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Time Value of Money

Financial Examples and Calculations Using Tables and Calculators

by

Eddy L. LaDue

Department of Agricultural, Resource and Managerial Economics
College of Agriculture and Life Sciences
Cornell University
Ithaca, NY

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TIME VALUE OF MONEY
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Eddy L. LaDue[1]

The time value of money is a very important concept in agricultural finance. It is widely used in investment evaluation, particularly for discounted cash flow analysis of investment alternatives. It is also very useful in determining values for financial transactions. For these transactions, the interest rate reflects the time value of money and the monetary values used are determined by time value concepts.

This publication provides a discussion of the time value of money concepts and their application to financial transactions.[2] A series of examples are presented to illustrate the application and calculations for each concept. Example calculations are given in detail to allow readers to learn the calculation procedure. Agricultural finance examples are used throughout.

Two approaches to the calculation are provided. One approach uses tabled values of time value functions. Specifically, the tables provided in "Present Value, Future Value and Amortization: Formulas and Tables" Cornell University Agricultural Economics Extension 90-17 are used. Whenever a table number (i.e. Table 6) is used in calculation, reference is to a table in that publication.

In the second approach, calculations are made using a calculator with financial functions. The calculator referred to is a Texas Instruments BAII Plus. The calculations can, of course, be made with other calculators, usually with slightly different key identification and key stroke configuration. In some cases, more complex variations of a problem that cannot be handled with the tables because table values are presented for only selected situations, are illustrated using the calculator. The financial function keys referred to in this publication are:

Key	Definition
N	Number of payments (or number of compoundings if no payments are involved)
I/Y	Interest rate per year

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[1] Professor of Agricultural Finance, Cornell University

[2] For discussion of time value as applied to investment decision see Casler, Anderson and Aplin, "Capital Investment Analysis Using Discounted Cash Flows."

PV	Present value (value now)
PMT	Payment (per month, year, quarter, etc.)
FV	Future value (value at the end of the period under consideration)
P/Y	Payments per year
BEG	Payments occur at the beginning of each period (month, year, etc.)
END	Payments occur at the end of each period (month, year, etc.)
CY	Compoundings per year (automatically equals payments per year unless specifically set).

It is assumed that the number of compoundings per year equals the number of payments per year and that payments all occur at the end of the period (year, month, etc.) unless specifically indicated otherwise (BEG is assumed to equal END unless specifically indicated otherwise).

A reader who plans to use the tables can use this publication by skipping the sections labeled "Using Calculators". Similarly, a reader who plans to use a calculator can use this publication by skipping the sections labeled "Using Tables".

Time value relationships can be divided into two basic categories: (1) the **future value** of a monetary amount or series of amounts, and (2) the **present value** of a monetary amount or series of amounts. Although present value and future value represent a continuum of the same relationships, we are normally interested in determining a value for one point in time, usually the present, or some point in the future. **Amortization** is really an application of future value concepts. The discussion of these concepts is presented in three sections: (1) future value, (2) present value and (3) amortization.

FUTURE VALUE

Future value techniques are designed to determine the value of an amount, or series of amounts of money as of some fixed point in time in the future. Examples include the value of a savings account in five years or the value of a farm in 19 years when land values are increasing at five percent per year.

1. Future value of a present sum (Cornpound Interest)

When we are dealing with the future value of a single amount of money, we use compound interest, or the future value of a present sum. The future value of a present sum is calculated using equation (1):

$$FV = PV(1+i)^n \qquad (1)$$

Where: FV = Value in period n (n periods in the future)

PV = Value in period o (now)

i = Interest rate per conversion period

n = Number of conversion periods

If each period is a year, n is the number of years into the future on which the value is to be calculated, and i is the annual interest rate. In this case, the formula results in normal compound interest with annual compounding. If payments are made more frequently than annually, i is the annual rate divided by the number of payments per year and n is the number of payments per year multiplied by the number of years.

Example 1

If $1 is invested in a savings account with a fixed interest rate of eight percent and annual compounding, how much will be in the account in 10 years? Direct application of the equation results in the following:

$$\$1(1+.08)^{10} = 1(2.16) = \$2.16$$

Using Tables

The calculations can be made using the compound interest table (Table 1). The factor from Table 1 for 10 years at eight percent interest is 2.158924. Multiplying the factor times the investment gives the correct value.

$$2.158924(\$1) = \$2.16$$

Using Calculators

P/Y = 1
N = 10
F/Y = 8 FV = $2.16
P/V = 1
PMT = 0
FV = ?

If interest were compounded quarterly instead of annually:

P/Y = 4
N = 40 (4 payments per year x 10 years)
I/Y = 8
PV = 1
PMT = 0 FV = $2.21
FV = ?

Example 2

Pete now has a farm that is worth $200,000. If farm values inflate at an average rate of four percent over the next 15 years, what will the farm be worth at the end of the 15 year period?

Using (1): $\$200,000\ (1+.04)^{15} = \$360,189$.

Using Tables

$200,000 (1.800943) = $360,189

Table 1, 15 years, four percent

Using Calculators

P/Y = 1
N = 15
I/Y = 4
PV = 200,000
PMT = 0 FV = $360,189
FV = ?

2. Future Value of a Nonuniform Series

The future value of a series of different amounts can be determined using the future value of a series. The future value of a series of sums is calculated using equation (2).

$$FV = PV_0\,(1+i)^n + PV_1\,(1+i)^{n-1} + PV_2(1+i)^{n-2} + \ldots + PV_i \qquad (2)$$

Where:
- FV = Value in period (year) n
- PV_0 = Amount invested in period o (now)
- PV_1 = Amount invested in period 1
- PV_n = Amount invested in period n
- i = Interest rate
- n = Nurnber of years to future time for which the value is calculated

If you look at this equation closely, you will see that this is just a series of equation (1) calculations added together. Thus, Table 1 can also be used to determine the future value.

Example 3

Jason, a beginning high school freshman, is planning to save money from his weekend and summer work on the farm to buy a car to drive back and forth to Cornell while he is attending college. He invested $200 at the end of this summer (now) and expects he can invest $400 one year from now, $500 in three years, and $600 in four years (he plans to work on a midwest farm between his Sophomore and Junior years and will spend all he makes). Jason's father has agreed to borrow these funds and pay Jason 10 percent interest. How much money will Jason have at the end of four years?

$$FV_4 = 200(1+.10)^4 + 400(1+.10)^3 + 500(1+.10)^1 + 600$$

$$= 200(1.4641) + 400(1.3310) + 500(1.10) + 600$$

$$= 292.82 + 532.40 + 550 + 600$$

$$= \$1,975.22$$

Using Tables

FV = 200 (1.4641) + 400 (1.3310) + 500 (1.1) + 600

Table 1, 10%, 4 years Table 1, 10%, 3 years

= $1,975.22

Using Calculator

P/Y	=	1		P/Y	=	1		P/X	=	1	
N	=	4		N	=	3		N	=	1	
F/Y	=	10	FV = $292.82	I/Y	=	10	FV = $532.40	I/Y	=	10	FV = $550.00
PV	=	200		PV	=	400		PV	=	500	
PMT	=	0		PMT	=	0		PMT	=	0	
FV	=	?		FV	=	?		FV	=	?	

Value = 292.82 + 532.40 + 550 + 600 = $1,975.22

Example 4

Three years ago, Will invested $2,000 in an individual retirement account (IRA). Two years ago, he invested $1,800 and last year he invested $1,500. If this IRA earns interest at eight percent, how much is currently in the account?

Using Tables

Value = 2,000(1.259712) + 1,800(1.1664) + 1,500(1.08)

Table 1, 8%, 2 years

= 2,519.42 + 2,099.52 + 1,620

= $6,238.94

Using Calculators

P/Y	=	1		P/Y	=	1		P/X	=	1	
N	=	3	FV =	N	=	2	FV =	N	=	1	FV =
F/Y	=	8	$2,519.42	I/Y	=	8	$2,099.52	I/Y	=	8	$1,620
PV	=	2,000		PV	=	1,800		PV	=	1,500	
PMT	=	0		PMT	=	0		PMT	=	0	
FV	=	?		FV	=	?		FV	=	?	

Value = $2,519.42 + $2,099.52 + 1,600 = $6,238.94

3. Future Value of a Uniform Series

If the sum involved (invested) is the same each period (year), the future value can be determined more easily, with a single calculation, using equation (3).

$$FV = PMT\left[\frac{(1+i)^n - 1}{i}\right] \tag{3}$$

Where: FV = Value in period (year) n (future)

PMT = Amount invested each period (year, month, etc.)

i = Interest rate per period (year, month, etc.)

n = Number of periods (years or months or ...)

Using Tables

Values of the equation 3 expression in the brackets, for situations where the period is one year, are presented in Table 4. Thus, the future value can be calculated by simply multiplying the amount received each year by the appropriate value from Table 4.

Table 4 assumes that investments (payments) are made at the end of each year, including the final year (at the time for which the future value is being calculated). If this is not the case, adjustments must be made. If an investment is made at the beginning of the period (as well as the end), use the number of years **plus one** in selecting the coefficient from Table 4. If investments are made at the beginning of each year but none is made at the end of the period (the time for which the future values are being calculated), use the number of years in the period **plus one** in selecting the coefficient and subtract one (1.0) from the value of the coefficient found. When the first payment is made in one year and no payment is made at the end of the last year, subtract one (1.0) from the coefficient for the number of years involved. The following table indicates the appropriate values for different situations.

Payment Plan	Payment Now?	Payment at End of Period	Table Value to Use	Coefficient to Use
End of each year	No	Yes	n	Table
Beg. of each year	Yes	No	n+1	Table-1
Beg. and end	Yes	Yes	n+1	Table
End of each year except last	No	No	n	Table-1

Table 4 can also be used to determine the investment required each year to meet a goal or provide a given sum of money at the future point in time. This is accomplished by dividing the coefficient from Table 4 into the amount required (the goal).

Example 5

Loren's banker has required that Loren develop a sinking fund to provide the funds for a new orchard sprayer which he will need in five years.

a. If he invests $5,000 per year at the end of each year for the next five years and the funds earn interest at 12 percent, how much money will be available for purchase of the sprayer?

Using Tables

$5,000(6.3528) = $31,764

Table 4, five years, 12 percent

Using Calculators

P/Y	= 1	
N	= 5	
I/Y	= 12	FV = $31,764
PV	= 0	
PMT	= -5,000	
FV	= ?	

If interest were compounded monthly instead of annually:

P/Y	= 1	
C/Y	= 12	
N	= 5	
I/Y	= 12	FV = $32,198
PV	= 0	
PMT	= -5,000	
FV	= ?	

b. If Loren expects the sprayer to cost $50,000 in five years, and funds are put in the same 12 percent account with annual compounding, how much must he invest each year?

Using Tables

$\frac{\$50,000}{6.3528} = \$7,871$

Using Calculators

P/Y = 1
N = 5
F/Y = 12 PMT = $7,870.49
PV = 0
FV = 50,000
PMT = ?

If interest were compounded monthly instead of annually:

P/Y = 1
C/Y = 12
N = 5
I/Y = 12 PMT = $7,764.51
PV = 0
FV = 50,000
PMT = ?

c. If investments were made at the beginning of each year instead of the end (annual compounding), how much must be invested each year?

Using Tables

$\frac{\$50,000}{7.1152} = \$7,027$

(Table 4, six years, 12 percent) - 1

Using Calculators

BEG = BEG
P/Y = 1
N = 5 PMT = $7,027
I/Y = 12
PV = 0
FV = 50,000
PMT = ?

d. If payments were made now and at the end of each year, how much must be invested?

Using Tables

$\frac{\$50,000}{8.1152} = \$6,161$

Using Calculators

Making payment now is the same as making a payment at the end of last year (year 0). Thus, this payment is like adding another year (year 0) to the front end of the payment stream. The number of years (periods) becomes 6.

BEG = END
P/Y = 1
N = 6
I/Y = 12
PV = 0 PMT = $6,161
FV = 50,000
PMT = ?

Example 6

If Joe invests $2,000 in an IRA now and at the end of each year for the next 15 years, how much will be in the account after he makes his 15 year payment if the account earns nine percent per year?

Using Tables

$2,000(33.0034) = $66,007

Table 4, 16 years, nine percent

Using Calculators

P/Y = 1
N = 16
I/Y = 9 FV = $66,007
PV = 0
PMT = -2,000
FV = ?

If interest were compounded quarterly:

P/Y = 1
C/Y = 4
N = 16 FV = $67,764
I/Y = 9
PV = 0
PMT = -2,000
FV = ?

Example 7

Sandy is planning to start farming in 10 years. To get the initial funds needed, she is working for Farm Credit and plans to set aside $4,000 per year starting now from her salary in a mutual fund account that she expects to earn 11 percent per year. How much money will she have for farm investment in 10 years if moving and other costs keep her from investing at the end of the 10th year?

Using Tables:

$4,000(18.5614) = $74,246

(Table 4, 11 years, 11 percent) - 1

Using Calculators
BEG = BEG
P/Y = 1
N = 10
I/Y = 11 FV = $74,246
PV = 0
PMT = -4,000
FV = ?

If she knows she will need $100,000 to start, how much will she need to invest each year?

Using Tables:

$$\frac{\$100,000}{18.5614} = \$5,387.52$$

Using Calculators:

BEG = BEG
P/Y = 1
N = 10 PMT = $5,387.52
I/Y = 11
PV = 0
FV = 100,000
PMT = ?

PRESENT VALUE

Present value or "discounting" is essentially the reverse of future value. Present value determines the current value of an amount or series of amounts to be received in the future. In general terms, present value is the amount that you could be paid now and be equally well off compared to receiving a specified payment or series of payments at some specific time in the future.

4. Present Value of a Future Sum

Present value of a future sum is used to calculate the amount that one could receive now that would be equal in value to the receipt of a specified single amount at some point in the future. Remember, the compound interest formula told us the **future value of a current sum**. Well, the present value of a sum tells us the **current value of a future sum**. We can modify the compound interest formula,

$$FV = PV(1+i)n$$

by dividing both sides by (1+i)n, to get the present value formula.

$$PV = \frac{Vn}{(1+r)\,n} \tag{4}$$

This can be reformulated as:

$$PV = FV\,(1+i)^{-n} \tag{5}$$

Using Tables

The bracketed value $(1+i)^n$ has been placed in tabular form in Table 2. Returning to our Example 1 situation, if we will receive $2.16 in 10 years and funds received now could earn eight percent over the next 10 years, the present value of the $2.16

= $2.16(.463193) = $1

Table 2, 10 year, eight percent

Note that $.463193 = \frac{1}{2.158924}$

Table 2 Table 1

Thus, if you have Table 2 you can calculate Table 1 values and vice versa.

$$\text{Table 1} = \frac{1}{\text{Table 2}}$$

$$\text{Table 2} = \frac{1}{\text{Table 1}}$$

Thus, the answer to Example 2 could be calculated using Table 2 values as shown below. The modest difference between the values results from rounding in the tables.

$$\$200{,}000 \left(\frac{1.0}{.555265}\right) = \$360{,}188$$

Table 2, 15 years, four percent

Example 8

To broaden the range of investment alternatives available, securities dealers have developed stripped treasuries where each of the interest payments and the final principal payment on a treasury note or bond are separated into separate instruments and sold individually. Thus, you can buy the next year's interest payment on a million dollar treasury note or the year 2000 principal payment on a $100,000 treasury bond. Similarly, zero coupon bonds represent an agreement of the issuer to pay the face amount of the bond at the maturity date. What is the present value of the $100,000 principal payment (or zero coupon bond) to be received in 12 years if you could earn nine percent interest on funds invested over the period?

Using Tables

$100,000 (.355535) = $35,553

Table 2, 12 years, nine percent

Using Calculators

P/Y = 1
N = 12
I/Y = 9 PV = 35,553
PMT = 0
FV = 100,000
PV = ?

Example 9

Two years ago John loaned his son, John Jr., a local grape grower, $20,000 to buy additional vineyard. The loan was for five years with interest compounded annually at 10 percent, but the loan was unamortized and no interest or principal was to be paid until the end of the five year period.

a. How much will John receive at the end of the five year period?

Using Tables

$20,000(1.61051) = $32,210

Table 1, five years, 10 percent

Using Calculators
P/Y = 1
N = 5
I/Y = 10 FV = $32,210
PV = 20,000
PMT = 0
FV =

b. If John were to sell the loan to an investor to obtain funds for other activities, how much could the investor afford to pay if she expected to earn 16 percent on all investments?

Using Tables

$32,210(.640658) = $20,636

Table 2, three years, 16 percent

Using Calculators

P/Y	= 1	
N	= 3	
I/Y	= 16	PV = $20,636
PMT	= 0	
FV	= 32,210	
PV	= ?	

5. Present Value of a Nonuniform Series

When a series of amounts are to be received in the future and the amounts are not all of the same magnitude, equation (6) can be used.

$$PV = \frac{PMT_1}{(1+i)^1} + \frac{PMT_2}{(1+i)^2} + \ldots + \frac{PMT_n}{(1+i)^n} \tag{6}$$

For ease in use, equation (6) can be reformulated as:

$$PV = PMT_1(1+i)^{-1} + PMT_2(1+i)^{-2} + \ldots + PMT_n(1+i)^{n} \tag{7}$$

In equation (7) form, it is easy to see that this is just the sum of a series equation 5 calculations.

Thus, the present value of the series is just the sum of the present value of the individual amounts in that series. That is, it is the sum of a series of calculations using equation 5.

Example 10

What is the value of a Federal Farm Credit Bond with a face value of $1,000, a 10 percent interest rate with annual payments and a three year maturity, if increasing interest rates have increased the normal yield on such bonds to 15 percent (or, you could earn 15 percent on available funds if invested elsewhere)?

You will receive the following amounts:

Year 1	($1,000 x .10)	= $ 100
Year 2	($1,000 x .10)	= 100
Year 3	($1,000 x .10) plus $1,000 face value	= $1,100

Using Tables

PV = 100(.869565) + 100(.756144) + 1,100(.657516)

Table 2
one year
15 percent

Table 2
three years
15 percent

= 86.96 + 75.61 + 723.27
= $885.84

If you paid $885.84 for this bond, how much would you earn? Obviously 15 percent.

Using Calculators

P/Y = 1
N = 3
I/Y = 15
PMT = 100
FV = 1000
PV = ?

PV = 885.84

If bond interest payments were semiannual:

P/Y = 2
N = 6
F/Y = 15
PMT = 50
FV = 1000
PV = ?

PV = 882.65

Why is the present value lower with more frequent payments? The semiannual discounting of the $1000 reduces the present value more than the earlier receipt of part of the interest payments increases the present value.

Example 11

In negotiating the sale of some machinery to his son, Harry agreed to accept interest and principal payments of $10,000 in one year; $15,000 in two years, and $22,000 in three years (in line with the son's ability to pay). If Harry can earn nine percent on invested funds, how much is he receiving for the machinery?

Using Tables

PV = $10,000(.917431) + 15,000(.841680) + 22,000(.772183)

= 9,174.31 + 12,625.20 + 16,988.03 Table 2, 9%, 3 years

= $38,787.54

Using Calculators

P/Y	= 1		P/Y	= 1		P/Y	= 1
N	= 1		N	= 2		N	= 3
I/Y	= 9	PV = 9,174.31	I/Y	= 9	PV = $12,625.20	I/Y	= 9 PV = 16,988.04
PMT	= 0		PMT	= 0		PMT	= 0
FV	= 10,000		FV	= 15,000		FV	= 22,000
PV	= ?		PV	= ?		PV	= ?

Value = 9,174.31 + 12,625.20 + 16,988.04 = $38,787.55

6. Present Value of a Uniform Series

If the amounts to be received in the future are the same for all consecutive years, the present value can be obtained more simply using equation (8).

$$PV = PMT\left[\frac{1-(1+i)^{-n}}{i}\right] \tag{8}$$

Where: PV = The value in period zero (now)

PMT = The amount received each period (year, month, etc.)

i = The interest rate
(annual interest rate divided by number of payments per year)

n = The number of periods the amount PMT is to be received

Example 12

Ann is an only child who became a banker rather than returning to the home farm. At the time her father retired from farming, he sold the farm to Dave on a land contract. The contract called for annual payments of $3,000 per year with interest at 10 percent. At the time of Ann's father's death, there were 10 years remaining on the contract and she could earn 16 percent on funds available for investment at the time. Dave offers $15,000 to pay off the contract. Should she accept Dave's offer?

Using Tables

The bracketed part of equation 8 has been calculated for a number of values of i and n, and presented in Table 3 for annual payments and table 5 for monthly payments. Present values are calculated by multiplying the amount received each year by the correct value from Table 3 or 5.

$3,000(4.83323) = $14,500

Table 3, 10 years, 16 percent

Using Calculators

P/Y = 1
N = 10
I/Y = 16 PV = $14,500
PMT = -3,000
FV = 0
PV = ?

Since she can earn 16 percent on money invested, the $3,000 per year for 10 years is equivalent to $14,500. Dave has offered $15,000. She should accept the offer.

Present value can be used to determine the principal outstanding on a loan. Discounting removes the earnings that could be obtained on funds if they are received now rather than in the future. For an amortized loan, the earnings generated for the holder of the loan is the interest to be received. Thus, discounting removes the interest from the loan payments and the present value is the current principal balance. Only the number of payments remaining, the amount of each payment and the interest rate must be known. Thus, the principal outstanding on Ann's contract can be calculated by discounting the payments at the interest rate specified by the contract (10 percent).

Using Tables

$3,000(6.14457) = $18,434 = principal owed

Table 3, 10 years, 10 percent

Using Calculators

P/Y = 1
N = 10
I/Y = 10 PV = $18,434 = principal
PMT = -3000
FV = 0
PV = ?

Example 13

Ten years ago, the Vegies National Bank made a 25 year loan at six percent interest to a local vegetable farmer to purchase a farm. The annual payments are $8,719.

1. The principal outstanding is:

Using Tables

$8,719(9.7122) = $84,681

Table 3, 15 years, six percent

Using Calculators

P/Y = 1
N = 15
I/Y = 6 PV = $84,681
PMT = -$8,719
FV = 0
PV = ?

2. If an insurance company (or someone else) that is currently earning 16 percent on newly invested funds were to buy this loan, the amount they can afford to pay for the loan is:

Using Tables

$8,719(5.5755) = $48,613

Table 3, 15 years, 16 percent

Using Calculators

P/Y = 1
N = 15
I/Y = 16 PV = #48,612
PMT = - 8,719
FV = 0 (results differ due to rounding on tabled values)
PV = ?

Example 14

Kelly borrowed the funds to buy new greenhouse space. The terms of the loan are:

$50,000 borrowed
15 percent interest
25 years repayment period
Annual payments

Using Tables

The annual payments calculated using Table 3 are:

$\frac{\$50,000}{6.4641} = \$7,735.03$

Table 3, 25 years, 15 percent

This result can be checked using amortization tables (Table 7) where the payment per $1 for loans for 15 percent for 25 years is $0.1547.

$.1547 x 50,000 = $7,735

The difference between the two results is due to rounding.

Example 15

In an effort to stimulate lagging farm machinery sales, John Deere recently offered a delayed payment plan which required no interest until March 15 and no payment until

December 15. The interest rate charged was 12 percent and interest was charged from the date of the purchase or March 15, whichever occurs last. The loan is for five years with six equal payments starting December 15. Mark purchased $100,000 worth of machinery on January 15, and took advantage of this special repayment plan.

Calculating the amount of each payment in this case requires first determining the amount outstanding on December 15. The interest can be calculated as,

$$\$100{,}000(.12)\frac{9}{12} = \$9{,}000$$

leaving a total of $109,000 outstanding on December 15.

Using Tables

Since the first payment is due on December 15, the Table 3 value must be modified to reflect the immediate payment. As of December 15, the present value coefficient for the December 15, payment must be 1.0, thus, adjusting for the immediate payment involves adding 1.0 to the coefficient for the remaining five payments. The annual payments are:

$$\frac{\$109{,}000}{4.60478} = \$23{,}671.05$$

(Table 3, five years, 12 percent) + 1.0

Clearly, if the finance plan required "no interest until December 15", the payment would be:

$$\frac{\$100{,}000}{4.60478} = \$21{,}716.56$$

Using Calculators

No Interest Until March 15

BGN = BGN
P/Y = 1
N = 6
I/Y = 12 PMT = 23,671.07
PV = 109,000
FV = 0
PMT = ?

No Interest Until December 15

BGN = BGN
P/Y = 1
N = 6
I/Y = 12 PMT = 21, 716.58
PV = 100,000
FV = 0
PMT = ?

The difference between values using calculator and tables is due to rounding of table values.

Example 16

The appropriate value to enter on the balance sheet for a **financial lease** is the present value of the future lease payments discounted at the firms incremental borrowing rate or the implicit rate on the lease. Thus, present value can be used to calculate the value of leases.

Jim is in the process of completing his 12/31 (end of year) balance sheet. On March 31, two years and nine months ago, Jim leased a farrowing unit from Telmark. The payments were $15,000 per year for seven years with all payments at the beginning of the year. The incremental borrowing rate for Jim's business was 12 percent. Three payments have been made (two years nine months ago, one year nine months ago, and nine months ago). The remaining four payments are due at the end of each of the next four years (ending March 31), so normal present values can be used for them. The present value of the remaining payments as of the last payment date (March 31, of the coming year) are:

Using Tables

$15,000(3.03735) = $45,560

Table 3, four years, 12 percent

Using Calculators

P/Y	= 1		
N	= 4		
I/Y	= 12		
PMT	= -15,000	PV	= $45,560
FV	= 0		
PV	=		

Example 17

Lynda is currently leasing a tractor for $500 per month. The implicit interest rate on the lease is 11 percent. She has 29 payments left to make. What is the present value of the lease?

Using Tables

$500(25.3641) = $12,682

Table 5, 29 months, 11 percent

Using Calculator

P/Y = 12
N = 29
I/Y = 11 PV = $12,682
PMT = -500
FV = 0
PV = ?

If the implicit rate on the lease were 11.23 percent, all entries are the same except I/Y = 11.23 and the present value of the lease is $12,648.

Example 18

A number of years ago, Dan borrowed funds at 13 percent interest to purchase an orchard sprayer. His payments are $157.60 per month. He still has 47 payments to make. What is the outstanding principal on the loan?

Using Tables

$157.60(36.6790) = $5,780.61

Table 5, 47 months, 13 percent

Using Calculators

P/Y = 12
N = 47
I/Y = 13 PV = $5,780.61
PMT = 157.60
FV = 0
PV = ?

Example 19

Mark is borrowing $10,000 to buy a pickup. If it is borrowed at nine percent for 30 months, how much will his monthly payments be?

Using Tables

$\frac{\$10,000}{26.7751} = \$373.48$

Table 5, 30 months, nine percent

Using Calculators

P/Y	= 12	
N	= 30	
I/Y	= 9	PV = $373.48
PV	= 10,000	
FV	= 0	
PV	= ?	

If Mark uses dealer financing he can get interest at 2.99 percent by giving up a $500 rebate. In this case, all entries would be the same except I/Y would equal 2.99 and PV would equal 10,500. The payment would be $363.68.

7. Present Value of a Uniform Infinite Series

If the same amount will be received each year forever, the present value can be calculated even more simply. In this case, the annual amount is divided by the interest or discount rate, equation (9).

$$PV = A/i \tag{9}$$

Example 20

One of the methods used to determine the value of farmland is to capitalize the annual rent the land is expected to earn (known as the income approach to appraisal). This is referred to as the capitalized value of land. If the net return to land for a particular apple orchard is estimated at $315 per acre and the appropriate capitalization rate is nine percent, the capitalized value of the land equals:

$$\frac{\$315}{.09} = \$3{,}500/\text{acre}$$

However, if the expected apple price drops $0.20/bushel, and the orchard has an average yield of 500 bushels, the expected annual income declines to $215 and the capitalized value equals:

$$\frac{\$215}{.09} = \$2{,}389/\text{acre}$$

AMORTIZATION

As implied earlier, the equation for the present value of a uniform series (equation 8) can be rearranged to provide an equation for calculating the payments required on a loan, or amortization values. The amortization equation, thus, is equation 10.

$$PMT = \frac{PV}{\left[\frac{1-(1+i)^{n}}{i}\right]} \tag{10}$$

Where PV = the amount to be amortized or the value of the loan in period zero (now).

PMT = the amount of each payment

i = the interest rate per period (annual rate divided by the number of payments per year)

n = number of payments (periods) the PMT amount is to be paid.

Using Tables

From equation 10 it is clear that the amount of the payments on a loan can be calculated by dividing the loan amount by the present value factor from tables 3 or 5 (as illustrated in Examples 14 and 15). It is also clear that the amount of payment per dollar of loan can be calculated as the reciprocal of the bracketed amount in equation 10.

To make amortization calculations more straightforward values of that reciprocal

$$\frac{1}{\left[\frac{1-(1+i)^{-n}}{i}\right]}$$

are calculated and presented in tables 7 and 8. These tables indicate the amount of payment per dollar of loan.

Example 21

Aaron is selling the farm real estate to his son, Alan. They have agreed that the price of the real estate is $450,000 and that Alan will pay 7.5 percent interest. How much are Alan's monthly payments if the money is paid back over 20 years?

Using Tables

450,000 (.008056) = $3,625.20

Table 8, 20 years, 7.5%

Using Calculators

P/Y = 12
N = 240
I/Y = 7.5 PMT = 3,625.17
PV = 450,000
FV = 0
PMT = ?

The difference between results determined using tables and those using calculators is due to rounding of table values.

Example 22

Steve is borrowing the money from his mother to get started in farming. She has agreed to lend him $75,000 and take a second priority lien on his line of equipment. They have agreed that he will repay the loan over 7 years at 6.5 percent interest with annual payments. How much are his payments?

Using Tables

75,000 (.182332) = $13,675

Table 7, 7 years, 6.5 percent

Using Calculators

P/Y = 1
N = 7
I/Y = 6.5 PMT = $13,675
PV = 75,000
FV = 0
PMT = ?

Example 23

Mary is planning to buy the home farm from her parents. The farm is valued at $500,000. They expect to ultimately will her half of the farm and plan to put the principal Mary repays in a separate account which, along with the remaining monthly payments, will be willed to Mary's nonfarming sister, Susan. Thus they want a loan that will amortize half of the principal (partial amortization) and provide Mary's parents with income on the complete farm value during their lifetimes. With this type of loan, how much will Mary's monthly payments be if the loan is for 25 years at 8.25 percent interest.

Using Calculators

P/Y = 12
N = 300 (25 x 12)
I/Y = 8.25 PMT = $3,690
PV = 500,000
FV = -250,000 (500,000 x .5)
PMT = ?

Example 24

George plans to lease his line of equipment to his daughter, Amanda. They have agreed that the equipment is now worth $150,000 and at the end of the 5 year lease period it will be worth about $25,000. If, through the lease, George wants to earn 7.8 percent interest on the money he has invested, and be paid for the machinery that is "used up", at what level should they set Amanda's monthly lease payments?

Using Calculators

P/Y = 12
N = 60
I/Y = 7.8 PMT = $2,685.10
PV = 150,000
FV = -25,000
PMT = ?

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