DYNAMIC MODELS OF AGRICULTURAL DEVELOPMENT WITH DEMAND LINKAGES

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March 1973

No. 73-7

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DYNAMIC MODELS OF AGRICULTURAL DEVELOPMENT WITH DEMAND LINKAGES[1]

by

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1. INTRODUCTION

Most of the empirical work describing agricultural development in the economically less developed countries (henceforth called LDC's) is concerned mainly with analyzing the acreage response to price changes and estimating price elasticities of various agricultural commodities. However, there are many other crucial variables which are not incorporated, or which are incorporated unsatisfactorily in agricultural development models. Some of these interrelated elements are farm-firm and farm-household interdependence; the aggregate supply of production inputs; the crucial role of money capital and credit; seasonal demands for physical resources and their corresponding supply restrictions; multiple production techniques and technological change; the ubiquitous presence of uncertainty in farm decision-making process; interactions between demand and supply of farm outputs and nonfarm inputs; linkages between farm and nonfarm sectors; the existence of government programs; and several other behavioral considerations which influence the decision environment of farmers and their eventual adjustments to these uncontrollable variables over time.

It has been observed in the Indian Punjab that there exists a cause and effect relationship of the cobweb type among acreage under

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[1]This paper is adapted from parts of author's doctoral dissertation submitted to the Graduate School at the University of Wisconsin, Mudahar [1972]. It was presented at the ADC Conference on "The Applications of Recursive Decision Systems in Agricultural Sector Analysis," Washington, D. C., November 1-3, 1972. I am indebted to Professor Richard H. Day, my supervising teacher, for his guidance and many valuable suggestions. The author gratefully acknowledges the financial support provided by ADC, the University of Wisconsin Graduate School and National Science Foundation during the initial stages of this study. The latter part was financed by the Ford Foundation through Ford Doctoral Dissertation Fellowships to the author. The final version of the paper was completed with partial support from the USAID project directed by Professor John W. Mellor.

sugarcane; production of gur and sugar; supply of sugarcane to mills; and prices of sugarcane, gur and sugar. Also, the current sale receipts earned by farmers depend not only on the total marketable surplus but also on the prices per unit which are simultaneously determined by a temporary equilibrium between the farm supply and nonfarm demand for farm commodities. However, in the conventional programming or econometric models of the agricultural sector, farm product prices are given exogenously without giving any importance to the corresponding demand which will clear the market for different farm commodities.

Engel observed that as development takes place the composition of consumption demand changes in favor of nonfarm-produced consumer goods. This implies that farm-produced consumer goods are substituted by nonfarm-produced consumer goods when farm income goes up. In the Punjab this has been found true of many farm commodities but the conspicuous example is the consumption of gur and sugar. Furthermore, in the case of some commodities, the production, consumption and marketing decisions are greatly influenced by the existing processing capacity for those farm commodities in the agro-industrial sector.

In the real world, current farm decisions are influenced by the consequences of past actions, present knowledge and future expectations. Furthermore, farmers in the LDC's seem to possess multiple goals which vary in importance from each other and correspond to a set of lexicographic ordering. These feedback effects and multiple goals are part of the decision environment of the farmers and must be incorporated in models designed to describe the development of agriculture in the LDC's.

All these interrelated elements circumscribe the decision-maker, influence his decision-making environment, constrain his set of choice alternatives and influence their corresponding payoffs. The general purpose of this study is to develop mathematical, dynamic and positive models of farm decisions which simultaneously incorporate all these interrelated microeconomic details of agricultural development and account for the saliant features of traditional agriculture in transition. Specific attention is directed to the substitution between farm and nonfarm-produced consumer goods and to exogenously versus endogenously determined product prices. These models are based on already tested notions of rationality in the LDC's and incorporate several behavioral considerations faced by the farmers. Furthermore, major emphasis is given to the linkages between farm and nonfarm sectors.

However, instead of mathematical analysis, this study explores the analytical properties of these models by computer simulation. Also, the simulation experiments are conducted to investigate their ability to represent farmers' behavior in a realistic manner and to project possible responses to different government programs, changed "environmental conditions" and "alternative model specifications."

To begin with, a microeconomic dynamic open sector model is developed for the agricultural sector of Punjab. The model is "open" in the sense that the commodity prices are given exogenously. It represents an intermediate stage between two extremes: (i) the analysis of the whole economy in the aggregate or multisector framework, and (ii) the analysis of individual economic units such as farm-firm or farm-household.

The open sector model is then extended to incorporate the nonfarm demand for farm commodities. The resulting product is a microeconomic dynamic "closed" sector model of farm decisions for Punjab agriculture. In the closed sector model the commodity prices are determined endogenously through a temporary equilibrium between farm supply and nonfarm demand for different farm commodities. The closed sector model concentrates on describing the historical development of the agricultural sector as it passes through different phases of development and the cyclical behavior of various farm commodities such as sugarcane cycles in the Punjab. However, both open and closed sector models are imbedded in banking sector feedback, and incorporate resource restrictions which are determined by the progress in the nonfarm sectors and eventually condition the progress in the farm sector.

Several alternative methodological approaches are evaluated elsewhere in the light of (i) criteria of suitability, and (ii) the microeconomic details of agricultural development, Mudahar [1971a]. Recursive programming seems to be an ideal tool which satisfies most of the criteria of suitability, and lends itself to incorporate most of the microeconomic details of agricultural development. It possesses all the advantages of mathematical programming. However, it is a positive rather than a normative technique. It makes use of input-output analysis to describe the technological structure of an industry or a sector. It makes use of regression analysis and the production function approach to determine various constraints and technological coefficients. As a consequence, other techniques do not compete with it, rather they supplement it by providing various kinds of information. It is a recent addition to the tool kit of economists and agricultural economists and has been developed by Day [1961, 1962, 1963].

Recursive programming is used to analyze the role of interrelated microeconomic variables in the dynamic context and specifically to analyze production, consumption, marketing and financial decisions with an explicit treatment of risk and uncertainty, several behavioral considerations, farm-nonfarm linkages and the implications of nonfarm demand for farm commodities. Recursive programming is a sequence of mathematical programming problems in which the parameters of a given problem are functionally related to the optimal or suboptimal variables (primal or dual) of the preceding problems in the sequence. It helps to analyze explicitly the aggregate implications of decision processes at the farm-firm and farm-household levels, and reflects the view that actual behavior is based on rationally planned actions.

It assumes that the decision-maker being studied knows the consequences of his past actions but is partly unsure of the mechanism that generates them. However, through feedback functions the researcher is able to study the effects of the consequences of past actions and future expectations on current decisions. The core of recursive programming is linear programming, which provides a description of the technological structure that presents decision-makers with alternative economic opportunities for change. This structure is then augmented by behavioral relationships that describe how given changes are determined. The result is a model that is ideally suited for the study of traditional agriculture in transition. Recursive programming is thus a positive means of incorporating and analyzing the influence of several interrelated microeconomic details of economic development over time on the decision-making environment under the assumption of explicit optimization.

One of the major features of recursive programming is the existence of multiple phases, which implies that the temporal behavior of the system is determined by distinct phases which are described by a distinct set of difference equations. The switch from one phase to another is governed by the optimizing functions in the recursive chain of mathematical programs.

Furthermore, recursive programming has the following distinct features: (i) it explicitly incorporates the optimization behavior of individual decision-makers when the decisions are subject to various technological and behavioral constraints; (ii) the planning horizon is very short and plans are made period after period based on the information generated from past actions, present knowledge and future expectations - the end results of which are rolling plans; (iii) it represents production, consumption, marketing, investment, financial and other economic opportunities of the farm sector by a detailed activity analysis; (iv) it incorporates realistic hypotheses of actual behavior such as rules-of-thumb, learning and cautious adjustment to currently attractive but uncertain future opportunities; and finally (v) it links aggregate regional behavior to nonfarm economy through various feedback functions and exogenously given or endogenously determined resource restrictions.

However, recursive programming does not always satisfy the principle of optimality and the solution which is <u>ex ante</u> optimal may or may not be <u>ex post</u> optimal. The dual variables represent the shadow prices or the opportunity cost of scarce resources and can be used to test several hypotheses, such as the existence of disguised unemployment in the rural sector of the LDC's. Finally, recursive programming models are highly nonlinear and use various kinds of "ad hoc" rules-of-thumb to represent the behavior of the decision makers.

Day [1963], Heidhues [1966], Schaller [1968] and Schaller-Dean [1965] have applied recursive programming to analyze various aspects of commercial agriculture. These models, however, do not take into account several important attributes of traditional agriculture and hence cannot be directly applied to describe agricultural development in the LDC's. Day-Singh [1971], Mudahar [1970] and Singh [1971] have developed recursive programming models of the agricultural sector in the LDC's and have applied them to the Indian Punjab. The models reported in this paper are simple and similar in spirit. However, these models emphasize (i) the crucial role of demand linkages, (ii) the substitution between farm-produced and nonfarm-produced consumer goods, and (iii) the theoretical and empirical implications of exogenously versus endogenously determined product prices.

The basic model of farm decisions is developed in section 2. This model is dynamized in section 3. Sections 4 and 5 deal with the existence of farm commodity cycles and the generalized cobweb theory, respectively. The 'open' sector model of section 3 is 'closed' by introducing 'inverse demand functions' in section 6. Section 7 analyzes the dynamic coupling and recursive interdependence among different farm decision units. The estimation of these models and their evaluation are discussed in sections 8 and 9, respectively. The empirical results are reported in section 10. Various conclusions and their policy implications are analyzed in section 11. Section 12 deals with the comparative performance of the open and closed sector models. The paper concludes with a note on the research work currently in progress.

2. THE MODEL OF FARM DECISIONS

The linear programming model used to represent farm decisions consists of an activity set, including various firm and household alternatives, a structure of constraints on the choice amongst these alternatives and an objective function that is maximized subject to several constraints. These three model components are discussed in turn.

2.1 The Activity Set

The representative farm is assumed to be engaged in (i) production activities; (ii) subsistence consumption activities; (iii) sales activities; (iv) labor hiring activities; (v) purchase activities; and, (vi) financial activities. Each of these activities is discussed in detail below.

2.1.1 Production Activities: Production activities transform production inputs into final or intermediate outputs. Final outputs include maize, cotton, sugarcane and wheat, produced for consumption and direct sale whereas intermediate output refers to a crop which is later on processed into final output, such as the manufacture of gur (brown sugar) from sugarcane. These outputs are further classified into summer (kharif), winter (rabi) and annual crops such as sugarcane. The production activities for time "t" are represented by $q_j(t)$, $j \varepsilon P$, where P is the set of crop names. The production of by-products of farm crops is not taken into account. Rather, it is assumed that by-products are retained at the farm for the consumption of livestock and for household consumption.

Furthermore, it is assumed that sugarcane is not grown as a ratoon crop. This is a simplifying assumption and is justified on two grounds: (i) the number of ratoon crops grown varies from one farm to another and from one sugarcane variety to another; and (ii) net returns (monetary) of planted sugarcane crop are not much different than the ratoon crop since fall in yield of ratoon crop is offset by a fall in its variable expenses, i.e. no seed costs.

2.1.2 Subsistence Consumption Activities: Subsistence consumption activities describe the consumption of farm produced commodities by the farm-household. Farmers in the LDC's grow enough of each food crop to satisfy their household consumption requirements. The consumption decisions are assumed to be made at the beginning of each production period and serve as constraints on production plans. In the present version of the model, the minimum household consumption requirements for each farm commodity are determined exogenously from the farm-household budget studies in the Punjab. The planned subsistence consumption for j^{th} farm commodity in "t" is represented by $\tilde{c}_j(t)$, $j \varepsilon P$.

In some cases, consumption requirements are satisfied by both farm-produced and nonfarm-produced commodities. It has been observed in various parts of the world that when development takes place, consumers tend to shift their preferences from farm-produced to nonfarm-produced consumer goods. This is true in case of sugar consumption i.e., the sugar requirements are met by the consumption of gur (brown sugar), refined sugar and sweets. As a result, the consumption of sugar is decomposed[2] into consumption of gur (c_g), consumption of refined sugar (c_s) and

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[2]The consumption decompositional analysis (between farm and nonfarm-produced consumption goods) is considered only in the case of sugarcane products, but can easily be extended to other consumer goods.

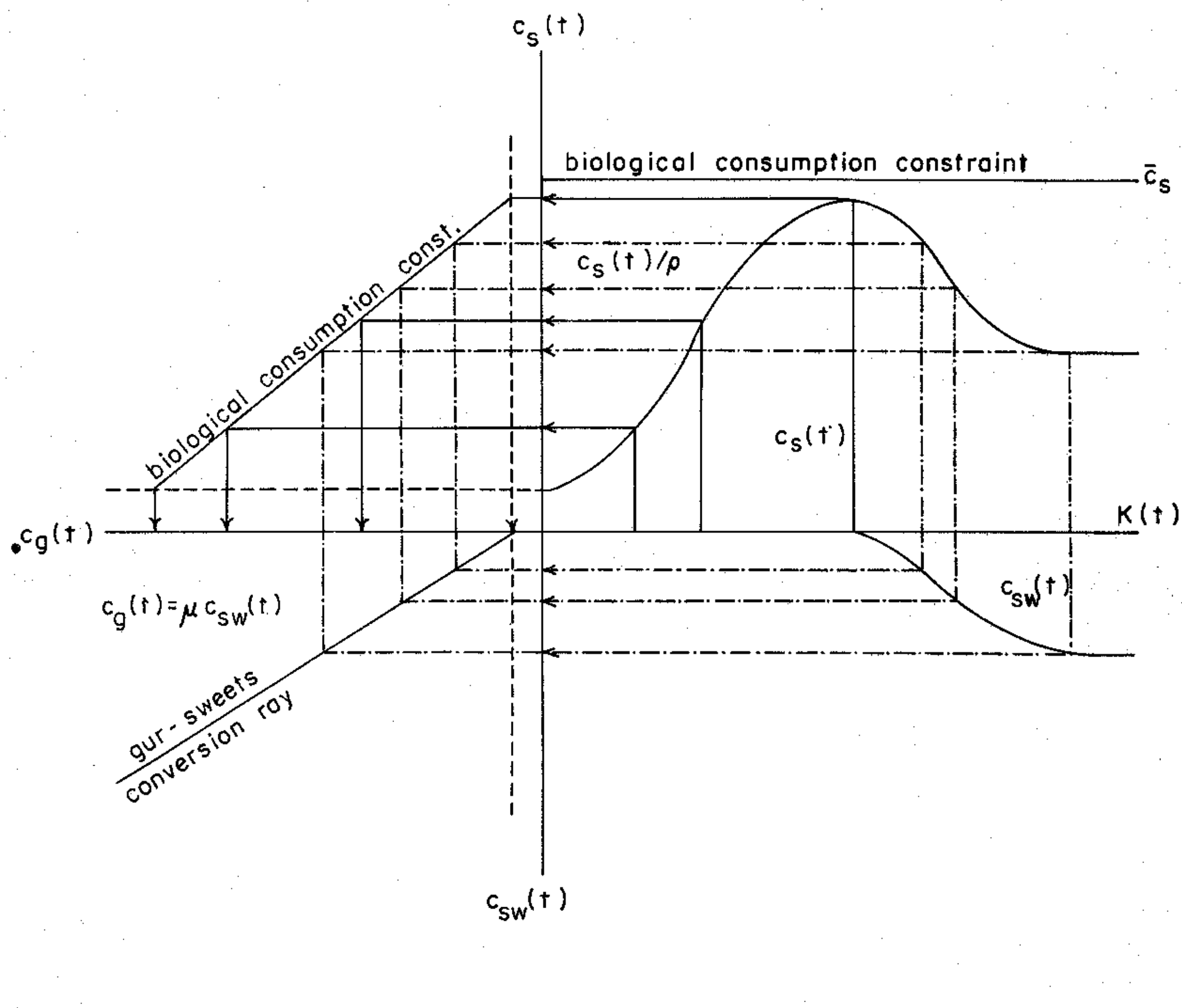
consumption of sweets (c_{sw}). As has been revealed by the farm-household budget studies in the Punjab, the total consumption of sugar is more or less stable over time, Mudahar [1972]. For simplicity, it is assumed that the consumption of sugar remains fixed at a certain 'biological optimum' level over time.

Gur is manufactured by farmers on their own farms, whereas sugar and sweets are manufactured by the nonfarm sector. Gur, refined sugar and sweets are very close substitutes for each other. However, sugar is superior to gur and sweets are superior to sugar in both quality and nutritional value. In the Punjab, the average per capita consumption of gur is decreasing and the consumption of refined sugar is increasing for the same period, Mudahar [1972]. This suggests that the consumption of gur is being substituted by refined sugar over time. In order to simplify the consumption analysis it is assumed that gur and sweets can be converted into refined sugar by using appropriate conversion factors.

It is postulated that, ceteris paribus, the consumption of sugar and sweets increases nonlinearly with an increase in farm family cash income, K(t). As has been revealed by the farm family budget studies in the Punjab, farmers tend to substitute sugar for gur as their income goes up. Since the consumption of sweets is almost negligible, it is assumed that farmers substitute sweets for gur only at a high income level when gur is almost completely substituted for sugar. At a very high income level, the consumption of sweets approaches an asymptote, indicating that no further substitution between sugar and sweets takes place. The substitution process and the composition of gur, sugar and sweets in the sugar products consumption set, at different levels of income, is shown in Figure 1.

A continuous increase in sugar consumption, fall in gur consumption, almost negligible amounts of sweets consumption, low but increasing income levels and the subsistence nature of farming suggest that the majority of farmers are in their initial stages of the sugar-substitution process. Although the inclusion of gur, sugar and sweets is of some theoretical interest, in order to keep the model simple it has been assumed that the consumption of sweets is nil. The total household consumption for sugarcane products (gur and sugar) can now be expressed as

FIGURE I. THE COMPOSITION OF GUR, SUGAR AND SWEETS IN THE SUGAR PRODUCTS CONSUMPTION SET AND THEIR SUBSTITUTION FOR EACH OTHER AT DIFFERENT LEVELS OF INCOME



(1) $\bar{c}_s = \frac{1}{\rho} c_g(t) + c_s(t),$ $(c_s(t) < \bar{c}_s)$

where $\bar{c}_s$ is the fixed amount of sugar in refined-sugar units consumed by the household (= biological optima) and ρ is the conversion factor to convert gur into refined sugar units.

The consumption possibilities and the relative amounts of gur and sugar consumed by the household at different income levels are shown in Figure 2. In the diagram, $c_{g\ max}$ and $c_{s\ max}$ represent the maximum amounts of gur and sugar whereas $c_{g\ min}$ and $c_{s\ min}$ are the minimum amounts of gur and sugar respectively. The maximum amounts can be interpreted as the biological optimum to be consumed by the household. Most of the farm-households in the Punjab are somewhere in the upper left part of the biological consumption constraint.

As has been hypothesized earlier the consumption of refined sugar increases nonlinearly with an increase in the household's income. Refined sugar being a superior good, it is expected that the composition of gur and refined sugar in the fixed consumption space will change in favor of refined sugar over time with an increase in household's income. The nonlinear refined sugar consumption function can be approximated with three linear segments (Figure 3), thereby dividing the area under refined sugar consumption function into three distinct regions. The k^{th} linear segment can be expressed with a different linear refined sugar consumption function as

(2) $c_s(t) = c_{sk}(t) = \min \{a_{sk} + b_{sk}K(t)\},$

$$a_{s,k+1} > a_{s,k} > 0,\ k = 1, 2$$

$$0 \leq b_{s,k+1} < b_{s,k},\ k = 1, 2$$

$$\text{if } 0 \leq K(t) \leq \bar{K}_1(t) \Rightarrow k = 1$$

$$\text{if } \bar{K}_1(t) < K(t) \leq \bar{K}_2(t) \Rightarrow k = 2$$

$$\text{if } \bar{K}_2(t) < K(t) < \infty \Rightarrow k = 3$$

where $c_{sk}(t)$ is refined sugar consumption in the k^{th} region in t, and

FIGURE 2. MAXIMUM AND MINIMUM CONSUMPTION LEVELS FOR GUR AND SUGAR

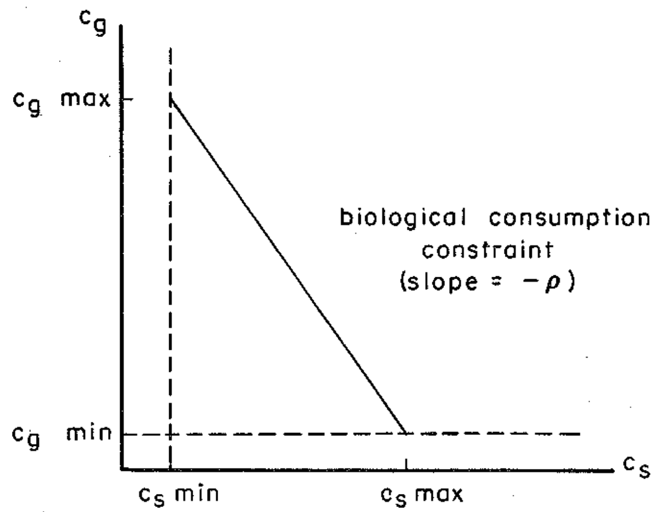
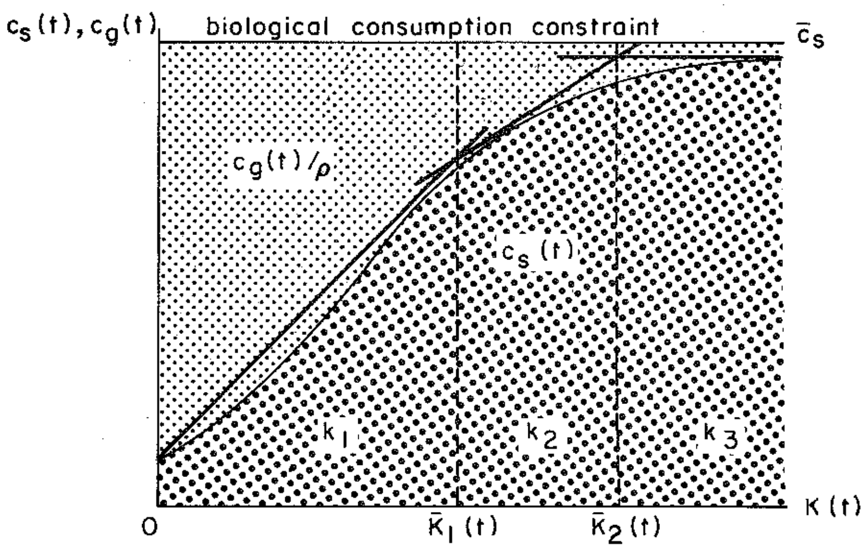


FIGURE 3. LINEARIZATION OF THE NONLINEAR SUGAR CONSUMPTION FUNCTION



$K(t)$ is cash income available with the household in the beginning of period t.

At certain levels of income in region k_3, the consumption of gur will almost be completely substituted with refined sugar. Since tastes for food commodities don't change completely it is possible that the household will still consume certain minimum amount of gur even at a very high level of income. The consumption function for gur can now be derived by combining equations (1) and (2) as

$$(3) \qquad c_g(t) = \rho \bar{c}_s - \rho \min \{a_{sk} + b_{sk} K(t)\}, \qquad k = 1, 2, 3$$

2.1.3 Sales Activities: The marketable surplus of various farm commodities is determined as a residual of production and planned consumption decisions made by the farm-firm and farm-household. Since the consumption of different farm commodities is determined exogenously, the marketable surplus function for the representative farm-firm can be derived as

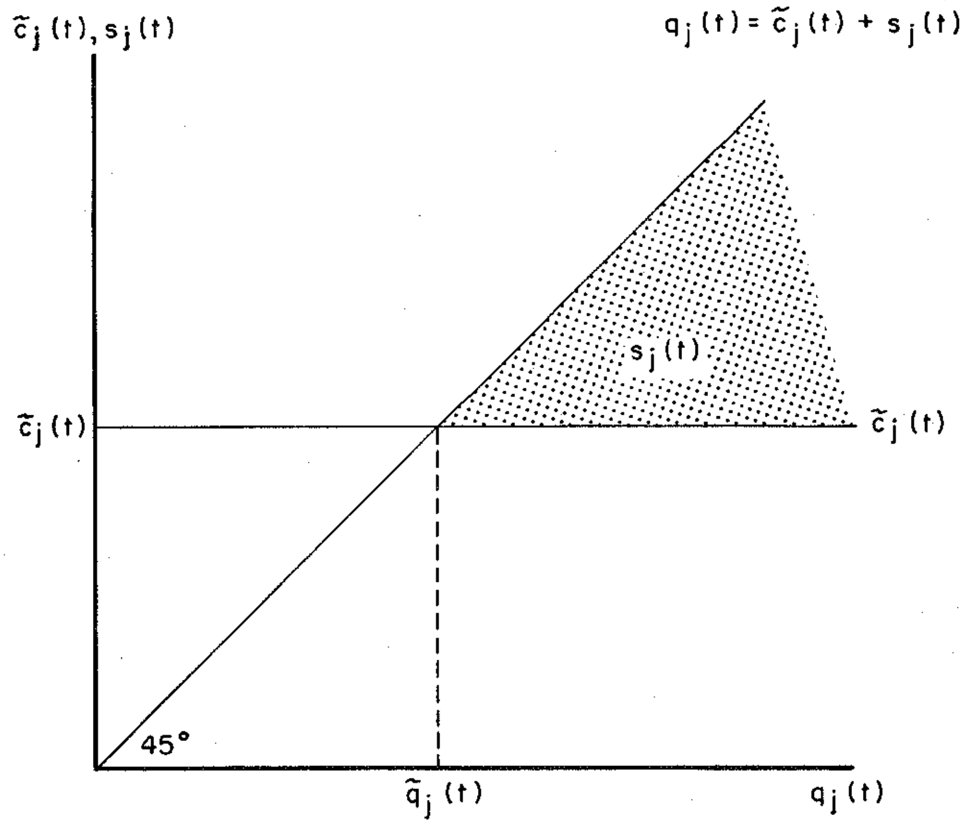
$$(4) \qquad s_j(t) = q_j(t) - \tilde{c}_j(t),$$

and since $q_j(t) \geq \tilde{c}_j(t) \Rightarrow s_j(t) \geq 0$,

where $q_j(t)$ is the total output of j^{th} farm commodity in t, $s_j(t)$ is the marketable surplus of j^{th} farm commodity in t, and $\tilde{c}_j(t)$ is the planned subsistence consumption of j^{th} farm commodity which is to be consumed in t + 1. Graphically, the derivation of marketable surplus for the j^{th} farm commodity from its production and consumption decisions is reported in Figure 4.

The marketable surplus is sold in a nearby market which is assumed to be perfectly competitive. The farm commodities brought to the market can be purchased by either (i) consumers in the nonfarm sector, or (ii) commodity traders from the Punjab, and/or (iii) the Punjab government to build its own buffer stocks. The gross price for each commodity is assumed to be determined by a temporary equilibrium between the nonfarm demand and the existing farm supply. The net price received by farmers is then derived by subtracting marketing and transportation charges from the gross price.

FIGURE 4. MARKETABLE SURPLUS FUNCTION FOR A PARTICULAR FARM COMMODITY



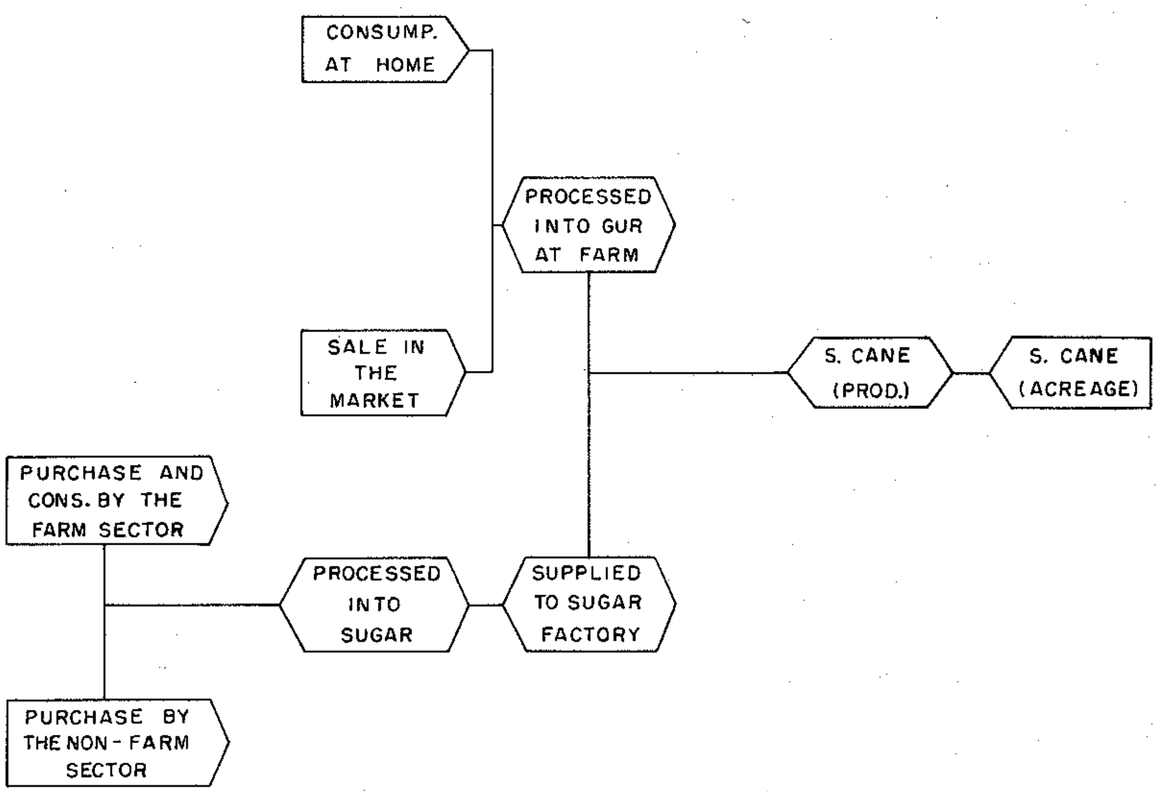
In case of sugarcane, however, farmers have a choice between either selling sugarcane to the factories or processing it into gur. Since the production of sugarcane cannot be altered significantly after the production decisions are made, the farm and nonfarm demand for sugarcane, gur and refined sugar plays a significant role in changing their relative price levels and hence influences the marketing behavior of farmers. The sequence of production, consumption, processing and marketing decisions by farmers for sugarcane and sugarcane products is shown schematically in Figure 5.

2.1.4 Labor Hiring Activities: There are two main sources of agricultural labor: family labor and hired labor. Farmers hire labor from outside only when the household (family) labor is not enough to perform all the agricultural operations in any particular month of the year. In all, there are twelve labor hiring activities i.e., one for each month. Consequently, labor hiring activities deal with hiring labor from outside at the prevailing wage rate and are included in the model through transfer activities. The total amount of labor hired depends on the level of production activities and the amount of family labor available from the farm-household. The monthly wage bill paid to the hired agricultural labor is an important part of the annual variable farm cash expenditure.

2.1.5 Purchase Activities: Purchase activities are divided into two categories: (i) the purchase of agricultural inputs for the firm's production process, and (ii) the purchase of nonfarm durable and non-durable consumer goods and services to be consumed by the farm-household.

Agricultural inputs are divided into two groups. The first group deals with the purchase of variable factors of production which include chemical fertilizers, insecticides, certified seed of high yielding crop varieties, electricity, diesel oil, etc. Purchases of these variable inputs are not included as explicit activities but the money capital needed to purchase them is specified as capital-input coefficients. The amount of money capital required to finance the purchase of variable production inputs depends on the levels of production activities since money capital coefficients are associated with each one of the production

FIGURE 5. FLOW DIAGRAM SHOWING THE PRODUCTION, CONSUMPTION, PROCESSING AND MARKETING DECISIONS BY FARMERS IN CASE OF SUGARCANE AND SUGARCANE PRODUCTS IN THE PUNJAB



activities.[3] The second group deals with the purchase of fixed and quasi-fixed factors of production. Items included in this group are farm machinery, other farm equipment and cash outflows which are more or less fixed in nature such as land revenue, taxes, contingency payments, etc. In this particular model investment activities are not incorporated explicitly in the model. However, the money capital required to finance these investments does not vary with the level of the production activities and hence it is included in the money capital constraint.

The purchase of consumer goods and services are divided into three groups. The first group deals with the purchase of refined sugar in the beginning of each period. The cash expenditure on sugar varies from period to period depending upon the price per unit and the quantity purchased. The second group deals with cash expenditure on items such as food, education, health, medical services, clothing, transportation, fuel, electricity, religious, social, recreational and miscellaneous consumer goods and services. The expenditure on these consumer goods is determined exogenously from the farm-household budget studies in the Punjab. At a later stage, however, explicit expenditure functions will be used to generate these variables endogenously. The third group includes cash expenditure on items which are durable and more or less fixed in nature such as insurance, taxes, housing, other consumer goods of fixed nature, etc. The total cash expenditure function $(v(t))$ for the firm and household can now be expressed as

(5) $$v(t) = v_1(t) + v_2(t),$$

(6) $$v_1(t) = \Sigma_j k_j(t) q_j(t) + \Sigma_i w_i(t) h_i(t) + f_1(t), \qquad i = 1, \ldots, 12$$

(7) $$v_2(t) = c_s(t)\hat{p}_s(t) + e(t) + f_2(t),$$

where $v_1(t)$: total cash expenditure on purchased agricultural inputs in t,

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[3] In the computation of capital-input coefficients no allowance has been made for the opportunity cost of family labor since annual net cash receipts expected from farming are attributed to family labor, management and fixed factors of production.

$v_2(t)$: total cash expenditure on purchased consumer goods and services in t,

$k_j(t)$: capital input-output coefficients for j^{th} production activity in t,

$w_i(t)$: wage rate in rupees per man day in i^{th} month of t,

$h_i(t)$: number of man days hired in i^{th} month of t,

$f_1(t)$: yearly cash expenditure on fixed factors of production in t,

$f_2(t)$: yearly cash expenditure on consumer goods and services which are fixed in nature in t,

$\hat{p}_s(t)$: expected price of sugar per unit in t,

$e(t)$: yearly cash expenditure on nonfarm-produced consumer goods (except sugar) and services in t.

The total cash expenditure function for the firm and household can be rewritten by combining equations (5), (6) and (7) and defining $f(t) = f_1(t) + f_2(t)$ as

$$v(t) = \Sigma_j k_j(t)q_j(t) + \Sigma_i w_i(t)h_i(t) + c_s(t)\hat{p}_s(t) + e(t) + f(t). \tag{8}$$

2.1.6 Financial Activities: Financial activities are associated with the financial structure of the farm-firm and the farm-household.[4] These activities can be divided into three categories: (i) banking, (ii) short-term borrowing and (iii) loan repayment.

2.1.6.1 Banking: After meeting cash expenditure on fixed farm inputs and household consumption, the farm-firm has a choice between investing its remaining capital in farm inputs or depositing it in the bank or village cooperative credit society. The relative amount of money capital invested in each alternative depends on the internal rate of return and the bank rate respectively. In case the internal rate of return is greater than the bank rate it indicates scarcity of money capital. However, if the money capital is not scarce, banking provides

[4] A detailed discussion on short, medium and long term financial activities and constraints and the correspondence between financial and investment activities and constraints is available in Mudahar [1971b, 1972].

an alternative means to earn some returns on the surplus money capital. The surplus money capital here refers to the amount left over after meeting all the necessary financial obligations at the farm. In this model, $y_1(t)$ stands for saving, i.e., depositing cash in the bank by the firm in the beginning of each period and $i_1(t)$ for interest rate for a single production period.

2.1.6.2 Short-Term Borrowing: In case the cash income is not enough even to meet the cash expenditure of fixed nature, farm-firm has to borrow in order to stay viable in the business. It is specified that the farmer can obtain short-term loans at the beginning of the period up to a limit determined by fixed proportions of his past cash earnings and of his equity base. These proportions are dictated by the maximum credit limits which are determined by the lending institutions. The borrowing of short-term loan is represented by $y_2(t)$ and farm-firm has to pay interest rate equal to $i_2(t)$ for a single period with the condition that $i_1(t) < i_2(t)$.

2.1.6.3 Loan Repayment: The short-term loan is advanced only for a single period and the farm-firm has to repay it back within a specified time. Since only short-term loan is considered, it is assumed that all the loan is repaid at the beginning of the following production period before the surplus cash is saved. In case this overdue loan is not repaid, the farm-firm will be in default and won't be eligible to borrow for the next period.

2.2 The Constraint Structure

Agricultural production on the representative farm-firm is restricted by (i) the technological constraints, (ii) the financial constraints, (iii) the farm-household consumption constraints, (iv) behavioral constraints, and (v) miscellaneous constraints. Let us now discuss these constraints in detail.

2.2.1 Agricultural Land: Availability of operational land serves as an important constraint to limit the acreage under different crops on the farm-firm and to determine the cropping pattern of an agricultural region. The seasonal land constraints for the production activities on the farm-firm can be expressed as

$$(9) \qquad \Sigma_j \ell^s{}_j(t) q^s{}_j(t) \leq L^s(t),$$

$$(10) \qquad \Sigma_j \ell^w{}_j(t) q^w{}_j(t) \leq L^w(t)$$

where $\ell_j(t)$ is the land input coefficient for j^{th} production activity and $L(t)$ is irrigated land in acres in t. The superscripts s and w refer to summer and winter crops respectively. The land input coefficients are determined by the yield per acre which in turn depends on the land quality, weather, crop variety, irrigation facilities, technology and many other factors.

2.2.2 Agricultural Labor: The availability of agricultural labor serves another important constraint especially during peak labor demand months. The demand for agricultural labor depends upon the cropping pattern, intensity and time distribution of agricultural operations and the mode of agricultural technology. As a result, the time distribution of labor demand varies from one season to another in a particular agricultural year. The seasonal agricultural labor constraints are specified separately for (i) farm-household labor, and (ii) hired agricultural labor.

2.2.2.1 Farm Household Labor: A large proportion of the total farm labor demand in the LDC's is supplied by the farm-household. Moreover, most of the agricultural operations are season specific. These two assertions are taken into account by specifying monthly constraints for the farm-household labor as

$$(11) \qquad \Sigma_j m_{ij}(t) q_j(t) - h_i(t) \leq M_i(t), \qquad i = 1, 2, \ldots, 12$$

where $m_{ij}(t)$ refers to labor input coefficients for the j^{th} production activity during the i^{th} month of t, and $M_i(t)$ is the number of total man days available from the farm-household during the i^{th} month of year t.

2.2.2.2 Hired Agricultural Labor: Since farm-household labor is not always enough, especially during the peak labor demand months, to perform all the agricultural operations, farmers hire labor from outside and pay them the existing wage rate. However, the availability of outside agricultural labor is limited and farmers cannot hire as much labor as they want. This is incorporated in the model by specifying hired agricultural labor constraints as

$$(12) \qquad h_i(t) \leq H_i(t), \qquad i = 1, 2, \ldots, 12$$

where $h_i(t)$ and $H_i(t)$ refer to the outside agricultural labor hired and available respectively during the i^{th} month of t.

2.2.3 Financial Constraints: As has been discussed under the financial activities, cash is required to finance the purchase of factors of production, consumer goods and services and to repay the old debt. If still there is some cash left over it is deposited in the bank. The level of these activities is restricted by the amount of money capital, K(t), available to the farm in the beginning of each period. These financial constraints are divided into (i) money-capital constraint, (ii) borrowing constraint, and (iii) loan repayment constraint. Each of these constraints is discussed separately.

2.2.3.1 Money Capital Constraint: The money capital available in time "t" is assumed to be generated from two different sources on the farm: (i) total cash receipts from past crop sales, R(t-1), and (ii) past household savings including interest less debt repayment. The annual money-capital constraint for the farm-firm is

$$(13) \qquad \Sigma_j k_j(t)q_j(t) + \Sigma_i w_i(t)h_i(t) + c_s(t)\hat{p}_s(t) + y_1(t) - y_2(t) \leq K(t) - e(t) - f(t),$$

$$(14) \qquad K(t) = R(t-1) + (1+i_1)\, y_1(t-1) - (1+i_2)\, y_2(t-1),$$

$$(15) \qquad R(t-1) = \Sigma_j \{p_j(t-1) - \bar{m}_j(t-1)\}\, s_j(t-1) ,$$

where $p_j(t-1)$ and $\bar{m}_j(t-1)$ stand for the harvest price and the marketing plus transportation charge, respectively, per unit for j^{th} sales activity in (t-1).

The money capital constraint can further be extended to account for agricultural subsidies provided by the state and/or federal governments. Since subsidies are advanced free of cost, farmers are not required to repay these subsidies. Equation (14) now looks as

$$(16) \qquad K(t) = R(t-1) + (1+i_1)y_1(t-1) - (1+i_2)y_2(t-1) + g(t)$$

where g(t) is the amount of subsidy or say free financial support.

2.2.3.2 Borrowing Constraint: In case farmers' own money capital is not enough to meet the production and consumption cash expenditure they can borrow short-term loan from government and/or private lending institutions. The total amount he can borrow is limited by his repayment capacity and the maximum credit limit. This borrowing constraint can be specified as

$$(17) \qquad y_2(t) \leq r_1(t)K(t) + r_2(t)E(t),$$

where r_1 is the short-term maximum credit limit coefficient out of cash income and is derived from farm practices data in consultation with the agricultural credit institutions in the Punjab, Mudahar [1967]. On the other hand, r_2 is the short-term maximum credit limit coefficient out of farmer's equity base and E(t) refers to the equity base, the value of which is approximated by the value of land owned by the farmer.

2.2.3.3 Loan Repayment Constraint: Farmers are required to repay their short-term loans in the beginning of the following production period before they are eligible to borrow for that period. This can be expressed as

$$(18) \qquad -d(t) \leq -(1+i_2)y_2(t-1),$$

where d(t) is the amount of short-term loan to be repaid during time t. This constraint will always be tight since farmers are not going to repay more than what they really owe to the lending institutions.

2.2.4 Household Consumption Constraints: The household and farm-firm interdependence, a peculiar characteristic of subsistence agriculture, can be represented by using household's consumption requirements for any food crop in t+1 as a constraint to the corresponding production activity in t. In general, the household's consumption constraint for the j^{th} production activity can be expressed as

$$(19) \qquad -q_j(t) + s_j(t) \leq -\tilde{c}_j(t).$$

2.2.5 Sugarcane Supply Constraints: In situations when the supply of sugarcane (s_c) is inadequate to keep the factories running at a normal capacity, government assures the factory owners a supply of certain minimum amount of sugarcane. This is accomplished by partially banning the manufacture of gur by power cane crushers at the farms. This behavior is incorporated in the model by specifying a lower limit on sugarcane supply and can be expressed as

$$(20) \qquad -s_c(t) \leq -\underline{s}_c(t),$$

where $\underline{s}_c(t)$ is the minimum amount of sugarcane which has to be supplied by the farmers to the sugar mills in t.

On the other hand, the total capacity of sugar mills is limited and only a certain amount of sugarcane can be processed each season. If farmers grow too much cane they may have to manufacture gur even when they would prefer to sell cane to the mills. As a result, there exists an upper limit on the total installed processing capacity of sugar mills which can be expressed as[5]

$$(21) \qquad s_c(t) \leq \bar{s}_c(t),$$

where $\bar{s}_c(t)$ is the maximum amount of sugarcane which can be processed by the sugar mills in t.

2.2.6 Balance Equations: Balance equations are used in the case of sugarcane and gur and can be represented as

$$(22) \qquad -q_c(t) + s_c(t) + \gamma q_g(t) \leq 0,$$

$$(23) \qquad -q_g(t) + s_g(t) + c_g(t) \leq 0,$$

where subscripts 'c' and 'g' refer to sugarcane and gur respectively. Equation (22) implies that the total production of sugarcane is exhausted by the manufacture of gur and/or by the sale of sugarcane. Equation (23), similarly, implies that the total production of gur is exhausted by its consumption and/or sale. Where γ refers to the amount of sugarcane required to produce one unit of gur. By combining equations (1) and (23), the overall consumption constraint for sugar (gur units) can be expressed as

$$(24) \qquad -q_g(t) + s_g(t) - \rho c_s(t) \leq -\rho\bar{c}_s, \qquad (\rho c_s(t) < \rho\bar{c}_s)$$

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[5]One can easily extend this analysis to other agricultural processing industries such as cotton processing and ginning factories, wheat and corn flour mills, oil seed processing industries etc.

2.2.7 Flexibility Constraints: These constraints represent that in any one period only a limited change from the previous period's acreage under various farm crops can be expected. This hypothesis, thus, implies a limited flexibility in the established cropping patterns in any agricultural region. Farmers exercise this caution of limited flexibility in order to account for risk and uncertainty related to farm prices, yield expectations, government programs and restrictions on the aggregate supply of production inputs. This idea was suggested by Henderson 1959 and has been analyzed in detail by Day [1961, 1963]. Flexibility constraints on the production of j^{th} activity are specified as

$$(25) \qquad q_j(t) \leq (1+\bar{\beta}_j)q_j(t-1),$$

$$(26) \qquad -q_j(t) \leq -(1-\underline{\beta}_j)q_j(t-1),$$

where $\bar{\beta}_j$ and $\underline{\beta}_j$ are the coefficients which define, respectively, the upper and lower bounds on the production of j^{th} activity, $q_j(t-1)$ is the realized level of j^{th} production activity in t-1. The use of these behavioral constraints is, however, only one way to account for risk and uncertainty in agriculture.[6]

2.2.8 Non-negative Constraints: None of the activities discussed above can be operated at negative levels. This fact is expressed by the following non-negativity constraints:

$$(27) \qquad q_j(t) \geq 0,\ s_j(t) \geq 0,\ h_i(t) \geq 0,\ c_s(t) \geq 0,$$

$$y_1(t) \geq 0,\ y_2 \geq 0.$$

2.3 The Objective Function

The farmers in the LDC's possess multiple goals which need to be satisfied. These goals include meeting nonfarm cash consumption expenditure, satisfying subsistence consumption requirements, maximizing expected annual total cash receipts etc. These goals are arranged in a specific order of preferences and can be represented by lexicographic ordering.

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[6] Several alternative approaches to incorporate risk and uncertainty in dynamic farm decision models are discussed in Mudahar [1972].

In this model all but one (expected annual total cash receipts) of the potential farm goals are incorporated in the form of constraints. Consequently, the main objective of the representative farm-firm is to maximize expected annual total cash receipts subject to technological, financial, consumption, behavioral and other constraints. The linear objective function in this decision model can be defined as

$$(28) \qquad \Pi^*(t) = \max \left[\Sigma_j \hat{z}_j(t) s_j(t) + (1+\hat{i}_1) y_1(t) \right] - (1+\hat{i}_2) y_2(t),$$

$$(29) \qquad \hat{z}_j(t) = \hat{p}_j(t) - \bar{m}_j(t),$$

where $\Pi^*(t)$: expected optimal level of annual total cash receipts in t,

$\hat{z}_j(t)$: expected unit total cash receipts from j^{th} sales activity in t,

$\hat{p}_j(t)$: expected unit gross price of j^{th} sales activity in t,

$\hat{i}_1(t)$: expected rate of interest on savings in t,

$\hat{i}_2(t)$: expected rate of interest on short-term loan in t.

The specification of the objective function looks different but leads to the same conclusions as the standard formulation of the objective function[7] in which $\hat{z}_j(t) = \hat{p}_j(t) - k_j(t) - \bar{m}_j(t)$. The activity set, the constraint structure and the objective function discussed above for the representative farm-firm in time "t" represent a complete linear programming model. The farm-firm is concerned with maximizing the linear objective function subject to several linear constraints discussed above.

Assuming that each farmer (i) expects the same output prices, (ii) incurs the same per unit production and marketing costs, (iii) possesses initial endowments of land, labor and money capital in the same proportion, and (iv) responds the same way to price and income changes in making production and consumption decisions, the above linear programming model can be used to represent the sum of the decisions for all the farm-firms in a particular homogeneous region. This model is developed in reference to the Punjab state in India. Consequently,

[7] This assertion is formally stated as an "objective function theorem" and proved in Mudahar [1972].

$L(t)$, $M(t)$, $H(t)$, and $K(t)$ will represent aggregate amounts of irrigated land, farm-household labor, hired agricultural labor and money capital available to farmers in the Punjab in t. $\tilde{c}_j(t)$ now stands for the aggregate planned household consumption requirements for j^{th} food crop for the total farming population in the Punjab. The input-output coefficients now represent the technical structure at the regional level. All the production, consumption, sales, labor hiring, purchase and financial activities are assumed to be carried out at the regional level.[8] A tabular presentation of the structure of the decision model of the farm sector, including the activities and constraints considered in the present version of the model, is given in Table 1.

3. ENVIRONMENTAL FEEDBACK AND THE 'OPEN' RECURSIVE DECISION MODEL

The above linear programming decision model gives ex ante optimal solutions (affixed with star) for a single period, say t. However, the solution in "t" is influenced by (i) the past output prices, (ii) the money capital which in turn is determined by past year's realized sales and savings, (iii) the consumption of sugar which is determined by past year's realized sales receipts, (iv) the outstanding short-term loan to be repaid, (v) ability to borrow short-term loans, and (vi) the ex post optimal or sub-optimal levels of each production activity in the previous period.

All these factors influence the farmer's decision environment significantly. Moreover, some of them cannot be controlled by individual farmers but rather depend upon the market behavior and the interactions of nonfarm demand and farm supply. However, farmers adjust and revise their own production plans in response to these environmental influences. These plans, once acted upon, interact with the decision-maker's environment and generate new information upon which succeeding

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[8] The more realistic approach, of course, would be to disaggregate the model into several homogeneous agricultural regions with explicit emphasis on the role of various sized farm-firms and soil classifications. However, the above simplifying assumptions are made only as first approximation and the larger model which is currently in progress incorporates various sized farms and several land categories.

TABLE I. STRUCTURE OF THE MICROECONOMIC DYNAMIC MODEL OF FARM DECISIONS: THE PUNJAB MODEL

Activities / Constraints			Production					Purchase	Labor-Hiring		Sales					Financial			RHS
			MAZ	COT	SCN	GUR	WHT	SUG	JNLB ...	DELB	MAZ	COT	SCN	GUR	WHT	DEPSAV	BORCRT	REPCRT	
			A_1	A_2	A_3	A_4	A_5	A_6	A_7 ...	A_{18}	A_{19}	A_{20}	A_{21}	A_{22}	A_{23}	A_{24}	A_{25}	A_{26}	
Obj. Fcn. Coeff.		(MAX)	0	0	0	0	0	0	0	0	$\hat{z}_1(t)$	$\hat{z}_2(t)$	$\hat{z}_3(t)$	$\hat{z}_4(t)$	$\hat{z}_5(t)$	$(1+\hat{i}_1)$	$-(1+\hat{f}_2)$	0	
1. Land	SUMLND	R_1	$\ell_{11}(t)$	$\ell_{12}(t)$	$\ell_{13}(t)$														$\leq L(t)$
	WINLND	R_2			$\ell_{23}(t)$		$\ell_{25}(t)$												$\leq L(t)$
2. Family Labor	JAFLBR	R_3			$m_{33}(t)$	$m_{34}(t)$	$m_{35}(t)$		-1										$\leq M_1(t)$
	FEFLAB	R_4			$m_{43}(t)$	$m_{44}(t)$	$m_{45}(t)$												$\leq M_2(t)$
	MRFLBR	R_5			$m_{53}(t)$	$m_{54}(t)$	$m_{55}(t)$												$\leq M_3(t)$
	APFLBR	R_6			$m_{63}(t)$		$m_{65}(t)$												$\leq M_4(t)$
	MAFLBR	R_7	$m_{71}(t)$	$m_{72}(t)$	$m_{73}(t)$		$m_{75}(t)$												$\leq M_5(t)$
	JUFLBR	R_8	$m_{81}(t)$	$m_{82}(t)$	$m_{83}(t)$														$\leq M_6(t)$
	JLFLBR	R_9	$m_{91}(t)$	$m_{92}(t)$	$m_{93}(t)$														$\leq M_7(t)$
	AUFLBR	R_{10}	$m_{10,1}(t)$	$m_{10,2}(t)$	$m_{10,3}(t)$														$\leq M_8(t)$
	SEFLBR	R_{11}	$m_{11,1}(t)$	$m_{11,2}(t)$	$m_{11,3}(t)$														$\leq M_9(t)$
	OCFLBR	R_{12}	$m_{12,1}(t)$	$m_{12,2}(t)$	$m_{12,3}(t)$		$m_{12,5}(t)$												$\leq M_{10}(t)$
	NOFLBR	R_{13}	$m_{13,1}(t)$	$m_{13,2}(t)$	$m_{13,3}(t)$		$m_{13,5}(t)$												$\leq M_{11}(t)$
	DEFLBR	R_{14}			$m_{14,5}(t)$		$m_{14,5}(t)$			-1									$\leq M_{12}(t)$
3. Hired Labor	JAHLBR	R_{15}							+1										$\leq H_1(t)$
	.	.																	. .
	.	.																	. .
	.	.																	. .
	DEHLBR	R_{26}								+1									$\leq H_{12}(t)$
4. Money Capital	MONCAP	R_{27}	$k_1(t)$	$k_2(t)$	$k_3(t)$	$k_4(t)$	$k_5(t)$	$\hat{p}_s(t)$	$w_1(t) \ldots w_{12}(t)$							+1	-1	+1	$\leq K(t)-e(t)-f(t)$
	BORCAP	R_{28}															+1		$\leq r_1(t)K(t)+r_2(t)E(t)$
	REPCAP	R_{29}																-1	$\leq -(1+i_2)y_2(t-1)$
5. Household Consumption	CONMAZ	R_{30}	-1								+1								$\leq -\bar{c}_1(t)$
	CONCOT	R_{31}		-1								+1							$\leq -\bar{c}_2(t)$
	CONSGR	R_{32}				-1		$-\rho$						+1					$\leq -\rho\bar{c}_4(t)$
	CONWHT	R_{33}					-1								+1				$\leq -\bar{c}_5(t)$
6. Sugarcane Supply	SCNSPU	R_{34}											+1						$\leq \bar{s}_c(t)$
	SCNSPL	R_{35}											1						$\leq -\underline{s}_c(t)$
7. Balance Equations	SCNBAL	R_{36}			-1	γ							+1						≤ 0
	SGRBAL	R_{37}						+1											$= c_s(t)$
8. Flexibility	PMAZUB	R_{38}	1																$\leq (1+\bar{\beta}_1)q_1(t-1)$
	PMAZLB	R_{39}	-1																$\leq -(1-\underline{\beta}_1)q_1(t-1)$
	PCOTUB	R_{40}		1															$\leq (1+\bar{\beta}_2)q_2(t-1)$
	PCOTLB	R_{41}		-1															$\leq -(1-\underline{\beta}_2)q_2(t-1)$
	PSCNUB	R_{42}			1														$\leq (1+\bar{\beta}_3)q_3(t-1)$
	PSCNLB	R_{43}			-1														$\leq -(1-\underline{\beta}_3)q_3(t-1)$
	PWHTUB	R_{44}				1													$\leq (1+\bar{\beta}_5)q_5(t-1)$
	PWHTLB	R_{45}				-1													$\leq -(1-\underline{\beta}_5)q_5(t-1)$

production plans can be based. As a consequence, farmers' decisions in a given period depend recursively on the previous periods' solutions. This intertemporal recursive interdependence generates environmental feedback functions which, once explicitly included, make the above static linear programming model a 'short-sighted' dynamic model of farm decisions. This model is, thus, useful in describing the evolution of agricultural growth process and structural change. The basic steps involved in obtaining the final version of the 'open' recursive decision model of the farm sector are systematically discussed below.

3.1 Price Expectations Behavior

The original cobweb theory is based on the assumption that producers are naive forecasters and use last period's price as their forecast for its immediate future value. This implies that farmers respond to output prices prevailed in the immediate past period and use them as a basis to make the current production decisions. Ezekiel [1938], Nerlove [1958] and Waugh [1964] have suggested that in certain agricultural commodities there exist more than one period price lag on the supply side. The influence of lagged prices goes on diminishing as the lag gets longer and may vary from one commodity to another. However, there is a very limited amount of empirical work done on determining the price-lag structure and marketing behavior of peasants and it might vary from one agricultural region to another. Farmers' price expectation behavior in the LDC's can be represented by a more general price expectation hypothesis which can be specified as

$$(30) \qquad \hat{p}_j(t) = \Sigma_{i=1}^{\tau} \lambda_i p_j(t-i), \qquad i = 1, \ldots, \tau, \; \Sigma_{i=1}^{\tau} \lambda_i = 1, \; \lambda_i \geq 0$$

where $\hat{p}_j(t)$ is the expected price for j^{th} farm commodity in t and $p_j(t-1)$ is the realized price for j^{th} farm commodity in (t-1).

Given the fact that majority of the farmers in the LDC's keep very little, or not at all, farm records and accounts, it is not possible for them to recall the realized price levels which prevailed more than a few years in the past. For simplicity, therefore, it is hypothesized that farmers make their production plans based on the weighted average of the preceding two period's realized price levels for different farm commodities. The naive price expectation behavior of farmers for j^{th} farm commodity can be represented as

(31) $\hat{p}_j(t) = \lambda p_j(t-1) + (1-\lambda)p_j(t-2).$

In the original cobweb theory $\lambda = 1$. When $\lambda = 1$, the second term in equation (31) drops out. In order to compare the implications of the price expectation behavior of farmers, both the hypotheses ($1 > \lambda > 0$ and $\lambda = 1$) are analyzed separately in the final model keeping everything else unchanged.

Similarly, the decision to save money and deposit it in the bank is determined by the expected rate of interest. In order to simplify the analysis it is assumed that the rate of interest on farm savings is constant over time implying

(32) $\hat{i}_1(t) = i_1(t-1) = i_1(t),$

where $\hat{i}_1(t)$ and $i_1(t)$ are the expected and actual rates of interest respectively. The same assumption is assumed to hold for interest rate on short-term loans.

3.2 Environmental Feedback Mechanism

The environmental feedback in the linear programming model enters through (i) the flexibility constraints, (ii) the objective function coefficients, and (iii) the money capital constraint. The flexibility constraints have already been specified by equations (25) and (26). By substituting for $\hat{p}_j(t)$ in equation (29), the objective function coefficients for the j^{th} farm commodity in t can be expressed as

(33) $\hat{z}_j(t) = [\{\lambda p_j(t-1) + (1-\lambda)p_j(t-2)\} - \bar{m}_j(t)].$

However, if $\lambda = 1$ the above simplifies to

(34) $\hat{z}_j(t) = [p_j(t-1) - \bar{m}_j(t)].$

The money capital supply function (equation (14)) can now be rewritten to incorporate the ex post solutions for commodity sales, realized commodity prices and the savings derived from the linear program in t-1. The modified money capital supply function is

(35) $$K(t) = \Sigma_j [\{p_j(t-1) - \bar{m}_j(t-1)\}s_j^*(t-1)] + (1+i_1)y_1^*(t-1) - (1+i_2)y_2^*(t-1).$$

Equation (35) can be further modified to incorporate financial subsidies.

By substituting equations (2), (25), (26), (33) and (35) into the above linear programming model, it is clear that the parameters of a linear program in "t" depend on the optimal or sub-optimal solutions derived from the preceding years' linear programs. This reflects the view that the actual behavior of farmers is based on rationally planned actions. The decision-maker being studied knows the consequences of his past actions but is partially unsure of the mechanisms that generate them. Consequently, the recursive dependence of one year's decision problem on previous years' solutions makes the above model a recursive programming model. Since product prices are given exogenously and decisions are made period after period, we prefer to call it the 'open' recursive decision model of the farm sector or just the open sector model of farm decisions.

3.3 Viability, Feasibility and the Safety-First

In the money capital constraint equation, the annual cash expenditures on (i) nonfarm-produced consumer goods (except sugar) and services, (ii) fixed cash consumption obligations, and (iii) fixed factors of production, are always positive in value i.e. $[e(t) + f(t)] > 0$. The 'open' recursive decision model of the farm sector will be viable only in case $[K(t) - e(t) - f(t)] > \theta$, where θ is the amount of money capital required to produce farm commodities in order to satisfy subsistence consumption constraints.

In case $[K(t) - e(t) - f(t)] = \theta$, the system will be feasible during "t" and infeasible during t+1 and thereafter due to shortage of money capital to finance the variable and fixed production and consumption cash expenditure and to satisfy subsistence consumption constraints. Secondly, if $0 \leq [K(t) - e(t) - f(t)] < \theta$, the system will be infeasible during "t" and inviable thereafter due to shortage of money capital to finance the production of farm commodities in order to satisfy the subsistence consumption constraints.

Finally, if $[K(t) - e(t) - f(t)] < 0$, the system will be absolutely inviable in "t" since it will not be able to finance even the fixed amounts of production and consumption cash expenditures. The only way to save this system from collapsing under all these <u>three</u> situations is to inject subsidies, provide relief funds or any other kind of free financial support and/or allow farmers to borrow short-term loans.

From the theory-of-choice point of view, θ can be considered a 'disaster level of income'. This concept was introduced by Roy [1957] in his safety-first principle. The safety-first principle reflects an alternative approach to account for uncertainty. It involves minimizing the probability such that the annual profit level falls below an exogenously specified 'disaster level of income'. As a result, this principle captures the behavior of farmers in the LDC's whose first major aim is to survive in the business. In addition to the flexibility constraints, uncertainty in farm business is taken into account by the subsistence consumption constraints.

4. EXISTENCE OF FARM COMMODITY CYCLES

The existence of farm commodity cycles such as corn cycles, sugarcane cycles, wheat cycles, hog cycles, etc., is inherent in the process of agricultural development. These cycles result from the imbalances in the demand and supply of a particular commodity. The relative profitability of different agricultural commodities is changing rapidly due to the adoption of new farm technology and changes in the input-output price structure over time. This, in turn, leads to changes in the cropping pattern of a particular region, and thereby alters the supply of various farm commodities. However, it has been maintained in the literature that the demand for various farm commodities is less elastic than industrial goods. This imbalance between the supply and demand for farm commodities leads to rapid price fluctuations. As has been assumed in the original cobweb theory, it is plausible that farmers are naive forecasters and use the immediate past prices as a guide to allocate their existing resources among different current agricultural activities. This type of behavior on the part of farmers thus leads to farm commodity cycles. A specific example of this kind of cyclical behavior is found in the case of sugarcane in the Punjab.

Sugarcane is one of the most important high income enterprises of Punjab agriculture. Although the area under sugarcane is only about 3 percent of the total cropped area, it occupies an important place in the product-mix of Punjab agriculture. Approximately 60 percent of the total sugarcane production of the Punjab is processed by farmers themselves into gur. The rest is supplied to sugar factories. The price of gur fluctuates more than that of sugarcane because of the fact that gur price is determined independently by market forces, whereas sugarcane prices are either fixed statutorily or greatly influenced by government farm policies. Current prices of gur and sugarcane in relation to prevailing prices of other farm commodities determine the acreage sown under sugarcane in the next year. The amount of sugarcane to be processed into gur depends primarily on the sugarcane price, and the difference between cost of manufacturing gur and price of gur in the market on a per unit basis. Thus there exists a cause and effect relationship of the cobweb type among the acreage under sugarcane, production of gur, supply of sugarcane to factories and prices of gur and sugarcane.

However, the production of sugarcane and gur is also influenced by the relative profitability of competing farm enterprises, the amount of gur to be consumed at home, the demand for gur, sugarcane and other farm commodities within and outside the farm sector, and the supply of nonfarm inputs by manufacturing and banking sectors and the availability of technological possibilities at the farm-firm. Therefore, in a rapidly changing farm economy the response of sugarcane and gur production to various stimuli from outside and within the agricultural sector must be obtained and analyzed. There is, thus, a need to incorporate all these interrelated elements determining demand and supply of gur, sugarcane and other competing farm commodities into a model of farm decisions.

5. GENERALIZED COBWEB THEORY

The cause and effect relationship between market price and production of different farm commodities was observed by many economists in the beginning of the twentieth century. This includes the empirical work done by Henry Moore and Arthur Hanau in the 1920's. A theoretical explanation of this type of phenomena was given independently at the same time during the 1930's by Jan Tinbergen in Holland, Henry Schultz in the U.S. and Umberto Ricci in Italy. The name "cobweb" was suggested by an English economist Nicolas Kaldor in 1934. The stability conditions of cobweb models were first worked out by Leontief [1934]. Ezekiel [1938] gave a more systematic treatment to cobweb theorem and applied it to explain commodity cycles in the U.S. Further generalizations of cobweb models, their role in economic theory and their relevance to government programs was analyzed by Waugh [1964].

The original cobweb theory, which was designed to explain cyclical behavior, is based on the assumption that current demand determines the current price, which in turn determines the supply in the next period. This implies that there exists one period price lag on the supply side and suggests that producers are naive forecasters. In certain commodities, there exists more than one period price lag on the supply side and the influence of lagged prices goes on diminishing as the lags get longer, Ezekiel [1938], Nerlove [1958] and Waugh [1964]. The cyclical behavior of cobweb phenomena is, however, determined by the relative elasticities of the demand and expected supply functions.

In actual situations, however, the supply and demand functions are determined by several other variables in addition to the price level which are not taken into account by the conventional cobweb theory. Waugh [1964] and Wold - Jureen [1953] suggested that the effect of variables other than price can be captured by introducing random variables in the demand and supply functions. This stochastic nature of cobweb theory, however, ignores the subsistence nature of agriculture which is a peculiar characteristic of agriculture in the LDC's. Also, it sums up the influence of numerous other important variables and there is no way to determine the contribution of each individual variable. Furthermore, it does not involve a choice between alternative farm opportunities subject to certain restrictions which are an important part of the farmer's decision environment. Finally, it ignores the role played by the seasonal availability of technological and financial resources, the existence of government programs and the uncertain future prospects faced by the farmer. Waugh [1964] has correctly pointed out that

> "Any kind of economic planning requires some sort of recursive analysis. How will this year's plans, policies, programs, affect next year's output, prices, consumption? This is especially important in agriculture, where programs and policies are being constantly debated and changed. And because of this, agricultural economists are being asked for long-term economic 'projections', indicating what agricultural output, prices, consumption...

would be under various programs. For this purpose we certainly need good cobweb models and more elaborate recursive systems" [pp. 749-50].

Day-Tinney [1969] generalized the original cobweb theory by replacing the expected supply function by an explicit farm decision model and determining inverse demand functions endogenously. This approach can be used not only to analyze the cyclical behavior of farm commodities but also to describe the growth process of the agricultural sector. From another point of view the model developed here is an extension and modification of the generalized version of the cobweb theory and is applied to describe the process of agricultural growth and commodity cycles in the Punjab.

6. MARKET FEEDBACK AND THE 'CLOSED' RECURSIVE DECISION MODEL

The 'open' recursive decision model incorporates various 'forward' and 'backward' linkages between farm and nonfarm sectors through (i) linking the farm sector with the banking sector through various financial activities and constraints, (ii) the use of resource restrictions which are determined by the progress in the nonfarm sectors and which eventually condition the progress in the farm sector. The purpose of this section is to develop a framework to introduce demand linkages by imbedding the farm sector in the product market environment of the nonfarm sector. Furthermore, the open sector model of farm decisions is extended and modified (i) to develop a theory of farm policies mainly price support programs, (ii) to determine product prices endogenously through inverse demand functions, (iii) to analyze the relevance of nonfarm demand for farm commodities in agricultural development models of this nature, and (iv) to analyze the cyclical behavior of different farm commodities, specifically sugarcane cycles by using a 'generalized' cobweb theory.

6.1 Market Demand and Price Regulations

Market demand functions can be divided into (i) nonfarm demand for farm products, and (ii) total demand for nonfarm consumer goods and services. The marketable surplus generated by the production and consumption decisions in the agricultural sector is sold in the market which is assumed to be perfectly competitive. This brings a temporary equilibrium between nonfarm demand and existing farm supply of different agricultural commodities which are then sold at a uniform price determined by the market equilibrium. The inverse nonfarm demand function for j^{th} farm commodity that describes this temporary equilibrium can be defined as

$$(36) \qquad \tilde{p}_j(t) = \psi\{s^*_j(t), s^*_{i\neq j}(t), x(t), A(t)\},$$

where $\tilde{p}_j(t)$ is the price of j^{th} commodity determined by the market mechanism in t, A(t) is the size of nonfarm population, $s^*_j(t)$ and $s^*_{i\neq j}(t)$ are, respectively, the actual farm supply of j^{th} and i^{th} farm commodities in t and finally, x(t) is the per-capita income in the nonfarm sector. The first-order partial derivatives of the above nonfarm demand function are assumed to have the following signs: $\psi'_1 < 0$, $\psi'_2 \lesseqgtr 0$ (depending upon whether i^{th} and j^{th} goods are substitutes, complementary or independent); $\psi'_3 \gtrless 0$ (depending upon whether j^{th} good is normal or inferior consumer good); and $\psi'_4 > 0$.

Given the above inverse demand function it is possible that $\tilde{p}_j(t) \gtrless 0$. In order to avoid negative prices and to make sure that farmers receive at least their transportation plus marketing costs, the above demand function is augmented by a 'positivity' condition. The modified inverse nonfarm demand function can be specified as

$$(37) \qquad p_j(t) = \max[\bar{m}_j(t), \tilde{p}_j(t)], \text{ or}$$

$$(38) \qquad p_j(t) = \max[\bar{m}_j(t), \psi\{s^*_j(t), s^*_{i\neq j}(t), x(t), A(t)\}],$$

where $p_j(t)$ is the actual gross price realized by farmers.

Above market demand functions are assumed to be unknown to agricultural producers but are part of their decision environment and influence their future decisions through environmental feedback. Since the Punjab has always been a food surplus state, the question of importing food does not arise. However, food exports have been allowed in the model which are enacted by private trading agencies and/or directly by the state government or Food Corporation of India.

The above demand function can further be modified to incorporate government agricultural development policies such as price support programs, minimum and maximum price levels which are fixed by the government, etc. During the best agricultural years or glut seasons, farm supply tends to outweigh the nonfarm demand for different farm commodities. As a result, farm prices tend to fall far below the normal price levels. Under such circumstances, the government assures farmers certain minimum price levels through price support programs which in turn influence the farmer's decision environment. This idea can be incorporated as

$$(39) \qquad p_j(t) = \max\,[\delta_j(t),\ \psi\{s_j^*(t),\ s_{i\neq j}^*(t),\ x(t),\ A(t)\}],$$

where $\delta_j(t)$ is the minimum price level for the j^{th} farm commodity. In the presence of such imperfections, market clearance is assured through the purchase of surplus grains by the state government.

Similarly, from the point of view of the welfare of the farm and nonfarm consumer sector, the government can fix a maximum price level for scarce commodities and hence not allowing prices to exceed an upper limit in case the price determined by the market phenomena is very high. This type of regulation exists, if at all, in the case of commodity sale at the retail level or commodity rationing and very rarely in the case of unprocessed agricultural commodities. The government imposes such price controls in case of sugar whenever it is in short supply. The maximum price dictated by the government authority can, however, be incorporated in above formulation as

$$(40) \qquad p_j(t) = \min\,[\sigma_j(t),\ \max\,[\bar{m}_j(t),\ \psi\{s_j^*(t),\ s_{i\neq j}^*(t),\ x(t),\ A(t)\}]],$$

where $\sigma_j(t)$ is the maximum price level dictated by the government for j^{th} commodity. Both maximum and minimum price policies can also be introduced jointly in the same equation as

$$(41) \qquad p_j(t) = \min\,[\sigma_j(t),\ \max\,[\delta_j(t),\ \psi\{s_j^*(t),\ s_{i\neq j}^*(t),\ x(t),\ A(t)\}]]$$

These modified nonfarm inverse demand functions for farm commodities represented by equations (39), (40) and (41) are shown graphically in

Figure 6. However, in these diagrams, all other variables except $s_j^*(t)$ are assumed constant.

In case of total demand for nonfarm produced consumer goods same general relationship is assumed. Since only the purchase of refined sugar is considered explicitly, the inverse demand function for refined sugar, augmented by the 'non-negativity' conditions, is specified as

$$(42) \qquad p_s(t) = \max\,[0, \theta\{s_c^*(t),\ s_g^*(t),\ \bar{x}(t),\ F(t)\}],$$

where θ is the inverse demand function. $\bar{x}(t)$ and $F(t)$ stand for per capita income in the economy and total population respectively.

These inverse demand functions establish an environmental feedback mechanism which links the present farm decisions with the past solutions. This market mechanism and behavioral interdependence is very crucial to describe the behavior of a farm decision maker and the process of agricultural growth.

6.2 Market Feedback Mechanism

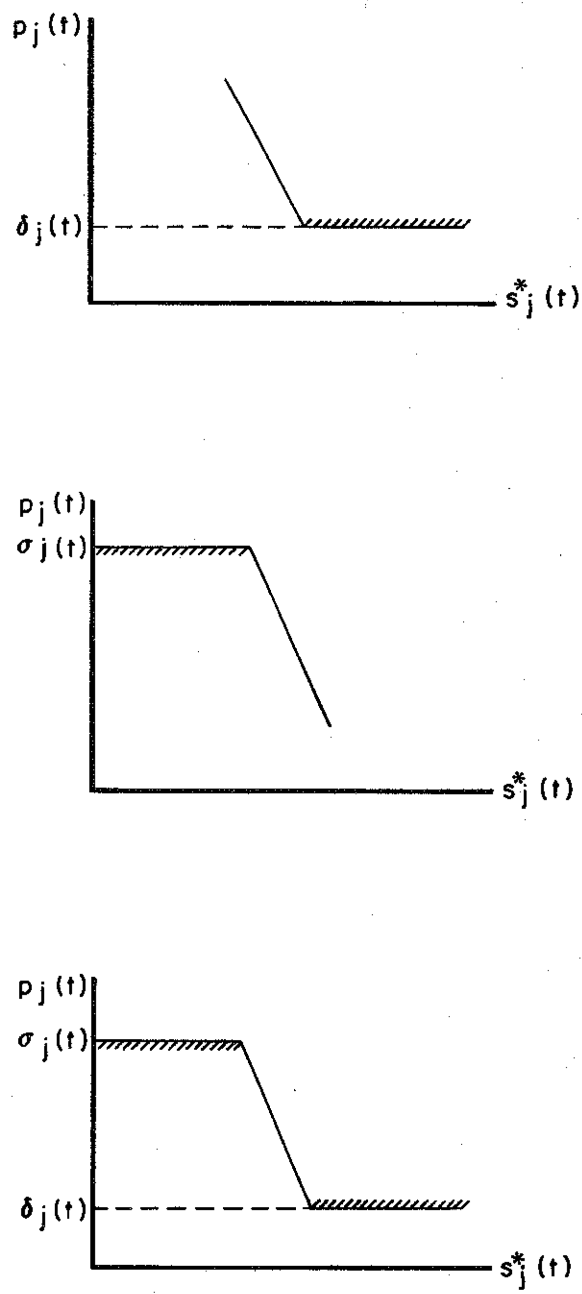
The market feedback in the linear programming model enters through (i) $\hat{z}_j(t)$, (ii) $K(t)$, (iii) $c_s(t)$, and (iv) $p_s(t)$. Substituting for $p_j(t-1)$ and $p_j(t-2)$ into (33); for $p_j(t-1)$ into (35), we obtain

$$(43) \qquad \hat{z}_j(t) = [\lambda[\max\,[\bar{m}_j(t-1),\ \psi\{s_j^*(t-1),\ s_{i\neq j}^*(t),\ x(t-1),\ A(t-1)\}]] + (1-\lambda)\,[\max[\,\bar{m}_j(t-2),\ \psi\{s_j^*(t-2),\ s_{i\neq j}^*(t-2),\ x(t-2),\ A(t-2)\}]]] - \bar{m}_j(t),$$

$$(44) \qquad K(t) = \Sigma_j[\max\,[\bar{m}_j(t-1),\ \psi\{s_j^*(t-1),\ s_{i\neq j}^*(t-1),\ x(t-1),\ A(t-1)\}] - \bar{m}_j(t-1)]\ s_j^*(t-1) + (1+i_1)\ y_1^*(t-1) - (1+i_2)\ y_2^*(t-1).$$

Similarly, equations (43) and (44) can be modified to incorporate government policies such as price support programs and farm subsidies, etc. Equation (44) can be extended to incorporate two period's distributed lagged price expectation hypotheses and equation (43) can be simplified by assuming $\lambda = 1$ i.e. by assuming single period price expectation hypothesis.

FIGURE 6 MINIMUM AND MAXIMUM SUPPORT PRICE PROGRAMS FOR AGRICULTURAL COMMODITIES AND CONSUMER GOODS



By substituting equations (2), (25), (26), (42), (43), and (44) into the basic linear programming model, it is clear that the parameters of a linear program in "t" depend on the optimal or sub-optimal solutions derived from the preceding years' linear programs. Consequently, this recursive interdependence of one year's decision problem on previous years' solutions and the endogenous determination of product prices makes the linear programming model the 'closed' recursive decision model of the farm sector or just the closed sector model of farm decisions.

Like the open sector model, the closed sector model of farm decisions will be feasible and viable thereafter only in case $[K(t) - e(t) - f(t)] > \theta$, where θ is described in section 3.3. Otherwise the system will go infeasible and inviable thereafter due to lack of financial resources. However, the system can be saved from collapsing by injecting free financial support from outside and/or allowing for short-term borrowings.

Some of the essential characteristics of the present model are (i) it explicitly incorporates the linkages between farm and nonfarm sectors and the interdependence between farm-firm and farm-household; (ii) it includes seasonal resource restrictions such as land, labor and capital; (iii) it allows borrowing and repayment of short-term loans when the farm-firm's own money capital is in limited supply; (iv) it includes various government programs and development policies; (v) it incorporates conventional cobweb theory and allows a choice among many alternative economic activities available to the decision maker; (vi) it takes into account the present knowledge, future expectations and the consequences of past actions; (vii) the decision problem is reformulated and solved at the beginning of each production period which is very short relative to the economic process as a whole; and (viii) it accounts for risk and uncertainty through behavioral constraints. The resulting product is a "short-sighted" dynamic model or "recursive programming" model. The model is similar in spirit to various other models such as Leontief's [1958] theory of economic growth, the cobweb theory, Goldman's [1968] "continual planning" model, and Waterson's [1965] concept of "rolling plans". In some respects the model is similar to the model developed by Day-Tinney [1969].

7. DYNAMIC COUPLING AND RECURSIVE INTERDEPENDENCE

The market feedback mechanism, dynamic coupling and the recursive interdependence between farm-household, farm-firm and commodity market and the working of the micro-economic dynamic "closed" sector model of farm decisions is described diagramatically[9] in Figure 7. There exists a hierarchy in the decision process. It is comprised of a chain of decision sequences. In the beginning of any time period, say t, the sequence of decisions goes from farm-household to farm-firm and finally to the commodity market. The consequences of past actions condition present decisions, the outcomes of which in turn determine the future actions on the part of the household and the firm. This dynamic interdependence between household, firm and market is connected by a mechanism called dynamic coupling, Day [1964] and Goodwin [1947]. Using the concept of dynamic coupling between interdependent decision units, the above model is decomposed into farm-household and farm-firm sub-models. The household's consumption requirements are determined exogenously. These planned consumption requirements are then used as constraints in the programming model of the farm-firm involving explicit optimization.

The farm-household's decisions consist of determining the planned consumption requirements, the purchase of consumer goods, the supply of family labor and savings to the farm-firm. The farm-firm, on the other hand, is concerned with making production, labor hiring, borrowing, sales and investment decisions. The firm's decisions, in turn, are determined by the outcome of the household's decisions, physical resource restrictions, availability of money capital, expected per unit cash returns (expected prices) on different farm activities and government programs. The availability of money capital depends upon borrowings, past savings and commodity sales. The expected prices of different farm commodities are the weighted average of their past prices. At the end of the planning horizon, a part of the total production of each crop is retained for household's consumption in the next period and the rest is sold. The

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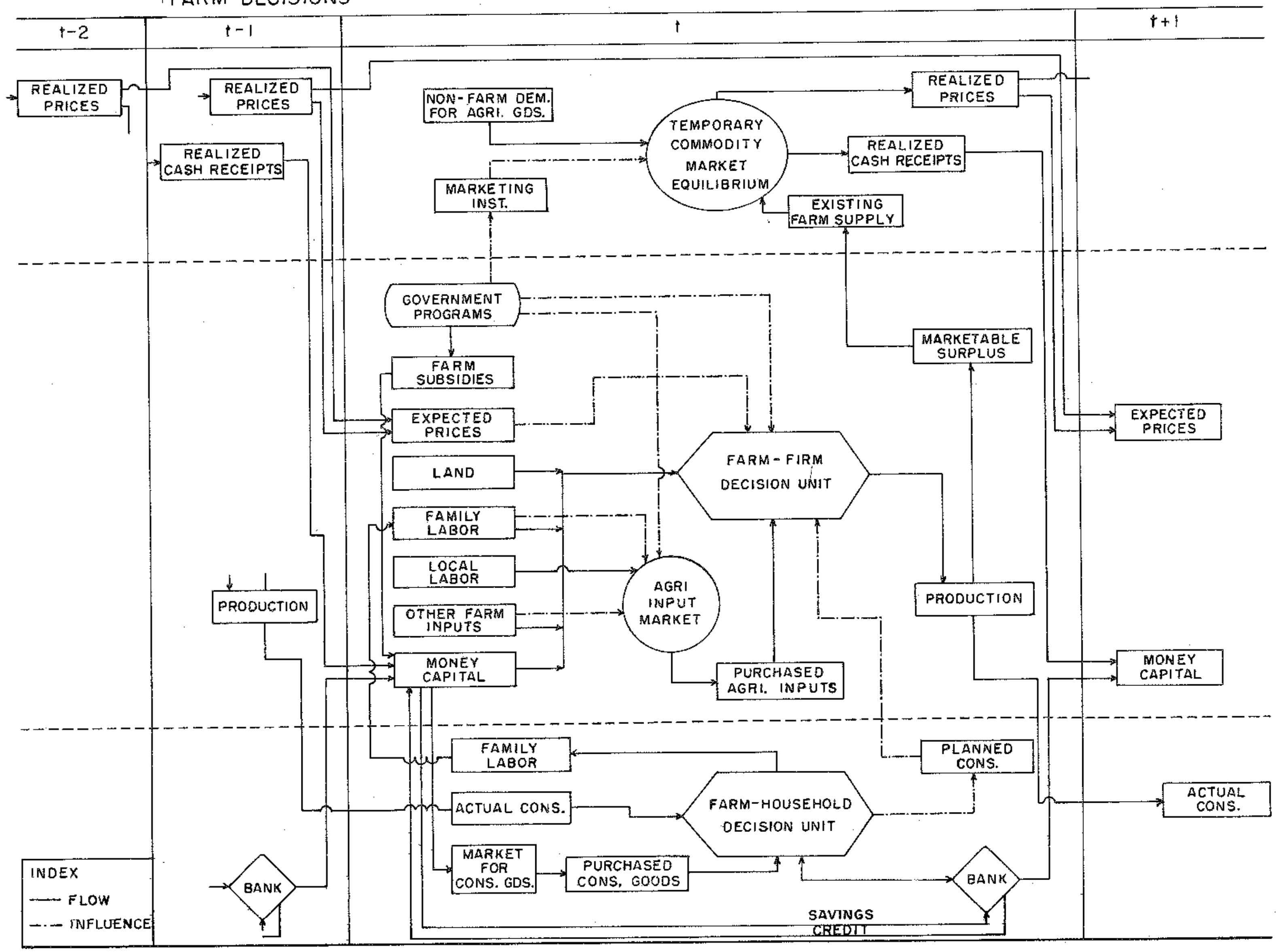
[9] The mechanism of dynamic coupling and the recursive interdependence for the "open" sector model of farm decisions is same except that product prices are given exogenously.

FIGURE 7 SCHEMATIC PRESENTATION OF THE MICROECONOMIC DYNAMIC CLOSED SECTOR MODEL OF FARM DECISIONS



marketable surplus or the farm supply to the nonfarm sector, thus consists of the difference between the realized production and planned consumption.

The existing farm supply and exogenous nonfarm demand for farm products determine prices through a temporary equilibrium in the commodity market which is assumed to be perfectly competitive. These actual prices result in a particular level of actual total cash returns. The actual cash income, prices, production and savings determine the household's and the firm's decisions in the next period and the cycle repeats thereon. The connection between each period's decisions and past consequences is provided by the market and environmental feedback mechanisms.

In addition to the market feedback, environmental feedback and dynamic coupling mechanisms, some of the exogenous forces which influence the household's and firm's decisions have also been taken into account. These exogenous forces include government programs, private and government marketing institutions, agricultural input markets, consumer goods markets and the availability of outside farm labor and other physical resources.

8. ESTIMATION AND SIMULATION

The 'open' and 'closed' recursive decision models of the farm sector are estimated and tested by using the same set of real data on various aspects of agriculture from the erstwhile Indian Punjab. The time series data is for the twenty years from 1951-52 to 1970-71. The data on input-output coefficients, constraints, parameters, and other variables used to test these models is reported in Mudahar [1972]. All the empirical results are obtained by using RDS*PRØCESSØR, an algorithm developed by Muller [1970] for the UNIVAC 1108 computer at the University of Wisconsin.

The empirical part of these models is scaled down in size and is representative of the actual farm decisions and full-blown models of the agricultural sector. These scale representative models not only help to clarify the analytical description of their qualitative properties, these are easy and relatively inexpensive to simulate. The preliminary results are obtained by assuming no investments in durable farm assets and no technological change. Various agricultural operations are performed by using one technology which is "hybrid" in nature. The technology is "hybrid" in the sense that it allows for the use of both traditional and modern technologies. This is not explicitly incorporated in the model and is derived from actual farm practices and operations. The use of modern inputs such as fertilizers, insecticides, new seed varieties, etc., is allowed. The model is limited to four major crops and all the land is irrigated.

The basic models are tested by using realistic flexibility coefficients, the actual sugarcane processing capacity, a single period price expectation hypothesis, realistic interest rates on farm savings and short-term credit, realistic price support programs, the realistic demand for farm products and the realistic maximum credit limit. The empirical results are then obtained for production and marketable surplus of different crops, consumption of gur and sugar, savings, borrowings, repayments, purchase of sugar, purchase and utilization of hired agricultural labor, utilization and demand for seasonal land and monthly family labor, demand for money capital and utilization of existing processing capacity of sugar mills. In the open sector model, product prices are given exogenously. However, in the case of the closed sector model, product prices and the price of sugar are determined endogenously by using inverse demand functions.

Given the initial conditions, the models are solved to obtain <u>ex ante</u> optimal solutions for period t=0. The information obtained from <u>ex post</u> solutions in t=0 is then used to set up initial conditions for t=1 through the 'feedback operators'. The rest of the required information is supplied exogenously. The model is then solved for period t=1. This information is then used by the 'decision operators' in period t=2, and the cycle repeats thereon from t=0, 1, ..., 19, for both the open and the closed sector models.

After testing the open and the closed sector models with real data the comparative dynamic and sensitivity analysis is conducted for both the models by incorporating and modifying a wide variety of policy variables so as to determine the performance of the open and the closed sector models under several 'government policy regimes' and programs, 'alternative model specifications' and 'changed environmental conditions'. The simulation experiments for the open and the closed sector models are conducted by (i) widening the zone of flexibility constraints, (ii) an increase in the sugarcane processing capacity, (iii) the use of two period price expectation hypothesis, (iv) changes in interest rates on savings and short-term credit, and (v) changes in the maximum credit limit coefficients.

Furthermore, in the closed sector model the simulation experiments are also conducted by changing (i) price support programs, and (ii) market demand for farm products. Finally, the inverse demand functions for various farm and nonfarm commodities are assumed to be linear and have the following functional form:

$$(45) \qquad p_j(t) = \max \{\bar{m}_j(t),\ a_j(t) + b_j s_j^*(t)\},\ a_j(t) > 0,\ b_j \leq 0.$$

This simple linear form is only a rough approximation to non-linear demand functions and ignores direct income and cross price effects. However, $a_j(t)$ changes over time. It is assumed that $a_j(t) = a_j(0)\ \{1 + \mu_j\}^t$, $t = 0, 1, \ldots, 19$, where μ_j is the demand "shift parameter" and has the effect which is equivalent to "income" and "population" effects. In the case of refined sugar $\bar{m}_j(t) = 0$.

The exogenous data used to test the closed sector model is same as has been used to test the open sector model, except for (i) the money capital coefficients refer to 1966-67 and are assumed constant over time, and (ii) the sugarcane processing capacity refers to 1966-67 and is assumed same for all the years.

Some of the important empirical results obtained from both the 'open' and the 'closed' sector models are reported in the following sections. However, before one can draw some policy implications and compare the performance of both the models, one must evaluate the performance of these models as to their ability to track the past history. The following section deals with the evaluation of the results obtained from the basic models.

9. MODEL EVALUATION

Unlike the statistical and econometric models, the empirical results obtained from the mathematical programming models are not accompanied by various statistics such as R^2, F ratio, t value, etc., which could be used to draw some inferences as to the validity of these models and to develop some "confidence" in the empirical results. However, some unconventional statistical methods have been suggested to evaluate the performance of mathematical programming and simulation models.[10]

Some of the important and most relevant (in this context) model evaluation criteria include (i) prediction-realization diagrams, (ii) prediction of turning points, (iii) information inaccuracy statistics, and (iv) Theil's U-statistic. These tests are not necessarily the best available to evaluate the performance of a particular model, these do help to compare the performance of the open and the closed sector models which applied to the same set of data. These evaluation tests are performed on the observed and predicted acreage and production for maize, cotton, sugarcane and wheat.

9.1 Prediction-Realization Diagrams

The main purpose of these diagrams is to give a visual description of the magnitude and direction of change predicted by the model and those of reality. The predicted percentage change over time, $M_i(t)$, is plotted against actual percentage change over time, $A_i(t)$. The model will be perfect in tracking the past history in case $M_i(t)$ and $A_i(t)$ fall on the 45° line passing through the origin in the first and third quadrants. However, this is highly improbable. Preferably, these points should not fall in the second and fourth quadrants. The prediction-realization diagrams for maize, cotton, sugarcane and wheat for the open and the closed sector models are displayed in Figures 8 and 9 respectively. These models have a tendency to over or under predict the production levels so far as the magnitude is concerned but has done a good job in predicting the direction of change of the production levels over time.

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[10] For the relevant literature and detailed discussion on various model evaluation tests see Austin [1970], Day-Nelson [1971] and Day-Singh [1971].

FIGURE 8 PREDICTION-REALIZATION DIAGRAMS FOR THE PRODUCTION OF DIFFERENT FARM COMMODITIES IN THE OPEN SECTOR MODEL OF FARM DECISIONS

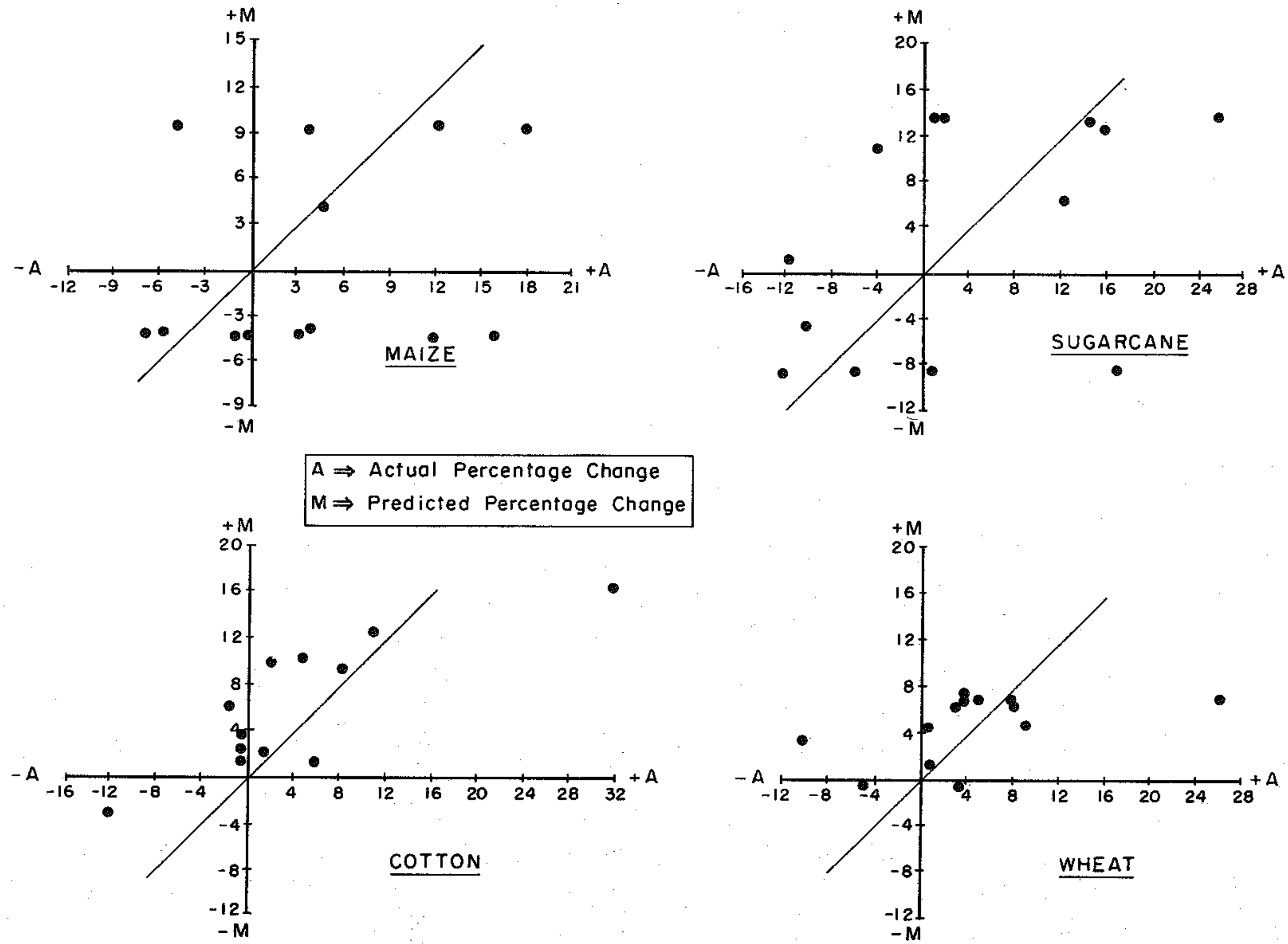
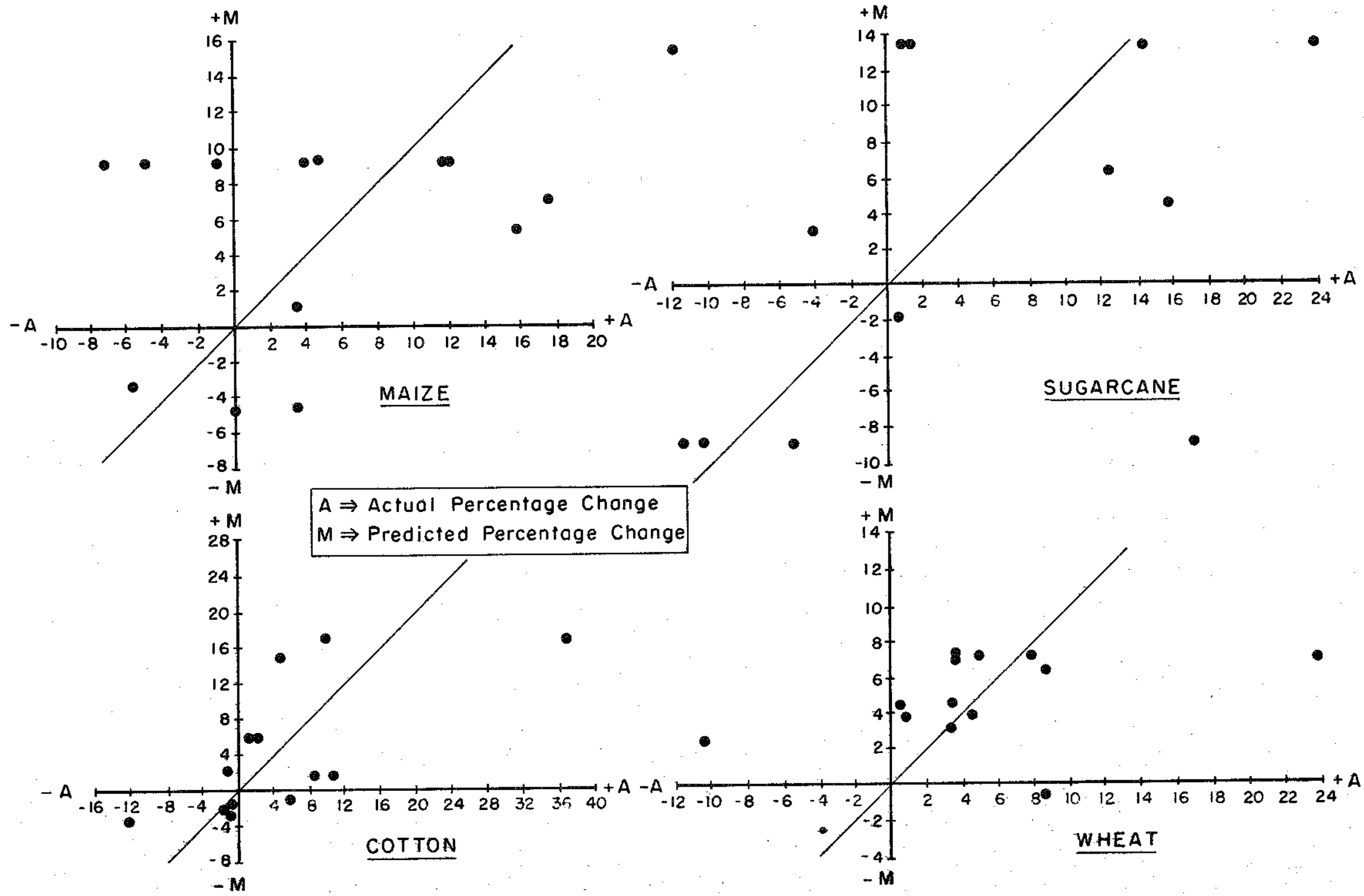


FIGURE 9 PREDICTION-REALIZATION DIAGRAMS FOR THE PRODUCTION OF DIFFERENT FARM COMMODITIES IN THE CLOSED SECTOR MODEL OF FARM DECISIONS



9.2 Prediction of Turning Points

This test gives some idea about the ability of the model in predicting the direction of change in actual production series. This seems to be an important test for short-term planning models. The prediction of turning points by the open and the closed sector models for different crops is reported in Table 2. The open and the closed sector models have predicted 65.38 percent and 76.9 percent of the turning points correctly, respectively. The closed sector model has performed slightly better.

9.3 Information-Inaccuracy Statistic

The information inaccuracy statistic was developed by Theil [1966, 1967] and it measures the amount of information contained in the actual proportions that is lost in the predicted proportions. The test can be represented as

$$(46) \qquad I_j(t) = \sum_{i=1}^{n} WA_{ij}(t) \log_e \left[\frac{WA_{ij}(t)}{WP_{ij}(t)} \right],$$

Where $I_j(t)$ is the information inaccuracy statistic for j^{th} season, $WA_{ij}(t)$ is the actual share of the i^{th} crop in the total acreage sown to all the crops in season j of t, and finally the $WP_{ij}(t)$ is the predicted share of i^{th} crop in the total acreage sown to all the crops in season j of t. In case

$$WA_{ij}(t) = WP_{ij}(t) \Rightarrow WA_{ij}(t)/WP_{ij}(t) = 1.$$

We know that $\log_e 1=0$. This implies that $I_j(t)=0$, i.e., no information is lost and the model results are able to track the actual history. This, however, is only a matter of coincidence. The information inaccuracy statistic for summer and winter crops for the open sector model is reported in Table 3 and for the closed sector model is reported in Table 4.

The information inaccuracy statistic allows us to take into account the relative importance of different crops in the cropping pattern of any cropping season. Based on an average over fifteen years, there is a slight indication that the open model has performed well in the winter season and the closed model has performed well in the summer season.

9.4 Theil's U-Statistic

This test has been developed by Theil [1966, 1967] and it measures

Table 2: Prediction of Turning Points (Percentage)

Crop	Open Model		Closed Model	
	Correct	Incorrect	Correct	Incorrect
Maize	46.15	53.85	69.2	30.8
Cotton	79.92	23.08	84.6	15.4
Sugarcane	61.54	38.46	69.2	30.8
Wheat	76.92	23.08	84.6	15.4
Average	65.38	34.62	76.9	23.1

the magnitude rather than the direction of change in model values compared to those in the actual values. It can be denoted as

$$(47) \qquad U^2 = \frac{\sum_{t=1}^{\tau} [P_i(t) - A_i(t)]^2}{\sum_{t=1}^{\tau} [A_i(t)]^2} ,$$

where $P_i(t) = M_i(t+1) - M_i(t)$ and $A_i(t) = V_i(t+1) - V_i(t)$. $M_i(t)$ and $V_i(t)$ are the model and actual values of production for the i^{th} crop in t. In case of perfect model predictions $P_i(t) = A_i(t)$, which implies that $U^2 = 0$. The U^2 calculated for the open and the closed sector model results for different crops is reported in Table 5. Again it indicates that the closed sector model has performed well over the open sector model.

Table 5: U-Statistic for Different Crops

Crop	Open Model	Closed Model
Maize	1.253	1.041
Cotton	0.489	0.467
Sugarcane	1.227	1.197
Wheat	0.642	0.610

10. EMPIRICAL RESULTS

The empirical results of the models are reported under two headings: (i) general results, and (ii) comparative dynamic results.

Table 3: Information Inaccuracy Statistic for Summer and Winter Crops in the Punjab: Open Sector Model

Year	Information Inaccuracy Statistic Summer	Winter	Average
1951-52	0.0005	0.0000	0.0002
1952-53	0.0541	0.0009	0.0275
1953-54	0.0038	0.0002	0.0020
1954-55	-0.0031	0.0002	0.0016
1955-56	0.0000	0.0009	0.0005
1956-57	0.0641	0.0014	0.0327
1957-58	0.0006	0.0041	0.0023
1958-59	0.0257	0.0087	0.0172
1959-60	0.0084	0.0002	0.0043
1960-61	0.0276	0.0014	0.0145
1961-62	0.0291	0.0068	0.0179
1962-63	0.0300	0.0000	0.0150
1963-64	0.0537	0.0004	0.0270
1964-65	0.0510	0.0006	0.0258
Total	0.3455	0.0258	0.1885
Average	0.0247	0.0018	0.0135

Table 4: Information Inaccuracy Statistic for Summer and Winter Crops in the Punjab: Closed Sector Model

Year	Information Inaccuracy Statistic Summer	Winter	Average
1951-52	0.0010	0.0009	0.00095
1952-53	0.0019	0.0007	0.0013
1953-54	0.0048	0.0072	0.0060
1954-55	0.0126	0.0022	0.0074
1955-56	0.0356	0.0105	0.0230
1956-57	0.0225	0.0236	0.0230
1957-58	0.0165	0.0138	0.0151
1958-59	0.0209	0.0173	0.0191
1959-60	0.0047	0.0049	0.0048
1960-61	0.0086	0.0044	0.0065
1961-62	0.0026	-0.0006	0.0016
1962-63	0.0061	0.0000	0.0030
1963-64	0.0152	-0.0007	0.0079
1964-65	0.0185	0.0137	0.0161
Total	0.1715	0.0992	0.13535
Average	0.0122	0.0071	0.0097

10.1 General Results

Some of the important results of the open and the closed sector models may now be summarized.

1. Although the predicted values are not extremely close to observed annual production and acreage figures, the models do reproduce the actual trends and general direction of past history as has been displayed in Figures 10 and 11 for the production of different crops. The predicted cyclical fluctuations in case of sugarcane are very much as observed.

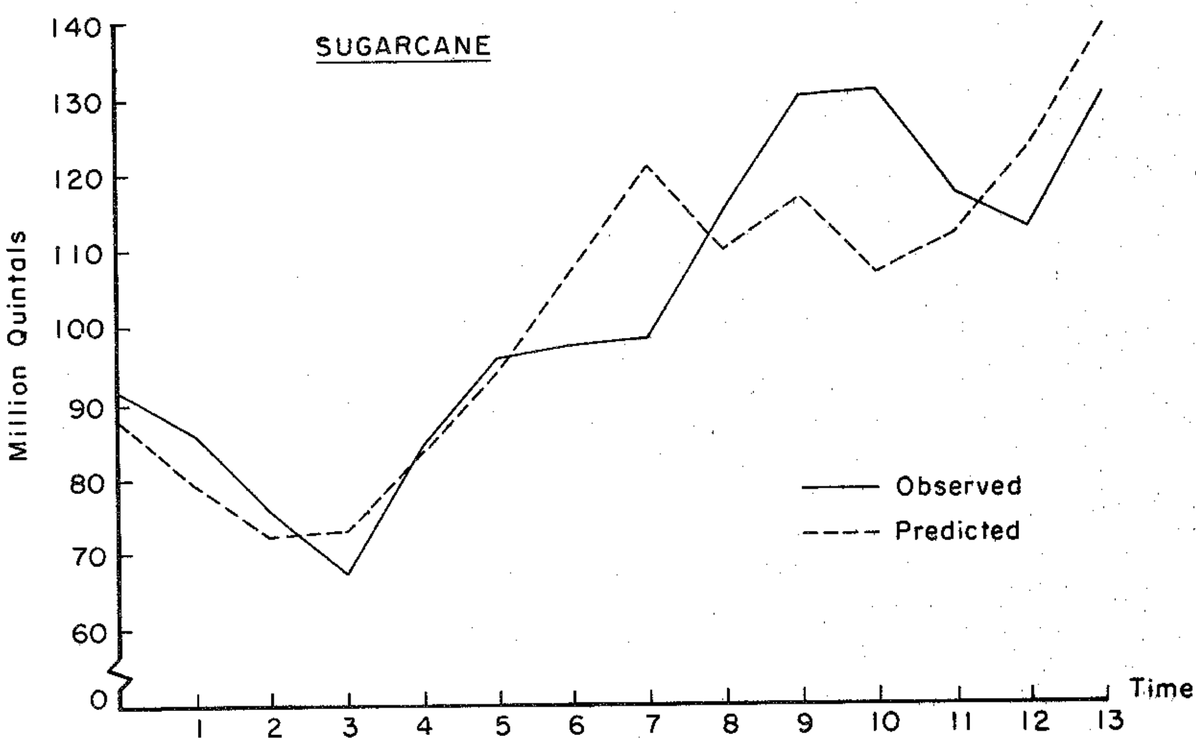
2. The demand for agricultural labor is highly seasonal. There does exist surplus labor during slack months but the contention that a part of the agricultural labor has zero marginal productivity throughout the year and hence can be pulled out to work in the industrial sector without reducing agricultural production is refutable. In some months the opportunity cost for labor is almost ten times the size of existing wage rates. This indicates that there do exist acute shortages of labor during certain months. The seasonal demand patterns of labor (family and outside) as determined by the open and the closed sector models are reported in Tables 6 and 7 respectively.

3. As the farm cash income goes up, farmers substitute gur with sugar for their household consumption. This confirms the empirical observation made by Engel about a century ago about the consumption behavior of households.

4. Growth in nonfarm demand for various farm commodities is crucial for the growth of the agricultural sector as a whole. In the absence of growing demand, farm prices gradually decline over time and lead to a drop in farm income levels. On the other hand, growing demand for farm products enhances growth, increases farm income and prevents bankruptcy.

5. Money capital has a crucial effect on agriculture production. Its availability helps not only in the choice of crops and growth of agricultural industry but in the survival of the agricultural sector as a whole. The role of money capital as displayed by these models is completely opposite to the assumptions made by the entire family of dualistic models which maintain that capital is not an important input for agricultural production.

FIGURE 10 PRODUCTION OF DIFFERENT CROPS IN THE PUNJAB-
OPEN SECTOR MODEL OF FARM DECISIONS



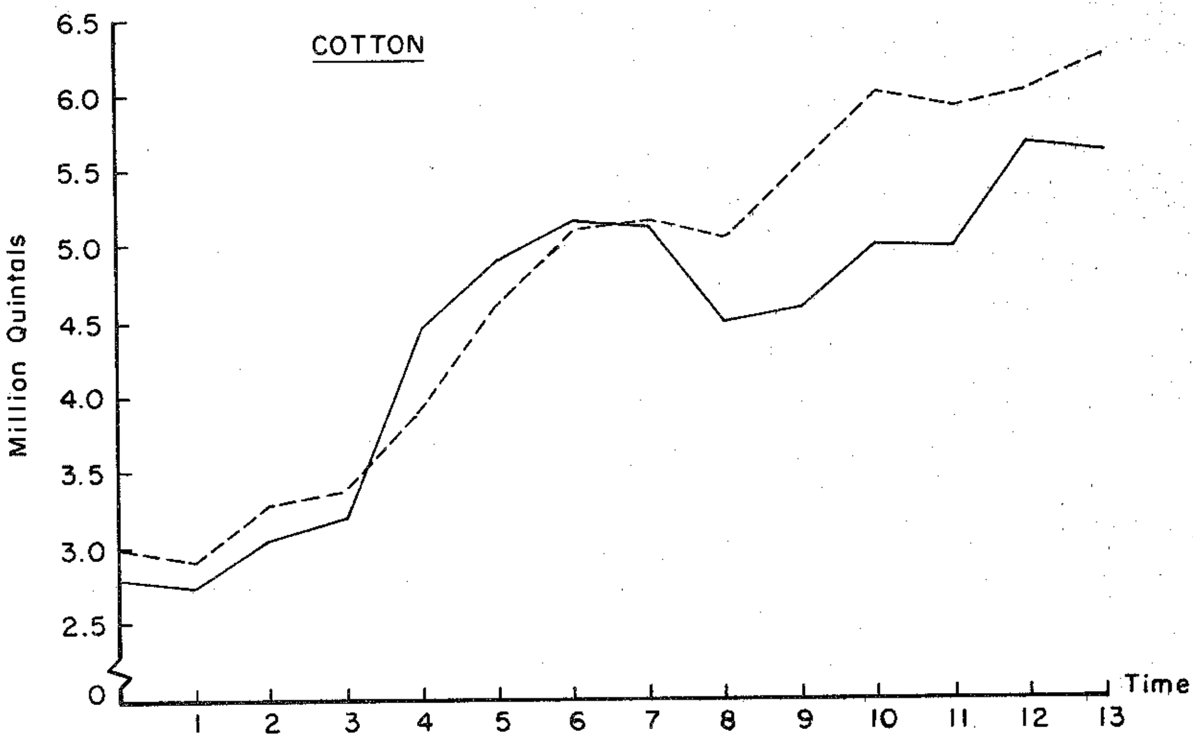


FIGURE 10. Cont.

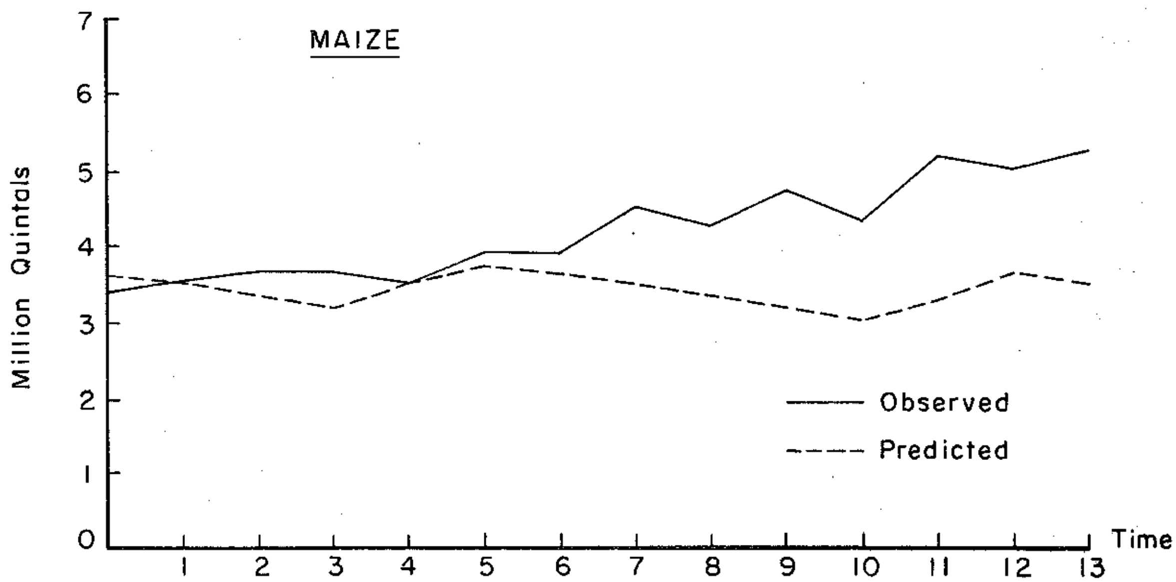
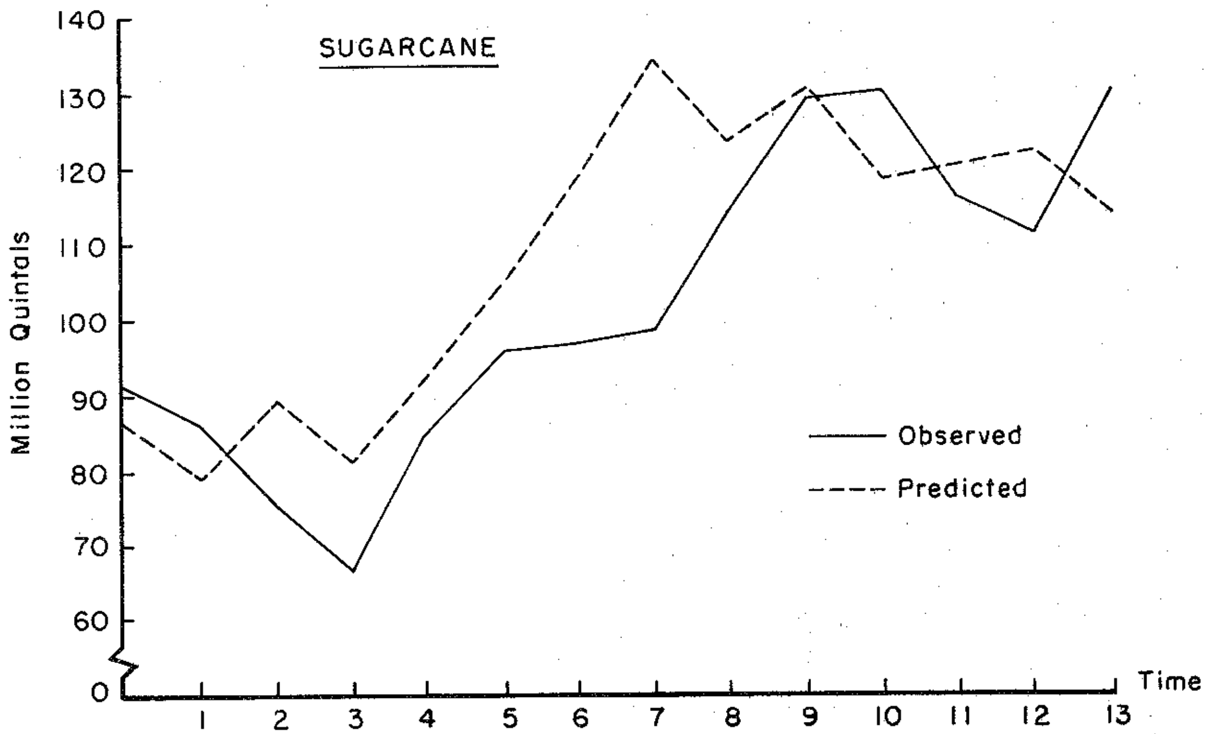


FIGURE 11 PRODUCTION OF DIFFERENT CROPS IN THE PUNJAB-CLOSED SECTOR MODEL OF FARM DECISIONS



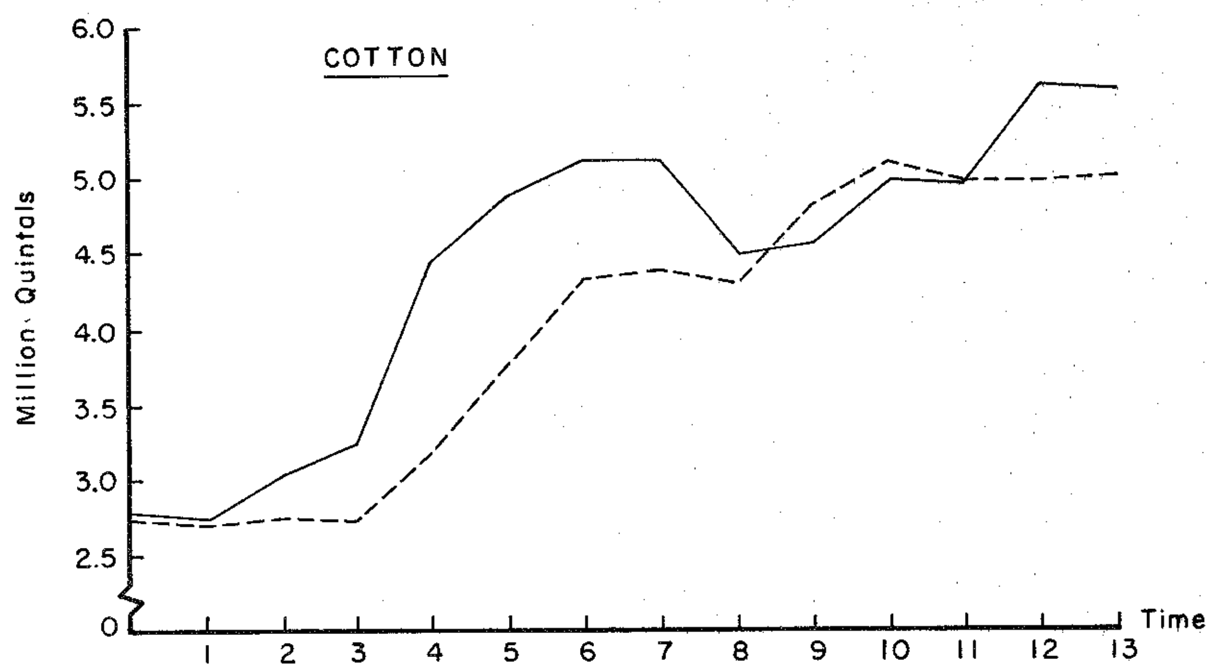
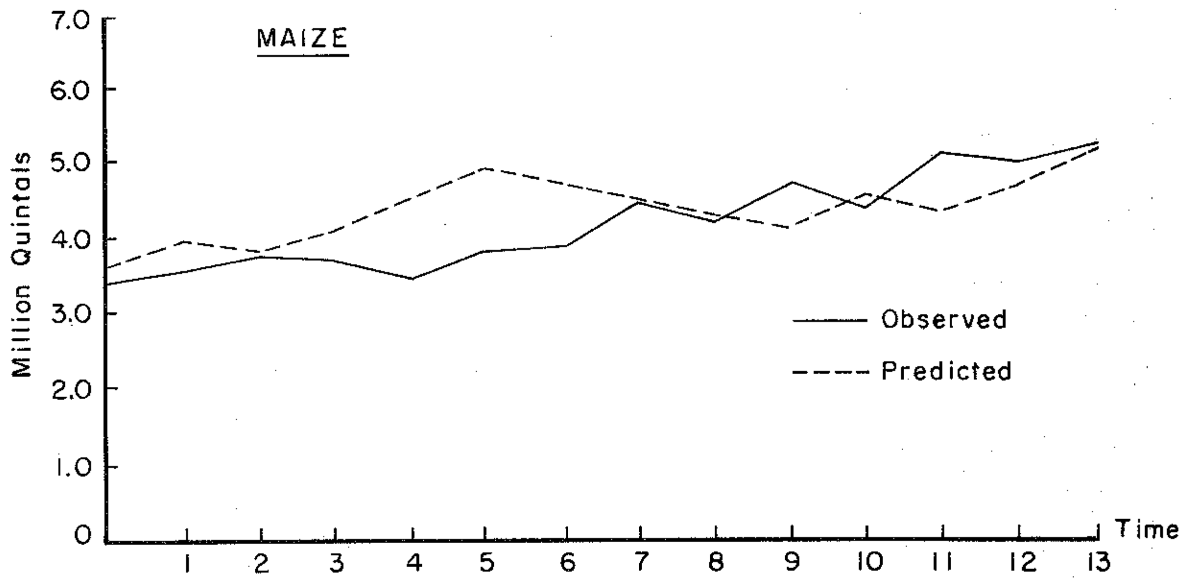


FIGURE 11 Cont.



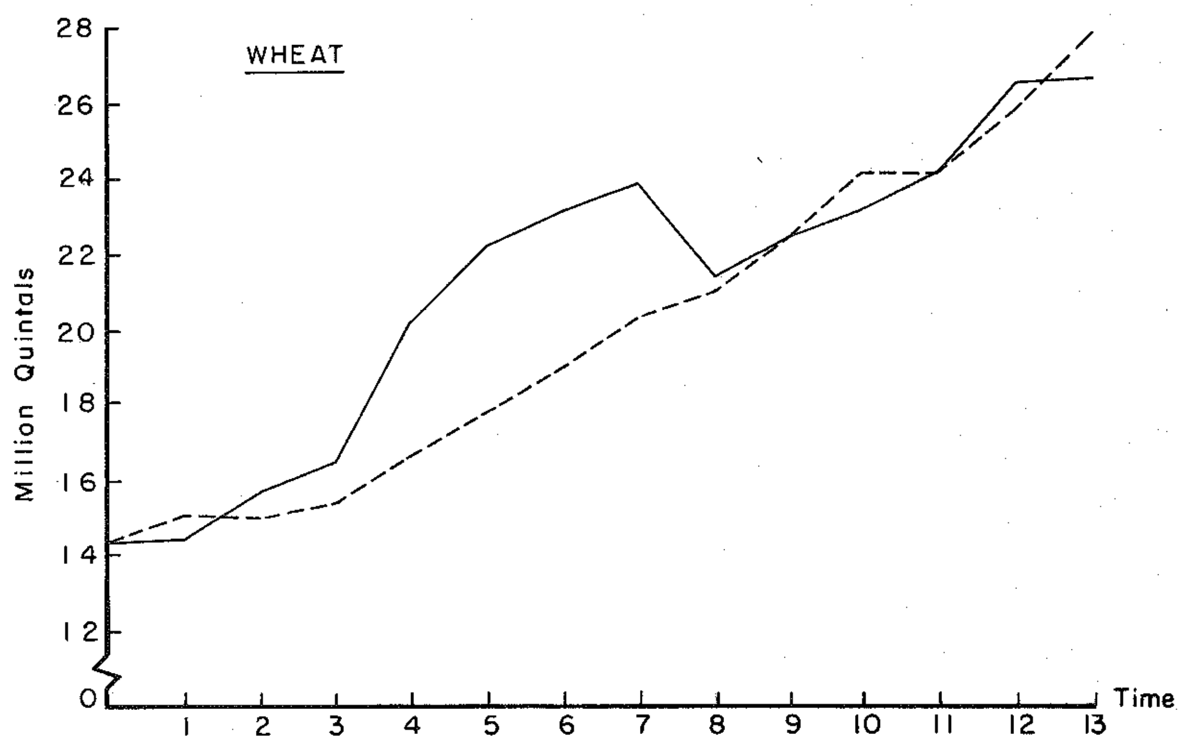


Table 6: Purchase of Labor from Outside at the Regional Level for the Punjab* in the Open Model of Farm Decisions (million man days)

Period	Year	January	February	March	April	May	July	October	November
0	1951-52	--	--	--	--	--	--	--	--
1	1952-53	--	--	--	--	--	--	--	--
2	1953-54	--	--	--	--	--	--	--	--
3	1954-55	--	--	--	--	--	1.164	1.164	--
4	1955-56	--	--	--	--	--	0.264	--	--
5	1956-57	--	--	--	--	--	1.002	0.660	--
6	1957-58	--	--	0.055	--	--	1.047	1.165	--
7	1958-59	--	0.584	2.378	--	0.774	1.479	1.819	--
8	1959-60	--	--	1.249	--	1.240	1.107	2.237	--
9	1960-61	--	--	0.421	--	0.701	0.251	2.399	--
10	1961-62	--	--	--	--	2.084	0.594	4.150	--
11	1962-63	--	--	1.294	1.555	3.991	2.550	5.998	1.564
12	1963-64	--	0.631	2.634	2.047	4.284	2.056	6.058	1.762
13	1964-65	1.838	2.550	4.348	1.916	4.196	3.073	5.775	1.037
14	1965-66	1.633	2.340	4.206	2.943	4.610	3.129	5.889	1.848
15	1966-67	--	0.590	2.801	3.848	4.793	2.709	5.786	2.605
16	1967-68	--	0.677	2.925	4.725	5.055	2.801	5.682	3.216
17	1968-69	1.154	1.870	3.987	5.582	5.372	3.251	5.582	3.742
18	1969-70	1.028	1.752	3.916	5.483	5.286	3.663	5.483	3.937
19	1970-71	--	--	1.644	5.386	5.065	3.403	5.386	4.410

______ ⇒ Exhaustion (full employment) of available hired agricultural labor.

*There is no demand for hired labor during June, August, September and December.

Table 7: Purchase of Labor from Outside at the Regional Level for the Punjab* in the Closed Sector Model of Farm Decisions (million man days)

Period	Year	January	February	March	April	May	July	October	November
0	1951-52	--	--	--	--	--	--	--	--
1	1952-53	--	--	--	--	--	--	--	--
2	1953-54	--	--	--	--	--	--	--	--
3	1954-55	--	--	0.640	0.201	0.950	1.807	1.242	--
4	1955-56	--	--	--	--	--	0.887	--	--
5	1956-57	--	--	0.256	--	--	1.590	0.289	--
6	1957-58	--	--	1.512	--	0.178	1.798	1.238	--
7	1958-59	1.901	2.591	4.213	--	1.366	2.212	1.993	--
8	1959-60	0.551	1.180	2.975	0.286	1.867	1.798	2.581	--
9	1960-61	--	0.359	2.434	--	1.443	0.921	2.401	--
10	1961-62	--	--	0.714	0.280	2.695	1.567	4.366	--
11	1962-63	0.072	0.690	2.633	2.484	4.457	2.615	5.998	0.778
12	1963-64	--	0.320	2.409	3.126	4.706	2.576	6.058	1.169
13	1964-65	--	--	0.397	3.254	4.650	2.080	6.087	1.468
14	1965-66	--	0.469	2.650	4.189	5.016	2.936	5.889	1.905
15	1966-67	--	--	2.301	5.114	5.269	2.649	5.786	2.388
16	1967-68	--	--	1.467	5.682	5.367	2.646	5.682	2.977
17	1968-69	--	0.690	2.956	5.582	5.379	3.570	5.582	3.129
18	1969-70	--	--	0.776	5.483	5.153	2.478	5.483	2.861
19	1970-71	--	0.712	3.033	5.386	5.216	3.847	5.386	2.944

_______ => exhaustion (full employment) of total supply of outside labor.

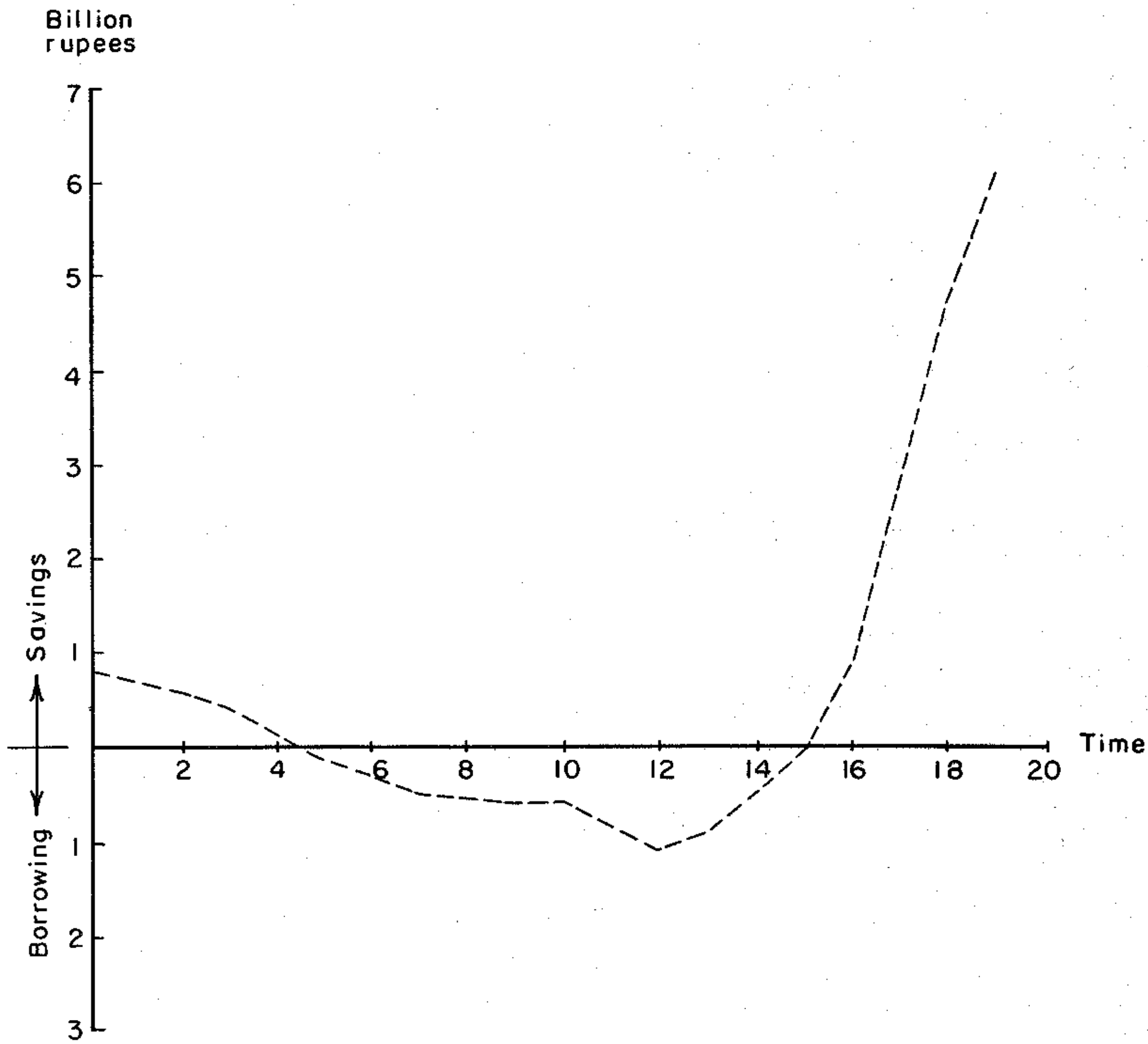
*There is no demand for hired labor during June, August, September and December.

6. The results of borrowings and savings are really interesting. The total amount of money capital the farmers can borrow each year depends partly on their own income which in turn is determined by prices, yields per acre, cash consumption expenditure etc. The cash consumption expenditure and production expenses were gradually increasing whereas farm prices were declining in the fifties. As a result, the total income was declining, forcing farmers to borrow to meet the financial obligations of the firm and the household. As determined by the open sector model, farmers start borrowing in 1956-57 and keep borrowing until 1966-67 and never borrow thereafter. These results seem to be more than a mere coincidence with the real situation in Punjab and in India. The Indian government realized the deteriorating financial conditions of farmers in the early fifties and set up a commission to study their plight. The commission came up with a report called "All India Rural Credit Survey Report" in 1956. Based on the recommendations of the commission the Indian government began advancing loans to farmers on a large-scale through both private and public rural credit institutions. The demand for short-term credit in the late sixties might change, however, once we incorporate investment activities explicitly. The total borrowings and saving for the Punjab as determined by the open sector model is displayed in Figure 12.

7. Land is not always fully used and its use varies from one season to another. In the middle fifties, the money capital and borrowings were not enough to plant crops in all of the land. On the other hand, in the late sixties the availability of both family and hired labor becomes an effective constraint and all of the available land is not fully utilized. However, when prices are favorable and money capital or labor are not effective constraints, the opportunity cost of land is a lot more than the existing land rent.

8. The <u>ex ante</u> optimal value of the objective function (in both the open and the closed sector models) is rarely equal to its <u>ex post</u> realized level. The <u>ex ante</u> levels are over estimated when farm prices decline over time and vice versa. This implies that farmers choose that cropping pattern which <u>ex ante</u> maximizes their objective but unlike in dynamic programming they rarely achieve it. This reflects the importance of the interrelated elements influencing the farm decision environment through feedback effects.

FIGURE 12 BORROWING AND SAVING BEHAVIOUR OF FARMERS IN THE OPEN SECTOR MODEL OF FARM DECISIONS (1951-52 to 1970-71)



10.2 Comparative Dynamic Results

The comparative dynamic results involve simulating the basic open and the closed sector models under alternative assumptions and policy programs. Some of the interesting results are summarized below.

1. One of the important features of the recursive programming models is the use of flexibility constraints to account for uncertain environment. The use of these constraints has received lots of criticism. The basis of the criticism is that the solution is always determined by the flexibility constraints. In other words, the solution is always equal to either upper or lower bounds of these constraints. As has been displayed in Table 8, this is true mainly in the case of maize. In all other crops these constraints are rarely effective. Even in the case of maize these bounds become ineffective when the zone of flexibility of these constraints is widened.

2. The use of two period distributed lag structure in price expectation functions tends to diminish oscillations in the acreage and production of various crops and hence stabilizes the cyclical phenomena.

3. The sugarcane processing capacity tends to be fully utilized even when it is arbitrarily increased by three times the existing actual capacity. This indicates (i) the selling of sugarcane is more profitable than processing it into gur, and (ii) the Punjab may have a shortage of sugarcane processing capacity. This confirms my own observations of (i) watching very long lines of trucks and bullock carts full of sugarcane often waiting the whole day and night to be unloaded, and (ii) watching farmers pull strings on (or even bribe) the administrative bureaucrats to obtain permission to sell their sugarcane to sugarmills.

4. The small changes in the interest rates have only insignificant effects on savings, borrowings and on the cropping pattern. However, the models seem to be more sensitive to changes in interest rate on loans than on savings implying high marginal utility derived from consumption expenditures.

5. The models go infeasible due to shortage of money capital when maximum credit limits on cash income and equity are reduced. However, an increase in these limits has only an insignificant effect on their savings, borrowings and cropping pattern. This, of course, may not hold

Table 8: Effectiveness of Flexibility Constraints for Different Farm Enterprises Under Alternative Assumptions in the Open Sector Model

		A								B							
		Maize		Cotton		Sugarcane		Wheat		Maize		Cotton		Sugarcane		Wheat	
Period	Year	U	L	U	L	U	L	U	L	U	L	U	L	U	L	U	L
0	1951-52		N		N		N		N		N		N		N		N
1	1952-53	N		N		N		N		N		N		N		N	
2	1953-54	N		N	N	N		N	N	N	N	N	N	N	N	N	N
3	1954-55	N		N	N	N	N	N	N	N	N	N	N		N	N	N
4	1955-56		N		N		N		N	N	N	N	N	N	N		N
5	1956-57		N		N		N		N	N	N	N	N	N		N	N
6	1957-58	N		N	N		N		N	N	N	N	N	N	N	N	N
7	1958-59	N		N	N		N		N	N	N	N	N	N	N	N	N
8	1959-60	N	N	N		N		N	N	N	N	N		N	N	N	N
9	1960-61	N		N	N	N	N		N	N		N	N		N	N	N
10	1961-62	N		N	N	N		N	N	N		N	N	N	N	N	N
11	1962-63		N	N	N	N	N	N	N	N	N	N			N	N	N
12	1963-64		N	N	N	N	N	N	N	N	N	N	N	N	N	N	N
13	1964-65	N		N	N	N	N	N	N	N		N	N	N	N	N	N
14	1965-66		N	N	N	N	N	N	N		N	N	N	N	N	N	N
15	1966-67		N	N	N	N	N		N	N	N	N	N	N	N	N	N
16	1967-68		N	N	N	N	N	N	N	N	N	N	N	N	N	N	N
17	1968-69		N	N	N	N	N	N	N	N	N	N	N	N	N	N	N
18	1969-70		N	N	N	N	N	N	N	N	N	N	N	N	N	N	N
19	1970-71		N	N	N	N		N	N	N	N	N	N	N	N	N	N

N ⟹ the constraint is not binding or effective

U ⟹ upper flexibility constraints

L ⟹ lower flexibility constraints

A ⟹ actual flexibility coefficients

B ⟹ three times the size of the actual flexibility coefficients true if we introduce investments in new technology and determine all the cash consumption requirements endogenously.

11. CONCLUSIONS AND POLICY IMPLICATIONS

Even these simple models lead to very important policy implications. Some of these are discussed below:

1. There exists seasonal unemployment in the rural sector. In order to provide employment to farm labor during the slack season, the establishment of agro-industries, public works programs in the rural sector and the creation of infrastructure in the rural areas is suggested. This not only would provide employment in the slack season but would also stimulate the demand for agricultural output and reduce marketing, transportation and storage costs of farm commodities. However, one can also think of mechanizing some of the farm operations during peak labor demand months and hence reduce the demand for labor during those months. By doing so we can pull some labor from the rural sector and provide employment by creating more employment opportunities in the urban-industrial sector.

2. The availability of money capital is too crucial for the survival and growth of the agricultural sector. The stories of little or no use of money capital in the agricultural sector of LDC's lead essentially to false assumptions. Government should provide timely and cheaper loans to farmers and encourage the establishment of private lending institutions, including cooperatives, in the rural sector. These institutions should be registered and their accounts should be audited annually by government auditors in order to make sure that they don't cheat farmers by imposing on them their own credit terms and charging unduly high interest rates.

3. Punjab being a surplus state in the supply of foodgrains, the government should provide genuine and effective price supports. Since government (Food Cooperation of India) is the major buyer of foodgrains at fixed procurement prices, these procurement prices should be announced before farmers make their decisions about acreages under various crops. The higher procurement prices or price supports for foodgrains might even provide an incentive to farmers to produce more foodgrains

than commercial crops and hence might help ease the food problem faced by some other States in India.

4. The results indicate that there exists a shortage of sugarcane processing capacity and an increased demand for sugar consumption in the Punjab state. This suggests that government should subsidize or encourage the establishment of more sugar mills to meet the demand for increased supply of sugarcane for processing, thereby meeting the increased demand for sugar consumption.

5. The production of various crops can be stabilized if farmers are made aware of past harvest prices and currently declared procurement and support prices for different crops. This can be accomplished through agricultural extension agencies, marketing intelligence services and radio announcements of price information before farmers make the sowing decisions for various crops.

6. The model results show that nonfarm demand for farm products plays a crucial role in the overall growth of the economy. Consequently, the government policies should be geared to create more and more nonfarm demand in order to absorb the increased supply of farm commodities.

7. As the sugar-gur consumption substitution process has shown, the consumption patterns of farmers are changing in favor of industrially-produced consumer goods. As a result, an effort should be made to increase the supply of these goods in order to meet the increased consumer demand.

8. A part of land is left unused when some of the physical resources are scarce. This can be avoided by making these resources available to farmers by the government--for example, by advancing more loans to farmers under long-term repayment plans. This is very important in those LDC's where density of population is high and population is growing at a faster rate than the supply of foodgrains, with the exception of the last few years.

12. PERFORMANCE COMPARISON BETWEEN THE OPEN AND THE CLOSED SECTOR MODELS

The main difference between the open and the closed sector models is the way by which the product prices are determined. In the open sector model product prices are given exogenously. However, in the closed sector model product prices are determined endogenously by a temporary equilibrium between farm supply and nonfarm demand in the agricultural market which is assumed to be perfectly competitive.

From analytical point of view the closed sector model is an improvement over the open sector model. This has been clearly displayed in the preceding sections. From empirical point of view there is a slight indication that the closed sector model has performed well in predicting crop acreages as compared to the open sector model. This could be due to its ability to capture the realistic price expectation behavior of farmers through feedback effects. However, due to slight differences in the data base and based on limited experimentation we are hesitant to conclude that the closed sector model is better suited to describe the process of agricultural development, as compared to the open sector model. We need to perform more simulation experiments in order to arrive at more definitive conclusions.

One of the main objectives of this paper was to develop a generalized cobweb model of agricultural development and to demonstrate whether it can be operationalized from the empirical point of view or not. This objective has been achieved with a fair amount of success. However, its ability to describe the process of agricultural development can be further improved by using more realistic inverse demand functions, introducing technological change and extending the model in some other important aspects. However, some of the theoretical extensions have already been developed and are reported below.

13. WORK IN PROGRESS

The contents of this paper are only an introduction to a more detailed 'recursive decision and risk programming' (RDRP) model of Punjab agriculture which is currently in progress. The most glaring weakness of the open and the closed sector models developed here is the absence of technological change. This was only a simplification. Furthermore, I have a strong suspicion that the inclusion of the technological change will not alter the conclusions and may even reinforce some of the arguments arrived in the paper. However, recursive programming is very flexible and highly suited to handle new technology and has been demonstrated by Day [1963] , Day-Singh [1971] , Heidhues [1965], Schaller-Dean [1965] and Singh [1971] in their empirical studies of the agricultural sector.

The open and the closed sector models have already been modified and extended theoretically in Mudahar [1972] to incorporate (i) short, medium and long term financial activities and constraints; (ii) a correspondence between alternative financial and investment activities and constraints; (iii) adoption and investment in new farm machinery; (iv) technological change; and (v) risk, uncertainty and alternative theories of choice. By incorporating these extensions, we obtain a complete theoretical and operational model of agricultural development. Currently, efforts are underway to estimate this larger model and then use it for making projections.

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