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A LABOR SUPPLY THEORY OF ECONOMIC DEVELOPMENT

by

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PREFACE

Recent technological breakthroughs in the agricultural sectors of a number of low income countries offer potential for acceleration in the rate of growth of employment and substantial broadening of the distribution of income. These complex processes are the subject of a Cornell University-USAID research contract the purposes of which are (1) to conceptualize the relationship between technological change in agriculture and employment and income distribution, (2) to empirically describe and analyze the relationships set forth in the conceptual framework, and (3) from these to develop policies for further increasing desirable effects on employment and income distribution.

In this paper we view the relationship between growth in potential for employment in the nonagricultural sector and the supply of food. Special attention is given to the varying effect of technological change in agriculture on the distribution of benefits of increased agricultural production to different income classes and the consequent differing proportions of production marketed. The model presented here provides a general equilibrium system for a dualistic economy in terms of the food and the labor market. The model concentrates attention on the role of technological change in food production and on the availability of food for the nonagricultural labor force. It analyzes these relations in the context of two markets - the food market and the labor market. A subsequent model will introduce a third market, the capital market, and view the effect of technological change on the availability of capital to the agricultural and nonagricultural sectors. In that model the complex interactions of the food, labor and capital markets will be studied with emphasis on the influential role of technological change in agriculture.

Issued concurrently with this paper is one entitled, "The Political Economy of Employment Oriented Development," by Uma J. Lele and John W. Mellor. In that paper we discuss the implications of our model to policy for development of the agricultural sector and to various other aspects of economic development -- including the choice of industrial structure, the choice of production technique, the domestic savings rate, the scale of industrial organization and the level and composition of trade.

A third paper entitled, "A Further Note on Dualistic Models," by Uma J. Lele points out the basic relevance of dualistic models to analysis of the implications of technological change in agriculture to overall development and then points out the basic shortcomings of these models as currently developed for the purposes of viewing contemporary technological change in the agricultural sector. That paper provides useful background for the basic model presented here.

A series of empirical studies are underway in which we are testing various parts of the formulation set forth in this model. These empirical efforts include studies of the technological bias of the new high yielding crop varieties and their first round effects on the distribution of income and the level of marketings. Another series of studies is examining the employment potentials in the agricultural sector as they relate to the new technologies in agriculture including both primary and secondary employment effects. Special attention is being given to the problem of allocating rural labor amongst various alternatives subject to the various restraints of food supply, capital and administrative talent. Other studies are concerned with the special problems of small farmers in a context of rapid technological change; the various factors which affect movement of rural labor to employment opportunities within the context of technological change in agriculture; and, the special relationships between developments in the rural sector and increased employment in small scale industry. Papers reporting the results of these studies will be issued in this series.

The papers in this series are part of a larger series which includes papers from a previous AID research contract concerned with the role and function of agricultural prices in economic development. Many of those papers, particularly those concerned with intersectoral resource transfers have relevance to the current research on employment and income distribution.

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A Labor Supply Theory of Economic Development

John W. Mellor and Uma J. Lele*

Macro planning models for low income countries typically emphasize growth in per capita consumption over time at the cost of immediate growth of employment. This generalization includes not only the earlier simple aggregative models of Harrod-Domar and Fel'dman-Mahalnobis, but a considerable number of the recent, more complex multisectoral, intertemporal models.[1] A low employment component in these models is a product of the assumptions on which they are built. For example, these models generally assume a low growth potential for exports, limited foreign aid and hence a balance of payments constraint. Most important, they assume a technologically stagnant agricultural sector, a sector which produces up to half the GNP and an even larger proportion of consumption goods. Further, these models often assume a domestic savings constraint or assume that output is only produced by capital. Employment thus becomes only a by-product of growth of output rather than being an explicit variable.

The employment question is relegated to the background by rigid assumptions about capital output ratios and composition of demand thus ignoring the implications to employment of chóice of technique and of industrial structure as well as of technological change. As a result of such assumptions these models may either produce import-displacing, capital intensive patterns of growth that have a low employment potential or recommend massive investments in agriculture to keep up with growth in the demand for food.[2]

The Indian development experience provides an excellent example of these assumptions and policies. The low employment, capital intensive, import-displacing pattern of industrial development in India seems to have been justifiable, expost facto. However, each of the justifying factors is in part a product of government policy and thus the premises of the planning models have been substantially self-fullfilling.

Per capita agricultural output did not increase significantly in the first twenty years of planning and showed substantial year to year fluctuations. This was followed by a failure to mobilize market suplus of food through the open market or through governmental policies of food procurement.[3] Given an essentially stagnant agricultural sector relative agricultural prices would have increased significantly if the growth of employment had been considerably more rapid than that actually accomplished in the past two decades.

Exports were stagnant not only because of the low growth potential for traditional exports but also because of failure to develop export markets for nontraditional manufactured goods. The latter itself was in part a consequence of a stagnant agriculture and a low rate of savings, and in part a product of the industrial structure implicit in the planning models. Small and uncertain per capita foreign aid reinforced the foreign exchange constraint.

Finally, the pattern of industrial development provided by the planning models failed to boost the saving rate among small savers or to effectively channel what small savings did occur.

Dualistic models of the Lewis type, which perceive economic development as synonymous with growth of non-agricultural employment provide a clear alternative to these capital oriented macro-models.[4] They have however, never received serious consideration at the policy level. This is because of the failure of the dualistic models to analyze realistically the mechanism of food transfers from the agricultural to the non-agricultural sector. The market mechanism to augment the food supplies required to feed the transferred labor does not work with a stagnant agriculture. And the political, administrative nightmare of forcibly extracting food surplus from an essentially stagnant agriculture rarely provides a viable alternative.

In a low income economy aggregate food production is generally highly inelastic to changing terms of trade between agriculture and industry, whereas the marginal propensity of the laboring class to consume food is high.[5] Because of these two features the food supply available to the nonagricultural sector constitutes a major constraint on growth of nonagricultural employment in the case of a stagnant agriculture. With a dynamic agriculture the rate of release of the food constraint is determined by complex forces of which the distribution of the agricultural product within that sector is of particular importance.

Given the possibilities of substantial increases in agricultural production as a result of the so-called "green revolution" there is a significant potential for release of the food constraint. Empirical evidence suggests considerable factor share bias of new agricultural technology. The extent of bias varies substantially among innovations and physical environments. Thus the nature of technological change in agriculture is of interest because of its effect on marketed surplus and labor mobilization for non-farm employment. These vary considerably, first, according to the factor share bias and second according to the demand elasticities of the various income classes.

For these reasons, we present a model which allows emphasis on the food transfer mechanism in a context of technological change in the agricultural sector. The approach has two major distinguishing features. First, rather than assuming that per capita agricultural output in the agricultural sector is jointly mobilized with labor, we treat the food market as an independent market and then examine the interaction between the food market and the labor market. More generally our model is labor supply and consumer goods oriented rather than capital supply and investment goods oriented. It is thus in sharp contrast to the Fel'dman-Mahalnobis models and their various sophisticated derivatives. Second, we explicitly allow for changing share of agricultural output between different classes and examine its effect on market supplies of food and hence on rate of growth of employment.

Our analysis has the following specific objectives:

1. It provides a general equilibrium system for a dualistic economy in terms of the food and the labor market.

2. It examines the effects of changes in: a) agricultural output and factor shares induced by technological change, b) population, and c) growth of capital stock in the non-agricultural sector on 1) the supply of marketed surplus, 2) the equilibrium level of non-agricultural employment, 3) the equilibrium terms of trade between agriculture and industry, and 4) the equilibrium real wage.

3. It analyzes a) the rate of growth of non-agricultural employment and its relationship with the growth of capital stock over time, and b) changes in terms of trade over time.

In the following section we discuss the assumptions on which our formulation is built. The discussion has a dual purpose. First, where we have departed from assumptions made in other labor surplus formulations we emphasize how our assumptions provide the model a current relevance. Second, we discuss how some of our assumptions help keep the mathematical formulation simple without substantially altering the relevant conclusions. For both these reasons the section should be of considerable interest from the viewpoint of policy analysis, although, to strict model builders, the discussion may seem only peripheral.

I. Assumptions

We assume that agricultural output is a function of labor, land and technological change. For the reasons outlined below, we further assume that per capita agricultural output can be increased only by technological change in agriculture.

In traditional agriculture, i.e., with no technological change, output increases through a direct input of labor or through land and capital which are largely a direct embodiment of labor. Such increased labor input is the result of an immiserizing process, added labor being employed in production at declining marginal product as population growth reduces per capita incomes, thus increasing marginal utility of the additional income.[6] For these and other reasons of logic supported by the empirical evidence, we assume aggregate agricultural production is highly inelastic with respect to the terms of trade between agriculture and non-agriculture.[7] These assumptions give primacy to technological change in agriculture in fostering a shift in the labor force to the non-agricultural sector.

Technological change in agriculture is often highly biased and varies greatly in the extent of bias. The same rate of increase in agricultural production in two successive periods may be brought about by two completely different technological changes, with highly different effects on marginal productivity of labor and labor use. Thus, although labor's share is determined at a given level of output by the marginal productivity of labor, its movement over time may be highly variable. We, therefore, examine the effect of changing labor share on the two market equilibria.

Owners of different factors of production evidence sharply differing consumption functions. For simplicity in dealing with this situation, our model divides the agricultural population into two classes--laborers and landowners. Laborers are assumed to have a positive income elasticity of demand for food of less than 1.00, but still substantially higher than that of landlords. Laborers are also assumed to have a negative price elasticity of demand. Landowners are assumed to consume a fixed amount of agricultural output per capita, regardless of its price or their income. Landowner's food consumption could also be expressed as a function of price and income changes just as in the case of laborers. However, empirical evidence shows that landowners with incomes well above subsistence have price and income elasticities of demand for food grains very close to zero.

Our formulation is intended to focus on intersectoral labor transfers. So we simplify our model by assuming that agricultural laborers consume all their income. In the case of landowners, the assumption is somewhat more involved. Since landowners are assumed to have zero income elasticity of demand for food grains the incremental share of landowners is marketed and a commensurate value of commodities purchased from the non-agricultural sector.[8]

These purchases include production inputs from the non-agricultural sector for use in the agricultural sector. Again in keeping with our focus on the labor transfer problem we do not include capital in our agricultural production function. In traditional agriculture, capital is essentially a direct embodiment of labor and, therefore, does not require separate treatment. Technological change generates sufficient increase in landowners' income to provide the required capital. This assumption is quite valid for the common case in which the bulk of increased capital associated with technological change is working capital for financing inputs purchased from the non-agricultural sector.

The sharp dichotomy between landowners and laborers is a very helpful simplifying assumption which distinguishes between those cultivators who predominently produce for the market as against those whose produce is mostly consumed domestically. The real world of peasant agriculture and gradation in size of farm is accommodated by viewing intermediate situations as appropriately weighted averages of landowners and laborers with a consequent weighted average set of demand elasticities. Since payments to laborers are assumed to be made in kind and since laborers are assumed to sell a portion of their receipts, this allows considerable further flexibility in the tenurial arrangements accommodated by the model.

Landowners are assumed to be fixed in number. This assumption can be modified to incorporate change in the population of landowners without altering the conclusions. Changing share of output, however, does illustrate many of the interesting results that could be derived by assuming changes in the population of landlords.

Production in the non-agricultural sector is assumed to be a function of labor and capital. No technological change is envisaged in the non-agricultural sector. Nevertheless, neutral technological change as assumed in other dualistic models could be incorporated easily in our system.

A closed economy is assumed. However, implications of this model for trade in non-agricultural commodities are discussed. The assumption of a closed economy focuses attention on the implications of price inelastic aggregate supply of agricultural commodities and the consequent key role of technological change in agriculture to overall economic growth. In practice individual small countries may encounter, through imports, an elastic supply of agricultural commodities. In those circumstances our model points to the desirability of trade if an adequate rate of technological change in agriculture cannot be achieved.

In the non-agricultural sector, the demand for agricultural commodities is assumed to be a function of the price of agricultural goods in terms of industrial goods and of the laborers real income. In equilibrium the wage of the laborers in the non-agricultural sector is equal to the per capita income of the agricultural laborers class.[9] Their income and price elasticities and budget shares are, therefore, assumed to be the same as those of the agricultural laborers. Further just as in the case of the agricultural laborers, non-agricultural laborers consume all their income. Demand for labor in the non-agricultural sector is determined by its marginal productivity. Non-agricultural profits are all saved and invested.

It is observed that rarely does the absolute size of the agricultural population decline prior to a major decline in the proportion of the population in agriculture and of the proportion of consumer expenditure on basic agricultural commodities. Thus, because our model refers to an early stage of development, we can assume that labor can be withdrawn from the traditional agricultural sector without reducing a) the absolute size of the agricultural labor force and b) per capita agricultural output. As long as industrial employment does not increase at a rate that more than absorbs increase in the population of agricultural laborers, the first condition will be fulfilled. It also seems apparent that with some reorganization of traditional agriculture, involving little additional capital input and marginal changes in techniques, it would be possible to withdraw a substantial amount of labor from agriculture without reducing per capita output. As we will see later this by no means assures constant terms of trade between agriculture and industry when labor is withdrawn from agriculture. These changes in terms of trade emphasize the potential limit imposed by the food market and the importance of viewing the two markets as separate but interacting entities.

Although we assume that the production of the basic foodgrains is inelastic, the production of other agricultural commodities may be quite elastic. This is because they occupy only a small proportion of the land area and use a much higher proportion of nonland inputs in their production. Thus in a practical treatment our agricultural sector would most usefully be defined to include only the basic foodgrains. As the food constraint is relaxed, production of high income elasticity, labor using agricultural commodities might expand in production through the supply of labor in a manner similar to that of industrial goods.

II. The Formulation

Our static model is comprised of a food market, a labor market, equilibrium in each and a general equilibruim as follows.

Notation:

A = agricultural output

l_A = agricultural labor input

$\bar{Z}$ = land

t = technology

M_S = marketed supply of food

M_D = market demand for food

$\bar{C}$ = total consumption of food by landowning classes

b = relative budget share allocated by laborers to the consumption of food

s = agricultural labor's relative share in agricultural output

P = price of agricultural goods in terms of non-agricultural goods

Y = per capita income of agricultural labor population in terms of agricultural goods

r = population of laborers in agriculture as proportion of total labor population

N = total labor force, i.e., labor force in the agricultural and non-agricultural sector

L_A = agricultural labor force

$= rN > l_A$

L_I = non-agricultural labor force

$= (1 - r)N$

W_D = demand price of labor in the non-agricultural sector in terms of non-agricultural goods

W_S = supply price of labor in the non-agricultural sector in terms of non-agricultural goods

X = non-agricultural output

K = non-agricultural capital stock

σ = elasticity of non-agricultural output with respect to capital

$1 - \sigma$ = elasticity of non-agricultural output with respect to labor

I = investment in the non-agricultural sector

The Food Market

Agricultural production is a function of labor, land and technological change. It is linear homogenous with respect to land and labor.

(1) $$A = f(l_A, \bar{Z}, t)$$

$$\frac{\partial A}{\partial l_A} > 0 , \quad \frac{\partial^2 A}{\partial l_A^2} < 0$$

The relative share of labor in aggregate agricultural output is determined by the marginal productivity of labor.

(2) $$s = \frac{l_A}{A} \cdot \frac{\partial A}{\partial l_A}$$

$$\frac{ds}{dt} \gtreqless 0$$

Marketed supply of food to the non-agricultural sector is the difference between output and consumption in the agricultural sector.

(3) $$M_S = A - \bar{C} - bsA$$

Budget share allocated by agricultural laborers to food is a function of terms of trade between agriculture and non-agriculture and their per capita income.

(4) $$b = f(P, Y)$$

such that $\frac{\partial b}{\partial P} < 0, \quad \frac{\partial b}{\partial Y} < 0$

Per capita income of agricultural laborers is equal to their share in the agricultural output divided by agricultural labor population.

(5) $$Y = \frac{sA}{rN} = \frac{sA}{L_A}$$

Market demand for food in the non-agricultural sector is equal to the budget share allocated to food consumption out of wage income by the non-agricultural laborers.

(6) $$M_D = b \cdot \frac{W}{P} \cdot L_I$$

The Labor Market

The production function for the non-agricultural sector is a Cobb Douglas linear homogeneous function of the first degree.

Thus:

(7) $$X = K^{\sigma} L_I^{1 - \sigma}$$

Demand for labor is a function of its marginal productivity.

(8) $$W_D = (1 - \sigma) \frac{X}{L_I}$$

Labor migrates from agriculture to the non-agricultural sector until the wage rate in the non-agricultural sector is equal to per capita income in the agricultural sector.

(9) $$\frac{W_S}{P} = \frac{sA}{rN}$$

Note that W_S is stated in terms of non-agricultural goods and, when deflated by the terms of trade index, shows the wage rate in terms of agricultural goods.

Investment in the non-agricultural sector is equal to the share of profits in non-agricultural output.

(10) $I = \frac{dK}{dt} = \sigma X$

Thus: $\frac{1}{K} \cdot \frac{dK}{dt} = \sigma \frac{X}{K}$

Equilibrium in the Food Market

(11) $M_S = M_D$

The equilibrium in the food market is graphically illustrated in figure 1.[10]

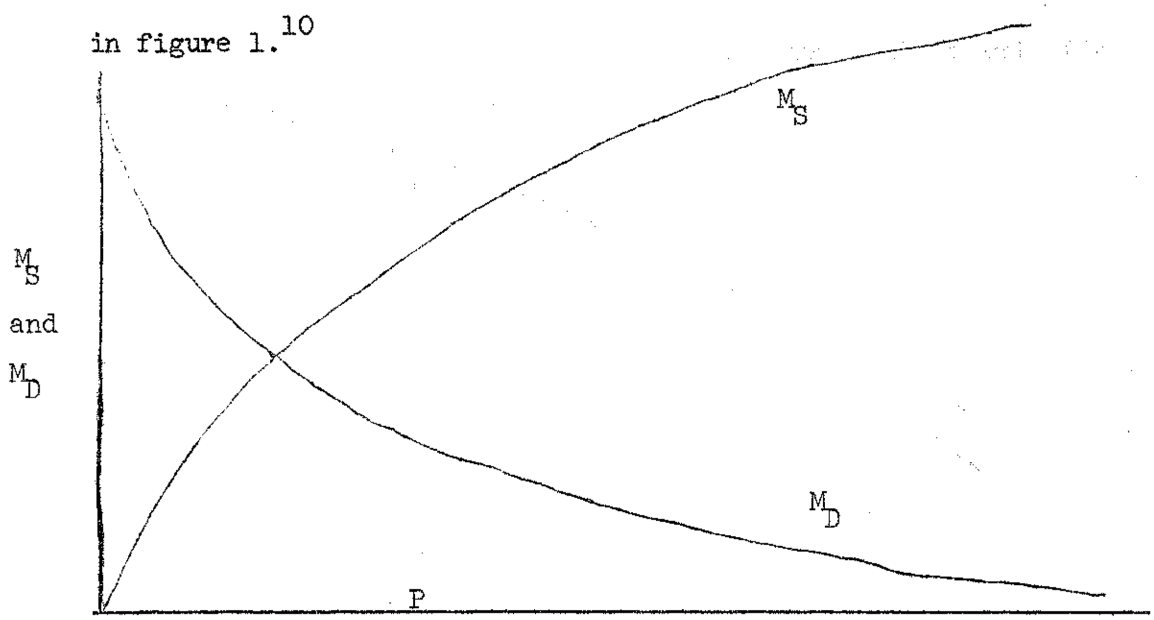

Figure 1: Equilibrium in the Food Market

Setting 3 equal to 6 it follows that:

(12) $A - \bar{C} - bsA = \frac{bsA\ (1 - r)}{r}$

12 can be restated as:

$$b(P,Y) = \frac{r(A - \overline{C})}{sA}$$

This will be referred to as an FF function representing equilibrium in the food market, It can be shown that for 12

$$\frac{\partial P}{\partial r} < 0$$

Equilibrium in the Labor Market

(13) $W_S = W_D$

The Equilibrium in the labor market is graphically illustrated in figure 2.

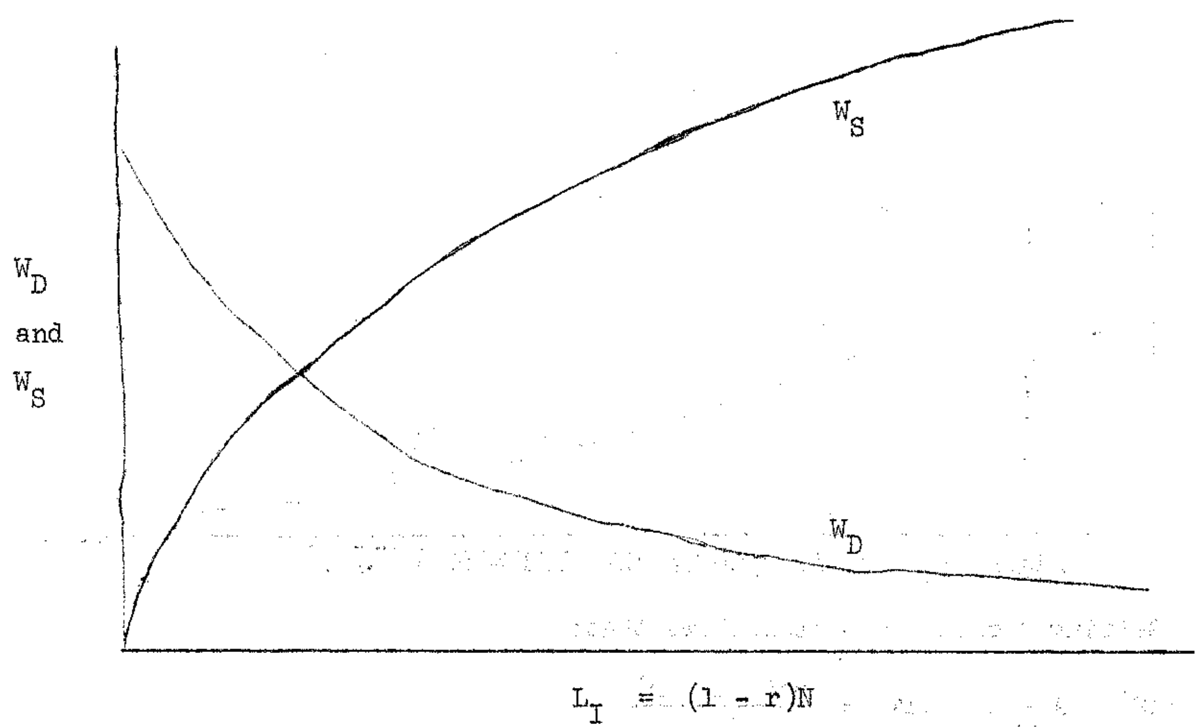

Figure 2: Equilibrium in the Labor Market

From 13 by setting 8 equal to 9 it follows that

$$(14) \qquad PY = \frac{(1 - \sigma)\, K^{\sigma}}{L_I^{\ \sigma}}$$

Substituting for L_I and Y 14 can be restated as:

$$P = \frac{(1 - \sigma)\, K^{\sigma}}{(1 - r)^{\sigma}\, N^{\sigma}} \cdot \frac{rN}{sA}$$

This will be referred to as the LL function representing equilibrium in the labor market. It can be shown that for 14

$$\frac{\partial P}{\partial r} > 0$$

<u>The General Equilibrium</u>

Since b is a function of P and Y, substituting

$$\frac{(1 - \sigma)\, K^{\sigma}}{(1 - r)^{\sigma}\, N^{\sigma}} \cdot \frac{rN}{sA}$$

for P from equation 14 and $\frac{sA}{rN}$ for Y from equation 5 into equation 12, we obtain a condition for a general equilibrium as follows:[11]

$$(15) \qquad r\left(\frac{A - \overline{C}}{A}\right) - b\left(\frac{rN}{sA}\,\frac{(1 - \sigma)\, K^{\sigma}}{(1 - r)^{\sigma}\, N^{\sigma}},\ \frac{sA}{rN}\right) s = 0$$

The general equilibrium derived from 12 and 14 may be shown graphically as follows:

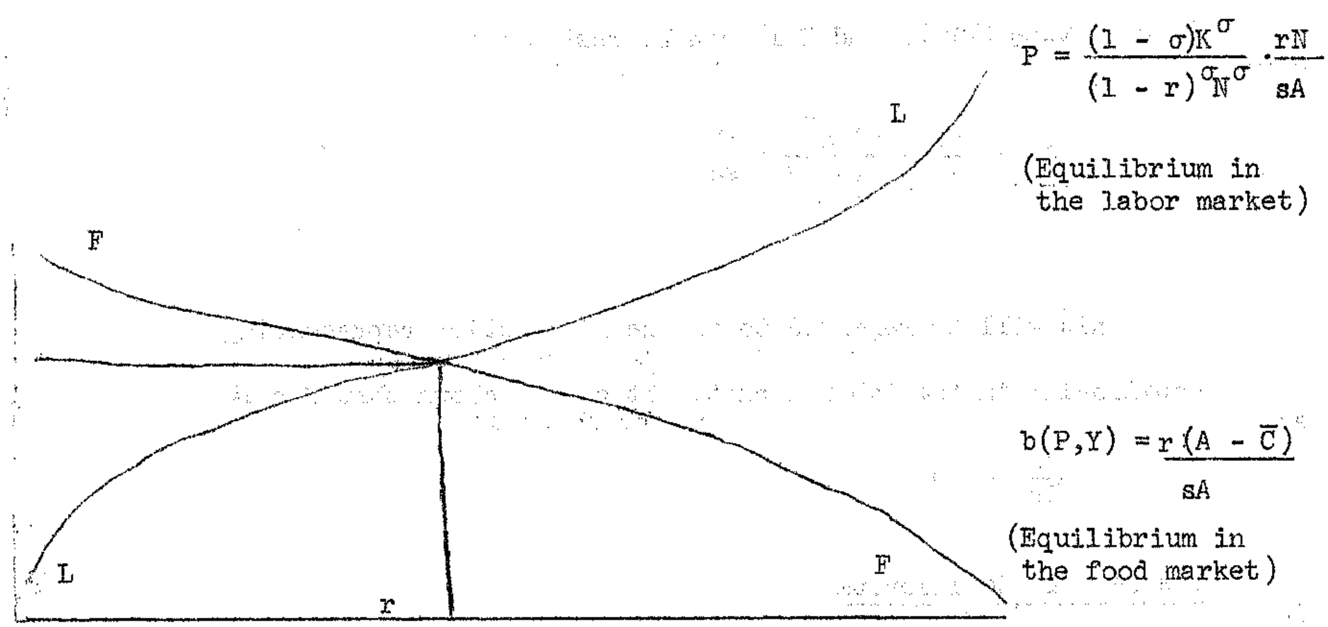

Figure 3: General Equilibrium in the Food and the Labor Market

IV. Sensitivity Analysis

We will now proceed to investigate the sensitivity of r, P, M_S and W/P with respect to A, N, and K. This is done by partial differentiation of the solution equations for each of the four endogenous variables in the model with respect to A, N, and K. Changes in A (agricultural output) which increase, decrease or leave labor share unchanged are each examined. The signs in the sensitivity matrix presented below are based on the ranges of all possible numerical values that the variables and parameters are likely to take in a dualistic economy.

	A			N	K
	$\frac{\partial s}{\partial A} > 0$	$\frac{\partial s}{\partial A} = 0$	$\frac{\partial s}{\partial A} < 0$		
r	$\pm$	$-$	$-$	$+$	$-$
P	$\pm$	$-$	$-$	$+$	$+$
M_S	$\pm$	$+$	$+$	$-$	$+$
$\frac{W}{P}$	$\pm$	$+$	$\pm$	$-$	$+$

The results in the sensitivity matrix are immensely interesting. They show that when the increase in the agricultural output is brought about without changing labor's relative share, as in the case of a neutral technological change, the effect of change in agricultural output on r, P, M_S and W/P can be determined unequivocally for all likely values of variables and parameters in a dualistic economy. The matrix shows similar unequivocal results in the case of effects of changes in N and K on r, P, M_s and $\frac{W}{P}$.

However, the most interesting results are obtained in the case of an increase in the agricultural output that changes relative factor shares. This may happen, either when technological change is biased or when the increase in the agricultural output is brought about mainly through increased labor input. The sensitivity analysis emphasizes that when factor shares in the agricultural sector change, what happens to r, P, M_s and $\frac{W}{P}$ as a result of increase in the agricultural output depends very much on the relative magnitude of the various counterbalancing forces.

For example, except in the case of W/P, all the other results obtained for neutral technological change are reinforced when labor's share declines as a result of an increase in the agricultural output. However, in the case of W/P, the effect of increased agricultural output accompanied by labor's declining share is indeterminate. This is because of the following factors. The equilibrium wage rate in the non-agricultural sector, when measured in terms of agricultural goods, is equal to the per capita income in the agricultural sector. A decline in the labor's share causes a decrease in the proportion of population in the agricultural sector, thus increasing per capita income of the existing agricultural labor force. While at the same time, a decline in the labor's share also pushes the per capita income of the agricultural labor force downward directly. The resultant wage rate in the non-agricultural sector is thus a combined effect of these two mutually opposite forces and depends upon their relative magnitudes.

When increase in agricultural output is accompanied by increase in labor's relative share, the effect on r, P, M_S and W/P may go in either direction. If labor's relative share increases only slightly, relative to the increase in the agricultural output, the effect of increased agricultural output on r, P, M_S and W/P will be greater relative to that of increasd labor's share. However, if the labor's share increases substantially as a result of the increased agricultural output the effect on r, P, M_S and W/P may be opposite to that when increased agricultural output is not accompanied by changing factor shares.

These interactions will be discussed in the dynamic analysis in the next section. The preceeding discussion does suggest that in the context of growth the most interesting results in the sensitivity matrix are those relating to labor's share in agricultural output. They show that with increased labor'share, as for production increases in traditional agriculture, proportion of population in the non-agricultural sector may decline, terms of trade may move in favor of the agricultural sector, the marketed surplus of food may decline and the real wage in the non-agricultural sector may increase. Converse changes may be expected when technological change decreases labor's share in agricultural output. The factor shares in the agricultural sector are thus of crucial importance in the growth of the non-agricultural sector.[12]

V. Dynamic Analysis

Growth of Employment and Capital Stock Over Time

Equation 15, may be differentiated with respect to time, set equal to zero and income elasticity of demand for food substituted for

$$\frac{\frac{1}{\bar{C}_F}\cdot\frac{d\bar{C}_F}{dt}}{\frac{1}{Y}\cdot\frac{dY}{dt}} \quad \text{where} \quad \frac{1}{\bar{C}_F}\cdot\frac{d\bar{C}_F}{dt} = \frac{1}{(A - \bar{C})}\cdot\frac{d(A - \bar{C})}{dt} - \frac{1}{N}\cdot\frac{dN}{dt}$$

It can be shown that the rate of growth of per capita real income of the agricultural labor force bears the following relationship with the capital-labor ratio in the non-agricultural sector

$$(16) \qquad \frac{1}{Y}\cdot\frac{dY}{dt} = \sigma\,\frac{1}{k}\cdot\frac{dk}{dt}$$

$$\text{where} \quad \frac{1}{k}\cdot\frac{dk}{dt} = \left(\frac{1}{K}\cdot\frac{dK}{dt} - \frac{1}{L_I}\cdot\frac{dL_I}{dt}\right)$$

Equation 16 shows that as long as per capita income of the agricultural laborers increases the capital-labor ratio in the non-agricultural sector also increases. Also, since $\sigma < 1$ the capital-labor ratio increases more rapidly than the rate of growth of per capita income. It is interesting to note here that, since $Y = \frac{sA}{rN}$, per capita income in the agricultural sector may increase, not only because of increase in agricultural output but, also because of increase in labor's share or due to decline in the labor force in the agricultural sector. It, therefore, seems highly probable that the capital-labor ratio in the industrial sector would rise over time, for even if agricultural output increases only as rapidly as the population growth, and even if labor's share does not increase, just the withdrawal of population from the agricultural sector would cause an increase in per capita income of agricultural laborers.

It is of considerable interest to examine further the factors that would determine the rate of growth of employment in the non-agricultural sector. Solving equation 15 for L_I and differentiating with respect to L_I gives us

$$(17) \quad \frac{dL_I}{dt} = \frac{\partial L_I}{\partial A}\cdot\frac{dA}{dt} + \frac{\partial L_I}{\partial N}\cdot\frac{dN}{dt} + \frac{\partial L_I}{\partial K}\cdot\frac{dK}{dt} + \frac{\partial L_I}{\partial s}\cdot\frac{ds}{dt}$$

Thus:

$$(18) \quad \frac{1}{L_I}\cdot\frac{dL_I}{dt} = \frac{1}{N}\cdot\frac{dN}{dt} + \frac{\frac{\alpha r\bar{C}}{A} - sb\{-(nA-1-\epsilon_{AA})(\mu-\alpha) + \epsilon_{AA}\sigma(\frac{\sigma X}{K}-\mu) - \beta(\epsilon_{AA}-n_A)\}}{(1-r)\left(\frac{A-\bar{C}}{A} - \frac{sb}{r}\left\{\frac{\epsilon_{AA}(1-r+\sigma r)}{(1-r)} - (n_A-1)\right\}\right)}$$

where η_A = income elasticity of demand for food

ϵ_{AA} = price elasticity of demand for food of agricultural laborers when payments are made in kind.[13]

μ = rate of growth of total labor force

α = rate of growth of agricultural output

$\beta = \frac{1}{s} \cdot \frac{ds}{dt}$ = rate of change of labor's relative share in the agricultural output

It can be shown from 18 that $\frac{1}{L_I} \cdot \frac{dL_I}{dt}$ is positive for all values of $\alpha > \mu$.

Further, from equation 18 may be noted the various factors that influence the magnitude of $\frac{1}{L_I} \cdot \frac{dL_I}{dt}$. For example, it may be noted that the larger the value of α, i.e., the greater the rate of growth of agricultural output, the faster the rate of growth of non-agricultural employment. It should also be noted that the rate of growth of employment is inversely related to the movement of labor's share in agricultural output. When labor's share decreases $\frac{1}{L_I} \cdot \frac{dL_I}{dt}$ increases. It is also clear from 18 that the larger the share of profits in the non-agricultural output the greater the rate of growth of employment in that sector.

These relationships are of immense interest in the policy context. They indicate that to the extent that technological change in the agricultural sector is accompanied by increased labor's share in output, it would provide a dampening effect on the growth of non-agricultural employment. This would occur

1) through its unfavorable effect on marketed supply of food

2) through its effect on the level of industrial wages required to withdraw labor from agriculture to the non-agricultural sector.

In fact, growth in agricultural output may be completely compensated by increased share of agricultural laborers with no effect on the growth of non-agricultural employment. By the same token technological change that brings about a movement in the distribution of agricultural output against the laboring classes, may enhance the growth of non-agricultural employment. This crucial relationship between distribution within the agricultural sector and its effect on non-agricultural employment through wage rate and through mobilization of marketed surplus is neglected by the existing dualistic models.

These results are of even further interest due to their implications for the magnitude of the capital-labor ratio over time. They show that although the capital labor ratio in the non-agricultural sector will increase with increase in the per capita income in the agricultural sector, the actual magnitude of the capital-labor ratio is contingent upon the rate of growth of agricultural output and the changes in relative factor shares in the agricultural sector. Thus the capital-labor ratio will increase less rapidly if agricultural output grows at a high rate than if it does not. It may increase even less rapidly if increase in agricultural output is accompanied by a decline in labor's share in agricultural output. This is because the opportunity cost of labor to the non-agricultural sector is dependent on per capita income in the agricultural sector, which is a function, not only of agricultural output but also of relative factor shares in the agricultural sector. These complex conclusions are at variance with the simplistic treatment and conclusions concerning capital-labor ratios in the Jorgenson and Fei-Ranis treatment of dualistic models.

These results also have significant implications for trade. A labor augmenting technological change in the agricultural sector, by keeping the capital-labor ratio in the non-agricultural sector from rising as rapidly as it would otherwise may provide considerable continuing comparative advantage in the production and export of labor intensive commodities in a dualistic economy such as ours.

<u>Change in Terms of Trade Over Time</u>

Movements in the terms of trade over time may be analyzed by differentiation of P with respect to A, s, N, and K.

$$(19) \quad \frac{dP}{dt} = \frac{\partial P}{\partial A}\cdot\frac{dA}{dt} + \frac{\partial P}{\partial K}\cdot\frac{dK}{dt} + \frac{\partial P}{\partial N}\cdot\frac{dN}{dt} + \frac{\partial P}{\partial s}\cdot\frac{ds}{dt}$$

$$(20) \quad \frac{1}{P}\cdot\frac{dP}{dt} = a_1\cdot\frac{1}{A}\cdot\frac{dA}{dt} + a_2\cdot\frac{1}{K}\cdot\frac{dK}{dt} + a_3\cdot\frac{1}{N}\cdot\frac{dN}{dt} + a_4\cdot\frac{1}{s}\cdot\frac{ds}{dt}$$

where a_1, a_2, a_3, a_4 are respectively elasticities of price with respect to change in agricultural output, capital stock, population and labor's share in agricultural output.

Thus:

(21)
$$a_1 = \frac{\partial P}{\partial A} \cdot \frac{A}{P}$$

$$= \frac{-\left[\frac{r\bar{C}}{A} - sb\ (-\epsilon_{AA} + n_A - 1)\right]}{\frac{(A - \bar{C})\ r\ (1 - r)}{A(1 - r + r\sigma)} - sb\epsilon_{AA}} < 0$$

(22)
$$a_2 = \frac{\partial P}{\partial K} \cdot \frac{K}{P}$$

$$= \frac{b\ \sigma\ \epsilon_{AA}}{\frac{(A - \bar{C})\ r\ (1 - r)}{A\ (\ - r + r\sigma)} - sb\epsilon_{AA}} > 0$$

(23)
$$a_3 = \frac{\partial P}{\partial N} \cdot \frac{N}{P}$$

$$= \frac{sb\ \{\epsilon_{AA}(1 - \sigma) - (n_A - 1)\}}{\frac{(A - \bar{C})\ r\ (1 - r)}{A\ (1 - r + r\sigma)} - sb\epsilon_{AA}} > 0$$

(24)
$$a_4 = \frac{\partial s}{\partial P} \cdot \frac{P}{s}$$

$$= \frac{s\ b\ (n_A - \epsilon_{AA})}{\frac{(A - \bar{C})\ r\ (1 - r)}{A\ (1 - r + r\sigma)} - sb\epsilon_{AA}} > 0$$

It can be shown that depending upon whether

$$s\ b\ (n_A - {}_{AA} - b\ \sigma\ \epsilon_{AA}) \gtreqless \frac{r\bar{C}}{A}$$

$\frac{1}{P} \cdot \frac{dP}{dt}$ will be $\gtreqless 0$ and prices will increase, remain constant or decrease over time. The movement of terms of trade between agriculture and industry over time are thus dependent upon a complex set of factors and may move in either direction depending upon the magnitudes of these several parameters and variables. It should be noted that the terms of trade are determined by the price, income and output elasticities on the one hand and by the factor shares in the agricultural sector and average propensities to consume of the two income classes on the other.

VI. Conclusion

The system presented in this paper fills a major gap in the theory of a dualistic economy. It examines the functioning of the food and labor market as two independent markets and examines the interaction between the two and its effect on labor mobilization. It explicitly allows for a) the varying share of labor in the total agricultural output b) for the varying response of food consumption to income and price changes for the two classes in the agricultural sector. This is a significant improvement over the existing dualistic theories that treat growth of agricultural output as being synonymous with growth of agricultural marketings. This model therefore, analyzes the rate of growth of non-agricultural employment in terms of the growth rate of agricultural output as well as of changing factor shares in the agricultural sector.

It must, however, be emphasized that this formulation is only a first step forward from the existing dualistic models. It does provide considerable scope for incorporating the third major market in the general equilibrium system, namely the intersectoral capital market. The existing theories, including this one, assume that nonagricultural investment occurs only through savings in that sector. The evidence from Japan and Taiwan and more recently from India indicates that savings in the traditional sector provide a considerable scope for industrialization particularly of the small scale type which does not require lumpy capital investments. In dualistic economies, in which the capital market is ill-organized and inefficient, considerable interest must lie in examining the role of the intersectoral capital market in the pace of industrialization. It is clear that technological change in agriculture with its varying factor bias will be an important determinant of the sign and direction of intersectoral capital transfers.

Further, this model, like its predecessors assumes that supply of consumers goods other than food is highly elastic and that no serious bottlenecks in industrialization occur due to changes in the prices of these goods. It may be worthwhile to examine the effect of varying degrees of elasticity of supply of consumer goods other than food on labor mobilization.

The analysis thus provides a variety of possibilities for developing a more realistic model of industrialization. It also suggests areas of empirical investigation of relationships that are crucial to this analysis and that have been largely neglected in the past. These include:

1. Changes in factor shares resulting from various types of new agricultural technology, e.g. effects of improved varieties, multiple cropping, irrigation, and mechanization, on factor shares in the agricultural sector. Our model provides a relevant framework in which to view the indirect effects of changing factor shares from different technologies.

2. The response of different income groups to price and income changes in terms of domestic consumption and marketing of food. This is of particular relevance in the context of technological change which results in considerable changes in the distribution of the physical product.

3. The capital-labor ratios and the capital-output ratios implicit in different types of non-agricultural investments--both in the manufacturing sector as well as in the development of infrastructure. By emphasizing labor as a scarce resource when combined with food, our model emphasizes a need for an optimal combination of industries with varying capital-labor ratios. Although not explicit in the presentation of the model, it is apparent that if the rate of increase of the marketed supply of food increases, a short run disequilibrium between capital and labor supplies will be created. This calls for search for

means of reducing capital-labor ratios. The potentials for reducing capital-labor ratios through restructuring of industry needs to be examined. Trade and a new structure of domestic demand incident to redistribution of income also will have considerable implications for the structure of industry.

4. Implications of rapid industrialization to trade patterns. The effect of accelerated growth of marketed food supply on the structure of industrialization on factor intensity and hence on comparative advantage needs increased attention.

5. The balance in foreign aid between consumer goods, including food aid, and capital goods. Past models of growth have favored a major emphasis on capital goods with a resultant small employment component in aid induced growth. Renewed study is needed of the relation and balance between consumer goods, including food, labor mobilization and capital goods. There is implicit in this a concept of balanced aid between capital goods and consumer goods and an interesting set of questions concerning the relative merits of trade and aid from the point of view of employment policy.

All these relationships acquire a new significance in the wake of the new potentials for technological change in the agricultural sector and their implications for expansion of the non-agricultural labor force.

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FOOTNOTES

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1. See Jagdish Bhagwati and Sukhomoy Chakravarty for a review of substantial literature on planning models as applied to India. Also see Irma Adelman and Erik Thorbecke. Louis Lefeber's recent article does examine the question of employment. However, due to his rigid assumptions, his work, like others also reaches the dismal conclusion of a choice between present growth of employment vs. future level of consumption.

2. As an example of the latter see Sukhomoy Chakravarty and Louis Lefeber. At an operational level the planning models have proved unsatisfactory for a variety of reasons, among them not the least important is failure to incorporate fluctuations in agricultural production. See, for example, the various forumlations of Chakravarty and Lefeber and Richard S. Eckaus and Kirit S. Parikh for India.

3. A number of empirical generalizations are made at various points in this paper. They are in general drawn from the work of John W. Mellor et. al. and Uma Lele for India T. H. Lee for Taiwan, and Mellor, 1970 for various countries.
4. See W. Arthur Lewis, Gustav Ranis and John C. H. Fei, and Fei and Ranis. For a critical comparison of these models with Dale Jorgensen, 1961-1965 as well as emphasis on the terms of trade problem, see John W. Mellor, 1967.
5. The scant empirical evidence that exists on the relationship of aggregate supply to price changes supports this contention. See for example, Rober Herdt and Howard Barnum.
6. For analysis of these complex firm-household relationships see Mellor, 1963, and Amartya K. Sen.
7. Assuming a utility function with fixed coefficients and unit elasticity of substitution Sen (p. 437) shows that the response of output to price must be positive. He, however, admits that "without further empirical research we cannot say how realistic are the cases covered here." The exercise is, therefore, only esoteric. In absence of knowledge of the true shape of the utility function a judgement about the most likely supply response must depend heavily on empirical evidence such as that cited in footnote 5.
8. It will be noted that although the emphasis in our presentation is on the labor supply problem, the mechanism discussed includes an increase in demand for goods produced

in the non-agricultural sector, which itself may be an important dynamic of growth.

9. This assumption is different from the assumption in the classical dualistic model. Our assumption may lead to an increasing equilibrium real wage in the non-agricultural sector, as per capita income of the remaining agricultural laborers in the agricultural sector increases a) with withdrawal of labor, b) with increasing agricultural output, or c) with changing share of laborers in agricultural output. Labor migrates from the agricultural to the non-agricultural sector until wages in the non-agricultural sector equal per capita incomes in the agricultural sector. The simplicity of the assumption has increased the analytical facility of our formulation. However, maintenance of the conclusions only requires that wages in the non-agricultural sector be a function of the average income in agriculture. This assumption could be modified to contain the more complex formulation of the Todaro model without altering the conclusions of our analysis. For example, a high non-agricultural wage might be discounted by repeated unemployment.

10. The derivation of the supply and demand curves is based on the assumption that b is not linear homogenous with respect to P and Y.

11. Assuming a price adjustment in the food market and a quantity adjustment in the labor market and using the correspondence principle and Engel's law it can be shown that for the equilibrium to be stable where $\frac{ds}{dA} = 0$, it is necessary that $b_1 < 0$ where $b_1 = \frac{\partial b}{\partial P}$. We are grateful to Roger Selley for derivation of the stability conditions. See Appendix B to this paper.

12. Our exposition places perspective on the controversy between James Nakamura and others concerning the rate of growth of agricultural output in Japan during the Meiji period. Presumably during the Meiji period growth in output was increasingly derived from yield increasing technological change in contrast to the more labor intensive sources of growth in Tokugawa period (see Thomas Smith). The resultant change in factor shares would support a greater growth in non-agricultural employment and greater structural transformation for a given increase in agricultural output. Thus we may at least partially accept Nakamura's analysis of output data without modifying the earlier assertions concerning agriculture's increased contribution to economic growth. A similar argument could be made for Taiwan's accelerated growth in agricultural output in the 1920's and the 1950's.

13. It can be shown that this price elasticity is equal to $\epsilon^*_{AA} + n_A$ where ϵ^*_{AA} is the usual price elasticity of demand and n_A the income elasticity of demand for food. We are grateful to Roger Selley for the detailed derivation. See Appendix A to this paper.

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Appendix A

A Note on Demand Elasticities, Monetary Payment and Payment in Kind

by

Roger Selley

Consider the demand equation for good i

$$Q_i = Q_i\ (P_i, \ldots, P_N, Y),\ i = 1, \ldots, N \qquad (1)$$

where Q_i is the quantity demanded of good i, P_i is the price of good i, and Y is the quantity of the j^{th} good received as payment in kind.

Totally differentiating (1) results in

$$dQ_i = \frac{\partial Q_i}{\partial P_1}\ dP_1 + \ldots + \frac{\partial Q_i}{\partial P_N}\ dP_N + \frac{\partial Q_i}{\partial N}\ dY \qquad (2)$$

where dQ_i is the change in the quantity of good i consumed resulting from the changes dP_i ... dP_N, dY. If we consider a change in the price P_j while holding all other prices constant, upon dividing through by dP_j, (2) becomes

$$\frac{dQ_i}{dP_j} = \frac{\partial Q_i}{\partial P_j} + \frac{\partial Q_i}{\partial Y}\frac{dY}{dP_j}\ ,\ i = 1, \ldots, N. \qquad (3)$$

If in addition we take the monetary value of Y as constant, i.e., $P_jY = k$, we can totally differentiate $P_jY = k$,

$$(dP_j)\ Y + (dY)\ P_j = 0, \qquad (4)$$

solve (4) for dY/dP_j and substitute the result into (3) which after multiplying by P_j/Q_i becomes:

$$\frac{dQ_i}{dP_j}\frac{P_j}{Q_i} = \frac{\partial Q_i}{\partial P_j}\frac{P_j}{Q_i} - \frac{\partial Q_i}{\partial Y}\frac{Y}{Q_i}\ ,\ i = 1, \ldots, N. \qquad (5)$$

Expressing (5) in terms of elasticities results in:

$$\epsilon^*_{ij} = \epsilon_{ij} - n_i \tag{6}$$

where ϵ^*_{ij} is the price elasticity of demand for the i^{th} good with respect to changes in the j^{th} price, all other prices and the monetary value of the payment in kind, P_jK, held constant, i.e., this is the usual price elasticity of demand discussed in the literature; ϵ_{ij} is the price elasticity of demand for the i^{th} good with respect to changes in the j^{th} price where all other prices and payments in kind are held constant; n_i is the elasticity of demand for the i^{th} good with respect to changes in the payment in kind, all prices held constant. The price and income elasticities presented on page 20 can now be shown to have the following relation to the usual price elasticity of demand:

$$\epsilon^*_{AA} = \epsilon_{AA} - n_A \tag{7}$$

Appendix B

Stability Conditions and the Correspondence Principle

by

Roger Selley

Let us hypotnesize that the terms of trade increase over time if demand for the marketable surplus exceeds its supply,

$$\dot{P} = H\,[M_D - M_S]\ ,\ H' > 0 = \frac{dH}{dx} > 0,\ y = M_D - M_S \qquad (1)$$

and that labor migrates to the nonagricultural sector when the demand price for nonagricultural labor exceeds its supply price,

$$\dot{r} = G\,[W_D - W_S]\ ,\ G' = \frac{dG}{dy} < 0,\ x = W_D - W_S. \qquad (2)$$

Consider the linear Taylor expansion of the system (1) and (2):

$$\begin{bmatrix} \dot{P} \\ \dot{r} \end{bmatrix} = \begin{bmatrix} b_1 \\ b_2 \end{bmatrix} + \begin{bmatrix} \partial\dot{P}/\partial P & \partial\dot{P}/\partial r \\ \partial\dot{r}/\partial P & \partial\dot{r}/\partial r \end{bmatrix} \begin{bmatrix} P \\ r \end{bmatrix} \qquad (3)$$

where the partials of $\dot{P}$ and $\dot{r}$ with respect to P and r are evaluated at an equilibrium point (P^*, r^*). A necessary and sufficient condition for local stability of the system (1) and (2) is that the eigen values of the matrix of partials in (3),

$$\frac{1}{2}\left\{\frac{\partial\dot{P}}{\partial P} + \frac{\partial\dot{r}}{\partial r} \pm \sqrt{\left(\frac{\partial\dot{P}}{\partial P} + \frac{\partial\dot{r}}{\partial r}\right)^2 - \left(\frac{\partial\dot{P}}{\partial P}\frac{\partial\dot{r}}{\partial r} - \frac{\partial\dot{P}}{\partial r}\frac{\partial\dot{r}}{\partial P}\right)}\right\} \qquad (4)$$

have negative real parts.

For the eigen values to have negative real parts it is necessary and sufficient that

$$\frac{\partial\dot{P}}{\partial P} + \frac{\partial\dot{r}}{\partial r} = H'b_1\ sA/rN + G'(-f''K^2/N^2(1-r)^3 + PsA/r^2N) \qquad (5)$$

$$= -G'f''K^2/N^2(1-r)^3 + (H'b_1 r + P)\ sA/r^2N < 0$$

and

$$\frac{\partial \dot{P}}{\partial P}\frac{\partial \dot{r}}{\partial r} - \frac{\partial \dot{P}}{\partial r}\frac{\partial \dot{r}}{\partial P} = H'(b_1\ sA/rN)\ G'(-f''K^2/N^2(1-r)^3 + PsA/r^2N \tag{6}$$

$$- H'\ (sA/rN)\ (b_2\ sA/r^2N + b/r)\ G'\ (sA/rN) > 0$$

where output is specified by the general production relation

$X = L_I\ f[K/L_I] = L_I\ f[k]$,

$f' = \partial f/\partial k$,

and $f'' = \partial^2 f/\partial k^2$.

Dividing (6) by $H'G'\ sA/rN$ results in the condition

$$b_1\ (-f''K^2/N^2(1-r)^3 + P\ sA/r^2N) \tag{6a}$$

$$- b_2\ (sA/rN)^2/r + b\ sA/r^2N < 0.$$

By Engel's Laws the percentage importance of food expenditure declines as income increases, i.e., $b_2 < 0$. Assuming diminishing returns in production $f'' < 0$ and assumes Engel's Laws apply, it is necessary that $b_1 < 0$ for (6a) and therefore (6) to be satisfied. The partial $b_1 < 0$ is also sufficient for the satisfaction of condition (5). Equation (6) places a stronger condition on b_1 which depends upon the magnitude of the parameters and variables of the model. Applying Engel's Laws and Samuelson's correspondence Principle permits the unambiguous determination of all of the signs in the sensitivity matrix on page 16.

Since the relative budget share spent on food by laborers can be expressed as $b = A/Y$ where A is defined here as per capita consumption of food and Y is per capita income, the constraints placed here on b_1 and b_2 can be expressed in terms of elasticities as follows:

$$b_1 = \frac{\partial b}{\partial P} = \frac{1}{Y}\left(\frac{\partial A}{\partial P}\frac{P}{A}\right)\frac{A}{P} = \frac{b}{P}\,\epsilon_{AA} < 0 \qquad (7)$$

$$b_2 = \frac{\partial b}{\partial Y} = \frac{1}{Y}\left\{\left(\frac{\partial A}{\partial Y}\frac{Y}{A}\right)\frac{A}{Y} - \frac{A}{Y}\right\} = \frac{b}{Y}\,(n_A - 1) < 0 \qquad (8)$$

which can be restated as

$$\epsilon_{AA} < 0 \qquad (7a)$$

and

$$n_A < 1. \qquad (8a)$$

From the results derived in Appendix A, equation (7a) can be in turn restated in terms of the "usual" price and income elasticities as follows:

$$\epsilon_{AA} = \epsilon^*_{AA} + n_A < 0. \qquad (7a\text{-}1)$$