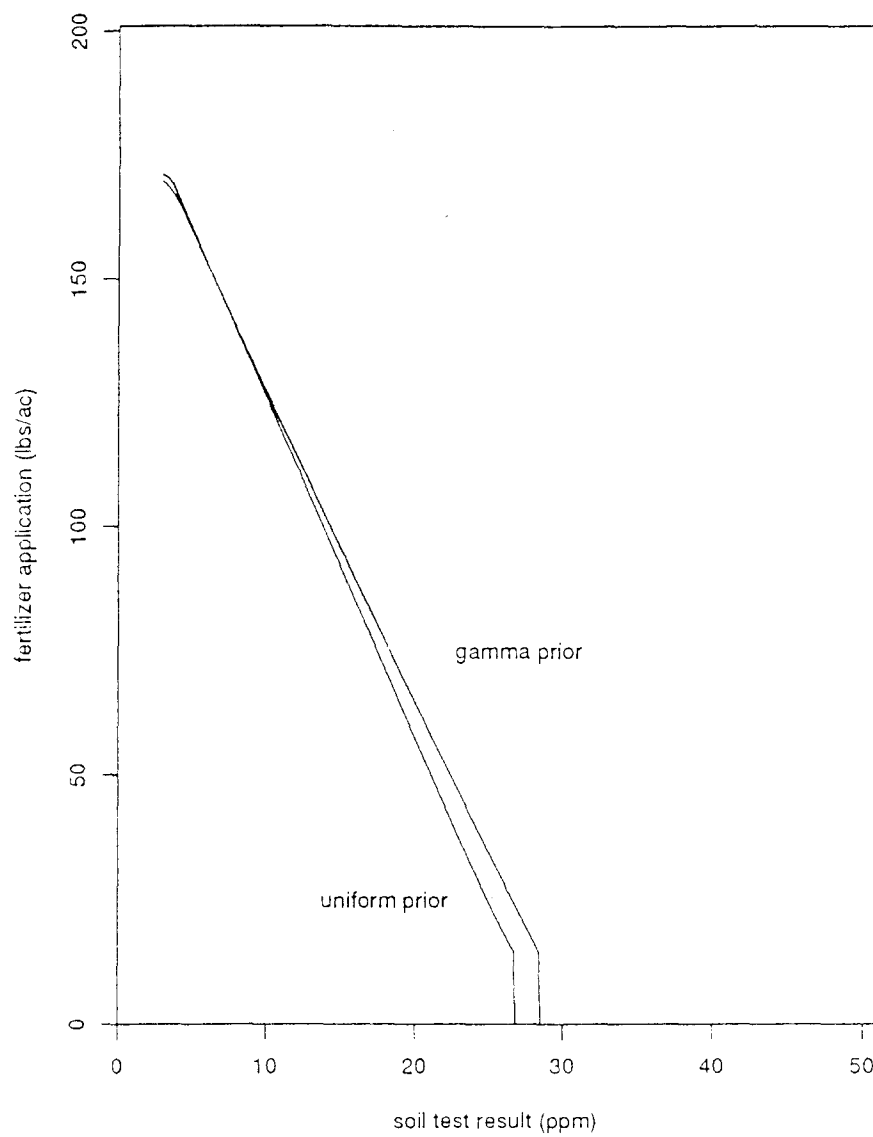


Figure 5: *Optimal fertilizer application rates as a function of soil test result under the two prior distributions.*

which nitrogen fertilizer is used.

The reductions in average fertilizer rates shown in Table 1 can only be achieved by increasing the variability with which nitrogen fertilizer is applied across one's field and over time. As can be seen in Figure 5, optimal rates vary from no fertilizer being applied to a maximum of 172 lbs/ac. One implication of Figure 5 is that regulations that limit maximum fertilizer application rates would decrease the incentive for voluntary adoption of the soil test because they would limit a producer's ability to apply large amounts of fertilizer when soil nutrients are low. A regulation that would be more consistent with variable rate fertilizer plans would be one that limits average applications of fertilizer rather than maximum rates. Limits on average rates would encourage adoption of variable rate plans because farmers would have an increased incentive to use their fertilizer efficiently.

The importance of accounting for residual risk with the soil test can be determined by comparing the results in Table 1 with results obtained under the assumption that the soil test reveals actual soil nitrate concentrations with no error. Under the assumption of perfect information, average application rates are 73.2 lbs/ac with the uniform prior and 121.4 lbs/ac with the gamma prior. The change in expected profits from adoption of the soil test is \$13.61/ac with the uniform prior, and \$6.34/ac with the gamma prior. Thus, not accounting for residual risk results in large overestimates of the changes that would result from adoption of the soil test. The change in average application rates would be overestimated by 16 percent with the uniform prior and by 36 percent with the gamma prior. The change in expected profits from adoption would be overstated by 36 percent for the uniform prior and by 116 percent for the gamma prior. The magnitude of these overestimates illustrate the importance of carefully determining the extent to which residual uncertainty remains after the adoption of uncertainty-reducing technologies.

## 5. Concluding Remarks

New production methods, like variable-rate fertilizer application, that involve increased acquisition of information and application of management skills, and decreased applications of chemical inputs are being promoted as one way of reducing pollution from agriculture. The degree to which this promise will be met voluntarily by producers depends on the profitability

and costs of the new approaches. Here we have considered one example, the late spring soil nitrate test used in dry-land corn production in the upper Midwest. The test involves minimal out-of-pocket investment, but requires farmers to make yearly adjustments in their fertilizer application rates. This paper uses Bayesian methods to interpret the soil test results and thereby determine optimal fertilizer application rates. The estimated value of the soil test is quite sensitive to prior information about soil nitrate levels and the accuracy of the soil test. Reductions in average nitrogen fertilizer application rates of between 15 and 40 percent are likely to result from adoption of the test for a continuous corn rotation. Whether the resulting cost savings are large enough to defray the costs of testing will determine the extent to which the test is adopted.

To take full advantage of the soil test requires that farmers vary their application rates from zero to 172 lbs/ac. Thus, any restriction on maximum allowable application rates will serve as a disincentive for farmers with respect to adopting the variable-rate approach. A restriction that places a ceiling on average application rates, however, would provide an incentive for farmers to determine the level of available soil nitrogen before application of fertilizer.

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