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Spatial Arrangements of Externality Generating and Receiving Activities

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Abstract

Many cases of externalities in agricultural production such as pesticide drift, cross-pollination, and offensive odors are attributable to the incompatibility of neighboring land uses. This paper offers an examination of when an efficient activity arrangement is compatible with free-market incentives. Also, free-market and socially efficient activity arrangements are characterized in terms of spatial concentration of the externality generating uses.

Keywords: externality, graph partitioning, land-use arrangement, spatial concentration.

SPATIAL ARRANGEMENTS OF EXTERNALITY GENERATING AND RECEIVING ACTIVITIES

Introduction

In recent years, agricultural markets analysts have paid increasingly more attention to the spatial concentration of production in both animal and crop agriculture. In particular, the geographic concentration of production of main field crops in several growing regions is a distinctive feature of the U.S. agricultural landscape. Geographic production patterns are shaped by a host of factors, including soil qualities, proximity to input markets, vertical integration, farm size, and the marketing environment. In this paper I will focus on another essential feature of the grower's decision environment: the presence of spatial externalities due to multiple land uses.

A number of externalities in agricultural production, such as pesticide drift, cross-pollination, invasion by foreign species and predators, offensive odors, industrial emissions and pollution, are well documented. For example, the negative impacts of livestock feeding operations on the values of residential properties located in the close vicinity are studied in Herriges, Secchi, and Babcock 2003. Damage to cotton due to the herbicide applied on rice planted in the surrounding area and damage to an olive crop produced in the vicinity of cotton are two examples of externalities in crop agriculture (Parker 2000). Another common occurrence of spatial externality is the danger of cross-pollination impairing the quality of the crop delivered by seed growers (Perkins 2003). The same is true for the potential contamination between non-genetically modified and genetically modified crop varieties (Belcher, Nolan, and Phillips 2003; Munro n.d.; Brasher 2003). For example, certified organic crop production may entail buffer zones with sizes varying depending on the types of crops grown on adjacent farms.

The damage from many types of externalities is frequently attributable to the incompatibility of neighboring land uses and declines with the distance between the externality generator and recipient (e.g., see Baumol and Oates 1998; Helfand and Rubin

1994; Albers 1996; Parker 2000). This implies that concentrating externality generators in a certain area may decrease the total cost imposed on the recipients. The concentration of generators has two counteracting effects on the distribution of damages across the recipients. On the one hand, the total number of immediate "border" neighbors with incompatible uses decreases. On the other hand, the "bordering" recipients are located next to a greater number of generators. The result is a more dispersed distribution of externality damages among the recipients. However, if the damage to a recipient from multiple externality generators accumulates quickly, a socially efficient arrangement of land uses may lead to spreading of the generators.[1] This is because a lower level of spatial concentration of generators spreads out the externality damages more evenly across the recipients.

Agricultural policymakers have developed a number of intervention tools designed to improve the efficiency of land-use arrangements, such as zoning orders, emission regulations, size restrictions, buffer zones, various environmental standards, and other types of legislation. In this light, it is interesting to examine when a given socially efficient land-use arrangement can be implemented through competitive markets by assigning the land uses (e.g., zoning orders) without any further regulation. The question of the choice of a policy instrument to correct for externalities when the damages must be spread out or concentrated in certain areas of the region is studied in Helfand and Rubin 1994; Baumel and Oates 1998; and Dosi and Tomasi 1994 in different settings.[2]

The goal of this paper is twofold. First, free-market and socially efficient land use arrangements are characterized in terms of spatial concentration of the externality generating uses. Second, a policy perspective is taken and production environments are found such that the efficient arrangements are implementable in a free-market setting. The two issues are related because the alignment of the free-market incentives with the efficiency considerations depends, in part, on the efficient level of generator concentration. While the implementability alone does not guarantee that the efficient arrangement will be realized in the free-market setting, it guarantees that, once the efficient arrangement is achieved, no further policy measures need to be taken.

The rest of the paper is organized as follows. First, a model is developed, and the conditions needed for the existence of equilibrium in the free-market setting are established. Then, geometric configuration patterns of the free-market arrangements are studied under different conditions on the externality dissipation across farms in the region. After that, the efficient arrangement that minimizes the total loss due to externality damages imposed on the recipients for a given amount of output is investigated in several special cases. The level of generator concentration is characterized, and a determination is made as to whether the efficient arrangement is implementable in the free-market setting. This analysis is followed by an inquiry into the effects of the rate of externality accumulation on the efficient arrangement, and concluding remarks.

Model

Let $N = \{1,...,n\}$ denote the set of farms (convex and non-overlapping plots of land) or agents located in a region described by a square $n \times n$ matrix $D^f = \{d_{ij}^f\}$, where $d_{ij}^f \in \{1,2,...,\overline{\rho}\}$ is the distance between farms i and j, $i \neq j$, with $d_{ii}^f = 0$, for $i, j \in N$.[3] Each agent operates one farm, all farms are of the same size, which is normalized to one, and each farm produces a unit of one of the two crops: the externality "recipient" crop, r, or the externality "generator" crop, g.[4] Farm i that produces the generator crop imposes a negative externality on all farms $j \neq i$ if they produce the recipient crop, and the damages decrease with the distance between the farms. At the farm level, the damage imposed by the generator j on the recipient i is given by $d_{ij} = \gamma(d_{ij}^f)$, where $\gamma(.) > 0$ is the externality dissipation function. It is a decreasing function of the ("geographic," "agronomic," or "economic") distance between the farm plots.[5] Let $D = \{d_{ij}\}$ denote the matrix of the potential individual externality damages between any two farms in the region with distance matrix D^f. The per acre cost of producing the externality receiving crop on farm i is then given by $c_i = C(\sum_{j=1}^{n} d_{ij}e_j - d_{ii}e_i)$, where e_i denotes the type of crop produced on farm i: $e_i = 1$ for the generator crop, and $e_i = 0$ for the recipient crop. The function $C(.)$ is (strictly) increasing, and reflects the rate of accumulation of the externality damage on an individual farm.

The per acre values of the two crops gross of all production costs except for the externality damages are v^r and v^g. These values are certain and common for all farms.[6] The premium net of the externality damage decreases with the output of the recipient crop (increases with the output of the generating crop): $v^r - v^g = v(s)$, where $s = \sum_{i=1}^{n} e_i$ is the amount of the generating crop. To assure that there is an incentive to produce both crops, we hold that $v(n) > \sum_{j=1}^{n} d_{ij} - d_{ii}$ for some i, and $v(0) < 0$.

Free-Market Equilibrium

In a free-market setting, farmers make their production decisions (choose the best response) in accordance with

$$\Pi(i) = \max[v^r - c_i, v^g], \tag{1}$$

so that $e_i^* = 1_{v^r - v^g - c_i \leq 0}$, where $1_{x \leq 0}$ is an indicator function. Therefore, in a pure strategy Nash equilibrium (PSNE), a set of farms that generates the externality is given by

$$G^* = \{i : v^r - v^g - c_i \leq 0\}, \tag{2}$$

where $c_i = C(\sum_{j=1}^{n} d_{ij} e_j^* - d_{ii} e_i^*)$, $v^r - v^g = v(n - \sum_{j=1}^{n} e_j^*)$, $e_j^* = 1$ if $j \in G^*$, and $e_j^* = 0$ otherwise. In general, one cannot guarantee the existence or uniqueness of the equilibrium set G^* defined by equation (2). The Nash equilibria in pure strategies exist if the (potential) costs imposed by generators on each other are sufficiently high compared with the costs imposed on any recipient. The spatial characteristics of the region must allow for a land-use arrangement with some degree of the concentration of generators.

Formally, this can be stated as follows. The existence of the PSNE is equivalent to the existence of a number $s^* \in \{2, ..., n-1\}$ and a permutation of farm indices $i \to \pi(i)$ such that $\sum_{j=1}^{s} d_{\pi(l),\pi(j)} - d_{\pi(l),\pi(l)} \geq \sum_{j=1}^{s} d_{\pi(h),\pi(j)}$ for any $1 \leq l \leq s^*$, $s^* + 1 \leq h \leq n$, and $v(s^*) \in [\underline{c}, \overline{c}]$, where $\underline{c} = \max_{i = s^*+1,..,n} C(\sum_{j=1}^{s^*} d_{\pi(i),\pi(j)})$ and $\overline{c} = \min_{i=1,..,s^*} C(\sum_{j=1}^{s^*} d_{\pi(i),\pi(j)} - d_{\pi(i),\pi(i)})$. Then the PSNE land-use arrangement is $e_{\pi(i)}^* = 1_{i \leq s^*}$. The required condition on the externality impacts across farms asserts that the assigned land uses are, in fact, the best responses for each farm. For example, for a given number of generators s, the PSNE exists if there is a permutation of the impact matrix D such that there is an s by $n-s$ zero submatrix below the main diagonal. Next, we characterize a region (impact matrix) where the PSNE with a certain arrangement of generators always exists, and furthermore, the number of generators is unique (possibly up to a scale parameter). (Proofs of results are available in the Appendix.)

RESULT 1. Suppose that $\sum_{j=1}^{s} d_{s+1,j} \le \sum_{j=1}^{s-1} d_{s,j}$ for all $s = 2,...,n-1$, and $d_{21} \le d_{12}$, Then there exists the PSNE with $e_i^* = 1_{i \le s^*}$, and s^* is uniquely determined by $v(s^*;\beta) = \beta\ \hat{v}(s^*) \in [\underline{c}, \bar{c}]$ for some scale parameter $\beta > 0$, $\underline{c} = C(\sum_{j=1}^{s^*} d_{s^*+1,j})$, and $\bar{c} = C(\sum_{j=1}^{s^*-1} d_{s^*,j})$.

The characteristics of the region's impact matrix guarantee that in any arrangement where the first s farms are generators and the last $n-s$ farms are recipients, the potential damage borne by each generator exceeds the actual damage borne by each recipient. This is illustrated using two impact matrices with a simple algebraic structure that will be frequently employed throughout the paper.

EXAMPLE 1. (Sum Impact Matrix) Let $d_{ij} = a_i + b_j$ for $i, j \in N$. Here the externality damage received by farm i from the externality generating farm j is the sum of the recipient's susceptibility, a_i, and the generator's intensity, b_j. The condition guaranteeing the existence of the PSNE becomes $\sum_{j=1}^{s}(a_{s+1} + b_j) \le \sum_{j=1}^{s-1}(a_s + b_j)$, or $a_s - a_{s+1} \ge (a_s + b_s)/s$ for all $s = 2,...,n-1$. That is, the susceptibility of each generator is larger than that of each recipient. Furthermore, the magnitude of the difference in susceptibilities increases with both the generator's susceptibility and intensity and decreases with the number of the operating generators. For example, let $a_i = b_i$ for all

$i \in N$. Then the condition becomes $(1-2/s)a_s \geq a_{s+1}$, that is, the region must be such that the farm "size" decreases with the farm index, and the magnitude of the fall in size decreases with the number of generators.[7]

EXAMPLE 2. (Product impact matrix) Let $d_{ij} = a_i b_j$ for $i, j \in N$. In this case, the externality damage received by farm i from the externality generating farm j is the product of the recipient susceptibility, a_i, and the generator intensity, b_j. The condition guaranteeing the existence of the PSNE becomes $\sum_{j=1}^{s} a_{s+1} b_j \leq \sum_{j=1}^{s-1} a_s b_j$, or a_{s+1} / a_s $\leq (1 - b_s / \sum_{j=1}^{s} b_j) < 1$ for all $s = 2, ..., n-1$. As before, the susceptibility of each generator is larger than that of each recipient. Furthermore, the magnitude of the fall in the susceptibility for two farms with consecutive indices, i and $i+1$, increases with the i's generating intensity and decreases with the generating intensities of farms with smaller indices.

Regions Where the PSNE Fails to Exist

In general, equilibrium with a strictly positive number of both recipients and generators may fail to exist for two distinct reasons. First, while the spatial distribution of costs is compatible with the PSNE, the price premium may not be (quantitative failure). Second, for any price premium, the spatial distribution of costs may not be compatible with the PSNE. In the former case, a tax or subsidy scheme common to all growers resolves the problem. In the latter case, the failure is more fundamental because any intervention needs to be heterogeneous at the grower level. For example, for any price premium $v(s)$, no equilibrium in pure strategies exists for the region with the impact matrix $d_{ij} = 1$ for $i \neq j$ and $d_{ii} = 0$, as depicted in Figure 1 for $n = 4$. Informally, the region must be sufficiently big so that the externality damages vary within the region, which may allow the "neighborhoods" of generators to form.

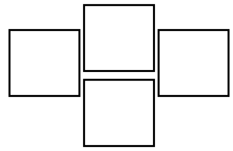

FIGURE 1. A "small" region where no PSNE exists

Basically, the equilibrium does not exist if it is impossible to form a configuration where farms of one type bordering the farms of another type have no incentive to switch. Next, we characterize one class of distance matrices that possess this property.[8]

RESULT 2. Suppose that $d_{ij}^f = |i - j|$ for all $i, j \in N$. Then there is no PSNE for any price premium $v(s)$ and externality dissipation function $\gamma(\rho)$.

The PSNE fails to exist because all ρ-distant neighbors of each farm are either (ρ-1)- or (ρ+1)-distant neighbors of an immediate neighbor of that farm. Furthermore, there is at most one ρ-distant neighbor of each farm that is also a (ρ+1)-distant neighbor of an immediate neighbor of that farm. Therefore, if two immediate neighbors have incompatible land uses, the externality damage to the recipient will exceed that of the generator. This is because the recipient always has at least one unit of damage more than the neighboring generator. For example, a region consisting of square lots located on a one-lot-wide strip of land, as in Figure 2, satisfies the condition in the result.

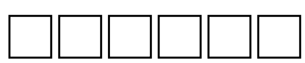

FIGURE 2. A "stretched" region where no PSNE exists

On the other hand, if the PSNE exists, there are areas with a high concentration of generators and areas with a high concentration of recipients because of the inverse relationship between the distance and the externality impact. Let $M_i(\rho) = \{j : d_{ij}^f = \rho\}$ denote the set of neighbors of farm i located at distance ρ. Suppose that the externality is local in the sense that $\gamma(\rho) = 0$ for all $\rho > \hat{\rho}$. Then, informally, in the PSNE generators are concentrated (agglomerated) in groups of "size" ρ because from $c_l > c_h$ for $l \in G^*$ and $h \notin G^*$ it follows that $\sum_{\rho=1}^{\hat{\rho}} \gamma(\rho)[\sum_{i \in M_l(\rho)} e_i^* - \sum_{i \in M_h(\rho)} e_i^*] \geq 0$. This implies that each generator has more ρ-distant neighbors than any recipient for some radius $\rho \in [1, \hat{\rho}]$, $\sum_{i \in M_l(\rho)} e_i^* \geq \sum_{i \in M_h(\rho)} e_i^*$. That is, generators are not "too" spatially dispersed and are, at least to some extent, clustered together in order to absorb more externality damage than the surrounding recipients. In this light, it is interesting to examine the geographical patterns of the PSNE land-use arrangements for different distance matrices, D^f, and for the externality dissipation functions, $\gamma(\rho)$.

Properties of Free-Market Arrangements When the Dissipation Function Is Concave

In this section, some geometric characterizations of equilibrium farm configurations are provided. To this end, let $U(i, j, \rho) = \{t : d_{ti}^f = d_{tj}^f + \rho\}$ denote the set of farms closer to j than to i by ρ (closer to i than to j by $-\rho$ if ρ is negative, or equi-distanced from j and i if ρ is zero). We hold that the distance matrix satisfies the condition that $\cup_{\rho=-1,0,1} U(i, j, \rho) = N$ for any i, j with $d_{ij}^f = 1$.

DEFINITION 1. A set L is said to be locally agglomerated if for any $i, j \in L$ and $h \notin L$ such that $d_{ih}^f = d_{jh}^f = 1$ it is true that $U(h,i,1) \cap [U(h, j,1) \cup U(h, j,0)] \neq \varnothing$ or $U(h, j,1)$ $\cap [U(h,i,1) \cup U(h,i,0)] \neq \varnothing$.

Consider a set of farms L, and an "outsider" farm (not in the set) that is located immediately next to two "insiders" in the set. The set L is locally agglomerated in the sense that the edge of the set is "somewhat" rounded toward the outside. That is, it is impossible to have a recipient that is surrounded by generators from the opposite sides, $U(h,i,1) \subseteq U(j,h,1)$ and $U(h, j,1) \subseteq U(i,h,1)$, and all three lie on the straight line. This is because at least some members of the set are closer to the "edge" members than to the outsider. Next, we ascertain that the equilibrium farm configuration possesses this weak property when the externality dissipates at an increasing rate as distance increases.[9]

RESULT 3. Suppose that the externality dissipation function $\gamma(\rho)$ is concave. Then the set of generators G^* is locally agglomerated in the PSNE.

Note that the dissipation function $\gamma(\rho)$ is held to be non-negative, non-increasing, and concave for all $\rho \in (0, \overline{\rho}]$. This implies that the externality generated from any point of the region is "felt" throughout the entire region (except for the edges of the region). To characterize global geometric properties of the equilibrium generator set, we make the following assumptions about the region where farms are located.

Consider a region with Euclidean distances between farms, which are represented by points in the plane. An example is a rectangular grid with farms located in the grid points. The Euclidean distance measure implies that any point k that lies on a straight line between points i and j is a convex combination of these points. Therefore, the distances from any arbitrary point x in the plane to points k, i, and j are connected by the inequality $d_{kx}^f \leq \lambda d_{ix}^f + (1-\lambda)\, d_{jx}^f$, where $\lambda = d_{jk}^f / d_{ij}^f \in [0,1]$ and $1-\lambda = d_{ik}^f / d_{ij}^f$ since $d_{ij}^f = d_{ik}^f + d_{kj}^f$. The following definition is a version of convexity suitable for a discrete finite set of points.

DEFINITION 2. A set of farms L is agglomerated if for any $i, j \in L$ with $d_{ij}^f = d_{ik}^f + d_{kj}^f$ it follows that $k \in L$.

That is, a set of farms is agglomerated if it contains all farms that lie on the shortest path (a line) between farms i and j in the set. The following result establishes that the equilibrium set of generators is agglomerated, if the externality dissipates "slowly" over the entire region, and the distances between farms are measured using the Euclidean metric.

RESULT 4. Suppose that the externality dissipation function $\gamma(\rho)$ is concave, and the distance matrix D^f is such that $d_{ij}^f d_{kx}^f \le d_{jk}^f d_{ix}^f + d_{ik}^f d_{jx}^f$ for any $i, j, k \in N$ with $d_{ij}^f = d_{ik}^f + d_{kj}^f$ and $x \in N$. Then the set of generators G^* is agglomerated in the PSNE.

On the other hand, in many agricultural contexts, the externality only impacts the immediate "border" neighbors and dissipates quickly, which is inconsistent with the global concavity of the dissipation function. The equilibrium land-use arrangements in these cases are investigated next.

Properties of Free-Market Arrangements When the Externality Impact Is Local

Suppose that generators impact only immediate neighbors within a unit radius, $\gamma(\rho) = 1_{\rho \le 1}$. And so, the externality damage imposed on a recipient is equal to the number of the neighboring generators, $z_i = \sum_{i \in M_i(1)} e_i^*$.[10] Let $D_4(a)$ and $D_8(a)$ denote the impact matrices corresponding to $a \times a$ square grids consisting of identical farms (cells) each having no more than, respectively, four and eight neighbors within a unit radius (with the exception of the cells at the edges of the region). In the case of D_4, a cell i has one immediate east, west, south, and north neighbor j if they share a common border, $d_{ij} = 1$; otherwise, $d_{ij} = 0$. In the case of D_8, two cells i and j are immediate neighbors, $d_{ij} = 1$, if they share either a common border or a common corner; otherwise, $d_{ij} = 0$.

A set of farms G is a neighborhood (a connected graph), if for any $i, j \in G$ there is a sequence of immediate neighbors in the set, $g_1, ..., g_k \in G$, such that $d_{g_t g_{t+1}}^f = 1$ for $t = 1, ...k-1$, $g_1 = i$ and $g_k = j$. The quick dissipation of the externality impact suggests that a plural number of generator neighborhoods may exist in equilibrium because farms that are not in the immediate vicinity of each other are effectively independent in terms of externality damage.

RESULT 5. Suppose that the externality is local, $\gamma(\rho) = 1_{\rho \le 1}$.

(a) Let $D = D_4(a)$ with $n = a^2$, $a \geq 3$. Then the generators are arranged in rectangular neighborhoods containing at least four farms, and $z_i \leq 1$ if $e_i^* = 0$ for all $i \in N$.

(b) Let $D = D_8(a)$ with $n = a^2$, $a \geq 3$. Then the generators are arranged in square neighborhoods containing exactly four farms, if $z_i \leq 2$ for all $i \in N$ with $e_i^* = 0$. Otherwise, the generators are arranged in the (irregular) octagon-shaped neighborhoods determined by the intersection of parallel vertical, horizontal, and diagonal lines, and $z_i \leq 3$ for all $i \in N$ with $e_i^* = 0$.

To prove Result 5, we take into account that farms located at the edge of the region have fewer neighbors than farms in the middle of the region. The limited number of possible local configurations implied by the simple spatial structure has an immediate consequence for the shape of the generator neighborhoods.[11] For example, Result 5 implies that, in case of $D_4\ (a)$, in any PSNE the number of generators, s, is not a simple number, $s \in [4, n-a]$. From Result 5 it also follows that for regions D_4 and D_8 each neighborhood of generators is agglomerated. For any $i, j \in G_m^*$ and $h \notin G^*$ with $d_{ih} = d_{jh} = 1$ it is true that $U(h,i,1) \subset [U(h,j,1) \cup U(h,j,0)]$ and $U(h,j,1) \subset [U(h,i,1)\ \cup U(h,i,0)]$, where G_m^* is a neighborhood of generators, $G^* = \cup G_m^*$, and $G_m^* \cap G_k^* = \varnothing$. An example of a region $D_8(11)$ with $n = 121$ cells and a PSNE with two neighborhoods of generators is depicted in Figure 3, where "x" cells are generators and "0" cells are recipients.

0	x	x	x	0	0	0	0	0	0	0
x	x	x	x	x	0	0	0	0	0	0
x	x	x	x	x	0	0	0	0	0	0
0	x	x	x	0	0	0	0	0	0	0
0	0	0	0	0	0	x	x	x	x	0
0	0	0	0	0	x	x	x	x	x	x
0	0	0	0	0	x	x	x	x	x	x
0	0	0	0	0	0	x	x	x	x	0

FIGURE 3. Octagon-shaped generator neighborhoods in $D_8(11)$

Efficient Land-Use Arrangement

In this section, we take a policy perspective and look for an efficient land-use arrangement that minimizes the total cost of producing the recipient crop for any given amount of total production, $T(e;s)=\sum_{i=1}^{n}(1-e_i)c_i$:

$$\min_{\{e_i\}} T(e;s) \text{ subject to } \sum_{i=1}^{n} e_i = s . \tag{3}$$

In a linear externality damage case, this is a well-known combinatorial problem that can be formulated as a graph partitioning problem, a specific instance of the quadratic assignment problem, or a quadratic optimization problem using graph-theoretic or matrix notation (Cela 1998; Burkard et al. 1998). This problem arises in a number of settings, including facility layout, manufacturing, circuit board and microchip design, parallel computing, and numerous other areas of engineering, physics, and management. The graph partitioning problem is NP-hard, that is, the time required to find an optimal solution grows exponentially with the size of the problem. The variety of the suggested solution algorithms based on different approaches can be grouped into four categories: spectral and geometric methods, multilevel algorithms, and discrete or continuous optimization-based methods (see Hager and Krylyuk 1999 and references therein). We follow the latter approach and consider several tractable special cases (Burkard et al. 1997).

We are interested in two properties of the optimal solution: the degree of concentration of generators, and the supportability of the efficient arrangement in a free-market setting. For the rest of the paper, we assume that the social planner has the ability to set the relative prices (common to all producers) for the externality generating and the externality receiving products. Then we say that an arrangement is implementable through free markets if the assigned land uses are such that the externality damage for any recipient is less than the externality damage for any generator.

To investigate the level of the generator concentration (externality absorption) in the efficient arrangement, it is more convenient to work directly with the impact matrix, D. We first consider a benchmark case with a linear externality damage and a symmetric "regular" impact matrix satisfying the condition $\sum_{j=1}^{n} d_{ij} = \bar{d}$ for all i. This can be interpreted to mean that the region has "rounded edges" and there are no farms in the "middle" of the region.[12] After that, the effects of the rate of externality damage accumulation and the spatial heterogeneities across the region on the efficient land-use arrangement are studied.

Linear Externality Accumulation and the "Regular" Impact Matrix

We will need a stronger notion of concentration as compared with the measures of concentration used to study the PSNE arrangements. Let $q(L_1, L_2) = \sum_{i \in L_1} \sum_{j \in L_2} d_{ij} - \sum_{i \in L_1} d_{ii}$ measure the potential externality damage imposed by farms in L_2 on farms in L_1. We say that a set L_1 is more concentrated than a set L_2 if $q(L_1, L_1) \geq q(L_2, L_2)$ and $|L_1| = |L_2|$. The efficient arrangement of generators is more concentrated than any other generator arrangement if the greatest degree of externality absorption by generators entails the least exposure by the recipients.

Divide the set of generators in the efficient arrangement into two non-overlapping sets L_1 and L_2, $L_1 \cup L_2 = G$, $L_1 \cap L_2 = \varnothing$, $G = \{i : e_i = 1\}$. In the case of a linear externality accumulation and a symmetric impact matrix, the total cost, $T(e;s)$, can then be written as (here, without loss of generality, we take $C(z) = z$)

$$T(e;s) = \sum_{i=1}^{n} \sum_{j=1}^{n} d_{ij} e_j (1 - e_i) = Q - Q^w - Q^a, \tag{4}$$

where $Q = q(N, L)$, $Q^w = \sum_{t=1}^{2} q(L_t, L_t)$, and $Q^a = 2q(L_1, L_2)$. Informally, the total cost is decomposed into three components. The first component, Q, represents the amount of the potential impact of generators on all farms in the region. The Q^w measures the degree of the concentration of generators within the subsets, and Q^a measures the degree of concentration across the subsets of generators. Next, we establish that the efficient arrangement of generators is always concentrated under certain conditions on the impact matrix.

Suppose that the impact matrix is symmetric, $d_{ij} = d_{ji}$, $d_{ii} = 0$, and regular, $\sum_{j=1}^{n} d_{ij} = \bar{d}$, for all i, j. Then minimizing (4) is equivalent to maximizing the total impact of generators on each other, $Q^w + Q^a$, because the potential impact is fixed, $Q = \sum_{j=1}^{n} e_j^* \sum_{i=1}^{n} d_{ij} = s\bar{d}$. And so, the efficient arrangement of generators, $G^e = \{i : e_i^e = 1\}$, is concentrated. Furthermore, any efficient arrangement satisfies $q(L_1, L_1) + 2q(L_1, L_2) + q(L_2, L_2) \geq q(L_1, L_1) + 2q(L_1, X) + q(X, X)$ for any $X \subset N$, $X \cap L_1 = \varnothing$ and $\sum_{i \in L_2} e_i^e = \sum_{i \in X} e_i$. Rearranging the last inequality yields $q(L_1, L_2) - q(L_1, X) \geq 0.5(q(X, X) - q(L_2, L_2))$, or $Q^a(G^e) - Q^a(G) \geq 0.5(Q^w(G^e) - Q^w(G))$. That is, there is a trade-off between achieving maximal concentration within the subsets of generators and across the subsets.

Incidentally, we find that under these conditions, any (individual) generator absorbs more externality damage than any recipient that is not affected by that particular generator. This property, of course, does not imply that the arrangement is implementable through free markets. Summarizing, we have the following.

RESULT 6. Suppose that the externality accumulation function C is linear and the impact matrix is symmetric, $d_{ij} = d_{ji}$, $d_{ii} = 0$, and $\sum_{j=1}^{n} d_{ij} = \bar{d}$ for all i, j.

(a) Then the efficient arrangement of generators is concentrated.

(b) The externality damage for the generator l exceeds that of the recipient h, $c_l \geq c_h$, if $d_{hl} = 0$.

To demonstrate part (b) of Result 6 we use a standard exchange argument. Observe that a generator not only increases the social (and private) costs since it imposes the cost on recipients but also decreases the social costs because it does not contribute to the total cost by not bearing the damage from other generators. In the efficient arrangement, the generating use is assigned to the set of farms that are both more susceptible to externality and generate it with lesser intensity as compared to any alternative rearrangement. The search for an optimal arrangement is greatly simplified if the impact matrix is separable in the sense that the generating intensity of each farm is recipient-invariant, while the susceptibility to externality of each farm is generator invariant. Impact matrices possessed of this property are considered in the next section.

Linear Externality Accumulation and Separable Impact Matrices

For the ease of exposition, we consider the case of sum and product impact matrices, $d_{ij} = a_i + b_j$ and $d_{ij} = a_i b_j$, separately. Any impact matrix with entries of the form $d_{ij} = \alpha_1 a_i + \alpha_2 b_j + \alpha_3 a_i b_j$, where $\alpha_1, \alpha_2, \alpha_3 \geq 0$, will possess the same properties. First, we analyze the efficient arrangement for impact matrices from Examples 1 and 2.[13]

EXAMPLE 3. (Sum and Product Impact Matrix) Let $d_{ij} = a_i + b_j$, $a_1 \geq a_2 \geq ... \geq a_n$, and $b_1 \leq b_2 \leq ... \leq b_n$. That is, it is held that farms can be ordered so that the farms with the largest susceptibility to externality are also those with the smallest externality generating intensity. After a little algebra, the total damage is $T(e;s) = s(\sum_{i=1}^{n} a_i \ - \sum_{i=1}^{n} a_i e_i)$ $+ (n-s)\sum_{i=1}^{n} b_i e_i$, where $\sum_{i=1}^{n} e_i = s$. The total damage to recipients is minimized when $e_i^e = 1_{i \leq s}$.[14] The efficient arrangement assigns generators to farms with the greatest susceptibilities, $a_1,...,a_s$, and the least intensities, $b_1,...,b_s$.

This arrangement is supportable through free markets if, in addition, $a_s - a_{s+1}$ $\geq (a_s + b_s)/s$ for all $s = 2,...,n-1$ (see Example 1). In the case of a product matrix, $d_{ij} = a_i b_j$, we have $T(e;s) = (\sum_{i=1}^{n} a_i - \sum_{i=1}^{n} a_i e_i) \sum_{i=1}^{n} b_i e_i$ and the optimal arrangement is unchanged. The additional condition required for the efficient arrangement to be implementable through free markets is stated in Example 2.

The efficient arrangement of generators may or may not be concentrated for both sum and product impact matrices. In case of a sum matrix, the degree of externality absorption is $q(G^e, G^e) = \sum_{i=1}^{s} \sum_{j=1}^{s} (a_i + b_j) - \sum_{i=1}^{s} (a_i + b_i) = (s-1)\sum_{i=1}^{s} (a_i + b_i)$, where $G^e = \{i : 1 \leq i \leq s\}$. Only if the sum of generating intensity and susceptibility decreases with the farm index, $a_s + b_s \geq a_{s+1} + b_{s+1}$, is the efficient arrangement of generators assuredly concentrated. For example, suppose that the intensities and susceptibilities are perfectly negatively correlated, $b_i = K - a_i$ for all i, $K \geq \max_i \{a_i\}$.

Then the measure of concentration is arrangement-invariant: $q(G,G)=(s-1)K$ for any G with $|G|=s$.

Next, we consider impact matrices where for each farm the (generator-invariant) susceptibility and (recipient-invariant) intensity coincide. Because in this case farms can no longer be ordered so that the degrees of susceptibility and intensity vary inversely, the efficient arrangement turns out to depend on the number of generators and the size of the region.

EXAMPLE 4. (Symmetric Sum Impact Matrix) Let $d_{ij}=a_i+a_j$, $a_1\ge a_2\ge ...\ge a_n$. Here the total cost is $T(e;s)=s\sum_{j=1}^{n}a_i+(n-2s)\sum_{j=1}^{n}a_ie_i$, and the optimal solution to (3) depends on the sign of $n-2s$. Because the susceptibilities (intensities) $\{a_i\}$ are non-increasing, the optimal arrangement is $e_i^e=1_{i\ge n-s}$ if $2s<n$, and $e_i^e=1_{i\le s}$ if $2s\ge n$.

If the number of generators is small, it is optimal to assign them to smaller farms because then the amount of the generated externality is also small. The situation is reversed if the number of generators is large because then it is optimal to reduce the overall recipient susceptibility. The efficient arrangement is not implementable through free markets in the former case, but it may be implementable in the latter (see Example 1).

The externality cost imposed on the recipient farm h ($e_h^e=0$) is $c_h=sa_h+\sum_{j=1}^{n}a_je_j^e$, while the potential cost for the generator l ($e_l^e=1$) is $c_l=(s-2)a_l+\sum_{j=1}^{n}a_je_j^e$. If $2s<n$ and $e_i^e=1_{i\ge n-s}$, the condition $c_h<c_l$ cannot be satisfied because it implies that $a_h<a_l$ for $h<l$, which contradicts the assumption. On the other hand, for $2s\ge n$ and $e_i^e=1_{i\le s}$, the condition $c_h<c_l$ is satisfied, if, in addition, $(1-2/s)\ a_s\ge a_{s+1}$ for all $s=[n/2],...,n-1$ (see Example 1).

Observe that the optimal degree of generator concentration, $q(G^e,G^e)=2(s-1)\sum_{i=n-s}^{n}a_i$, is the least for $2s<n$ with $G^e=\{i:i\ge n-s\}$, and is the largest for $2s\ge n$,

$q(G^e, G^e) = 2(s-1)\sum_{i=1}^{s} a_i$ when $G^e = \{i : i \le s\}$. To summarize, the efficient arrangement may be supportable if the number of generators is large because then it is optimal to assign the biggest farms to the externality generating use. Because the biggest farms are not only the "biggest" externality generators but also the "biggest" externality recipients (if assigned to the recipient use), the generator farms do not have an incentive to alter the assigned activity.

EXAMPLE 5. (Symmetric Product Impact Matrix) Now let $d_{ij} = a_i a_j$, $a_1 \ge a_2 \ge \ldots \ge a_n$. Here the total cost is $T(e;s) = \sum_{j=1}^{n} a_i z - z^2$, where $z = \sum_{j=1}^{n} a_i e_i$, $\sum_{i=1}^{n} e_i = s$. This quadratic function is minimized by $e_i^e = 1_{i \ge n-s}$ or $e_i^e = 1_{i \le s}$ depending on whether $\sum_{i=1}^{s} a_i \sum_{i=s+1}^{n} a_i > (\le) \sum_{i=1}^{n-s-1} a_i \sum_{i=n-s}^{n} a_i$.

It is easy to check that the last condition is equivalent to $2s < (\ge) n$. The optimal arrangement assigns the externality generating uses to the farms with the lowest (highest) capacity if less (more) than half of all farms are generators. Note that the products of partial sums in the optimality condition are minimized when the difference between the product terms is maximized because $\sum_{i=1}^{s} a_i + \sum_{i=s+1}^{n} a_i = \sum_{i=1}^{n-s-1} a_i + \sum_{i=n-s}^{n} a_i = \sum_{i=1}^{n} a_i$. Since farm capacity decreases with farm index, we have $| \sum_{i=1}^{n-s-1} a_i - \sum_{i=n-s}^{n} a_i | >$ $(\le) | \sum_{i=1}^{s} a_i - \sum_{i=s+1}^{n} a_i |$ as $2s < (\ge) n$. That is, the difference between the total generating intensities and recipients' susceptibilities is maximized when the generators have smaller intensities and the recipients are more susceptible, if the number of generators is small. In contrast, this difference is maximized when the generators have greater intensities and the recipients are less susceptible, if the number of generators is large.

As in the case of a symmetric sum impact matrix, the efficient arrangement is not implementable through free markets if $2s < n$, but it may be implementable if $2s \ge n$ (see Example 2). The externality cost imposed on the recipient farm h $(e_h^e = 0)$ is c_h $= a_h \sum_{j=1}^{n} a_j e_j^e$, while the potential cost for the generator l $(e_l^e = 1)$ is $c_l = a_l(\sum_{j=1}^{n} a_j e_j^e$

$-a_l)$, if $e_i^e = 1$. If $2s < n$ and $e_i^e = 1_{i \geq n-s}$, the condition $c_h < c_l$ cannot be satisfied because it implies that $a_h < a_l$ for $h < l$, which contradicts the assumption. On the other hand, for $2s \geq n$ and $e_i^e = 1_{i \leq s}$, the condition $c_h < c_l$ is satisfied, if, in addition, a_{s+1} / a_s $< 1 - a_s / \sum_{i=1}^{s} a_i$ for all $s = [n/2],...,n-1$. Also, observe that the optimal degree of generator concentration, $q(G^e, G^e) = 2\sum_{i=s}^{n} \sum_{j=i+1}^{n} a_i a_j$, is the least for $2s < n$ when G^e $= \{i : i \geq n-s\}$ and the arrangement is not implementable. The concentration, $q(G^e, G^e)$ $= 2\sum_{i=1}^{s} \sum_{j=i+1}^{s} a_i a_j$, is the largest for $2s \geq n$ when $G^e = \{i : i \leq s\}$ and the arrangement is implementable through free markets. The generators must be big (have high susceptibilities), and the recipients must be small (have low susceptibilities) to assure that the actual damages exceed the potential damages from changing the assigned use.

Convex and Concave Externality Damage Accumulation Functions

Now we turn to a more general form of the externality damage accumulation function. In the non-linear damage accumulation case, not only the sum of the private damages but also the distribution of damages among the recipients determines the social cost of the arrangement, $T(e;s)$. To compare the distributions of damages corresponding to the candidate land-use arrangements, we will need the following definitions commonly used to measure dispersion (Marshall and Olkin 1979).

DEFINITION 3. A vector $x = (x_1,...,x_N)$ is sub-majorized by the vector $y = (y_1,...,y_n)$ (denoted by $\prec_w$) if $\sum_{i=1}^{k} x_{[i]} \leq \sum_{i=1}^{k} y_{[i]}$ for $k = 1,2,...,n$, where $x_{[1]} \geq x_{[2]} \geq ... \geq x_{[n]}$ and $y_{[1]} \geq y_{[2]} \geq ... \geq y_{[n]}$ are their components in the decreasing order.

DEFINITION 4. A vector $x = (x_1,...,x_N)$ is super-majorized by the vector $y = (y_1,...,y_n)$ (denoted by $\prec^w$) if $\sum_{i=1}^{k} x_{(i)} \geq \sum_{i=1}^{k} y_{(i)}$ for $k = 1,2,...,n$, where $x_{(1)} \leq x_{(2)} \leq ... \leq x_{(n)}$ and $y_{(1)} \leq y_{(2)} \leq ... \leq y_{(n)}$ are their components in the increasing order.

As is well-known, the sub-majorization ($x \prec_w y$) and super-majorization ($x \prec^w y$) relations generate classes of the order-preserving functions: increasing Schur-concave and increasing Schur-convex functions. A function $f(x)$ is Schur-convex if $x \prec y$ implies that $f(x) \le f(y)$. Here $\prec$ denotes the usual majorization order obtained by requiring that $\sum_{i=1}^n x_i = \sum_{i=1}^n y_i$ in Definition 3 or 4. Suppose that the damage accumulation function, $C(z)$, is convex (concave). For a given number of generators, the total damage cost imposed on the recipients, $T(z_1,...,z_n) = \sum_{i=1}^n C(z_i)$, is increasing Schur-convex (Schur-concave) in the damages to recipients, $z_i(e) = \sum_{j=1}^n d_{ij} e_j (1-e_i)$, as a sum of increasing convex (concave) functions. Now imagine that we need to compare two arrangements e and e' with $\sum_{i=1}^n e_i = \sum_{i=1}^n e_i'$ in terms of social efficiency. If it is known that $z(e) \prec_w z(e')$ or $z(e') \prec^w z(e)$, then arrangement e (e') is welfare dominant depending on whether $C(z)$ is convex (concave). Summarizing, we have the following.

RESULT 7. (a) Suppose that there exists a permutation of farm indices $\phi(i)$ such that $\sum_{i=\phi(s+1)}^{\phi(k+s+1)} \sum_{j=\phi(1)}^{\phi(s)} d_{ij} \le \sum_{i \in S_k} \sum_{j \in L} d_{ij}$ for any $k = 1,...,n-s-1$, $S_k, L \subset N$, $|S_k| = k$, $|L| = s$, $S_k \cap L = \varnothing$.

(b) Suppose that there exists a permutation of farm indices $\varphi(i)$ such that $\max_{T_k} \sum_{i \in T_k} \sum_{j=\varphi(1)}^{\varphi(s)} d_{ij} \le \max_{S_k} \sum_{i \in S_k} \sum_{j \in L} d_{ij}$ for any $k = 1,...,n-s-1$, $S_k, L \subset N$, $T_k \subseteq \{\varphi(s+1),..., \varphi(n)\}$, $|T_k| = |S_k| = k$, $|L| = s$, $S_k \cap L = \varnothing$. Then the efficient arrangement is given by $e^e_{\pi^e(i)} = 1_{i \le s}$ for $i \in N$, where $\pi^e = \phi$ if C is concave, and $\pi^e = \varphi$ if C is convex. Observe that condition (a) implies that $\sum_{j=\phi(1)}^{\phi(s)} d_{\phi(h)j}$ $\le \sum_{j=\phi(1)}^{\phi(s)} d_{\phi(h+1),j}$ for any $h = s+1,..., n-1$, while condition (b) implies that $\sum_{j=\varphi(1)}^{\varphi(s)} d_{\varphi(h),j} \ge \sum_{j=\varphi(1)}^{\varphi(s)} d_{\varphi(h+1),j}$ for any $h = s+1,...,n-1$. If the impact matrix D is such that both permutations ϕ and φ satisfy conditions (a) and (b), respectively, we have

$\{c_{\phi(i)}\}_{i=s+1}^{n} \prec \{c_{\varphi(i)}\}_{i=s+1}^{n}$. That is, the optimal distribution of damages is more "spread out" if the externality damage accumulation function is convex.

It is easy to check that both conditions (a) and (b) are satisfied with the identity permutation $\phi=\varphi=i$ for the product and sum impact matrices considered in Example 3. Let the externality from a generator j to a recipient i be a sum or a product of the recipient's susceptibility and the generator's intensity, or a linear combination of these, $d_{ij} = \alpha_1 a_i + \alpha_2 b_j + \alpha_3 a_i b_j$, where $\alpha_1, \alpha_2, \alpha_3 \geq 0$. Suppose that the farms with the smallest generating intensities, b_i, have the greatest susceptibilities, a_i: $a_1 \geq a_2 \geq ... \geq a_n$ and $b_1 \leq b_2 \leq ... \leq b_n$. From Result 7 it follows that it is optimal to assign the externality generating uses to the farms with smallest generating intensities for any shape of the damage accumulation function. This is generalized in the following result.

RESULT 8. Let $d_{i,j} \leq d_{i,j+1}$ and $d_{i,j} \geq d_{i+1,j}$ for all $i, j \in N$. Then for any shape of the damage accumulation function the efficient arrangement is $e_i^e = 1_{i \leq s}$ for all $i \in N$.

The monotonicity conditions on the individual externality damages shift the "mass" of the impact out of the left bottom corner of the externality impact matrix.[15] Clearly, the product and sum matrices with the inversely ordered susceptibilities and intensities considered in Example 3 satisfy the monotonicity conditions.

Next, we inquire into some effects of the geographical features of a region on the efficient arrangement. The level of the concentration of generators in the efficient arrangement depends on both the type of curvature of the damage accumulation function and the specifics of the spatial interactions and externality dissipation in the region. For a concave accumulation function, it may be optimal to let a small number of recipients bear most of the externality damage while lowering the damage for other recipients. In contrast, for a convex accumulation function, it may be optimal to spread the externality damage more evenly among the recipients.[16] Therefore, it appears that in the efficient arrangement the degree of generator concentration should be greater in the former case than in the latter. As the following example demonstrates the specifics of the spatial structure of the region and the externality dissipation function may interact to make the effect of the accumulation function curvature on the efficient arrangement ambiguous.

EXAMPLE 6. (Tick-Tac-Toe) Consider a region $D_4(3)$ (see Result 5 for details), where only cells with a common border may impact each other. Farms are indexed as depicted in Figure 4a.

1	2	3
4	5	6
7	8	9

(a)

x	x	x
x	0	0
0	0	0

(b)

x	x	0
x	x	0
0	0	0

(c)

FIGURE 4. Efficient land-use arrangements in $D_4(3)$

Let there be four generators, $s = 4$, and $e^e_{\pi(i)} = 1_{i \le s}$ for $i \in \{1,...,9\}$. Consider the identity permutation, $\phi(i) = i$, and let $\pi^e = \phi$, so that the first s cells are generators ("x"), and the last $n - s - 1$ cells are recipients ("0") as in Figure 4b. Then the amounts of externalities (actual and potential dissipated among the farms) are $z_1^x = 2$, $z_2^x = 2$, $z_3^x = 1$, $z_4^x = 1$, $z_5^0 = 2$, $z_6^0 = 1$, $z_7^0 = 1$, $z_8^0 = 0$, and $z_9^0 = 0$. Consider another arrangement with $\varphi(i) = \{1,2,4,5,3,6,7,8,9\}$ as in Figure 4c, and let $\pi^e = \varphi$. Then the corresponding amounts of externalities are $z_1^x = 2$, $z_2^x = 2$, $z_4^x = 2$, $z_5^x = 2$, $z_3^0 = 1$, $z_6^0 = 1$, $z_7^0 = 1$, $z_8^0 = 1$, and $z_9^0 = 0$. Checking all possible permutations $\pi(i)$ reveals that condition (a) holds for the distribution in Figure 4b while condition (b) holds for the distribution in Figure 4c (up to the reordering of the costs). Note that the distribution of the amounts of the received externalities under $\pi^e = \phi$ is majorized ("more uneven than") by that under $\pi^e = \varphi$, $\{z^x_{\phi(i)}\}_{i=5}^9 \prec \{z^x_{\varphi(i)}\}_{i=5}^9$.

Turning to the level of generator concentration, note that the arrangement in Figure 4b is not concentrated, $q(G^e, G^e) = 6$, $G^e = \{1,2,3,4\}$, while the arrangement in Figure 4c is concentrated, $q(G^e, G^e) = 8$, $G^e = \{1,2,4,5\}$. In the case of a concave externality accumulation function, it is optimal to decrease the number of exposed recipients at the expense of a "double" exposure for one recipient. In the case of a convex externality accumulation function, it is optimal to "even out" the exposure among the recipients at the expense of increasing the number of the exposed ones.

If the externality accumulation function is concave, by Result 5a, the efficient arrangement is not supportable by a free market (note that $c_5^0 = 2 > 1 = c_3^x = 1$). On the other hand, in the case of a convex externality accumulation function, by Result 5a, the efficient arrangement is supportable by a free market (note that $c_j^0 = 1 < 2 = c_i^x$ for all j, i).

In general, the efficient arrangements in Result 7 is implementable through free markets if condition $\max_{i>s} \sum_{j=\pi(1)}^{\pi(s)} d_{\pi(i)j} \le \min_{i\le s} \sum_{j=\pi(1)}^{\pi(s)} d_{\pi(i)j}$ holds for $\pi = \phi, \varphi$. The conditions of efficiency and free-market implementability may or may not be compatible as demonstrated in Example 6. In the case of convex externality damage, the efficient allocation is more likely to be implementable if the number of generators is large relative to the number of recipients. Then the arrangement that is characterized by concentration of generators may be optimal even in the convex cost case if the "spreading" of generators across the region raises private costs for each recipient (see Example 6). Otherwise, the implementability may fail because condition (b) implies that $\max_{i\in H} c_i = \sum_{j=\pi(1)}^{\pi(s)} d_{ij}$ for $H = \{\pi(s+1),..., \ \pi(n)\}$ is low. This means that the externality impacts are "evenly" spread out across the recipients, which, in turn, implies a small degree of the concentration of generators and may violate the condition that the exposure to externality for each generator exceeds that for each recipient.

Conclusions

This paper takes a close look at the arrangement of externality generating and externality receiving activities in a region. We consider a distance-dependent externality with heterogeneous externality generating intensities and damage susceptibilities across the production units in the region. Two types of activity arrangements: free-market and efficient, are studied. The free-market arrangements arise as a result of independent decisionmakers playing the Nash game in pure strategies. The efficient arrangements minimize the total cost due to the externality imposed on recipients by generators for a given amount of output. The level of generator concentration in terms of the externality absorption, which may or may not coincide with the spatial concentration, is investigated for both free-market and efficient arrangements. Broadly speaking, the implementability of the efficient arrangement in a free-market setting depends on the spatial characteristics of the region, the number of generators, and the curvature of the externality accumulation function.

We first analyze the benchmark case when the individual damage from the externality accumulates linearly under certain restrictions placed on the spatial characteristics of the region. In particular, we consider a regular region where the sum of the distances from one farm to all other farms is invariant across farms (a regular graph). In this case, generators are concentrated in the efficient arrangement, and the free-market implementability property feasibly holds. We also investigate a case where the susceptibility (if in the receiving use) is invariant to the generator, and the generating intensity (if in the generating use) is invariant to the recipient for each farm in the region. Here, the efficient arrangement is easily determined, and is likely implementable as long as the level of intensity varies inversely with the level of susceptibility for each farm. If the generating intensity and susceptibility to the externality are perfectly correlated, the efficient arrangement depends on the number of generators and the size of the region. Then the arrangement may admit the free-market implementability only if the number of generators is large relative to the size of the region.

Another aspect of the environment under scrutiny is the effect of the rate of the externality accumulation on the efficient arrangement. To this end, we use the majorization orders, which capture the effect of the curvature of the objective function on the optimal arrangement. The conventional wisdom that associates the slow rate of damage accumulation with the optimality of concentrating damages and the fast rate of damage accumulation with the optimality of spreading damages may be overturned in several instances. In the case of the separable impact matrix with inversely related intensities and susceptibilities, the efficient arrangement is invariant to the curvature of the accumulation function. We also consider a simple example that illustrates that a higher level of generator concentration does not necessarily lead to a more “uneven” distribution of the externality exposure among the recipients. In fact, the distribution of the externality exposures among the recipients may be “more even” when the level of generator concentration is the highest.

The issue of the spatial arrangement of conflicting or benefiting activities is not confined to crop agriculture; in fact, it is pertinent in other areas of economics such as urban economics and social sciences (e.g., Berliant, Peng, and Wang 2000). For example, Calvo-Armengol and Zenou (2003) use a similar model to study the role of social networks in promoting criminal activities and explore the endogenous formation of a criminal network. In their framework, affiliated criminals impose a positive externality on each other by sharing “trade secrets.” In the agricultural contexts, the spatial connection of land lots is ordinarily exogenous to the model. However, some insights developed in this paper may be applicable, for example, to the inquiry into the efficiency of an allocation of police resources among city districts.

The model developed in this paper rests on a number of restrictive assumptions, such as perfect information among the agents, the observability of the production activities in the entire region, and the lack of countermeasures to combat the effects of the externality (e.g., pollution abatement) other than the choice of the production activity. The temporal dimension of the activity choice and, as a consequence, the fixed costs that are frequently associated with changing the land use, as well as the uncertainty of the future income flow contingent on the surrounding land uses, are completely left out of the model. Relaxing these and other assumptions regarding the participants’ behavior is likely to glean valuable insights into the problem of improving the efficiency of the spatial arrangement of externality generating and receiving activities.

Endnotes

1. Helfand and Rubin (1994) identify a number of “technical” and “psychological” sources of non-convexities in the externality damage accumulation function, which cause constant or diminishing marginal damage to the recipient of one more additional unit of the externality.

2. To cite one example of an applied study in this area, Ancev, Stoecker, and Storm (2003) investigate the optimal spatial allocation of waste management practices to reduce phosphorous pollution in a watershed.

3. To reflect the potential agronomic influence, the distance may not coincide with the Euclidean distance between the (land mass) centers of the two farm plots.

4. The model can be easily adjusted to account for the heterogeneity in farm size. For example, the differences in farm size can be reflected in the distance matrix.

5. Explicitly accounting for the variation in farm sizes complicates the exposition. This variation can be accommodated by allowing for asymmetry in the externality damages. Say, $d_{ij} > d_{ji}$ may be attributed to a bigger size of farm *j* relative to farm *i*.

6. The formulation where the externality affects yields rather than production costs is accommodated by positing $v_i^r = p^r(y - c_i)$, where p^r is the per unit price and *y* is the per acre yield for recipient crop.

7. Here, the association of the susceptibility and intensity parameter with the physical size of a farm is somewhat internally inconsistent because the output is held to be invariant across farms.

8. This type of distance matrix plays an important role in discrete optimization problems. The impact matrix generated from such a distance matrix, $D = \{d_{ij}\} = \gamma(|i - j|)$, is called a Toeplitz matrix (e.g., see Burkard et al. 1997).

9. This is a weak version of the separability implied by the hyperplane theorems for closed convex sets.

10. These kinds of environments were modeled using a cellular automata simulation program known as the game of life (e.g., see Parker 2000; Belcher, Nolan, and Phillips 2003.) Munro (n.d.) assigns a fixed number of externality generating uses in a random manner and considers upper and lower bounds on the total externality damage imposed on the recipients for $D_8(20)$.

11. The regions are held to be square mostly for the ease of exposition. In fact, in Figure 3, the region is rectangular.

12. A square matrix whose entries are non-negative, and whose rows and columns sum to one is called doubly stochastic. The corresponding graph with the doubly stochastic adjacency matrix (with the edge weights equal to the distances between any two farms) is called regular.

13. Note that the total externality damage $T(e;s)$ does not depend on the diagonal values of the impact matrix, d_{ii}.

14. In the context of the quadratic assignment problem, the results of this sort are surveyed in Burkard et al. 1997.

15. A matrix with this property is sometimes called left-lower graded. This result is an easy generalization of Theorem 3.2 in Burkard et al. 1997, p. 7, with one of the matrices having entries $m_{ij} = e_i(1-e_j)$, $e_i = 1_{i \le s}$ (right-upper graded).

16. Different circumstances that may lead to optimal concentration versus spreading damages are discussed in Helfand and Rubin 1994.

Appendix

Proofs of Results

Proof of Result 1

Suppose that generators are located in farms $i = 1,...,s$, and recipients are in the remaining farms, $i = s+1,...,n$. From the partial sum condition it follows that

$$c_h(s) = \sum_{j=1}^{s} d_{hj} \le \sum_{j=1}^{h-1} d_{hj} \le \sum_{j=1}^{h-2} d_{h-1,j} \le ... \le \sum_{j=1}^{l-1} d_{lj} \le \sum_{j=1}^{l-1} d_{lj} + \sum_{j=l+1}^{s} d_{lj} = c_l(s)$$

for any $l \le s < h$. Also, observe that $c_s(s) = \min_{i \le s} c_i(s)$ and $c_{s+1}(s) = \max_{i \ge s+1} c_i(s)$. Because both costs $c_s(s)$ and $c_{s+1}(s)$ decrease with s, while the premium $v(s)$ (smoothly) increases with s there exists unique s^* such that $\beta\hat{v}(s^*) \in [\, c_{s^*+1}(s^*),\, c_{s^*}(s^*)\,]$ for some $\beta > 0$. Note that the scaling parameter β is necessary only because s^* is an integer.

Proof of Result 2

The proof proceeds in two steps.

Step 1. Let $M_i(\rho) = \{ j : | i - j | = \rho \}$. Observe that for any i, j such that $i \in M_j(1)$ two properties are satisfied: (*a*) $M_i(\rho) \subset M_j(\rho - 1) \cup M_j(\rho + 1)$, and (*b*) $| M_i(\rho) \cap M_j(\rho + 1) | \le 1$ for any $\rho \in [0, n]$. From property (*a*) it follows that $M_i(\rho) = M_i(\rho) \cap (M_j(\rho - 1) \cup M_j(\rho + 1))$.

Step 2. Suppose that $\{e_i^*\}$ is a solution to (2), and pick h, $l \in N$ such that $e_h^* = 0$, $e_l^* = 1$, and $| h - l | = 1$. By Step 1, we write $c_h = C(\sum_{\rho=1}^{n} \sum_{i \in M_i(\rho) \cap M_j(\rho-1)} \gamma(\rho) e_i^*$

$$+ \sum_{\rho=1}^{n} \sum_{i \in M_i(\rho) \cap M_j(\rho+1)} \gamma(\rho) e_i^*) > C(\sum_{\rho=1}^{n} \sum_{i \in M_i(\rho) \cap M_j(\rho-1)} \gamma(\rho) e_i^*$$

$$+ \sum_{\rho=1}^{n} \sum_{i \in M_i(\rho) \cap M_j(\rho+1)} \gamma(\rho + 1) e_i^*) = C(\sum_{\rho=0}^{n} \sum_{i \in M_i(\rho+1) \cap M_j(\rho)} \gamma(\rho + 1) e_i^*$$

$$+ \sum_{\rho=2}^{n} \sum_{i \in M_i(\rho-1) \cap M_j(\rho)} \gamma(\rho) e_i^*) = C(\sum_{\rho=0}^{n} \sum_{i \in M_i(\rho+1) \cap M_j(\rho)} \gamma(\rho + 1) e_i^*$$

$-\sum_{\rho=1}^{n}\sum_{i\in M_i(\rho+1)\cap M_j(\rho)}\gamma(\rho)e_i^* + \sum_{\rho=1}^{n}\sum_{i\in M_j(\rho)}\gamma(\rho)e_i^*) \geq C(\gamma(1)+\sum_{\rho=1}^{n}\gamma(\rho+1)-\gamma(\rho)$ $+\sum_{\rho=1}^{n}\sum_{i\in M_j(\rho)}\gamma(\rho)e_i^*) = c_l$. The first inequality follows because $\gamma(\rho) > \gamma(\rho+1)$. The third equality follows because $e_h^* = 0$. The last inequality follows from Step 1, and because $M_h(1)\cap M_l(0) = l$ and $e_l^* = 1$. Hence, we have $c_h > c_l$, which contradicts the assumption. Therefore, no equilibrium solution to (2) exists.

Proof of Result 3

Suppose that there is $i, j, h \in N$ such that $e_i^* = e_j^* = 1$, $e_h^* = 0$, and $d_{ih} = d_{jh} = 1$, and $U(h,i,1) \subseteq U(j,h,1)$ and $U(h,j,1) \subseteq U(i,h,1)$. Observe that we can write $d_{ij}^f = 1_{j\in U(h,i,1)}$ $(d_{hj}^f - 1) + 1_{j\in U(h,i,0)} d_{hj}^f + 1_{j\in U(i,h,1)}(d_{hj}^f + 1)$. Using this decomposition we have c_x $= C(\sum_{l\in U(h,x,1)}\gamma(d_{hl}^f - 1)e_l + \sum_{l\in U(h,x,0)}\gamma(d_{hl}^f)e_l + \sum_{l\in U(x,h,1)}\gamma(d_{hl}^f + 1)e_l)$, where $x = i, j$. From equilibrium condition $c_x > c_h$ and the monotonicity of $C(.)$ it follows that $\sum_{l\in U(h,x,1)}(\gamma(d_{hl}^f - 1) - \gamma(d_{hl}^f))e_l > \sum_{l\in U(x,h,1)}(\gamma(d_{hl}^f) - \gamma(d_{hl}^f + 1))e_l$. By assumption it follows that $\sum_{l\in U(j,h,1)}(\gamma(d_{hl}^f - 1) - \gamma(d_{hl}^f))e_l > \sum_{l\in U(i,h,1)}(\gamma(d_{hl}^f) - \gamma(d_{hl}^f + 1))e_l$, and $\sum_{l\in U(i,h,1)}(\gamma(d_{hl}^f - 1) - \gamma(d_{hl}^f))e_l > \sum_{l\in U(j,h,1)}(\gamma(d_{hl}^f) - \gamma(d_{hl}^f + 1))e_l$. Adding the last two inequalities yields $\sum_{l\in U(j,h,1)}\Delta^2\gamma(d_{hl}^f - 1)e_l + \sum_{l\in U(i,h,1)}\Delta^2\gamma(d_{hl}^f - 1)e_l$, > 0 where $\Delta^2\gamma(\rho)$ $= \gamma(\rho+2) - 2\gamma(\rho+1) + \gamma(\rho)$ denotes the second-order difference operator. But this is a contradiction because $\Delta^2\gamma(\rho) \leq 0$ for all $\rho > 0$.

Proof of Result 4

Suppose that there are three farms $i, j, k \in N$ such that $e_i^* = e_j^* = 1$, $e_k^* = 0$, and k lies on the shortest path (or a line) between i and j, $d_{ij}^f = d_{ik}^f + d_{kj}^f$. Let $\lambda = d_{jk}^f / d_{ij}^f \in [0,1]$ so that $1-\lambda = d_{ik}^f / d_{ij}^f$. By assumption we have $d_{kx}^f \leq \lambda d_{ix}^f + (1-\lambda)$ d_{jx}^f for any $x \in N$. Because the externality dissipation function $\gamma(.)$ is decreasing and concave it follows that $\gamma(d_{kx}^f) \geq \gamma(\lambda d_{ix}^f + (1-\lambda)d_{jx}^f) \geq \lambda\gamma(d_{ix}^f) + (1-\lambda)\gamma(d_{jx}^f)$. Summing over all x with $e_x^* = 1$ yields $\sum_{x=1}^{n} \gamma(d_{kx}^f) e_x^* \geq \lambda \sum_{x \neq i} \gamma(d_{ix}^f) e_x^* + (1-\lambda) \sum_{x \neq j} \gamma(d_{ix}^f) e_x^*$, which implies that $\sum_{x=1}^{n} \gamma(d_{kx}^f) e_x^* \geq \min[\sum_{x \neq i} \gamma(d_{ix}^f) e_x^*, \sum_{x \neq j} \gamma(d_{ix}^f) e_x^*]$, or $c_k \geq \min[c_i, c_j]$ by the monotonicity of the externality accumulation function $C(.)$. But this is a contradiction.

Proof of Result 5

Part (a). It will be convenient to index cells in the square grid starting in the left-upper corner and going from the left to the right of each row. Then the horizontal and vertical locations of the cell i are given by $y_i = [(i-1)/a]$ and $x_i = i - ay_i$, where $[b]$ denotes the integer part of b. In the case of $D_4(a)$, the externality impacts are given by $d_{ij} = 1_{|x_i - x_j|=0, |y_i - y_j|=1} + 1_{|x_i - x_j|=1, |y_i - y_j|=0} = 1_{|i-j|=a} + 1_{|i-j|=1,\ [(i-1)/a]=[(j-1)/a]}$. To show that a generator can border, at most, one generator, suppose that $\max_i z_i (1 - e_i^*) > 1$. Then the corner farms must be recipients because $z_1 = z_a = z_{n-a} = z_n \leq 2$. But this implies that any farms located in the cells adjacent to the corners must be recipients because we also have $z_i \leq 2$ for $i = \{2, a+1, a-1, 2a, n-2a, n-a+1, n-1, n-a-1\}$. But the farms adjacent to these cells may have, at most, two neighbors that are generators and thus must be recipients as well. Continuing in this manner, we can show that all farms are recipients, which cannot be in equilibrium. Hence, in any equilibrium, $\max_i z_i (1 - e_i^*) \leq 1$.

Next, we show that the generators must be arranged in rectangular neighborhoods. Consider a two-by-two fragment of a square cell with farms $\{i, i+1, i+a, i+a+1\}$, where $i \neq ka$ for any $k = 1, ..., a$. Suppose that there are two generators that are corner neighbors, for example, $e_i^* = e_{i+a+1}^* = 1$. Then any recipient in $i+1$ or $i+a$ must border two generators, which is impossible. However, each generator must border at least two other generators. Therefore, generators must be arranged in rectangular neighborhoods of four or more generators.

Part (b). For $D = D_8(a)$, an analogous argument is used to show that $\max_i z_i (1 - e_i^*) \leq 3$. Suppose that $\max_i z_i (1 - e_i^*) > 3$. Then the corner farms must be recipients because $z_1 = z_a = z_{n-a} = z_n \le 3$. But the edge cells adjacent to the corner cells, $i = \{2, a{+}1, a{-}1, 2a, n{-}2a{+}1, n-a+1, n-1, n-a\}$, have, at most, four neighbors that can be generators. And so, they must be recipients as well. The same is true for all of the remaining edge cells, $i = \{3,...,a-2\}, \{3a+1,...,(a-3)a+1\}, \{3a,...,(a-3)a\}$, $\{(a-1)a+3,...,n-2\}$. Next, remove the (recipient) edge cells of the region and apply the same reasoning to the remaining cells. Continue in this manner until the region consists of the one cell, $i = [n/2]+1$, when n is odd, or the four adjacent cells in the middle, $i = \{(a-1)a/2,\ (a-1)a/2+1, (a-1)a/2+a\ (a-1)a/2+a+1\}$, when n is even. Hence, all farms are recipients, which cannot be in equilibrium. It is also clear that no PSNE exists if $\max_i z_i(1-e_i^*) = 1$. Suppose that a recipient i has one generator neighbor (corner or border). Then all neighbors of this generator must be recipients, which is impossible.

To investigate the geometric configuration of the generator neighborhoods, we need to consider two cases: (*i*) $\max_i z_i(1-e_i^*) = 2$, and (*ii*) $\max_i z_i(1-e_i^*) = 3$. In case (i), let $e_i^* = 0$ and consider cells surrounding the recipient in cell i, $B_i = \{k : |i-k| = 1$, or $|i-k| = a$, or $|i-k| = a+1$, or $|i-k| = a-1\}$, where $i \ne ka$ for any $k = 1,...,a$. There must be exactly two $g_1, g_2 \in B_i$ with $e_{g_1} = e_{g_2} = 1$, and suppose that $|g_1 - g_2| = n+1$. Then for $j \in B_{g_1} \cap B_{g_2}$, $j \ne i$ there must be exactly two $g_1, g_2 \in B_j$ with $e_{g_1} = e_{g_2} = 1$. But this implies that $z_{g_1}, z_{g_2} \le 2$, which is impossible. Note that neighborhoods of more than four generators cannot exist because this would imply that at least one recipient has three or more generator neighbors. Because for each generator there must be at least three cells $g_1, g_2, g_3 \in B_g$ with $e_{g_1} = e_{g_2} = e_{g_3} = 1$, generators must be arranged in square neighborhoods of exactly four generators.

In case (ii), we first make the following observations. Let $e_i^* = 0$ and consider eight cells surrounding cell i, $B_i = \{k : |i-k| = 1$, or $|i-k| = a$, or $|i-k| = a+1$, or $|i-k| = a-1\}$. The following observations regarding local configurations will be useful.

Observation 1: Suppose that $z_i = 1$. Then we have $\{k : e_k^* = 1, k \in B_i\} \in K = \{i-a-1, i-a+1,\ i+a-1,\ i+a+1\}$. Otherwise, we have $z_k \le 3$, which is impossible.

Observation 2: Suppose that $z_i = 2$. Then we have $\{k,l : e_k^* = e_l^* = 1, k \neq l, k,l \in B_i\} \in K$ $= \{k,l : |k-l| = 1, |k-l| = a, |k-l| = 2a+2 \text{ for } k,l \in B_i\}$. To prove, suppose that $e_j^* = 1_{j=k,l}$ for all $j \in B_i$ and some pair $\{k,l\} \notin K$. Then we have $\sum_{t \in B_x} e_t^* \geq 4$ for $x = k,l$. But this implies that there is a $j \in B_i$ with $z_j \geq 4$ and $e_j^* = 0$, which is impossible. Observation 3: Suppose that $z_i = 3$. Then we have $\{k,l,m : e_k^* = e_l^* = e_m^* = 1,\ k \neq l \neq m,\ \text{for } k,l,m \in B_i\} \in\ K = \{k,l,m : |k-l| = 1$ and $|k-m| = 1, |k-l| = 1$ and $|k-m| = a,\ |k-l|\ = a$ and $|k-m| = a, |k-l| = 1$ and $|k-m| = 2a+2, |k-l| = a$ and $|k-m| = 2a+2$ for $k,l,m \in B_i\}$. That is, the three generators are either located in one of the corners of B_i, or along one of the sides of B_i, or two generators are located at the diagonally opposite corners of B_i, and the third generator borders one of them. All other cases are ruled because $\sum_{t \in B_x} e_t^* \geq 4$ for $x = k,l,m$ implies that there is a $j \in B_i$ with $z_j \geq 4$ and $e_j^* = 0$, which is impossible.

Because $\min_i z_i e_i^* \geq 4$, there must exist at least one generator neighborhood G with $|G| > 4$. Pick a recipient i with $|B_i \cap G| \geq 2$ (there are always at least four recipients in cells 1, *a*, *n*-*a*+1, and *n*). It is possible, because if $|B_i \cap G| = 1$, by observation 1, there must be a recipient in cell $j \in B_i \cap B_g$ with $z_j \geq 2$, where $g \in B_i$ and $e_g^* = 1$. Draw a line passing through the common corners of cells $g \in G \cap B_i$ and cell i that will be shown to "separate" i and G. The line is horizontal if $z_i = 2$ and $|g_1 - g_2| = 1$ for some $g_1, g_2 \in B_i \cap G$, or $z_i = 3$ and $|g_1 - g_2| = 1$ and $|g_1 - g_3| = 1,$ for some $g_1, g_2, g_3 \in B_i \cap G$, so that $[(i-1)/a] > (<)[(g-1)/a]$ for all $g \in G$ when i is "below" and "above," respectively. The line is vertical if $z_i = 2$ and $|g_1 - g_2| = a$ for some $g_1, g_2 \in B_i \cap G$, or $z_i = 3$ and $|g_1 - g_2| = a$, $|g_1 - g_3| = a$ for some g_1, g_2, g_3 $\in B_i \cap G$, so that $i - [(i-1)/a]a > (<) g - [(g-1)/a]a$ for all $g \in G$ when i is "to the left" and "to the right," respectively. The line is diagonal if $z_i = 3$ and $|g_1 - g_2| = 1$,

$|g_1 - g_3| = a$ for some $g_1, g_2, g_3 \in B_i \cap G$, so that we have $g - [(g-1)/a](a+1) >$ $(<)[(n-i)/a-1)]$ or $g - [(g-1)/a](a-1) > (<)[(n-i)/(a+1)]$ for all $g \in G$ depending on the slope of the line and the location of i relative to G.

For concreteness, suppose that i is below G, $z_i = 3$, $|g_1 - g_2| = 1$ and $|g_1 - g_3|$ $=1$ for $g_1, g_2, g_3 \in B_i \cap G$. Then we need to show that $[(g-1)/a] < [(i-1)/a]$ for all $g \in G$. From observations 1, 2, and 3 applied to the recipients in $j \in L_1 = \{r : r \in B_i,$ $e_r^* = 0,\ B_r \cap G \neq \varnothing\ \}$, and the fact that G is connected, it follows that $[(g-1)/a]$ $< [(i-1)/a]$ for $g \in B_j \cap G$. To induct, assume that the same is true for $g \in B_j \cap G$, where $j \in L_t = \{r : e_r^* = 0,\ B_r \cap G = \varnothing\}$ such that $i \in L_t$, and consider $L_{t+1} = \{r : r \in L_t,$ $e_r^* = 0,\ B_r \cap G \neq \varnothing\ \}$ and generators $g \in B_j \cap G$. For $j \in L_{t+1} / L_t$ with $[(i-1)/a]$ $-[(j-1)/a] \geq 2$ the condition is trivially satisfied. We cannot have $[(i-1)/a]$ $-[(j-1)/a] < 0$ because of the induction assumption. Then, using observations 2 and 3, and the fact that G is connected, it follows that $[(g-1)/a] < [(i-1)/a]$ for $g \in B_j \cap G$, $j \in L_{t+1}$. Observe that $L_t \subseteq L_{t+1}$. Therefore, for some z we must have $L_z = L_{z+1}$ because the number of cells is finite. This means that i and G must lie on the opposite sides of the line. The cases where the recipient is in the corner or on the edge of the region are considered completely analogously. Furthermore, each generator neighborhood must have obtuse corners formed by the intersection of diagonal and vertical or diagonal and horizontal lines. Otherwise, there is a $g \in G$ with $z_g \leq 3$, which is impossible.

Proof of Result 6

Part (b). Suppose the distribution e_i^e is a socially efficient allocation with $e_h^e = 0$, $e_l^e = 1$. Then $T(e^e; s) \leq T(e; s)$ for any $e = e^e$ except for $e_h = 1$, $e_l = 0$. Upon substitution, $\sum_{i \neq h,l} C(\sum_{j \neq h,l} d_{ij} e_j^e + d_{il})(1 - e_i^e) + C(\sum_{j \neq h,l} d_{hj} e_j^e + d_{hl})$ $\leq \sum_{i \neq h,l} C(\sum_{j \neq h,l} d_{ij} e_j^e + d_{ih})\ (1 - e_i^e) + C(\sum_{j \neq h,l} d_{lj} e_j^e + d_{hl})$. Because $C'' = 0$, we have

$c_h - \sum_{i=1}^{n} d_{ih}(1-e_i^e) \leq c_l + d_{hl} - \sum_{i=1}^{n} d_{il}(1-e_i^e)$ for each $e_h^e = 0$, $e_l^e = 1$. Rearranging the last inequality yields $c_l - c_h \geq 0.5(\sum_{j=1}^{n} d_{lj} - \sum_{j=1}^{n} d_{hj} - d_{hl}) = 0.5 d_{hl}$.

Proof of Result 8

We need to check both conditions (a) and (b) in Result 7. Note that the monotonicity conditions imply that $c_i = C(\sum_{j=1}^{s} d_{ij}) \geq c_{i+1} = C(\sum_{j=1}^{s} d_{i+1,j})$ for $i = s+1,...,n-1$. And so, to check condition (*a*), let $\phi(i) = i$ for $1 \leq i \leq s$ and $\phi(i) = n - i + s + 1$ for $s+1 \leq i \leq n$. Then we have $\sum_{i \in S_k} \sum_{j \in L} d_{ij} \geq \min_{L} \sum_{i \in S_k} \sum_{j \in L} d_{ij}$ $= \sum_{i \in S_k} \sum_{j=1}^{s} d_{ij} \geq \sum_{i=\phi(s+1)}^{\phi(k+s+1)} \sum_{j=1}^{s} d_{ij}$ for any $k = 1,...,n-s-1$, $L \subset N$, $S_k \subseteq N \setminus L$, $|S_k(L)| = k$, $|L| = s$ since the elements entering the last summation are the smallest by the monotonicity conditions. To check condition (b), let $\varphi(i) = i$ for all $i \in N$, and proceed analogously.

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