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Spatial Production Concentration under Yield Risk and Risk Aversion

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Abstract

A study of equilibrium acreage allocation decisions at the farm and regional levels under risk aversion, yield uncertainty, and endogenous crop prices is undertaken in a two-crop, two-region setting. The main insight is that a partial specialization in one crop at a regional level may be an equilibrium dominant strategy relative to the more diversified crop mix produced on farm. This is due to the trade-offs among the effects of the "natural hedge" based on the negative price-yield correlation on the probability distributions of crop revenues and whole-farm revenue risk reduction through crop enterprise diversification. Another finding is that equilibrium in which each region grows only one crop is unlikely unless there are comparative advantages in production (or marketing) across regions. Other circumstances under which complete regional specialization is possible include a high level of producer risk aversion along with limited benefits derived through crop enterprise diversification due to high correlation of farm-level yields for different crops, and a low crop price elasticity. This applies to situations in which one can identify growing regions that are possessed of two features. The first feature is that growing conditions are relatively homogenous and farm-level yield co-variability is higher within each region as compared with that across the regions. The second feature is that the sizes of the regions are large enough to have a substantial impact on output prices.

Keywords: land allocation, spatial yield dependence, supermodular order.

SPATIAL PRODUCTION CONCENTRATION UNDER YIELD RISK AND RISK AVERSION

Introduction

In recent years, agricultural market analysts have increasingly paid more attention to the spatial concentration of production in both animal and crop agriculture. In particular, the geographic concentration of production of main field crops in several growing regions is a distinctive feature of the U.S. agricultural landscape. Geographic production patterns are shaped by a host of factors, including soil qualities, proximity to input and output markets, vertical coordination, farm size, and marketing environment. Here the focus is on another essential feature of the grower's decision environment: price and yield uncertainty and spatial yield co-variability.

The goal of this paper is to investigate circumstances under which regional specialization in a small number of crops substitutes for farm-level diversification among multiple crops as a risk management strategy when most of the price risk is attributed to output risk. It has long been observed that there exists a negative price-yield correlation when production is concentrated in one geographical area (Harwood et al. 1999; Hart 2003).[1] This seems to provide a "natural hedge" that insures producers against low revenues due to low yields given that individual yields are correlated with the national yield. Because of common soil conditions and weather patterns, the correlation among crop yields is typically higher when production is geographically concentrated in one area. This implies that for producers in a region where a small number of crops are produced, growing multiple crops may be an inferior risk diversification strategy. Introducing other sources of risk may increase the overall farm exposure compared with specializing in a few crops when the individual output is highly correlated with the total supply. However, the expected revenue for the crops with little correlation between price and producer-specific yield is also higher, keeping everything else equal.

And so, depending on the spatial pattern of crop production in the area, there appears to be two distinct approaches to managing risks through acreage allocation.[2] In a region with

multiple crops and dispersed production, growing multiple crops is likely preferred to specializing in one crop because whole-farm revenue risk is subdivided among several relatively independent risks. However, in a region where a small number of crops are grown, there may be enough built-in "natural insurance," and producers may prefer to face the one "insured" source of risk as opposed to multiple but relatively "uninsured" (from the producer's point of view) risks. In a two-crop setting, the trade-offs between these revenue risk management strategies and the resultant inter-regional cropland allocation are analyzed.[3]

This paper contributes to the literature on land allocation under uncertainty (e.g., Collender and Zilberman 1985) in two respects. First, the crop revenue risk is endogenously determined by both producer-specific and aggregate yield risks in a general equilibrium framework. Second, the spatial yield dependence structure is explicitly modeled in a setting with two crops and two regions in a transparent, albeit somewhat rudimentary, way. This study draws on the standard insights from the literature on optimal portfolio selection (e.g., Hadar and Seo 1990; Meyer and Ormiston 1994; Gollier 2001) in order to inquire into the effects of spatial yield dependence on cropland allocation decisions. The concern here is not with the comparative statics properties but rather with the interaction of the optimal land allocation by risk-averse producers dispersed across two regions and the aggregate yield risk. The study inspects the plausibility of the hypothesis that the observed distribution of crop production across geographical areas has, at least in part, a risk management leitmotiv. And so, the nature of crop revenue risk endogeneity modeled in this paper is not due to the input adjustment under general stochastic production technology studied by Chambers and Quiggin (2001) but rather is due to the aggregation of individual yields at the market level.[4] While the endogeneity in the amount of risk one is willing to accept has received much attention in the portfolio selection, self-insurance, and self-protection literatures, the focus here is on the endogeneity of price uncertainty as a result of the inter-regional cropland allocation and its impact on the producer-specific revenue risks.

To analyze the equilibrium level of regional specialization in one crop, the production and demand environments are restricted to attain symmetry across both crops and regions. This rules out any motives for asymmetric land allocation across regions stemming from comparative advantages in crop production or marketing, and allows us to focus on the effects of yield risk aggregation and inter-regional cropland distribution. The imposed

symmetry assures that crop revenues, given appropriate acreage allocation, have identical probability distributions, which is parallel to the standard assumption of ex-ante identical and independent assets used in the analysis of the investment portfolio selection problem. Environments in which regions may partially specialize in one crop are characterized by one or more of the following: a high level of producer risk aversion, a low crop price elasticity, a high co-variability of yields for different crops within a region, and a low variability of revenues under full diversification in both regions. Furthermore, the producers' welfare (in the ex ante sense) may be higher under incomplete specialization relative to full diversification in the environment analyzed in the paper.

An interesting implication of the analysis is that under certain conditions, more volatile prices and an uneven distribution of crop production across regions may contribute to stabilizing crop revenues "more" than would an even distribution of crop production and stable prices. This happens when price volatility is attributed to yield volatility in a way that dampens the fluctuations in the gross crop revenues received by growers but leaves enough upside potential. Producers may benefit from allocating their land between a relatively risky crop with high mean revenue and a relatively safe crop with low mean revenue as compared with the land allocation among crops with moderate risks and mean revenues. This insight complements the literature on government intervention in agricultural markets, in which the effects of price-stabilizing schemes are studied under the assumption that stochastic output price is exogenous (Chambers and Quiggin 2003).

The production environment studied here is plausible when a growing area can be divided into regions, each possessed of a relatively high systemic yield risk as compared with the level of systemic yield risk across the regions (Wang and Zang 2003).[5] Statistical literature and several branches of the economics of risk and uncertainty offer a number of multivariate dependence concepts that can be used to model systemic risk and positive dependence among random variables (Joe 1997). To model spatial yield dependence structure, the concept of dependence known as the supermodular stochastic order is employed. This concept of dependence is commonly used in a wide variety of contexts and is appealing on a number of grounds.

This approach to modeling spatial yield dependence is more general than the "linear additive model" based on the decomposition of the farm-level yield into the sum of systemic

and idiosyncratic components that is usually used in the context of area-yield insurance (Ramaswami and Roe 2004). As Ramaswami and Roe point out, this decomposition requires a considerable amount of structure on farm-level production functions and the implicit level of aggregation. In agricultural economics, the supermodular stochastic order was used by Hennessy, Saak, and Babcock (2003) to model dependence among crop revenues to study the choice between whole-farm and crop-specific revenue insurance.

The rest of the paper is organized as follows. The first section presents the multivariate dependence concept that is used to model spatial intra- and inter-regional farm-level yield dependence structure. Then the model of the acreage allocation by risk-averse producers in a two-crop, two-region setting with yield uncertainty and endogenous prices is developed. Next, it is shown that regions with similar growing conditions cannot specialize in one crop because producing a single crop is dominated by farm-level crop diversification when the price elasticity is sufficiently high, or the intra-regional dependence among yields for different crops is low (Result 1). After investigating some determinants of the expected crop revenues (Lemma 1), a convenient case is presented in which regions as well as the *ex ante* revenues for the two crops are symmetric in all respects. Once it has been established that in such an environment the 50/50 crop mix is always an equilibrium (Lemma 2), it is shown that this is the only possible equilibrium under risk neutrality (Result 2). In the rest of the paper the effect of the introduction of risk aversion on the optimal crop mix when prices and revenues are endogenous at the market level is examined. Following a demonstration that regions may completely specialize in one crop when farm-level yields for different crops are comonotic within each region (Result 3), incomplete specialization in less restrictive and more realistic environments is analyzed. A sufficient condition is provided in the case of mean-variance utility (Result 4), and a necessary and sufficient condition is provided for the existence of a utility function such that equilibria with incomplete specialization exist (Result 5). Some discussion of the limitations and practical aspects of the analysis is offered in the conclusions.

Multivariate Dependence Concept

To model spatial yield correlation we will use the following concept of positive dependence among multivariate random variables (e.g., see Shaked and Shanthikumar 1994).[6]

DEFINITION 1. (The Supermodular Stochastic Order) A multivariate probability distribution $G(\vec{x})$ is said to be smaller than the probability distribution $G'(\vec{x})$ in the supermodular stochastic order (denoted by $\prec_{sm}$) if $\int \phi(\vec{x})dG \le \int \phi(\vec{x})dG'$ for all supermodular functions ϕ for which the expectations exist, where $\vec{x} = \{x_1,...,x_n\}$.

A function ϕ is called supermodular (submodular) if for any pair x_i, x_j with evaluations $x_i' > x_i''$ and $x_j' > x_j''$ we have $\phi(x_1,...,x_i',x_j',...,x_n) + \phi(x_1,...,x_i'',x_j'',...,x_n) \ge (\le)\ \phi(x_1,...,x_i', x_j'',...,x_n) + \phi(x_1,...,x_i'',x_j',...,x_n)$. The supermodularity is equivalent to the "increasing differences" property: $\Delta_i^{\varepsilon}\Delta_j^{\delta}\phi(\vec{x}) \ge 0$ for $i, j = 1,2$, $i \ne j$, $\varepsilon > 0$, and $\delta > 0$, where $\Delta_i^{\varepsilon}\phi(x_1,...,x_n) = \phi(x_1,...,x_i + \varepsilon,...,x_n) - \phi(x_1,...,x_i,...,x_n)$.[7] In other words, the value of a supermodular function increases more with x_i when other $x_j, j \ne i$ take on high values.

The supermodular stochastic order extends the idea of capturing the strength of positive dependence in the bivariate (x_1, x_2) case: "big (small) values of x_1 go with big (small) values of x_2," to a multivariate case. Muller and Scarcini (2000) showed that the supermodular stochastic order satisfies the set of appealing axioms that define a multivariate positive dependence order (Joe 1997). One of the attractive features of the supermodular stochastic ordering is its immediate connection with a more familiar notion of correlation. Namely, let the vector of random variables $X_1,...,X_n$ and $Y_1,...,Y_n$ have the joint probability distribution $G(X)$ and $G'(Y)$, respectively. Then one can easily show that $G \prec_{sm} G'$ implies that $Cov(f(X_i), g(X_j)) \le Cov(f(Y_i), g(Y_j))$ for any $i, j = 1,...,n$, and functions f and g are monotonic in the same direction given that the covariances exist.

To further appreciate why this concept allows us to order the random vectors by the degree of interdependence (or systematic risk) consider the following. Imagine that the joint probability distribution is transformed using the procedure described next. The probability mass is shifted away from the realizations when some components of the vector have high values while others have low values to the realizations when all components of the vector are simultaneously high, or simultaneously low.

In the case of the bivariate random variables with two-point marginal distributions, this procedure is illustrated in Figure 1, where $\varepsilon > 0$ so that $G \prec_{sm} G'$. Note that the realizations when both x_1 and x_2 take "high" or "low" values are more likely while the realizations when x_1 and x_2 "mismatch" are less likely under the transformed distribution G' compared to G. The expectation of a supermodular function increases under the transformed joint probability distribution because supermodular functions "reward" the evaluations with relatively more aligned components ("all are low" or "all are high"). The supermodular ordering is possible only if the distributions $G(\vec{x})$ and $G'(\vec{x})$ are possessed of the same marginals (in what follows, this requirement will be satisfied by definition).

Model

There are two regions, A and B, each consisting of, respectively, n and m farms that produce two crops: c and s. The size of each farm is normalized to equal one unit of land. Yield for crop o in region r on farm i is a random variable $y_{ri}^{o} \in [\underline{y}^{o}, \overline{y}^{o}]$, where $o = c, s$, $r = A, B$, $i = 1,...,n(m)$. The joint probability distribution of farm-level yields for both crops in both regions is $F(y_{A1}^{c},...,y_{An}^{c}, y_{B1}^{c},...,y_{Bm}^{c},\ y_{A1}^{s},...,\ y_{An}^{s},\ y_{B1}^{s},...,y_{Bm}^{s})$ with support on $[\underline{y}^{c}, \overline{y}^{c}]^{nm} \times [\underline{y}^{s}, \overline{y}^{s}]^{nm}$. Let $F_r\ (y_{r1}^{c},...,y_{rn_r}^{c}, y_{r1}^{s},...,\ y_{rn_r}^{s})$ be the marginal joint distribution of farm-level yields for both crops in region r, $F_r^{o}(y_{r1}^{o},...,y_{rn_r}^{o})$ be the marginal joint distribution of farm-level yields for crop o in region r, and $F_{ri}^{o}(y_{ri}^{o})$ be the distribution of yields for crop o in region r on farm i, all consistent with F, where $n_A = n$ and $n_B = m$. We make the following assumptions about the nature of the spatial farm-level yield dependence structure in each region and across the regions.

$G(x_1, x_2)$	x_1^{low}	x_1^{high}		$G'(x_1, x_2)$	x_1^{low}	x_1^{high}
x_2^{low}	$p_{low,low}$	$p_{low,high}$	$\Rightarrow$	x_2^{low}	$p_{low,low} + \varepsilon$	$p_{low,high} - \varepsilon$
x_2^{high}	$p_{high,low}$	$p_{high,high}$		x_2^{high}	$p_{high,low} - \varepsilon$	$p_{high,high} + \varepsilon$

FIGURE 1. Increase in positive dependence

ASSUMPTION 1. (Spatial Yield Dependence Structure)

(a) $F = F_A \cdot F_B$ (independence across regions);

(b) $F^o_{r1} \cdot \ldots \cdot F^o_{rn_r} \prec_{sm} F^o_r$ for $o = c, s$ and $r = A, B$ (positive dependence across farm yields in each region for the same crop);

(c) $F^c_r \cdot F^s_r \prec_{sm} F_r$ for $r = A, B$ (positive dependence across farm yields in each region for different crops).

According to part (a), the yields are independent across regions but not necessarily across crops within a region. Namely, parts (b) and (c) state that the yields are positively correlated for the same crop as well as across crops within a region.[8] Part (c) implies that yields for different crops on an individual farm are also positively dependent, that is, $F^c_{ri}(y^c_{ri}) \cdot F^s_{ri}(y^s_{ri}) \prec_{sm} F_{ri}(y^c_{ri}, y^s_{ri})$. This spatial yield dependence structure is consistent with the condition that crop yields follow a finite-range positive dependent (f.r.p.d.) spatial process (Wang and Zhang 2003). A spatial yield process is f.r.p.d. if the correlation between yields on any two land plots is positive when the distance between the plots is within a certain range and is zero or negative otherwise.[9]

The inverse demand for crop o is $P_o(Y^o)$, where $Y^o = \sum_{r=A,B} \sum_j \alpha^o_{rj} y^o_{rj}$ is the aggregate output harvested in both regions, and α^o_{ri} is the share of farm i's acreage sown to crop o in region r, $\alpha^c_{rj} + \alpha^s_{rj} = 1$.[10] The per acre revenue for crop o in region r on farm i is $R^o_{ri} = P_o(Y^o)\ y^o_{ri} - c^o_{ri}$, where c^o_{ri} is the per acre production cost. Each producer owns one farm (a unit of land) and is a von-Neumann and Morgenstern expected utility maximizer. The producer's expected utility conditional on the acreage allocation is $U_{ri}(\alpha_{ri}) = Eu((1-\alpha_{ri})R^c_{ri} + \alpha_{ri} R^s_{ri})$, where u is a twice differentiable utility function with $u'_{ri} > 0$ and $u'' < 0$; $E[.]$ is the expectation operator with respect to the distribution F or the relevant marginals of F; and $\alpha_{ri} = 1 - \alpha^c_{ri}$.[11] Producers choose the acreage allocation between crops c and s, $\alpha_{ri} \in [0,1]$, to solve

$$\max_{\alpha_{ri}} U_{ri}(\alpha_{ri}), \tag{1}$$

with the interior optimality condition given by

$$V_{ri}(\alpha) = E[u'((1-\alpha)R_{ri}^{c} + \alpha R_{ri}^{s})(R_{ri}^{s} - R_{ri}^{c})] = 0 . \tag{2}$$

The problem is well behaved because $U_{ri}(\alpha_{ri})$ is strictly concave in α_{ri} when prices are exogenous to the producer's problem. The mix of the crops is produced if $V_{ri}(\alpha) = 0$ for $\alpha \in (0,1)$, and the farm is specialized in one crop with $\alpha = 0$ if $V_{ri}(0) \le 0$, or $\alpha = 1$ if $V_{ri}(1) \ge 0$. Next we verify that each region cannot be an exclusive producer of a crop when there are no comparative advantages in crop production across regions and growing a mix of crops lowers overall risk exposure of the operation. Proofs of results are in the Appendix.

RESULT 1. Suppose that Assumption 1 holds , and either (i) $P_o'(\sum_i y_{ri}^o) y_{rj}^o + P_o(\sum_i y_{ri}^o) \le 0$ for all evaluations y_{rj}^o, $o = c, s$; or (ii) $F_r\ (y_{r1}^c,..., y_{rn_r}^c, y_{r1}^s,..., y_{rn_r}^s) = F_r^c(y_{r1}^c,..., y_{rn_r}^c) \cdot F_r^s(y_{r1}^s, ..., y_{rn_r}^s)$, $r = A, B$, and (iii) $E[y_{Ai}^c]/E[y_{Bj}^c] = E[y_{Ai}^s]/E[y_{Bj}^s] = (c_{Ai}^c - c_{Ai}^s)/(c_{Bj}^c - c_{Bj}^s)$ for $i = 1,...,n$, and $j = 1,...,m$. Then both regions cannot specialize in equilibrium.

Result 1 presents sufficient conditions such that under complete regional specialization in one crop the expected revenue for the crop that is not locally grown warrants an adjustment in cropland allocation. This holds for any level of non-negative, farm-level yield dependence among different crops in each region as well as any degree of producer risk aversion.

Note that if the curvature properties of the inverse demand functions are known, that is, $P_o'' \le (>) 0$, condition (i) is satisfied if $P_o'(n\underline{y}^o)\underline{y}^o + P_o(n\underline{y}^o) \le 0$, or $e_o(n\underline{y}^o) \ge n$ (respectively, $P_o'(n\bar{y}^o)\underline{y}^o + P_o(n\underline{y}^o) \le 0$, or $e_o(n\bar{y}^o) \ge n(P_o(n\underline{y}^o)\bar{y}^o)/(P_o(n\bar{y}^o)\underline{y}^o)$), both of which are easy to verify, where $e_o(y) = -P_o'(y)y/P_o(y)$ is the price elasticity for crop o. Any one of these conditions assures that the expected marginal utility from switching to an alternative crop is sufficiently high when each region producers only one crop. This happens because the local yields for the crop grown elsewhere are uncorrelated with that crop's price, and the prices are sufficiently elastic to assure that "good weather is worse than bad weather." Because of the positive intra-farm yield correlation for different crops, high local farm-level

yields for a crop grown elsewhere coincide with high yields for the crop grown in the area that cause the crop's price to drop (highly elastic) and the marginal utility of gross farm income (that exclusively consists of the revenue from the locally grown crop) to rise. The converse is true when yields for the alternative crop are low. To recap, high local yields for the crop produced mostly elsewhere likely coincide with high marginal utility, and low local yields for the crop produced mostly elsewhere likely coincide with low marginal utility since farm-level yields for different crops are correlated. It is in this way that, under regional specialization, growing a mix of crops constitutes a form of protection against income variability. It works because the price for the other crop does not depend on the local yields.

Otherwise, a potentially onerous condition (i) can be dispensed with if the farm-level yields for different crops are assumed to be independent, as in condition (ii). Note that when we abandon these assumptions, in what follows, equilibrium with complete specialization will be shown to exist (see the section titled "Complete Specialization under Risk Aversion," Result 3). Condition (iii) assures that no farm or region has a "built-in" comparative advantage to specialize in a certain crop. Under these conditions, complete specialization in both regions is impossible under any level of risk aversion (including risk neutrality.). The intuition is as follows.

Suppose that both regions completely specialize in one of the crops, for example, region A is the sole producer of crop c, and region B is the sole producer of crop s. As is well known from the analysis of the standard portfolio problem, given the opportunity to subdivide risks, a risk-averter may avoid investing in one of the two independent risky assets only if the mean return for that asset is lower. Therefore, the expectation of crop s revenue in region A must be lower than that for crop c, while the expectation of crop c revenue in region B must be lower than that for crop s. But these conditions cannot hold simultaneously because, under Assumption 1 and complete specialization, the price risk is independent of the individual yield risk for the crop that is not grown in the region. The fact that the individual yield and the price as a function of aggregated individual yields are negatively correlated assures that for producers in region A the expected revenue for the alternative crop s will be higher than for crop c, and the converse is true for producers in region B. Because there is no regional comparative advantage, this is impossible in equilibrium with complete specialization. The following example demonstrates that even when complete specialization in one crop elimi-

nates any randomness in the crop revenue it cannot be in equilibrium, no matter what preferences toward risk are exhibited by producers.

EXAMPLE 1. Let $\Pr\{y_{ri}^{o} = y_{r}^{o}\} = 1$ so that all yield risk is systematic within a region, and there is no farm-specific risk. Furthermore, we hold that yields for different crops y_{r}^{c} and y_{r}^{s} are independent in each region, $E[y_{A}^{o}] = E[y_{B}^{o}]$ for $o = c, s$, the inverse demand functions are $P_{c}(y) = P_{s}(y) = 1/y$, and production costs are invariant across farms in both regions, and are normalized to zero, $c_{ri}^{o} = 0$. The crop revenues evaluated at $\alpha_{Ai}^{*} = 0$ and $\alpha_{Bj}^{*} = 1$ are $R_{Ai}^{c} = 1/n$, $R_{Ai}^{s} = y_{A}^{s}/(my_{B}^{s})$, $R_{Bi}^{s} = 1/m$, and $R_{Bi}^{c} = y_{B}^{c}/(ny_{A}^{c})$. Hence, using the proof of Result 1, complete specialization implies that $m/n > E[y_{A}^{s}/y_{B}^{s}]$ and $n/m > E[y_{B}^{c}/y_{A}^{c}]$. However, this is impossible since $E[y_{A}^{o}/y_{B}^{o}] > 1$ for $o = c, s$. This is because the expected revenue for the crop with price risk that is independent of the producer-specific yield risk is greater than that for the crop with a price that is negatively correlated with the farm-level yield.

To proceed with the analysis of the possible equilibrium levels of regional specialization, it is convenient to consider environments with homogenous producers in the manner of Example 1. The characterization of equilibrium land allocation is straightforward when there is no heterogeneity in production decisions within each region. To this end, we make the following.

ASSUMPTION 2. (Symmetry Across Farms within Each Region)

(a) $F_{r}(y_{r1}^{c},...,y_{rn_{r}}^{c}, y_{r1}^{s},...,y_{rn_{r}}^{s}) = F_{r}(y_{r\pi(1)}^{c},...,y_{r\pi(n_{r})}^{c}, y_{r\varphi(1)}^{s},...,y_{r\varphi(n_{r})}^{s})$ for $o = c, s$, $r = A, B$, and any permutations of indices π, φ;

(b) $c_{ri}^{o} = c_{r}^{o}$ for all $i = 1,...,n_{r}$, $r = A, B$ (production costs are invariant across farms);

(c) $u_{ri}(.) = u(.)$ for $i = 1,...,n_{r}$, $r = A, B$ (attitudes to risk are invariant across producers).

Part (a) of Assumption 2 states that the farms in each region are homogenous in the sense of having identical farm-level yield probability distributions for both crops so that, in particular, $F_{ri}(y^c, y^s) = F_{rj}(y^c, y^s)$ for any i, j, $r = A, B$. A commonly used restriction that yields are identically and independently distributed (i.i.d.) is a special case of this assumption since we allow for positive dependence among farm-level yields in each region (across crops and across farms). In addition, according to parts (b) and (c), production costs as well as risk attitudes are common for all farmers in the region. Because within a region all producers are (*ex ante*) identical by Assumption 2, and the solution of (1) is unique, in equilibrium we have $\alpha_{ri}^* = \alpha_r$, $r = A, B$.

In general, the extent of specialization as measured by the difference between the shares of acres under crop s in region A, α_A, and in region B, α_B, has an ambiguous effect on the crop revenues. This is because the yield risk enters the gross farm revenue, which is equal to the product of the crop price—a decreasing function of the aggregate output—and the individual farm output twice. Let us consider this point in more detail.

Expected Crop Revenues and Inter-regional Acreage Allocation

Suppose that a share of crop c produced in a region A is large and begins to decline. Then the negative price-yield correlation for crop c in region A is likely to weaken because a smaller share of the aggregate yield depends on the average yield in region A, which is correlated with an individual farm-level yield in region A. Therefore, the expected value of crop c revenue increases because a large individual yield for crop c in region A is less likely to be offset by a small price, and, conversely, a small yield is less likely to coincide with a high price. On the other hand, the volatility of the aggregate yield also declines because there is less "undiversified" systematic risk present in region A that enters the aggregate yield. This lowers the expected value of the revenue function if the crop's inverse demand is convex in the total output. This is formalized in the following lemma.

LEMMA 1. Suppose that Assumption 1 and part (a) of Assumption 2 hold, $\Pr\{\sum_i y_{Ai}^o \le Y\}$ $= \Pr\{\sum_i y_{Bi}^o \le Y\}$, and crop o is produced exclusively in region A. Then the expected revenue

for crop o in region A, ER^{o}_{Ai}, increases (decreases) with the share of the crop produced in region B depending on $-P''_o(y)y/P'_o \leq (>) 1$ for all $y \in [n\underline{y}^o, n\overline{y}^o]$.

Note that the expected revenue increases as the region gives up its exclusive position in production if the inverse demand function is concave. In this case both the decline in the aggregate output volatility (some of the region's systematic risk is now diversified away by the other region's systematic risk) and the weakening of the negative price-yield correlation work in the same direction.

In order to understand the role of risk diversification motives behind acreage allocation it appears useful to first consider equilibrium under risk neutraility and then study the equilibrium effects of risk aversion. However, Lemma 1 illustrates that, without some restrictions on the demand environment, the expected crop revenue differential, $E[R^c_{ri} - R^s_{ri}]$, is, in general, non-monotone in the extent of regional (farm-level) specialization in a certain crop, α^*_r. This may lead to the multiplicity of equilibrium levels of regional specialization caused by the special features of the demand and production environments as well as the consequences of expected utility maximization. Therefore, it may be difficult to disentangle the effect of risk management considerations (second-order effect) on equilibrium land allocation pattern when the expected incremental profit from switching crops (first-order effect) is non-monotone in the inter-regional crop production distribution.

To circumvent this difficulty and highlight the role of risk aversion in acreage allocation patterns, we consider a special benchmark case where equilibrium under risk neutrality is unique. It turns out that the expected crop revenue differential is monotone in the regional share of the total plantings when revenues are held to follow identical probability distributions for both crops in both regions. Note that this approach generates monotonicity without any restrictions on the inverse demand functions.

Identical Crops, Regions, and 50/50 Diversification Equilibrium

To proceed, the following symmetry assumption about the probability distributions of crop revenues is introduced. Throughout the rest of the paper, total acreage under each crop is kept constant and invariant across crops: $\alpha_A + \alpha_B = 1$ and $n = m$ (regions are of the same

size). Let $T(R_{ri}^c, R_{ri}^s; \alpha)$ denote the farm-level joint probability distribution of the crop revenues on farm i in region r conditional on the acreage allocation α, where $R_{ri}^o(\alpha)$ $= P_o(\alpha \sum_j y_{Aj}^o + (1-\alpha)\sum_j y_{Bj}^o) y_{ri}^o - c_{ri}^o$.

ASSUMPTION 3. (Symmetry across Crops and Regions)

(a) For any $\alpha \in [0,1]$ the crop revenues are exchangeable random variables: $T(R_{ri}^c, R_{ri}^s; \alpha) = T(R_{ri}^s, R_{ri}^c; \alpha)$, $r = A, B$ (symmetry across crops within each region);

(b) $F_A(y_{A1}^c, ..., y_{An}^c, y_{A1}^s, ..., y_{An}^s) = F_B(y_{B1}^c, ..., y_{Bn}^c, y_{B1}^s, ..., y_{Bn}^s)$ (symmetry across regions);

(c) $c_r^o = c^o$ for $r = A, B$ (production costs invariant across regions).

For example, part (a) of the assumption is satisfied under the following circumstances. The yields for the two crops differ by a scale parameter, say, $y_{ri}^c = a y_{ri}^s$, so that $F^c(y_{A1}^c, ..., y_{Bn}^c)$ $= F^s(y_{A1}^c / a, ..., y_{Bn}^s / a)$, the scale-parameter-adjusted inverse demands for crops coincide, $P^c(y) = aP^s(ay)$, and the per acre production costs are the same for both crops, $c^c = c^s$. However, such stringent conditions are not necessary in order for part (a) of Assumption 3 to hold. Part (b) stipulates that the yields for both crops are *ex ante* identical across regions in the sense of both having identical marginals and identical intra-regional dependence structure in each region. In other words, yields in region A are an independent replica of yields in region B. Part (c) assures that all aspects of the production environment are invariant across regions as well as across farms within each region.

Given that $\alpha_A = \alpha$ and $\alpha_B = 1-\alpha$, the expected utility for a producer in region A is $U_{Ai}(\alpha) = Eu((1-\alpha)\, R_{Ai}^c(1-\alpha) + \alpha R_{Ai}^s(\alpha))$, while that for a producer in region B is $U_{Bj}(1-\alpha) = Eu(\alpha R_{Bj}^c(1-\alpha) + (1-\alpha) R_{Bj}^s(\alpha))$.

LEMMA 2. Suppose Assumptions 1 through 3 hold. Then $U_{Ai}(\alpha) = U_{Bj}(1-\alpha)$ for all i, j, where $\alpha = \alpha_A = 1-\alpha_B$, and there always exists equilibrium with $\alpha_A^* = \alpha_B^* = 0.5$.

In light of Lemma 2, the main bite of Assumption 3 is that it allows us to characterize equilibrium using just one variable, α, since the regions are held to reciprocate each other's land allocation pattern. The focus of the inquiry is then solely on the regional specialization, $\alpha = \alpha_A = 1 - \alpha_B$, in which we dropped the subscript for the region. In addition, Assumptions 2 and 3 provide a very convenient calibration because there always exists equilibrium where producers in both regions follow a 50/50 crop enterprise diversification plan. Next we demonstrate that the 50/50 plan is the only possible equilibrium when producers are risk neutral.

Uniqueness of 50/50 Acreage Allocation under Risk Neutrality

To isolate better the role of risk aversion on the diversification behavior as revealed through the crop mix produced on farm, we consider the case of risk-neutral producers. Under risk neutrality, individual decisions of producers within a region are indeterminate, so we continue to refer to α $(1-\alpha)$ as the share of acreage allocated to crop *s* in region *A* (*B*) both at farm and regional levels.

RESULT 2. Suppose that Assumptions 1 through 3 hold and producers are risk neutral. Then equilibrium in which both regions allocate half of their acreage to each crop, $\alpha^* = 0.5$, is unique.

Note that part (a) of Assumption 1 can be weakened by only requiring that the positive dependence of yields within the region exceeds that across regions: $F^o_{Ai,Bj} \prec_{sm} F^o_{Ai,Ak}$ for all i, j, k, which will assure that $ER^c_{Ai}(1) < ER^s_{Ai}(0)$ and $ER^c_{Ai}(0) > ER^s_{Ai}(1)$, and the case is similar for region *B*. We find that under Assumptions 1 through 3, the expected incremental revenue from switching crops is monotone in the share of the regional acreage in the total acres under a crop. This is because the increase in the aggregate yield volatility due to a greater share of undiversified systematic risk present at the regional level has the same effect on the revenues from both crops. This allows us to isolate the effect of the increase in the price-yield correlation on the expected revenue, which has an unambiguous sign. While unrealistic, Assumption 3 is very useful in highlighting the role of risk aversion and risk

diversification motives in equilibrium acreage allocation patterns across different regions. This is the subject of the inquiry in the following sections.

Complete Specialization under Risk Aversion

In this section we decompose the yield risk into systematic (regional) and farm-specific components in the manner of Ramaswami and Roe (2004). Furthermore, we restrict part (c) of Assumption 1 by imposing a particularly strong dependence structure among the (average) yields for different crops within each region as follows.

ASSUMPTION 4. (Yield Decomposition and High Correlation among Yields for Different Crops in a Region)

(a) Yield for crop o in region r on farm i is $y_{ri}^{o} = \beta_{ri}^{o} y_{r}^{o} + \varepsilon_{ri}^{o}$, where $\beta_{ri}^{o} > 0$, $\sum \beta_{ri}^{o} = n$, and $E[\varepsilon_{ri}^{o} \mid y_{r}^{o} = y] = 0$ for all y, $o = c, s$, $r = A, B$, $i = 1,...,n$;

(b) Average yields in each region are "highly" correlated across crops, $F_r(y_r^c, y_r^s) = \min[F_r^c(y_r^c),\ F_r^s(y_r^s)]$, $r = A, B$ (comonotonic systemic components of yields for different crops within each region).

Part (a) of Assumption 4 is the standard "linear additive" formulation of spatial yield dependence, $E[\sum y_{ri}^{o}] = n y_{r}^{o}$. Part (b) postulates that in each region the average yields for the two crops are highly (in fact, "perfectly" in the sense of possible dependence structures) correlated. The distribution function of the form $\min[F_r^c(y_r^c),\ F_r^s(y_r^s)]$ is called the upper Frechet bound (Joe 1997). It is a well-known property of the upper Frechet bound that any probability distribution $H(x, y) \le \min[H_x(x),\ H_y(y)]$ where $H_x(x), H_y(y)$ are marginals of $H(x, y)$. The random variables with distribution function $H(x, y) = \min[H_x(x),\ H_y(y)]$ are called comonotonic and are possessed of the highest possible degree of dependence, that is, $H(x, y) \prec_{sm} \min[H_x(x),\ H_y(y)]$ for any distribution function $H(x, y)$. Observe that if $F_r^s(y) = F_r^c(y)$, then the imposed correlation structure implies that $\Pr(y_r^s = y_r^c) = 1$.

Next we determine conditions when complete regional specialization in one crop is possible in equilibrium under the assumption that the "average" yields for different crops in each

region are perfectly correlated. Let $Q_{ri}(w) = -u''_{ri}(w)/u'_{ri}(w)$ and $\hat{Q}_{ri}(w) = wQ_{ri}(w)$ denote the Arrow-Pratt measures of risk aversion and relative risk aversion, respectively.

RESULT 3. Suppose that part (a) of Assumption 1, part (a) of Assumption 3 at $\alpha = 0$ and $\alpha = 1$, and Assumption 4 hold. In addition, let (*i*) $\Pr(\varepsilon^o_{ri} = 0) = 1$ for all o, r, i; and (*ii*) $(\hat{Q}_{ri}(R^o_{ri}) + Q_{ri}(R^o_{ri})c^o_{ri})(1 - e_o(ny)) \geq 1$ for all $y \in [\underline{y}^o, \bar{y}^o]$, where $R^o_{ri} = P_o(ny)\beta^o_{ri}y - c^o_{ri}$, $o = c, s$. Then there exist equilibria in which each region produces only one crop, $\alpha^*_{Ai} = 0$ and $\alpha^*_{Bj} = 1$, or $\alpha^*_{Ai} = 1$ and $\alpha^*_{Bj} = 0$.

Note that in equilibrium where each farm grows only one crop, farmers have no incentive to grow both crops even though the expected revenue for the crop produced locally is less than that for the crop grown in the other region (see the proof of Result 4). In this case, the benefits provided by the "natural hedge" outweigh the benefits of whole-farm revenue risk reduction achieved through crop diversification.

Condition (i) of Result 3 states that there is no farm-specific yield risk for both crops.[12] This, combined with part (b) of Assumption 4, assumes away any benefits from crop diversification at the farm level. These restrictions are unrealistic and are made purely for the expositional convenience. However, note that we make no use of Assumption 2 that farmers are homogenous in the sense of having identical yield distributions and attitudes toward risk because the optimality conditions at the corner solutions hold as inequalities. Also, Assumption 3 that crop revenues have identical probability distributions conditional on symmetric acreage allocation can be easily relaxed. In this light, Result 3 emphasizes risk-aversion as the main driver behind regional specialization. The postulated very strong positive dependence among yields for different crops within the region allows us to find a simple condition for equilibrium regional specialization that relates to the degrees of risk aversion and relative risk aversion and has an interesting intuitive interpretation.

Because the diversification benefits associated with multiple crops are assumed away, the "natural hedge" provided by the negative price-yield relationship is of particularly high value to risk-averters. However, even in that case, the degree of risk aversion and/or relative risk aversion as well as production costs must be sufficiently high to counterbalance the

incentive provided by switching to the crop produced in the other region that has higher expected revenue for local producers. A sufficiently low price elasticity (less than one) guarantees that this, in fact, is the case.

Note that an increase in the price elasticity has an ambiguous effect on the incentive to specialize in one crop. On the one hand, the "natural hedge' effect is enhanced because of a better cap on the revenue when yields are low. On the other hand, the expected revenue from growing the alternative crop is also higher. When the price elasticity is sufficiently high (greater than one), the latter effect always dominates, and no equilibrium with complete specialization exists. Compare this finding with sufficient conditions (i) and (ii) for equilibrium crop diversification in Result 1.

Incomplete Specialization under Risk Aversion

In the previous section, the outcomes in which regions completely specialize in one of the crops were due to the limited benefits of growing a plural number of crops as a result of the imposed very strong positive dependence among the yields for different crops within the region. In this section we demonstrate that this is neither necessary nor sufficient to generate equilibria with incomplete diversification as opposed to the unique 50/50 diversification equilibrium under risk neutrality previously characterized. And so, in contrast to Assumption 4, we assume that yields for different crops are independent within a region (in addition to the independence across the regions in part (a) of Assumption 1).

ASSUMPTION 5. $F_r\,(y^c_{r1},...,y^c_{rn},y^s_{r1},...,y^s_{rn}) = F^c_r(y^c_{r1},...,y^c_{rn})F^s_r(y^s_{r1},...,y^s_{rn})$ for $r = A,B$.

In particular, Assumption 5 implies that yields for different crops on an individual farm are also independent, that is, $F_{ri}(y^c_{ri},y^s_{ri}) = F^c_{ri}(y^c_{ri})F^s_{ri}(y^s_{ri})$, and thus proffer a venue for an effective risk management through crop enterprise diversification.. By Lemma 2, the equilibrium with incomplete diversification, $\alpha \in (0,1)/\{0.5\}$, is given by[13]

$$V_{Ai}(\alpha) = Eu'((1-\alpha)R^c_{Ai}(1-\alpha) + \alpha R^s_{Ai}(\alpha))(R^s_{Ai}(\alpha) - R^c_{Ai}(1-\alpha)) = 0\,. \qquad (3)$$

In order to ascertain whether $\alpha^* = 0.5$ is the only possible equilibrium under risk aversion, we will need some properties of the interior optimality condition (3). The following lemma is

an immediate consequence of Result 1 as part (a) of Assumption 2 (farms are identical in each region), parts (b) and (c) of Assumption 3 (regions are identical), and Assumption 5 (yields for different crops within the region are independent) imply that conditions (ii) and (iii) in Result 1 are satisfied.

LEMMA 3. Suppose Assumptions 1 through 3 and 5 hold. Then $V_{Ai}(0) > 0$, $V_{Ai}(1) < 0$.

According to Lemma 3, equilibria where each region completely specializes in one crop are impossible under Assumptions 1 through 3, and 5. Also, Lemmas 2 and 3 imply that if $V'_{Ai}(0.5) > 0$ then there must exist equilibria with $\alpha \in (0,0.5) \cup (0.5,1)$. Furthermore, any equilibrium with $\alpha \neq 0.5$ is non-trivially different from complete diversification equilibrium because $V''_{Ai}(0.5) = 0$ since we have $V''_{Ai}(\alpha) = -V''_{Ai}(1-\alpha)$. After some algebra, differentiation of equation (3) and evaluation at $\alpha = 0.5$ yields

$$V'_{Ai}(0.5) = E[u'(w)((R^s_A)' - Q(w)(R^s_A - R^c_A)\ (R^s_A - R^c_A + (R^s_A)'))], \tag{4}$$

where $w = 0.5(R^c_A + R^s_A)$, $R^o_A = R^o_{Ai}(0.5)$ and $(R^s_A)' = P'_s(0.5\sum_{r,j} y^s_{rj})y^s_{Ai}(\sum_{r,j} y^s_{Aj} - \sum_{r,j} y^s_{Bj})$, in order to minimize notation. Expression (4) is difficult to sign for several reasons. One is the a priori ambiguous effect of the change in risk on the demand for risky assets in the presence of other risky investments (Gollier 2001). Another reason is, of course, due to the endogeneity of crop revenue risks at the market level. One way to proceed is to impose some restrictions on the properties of the utility function and the demand and production environments. We shall take this route in the next section.

Incomplete Specialization: Mean-Variance Preferences

Consider the case of the mean-variance preferences: $U_{Ai}(\alpha) = E[w] - \lambda Var[w]$, where $w = (1-\alpha)R^c_{Ai}(1-\alpha) + \alpha R^s_{Ai}(\alpha)$, and $\lambda \geq 0$ is the disutility from bearing a unit of risk.

RESULT 4. Suppose that Assumptions 1 through 3 and 5 hold, $U_{Ai}(\alpha) = E[w] - \lambda Var[w]$, and $Var[R_A^s] < -Cov[R_A^s, (R_A^s)']$. Then equilibria with incomplete diversification, $\alpha^* \in (0,0.5) \cup (0.5,1)$ exist if $\lambda > \hat{\lambda}$, where $\hat{\lambda} = E[(R_A^s)'] / 2(Var[R_A^s] + Cov[R_A^s, (R_A^s)'])$.

Note that the minimum required level of risk disutility, $\hat{\lambda}$, increases with the variance of crop revenues under complete diversification, $Var[R_A^s]$, because $E[(R_A^s)'] < 0$ (see the proof of Result 4). However, in general, the intuition behind the minimum required level of risk disutility needs to be viewed with caution because of the undesirable property of the quadratic utility function that the level of risk aversion, $Q(w)$, increases with wealth. Furthermore, from the proof of Result 2 we know that the expected crop revenue differential is a single-crossing function of α: $ER_{Ai}^c < ER_{Ai}^s$ for $\alpha < 0.5$, and $ER_{Ai}^c > ER_{Ai}^s$ for $\alpha > 0.5$.[14] Because both crops are produced in equilibria with $\alpha^* \in (0,0.5)$, the riskiness associated with the revenue for crop c in region A is also less than that for crop s, $Var[R_{Ai}^c] < Var[R_{Ai}^s]$.[15] The situation is reversed in equilibria with $\alpha^* \in (0.5,1)$. The trade-offs between the expected crop revenues and risks in region B are analogous.

Observe that the producer welfare measured by the expected utility of farming may be higher when $\alpha^* \in (0,1)/\{0.5\}$ (and may, therefore, provide a means of redistributing welfare in the society from consumers to agricultural producers). Under partial specialization, by symmetry, the expected utility must have at least three local optima ($\alpha_1^* < \alpha_2^* = 0.5 < \alpha_3^*$). Among these there must be equilibria in which the producer's expected utility is greater than that in the full diversification equilibrium. This is because the expected utility increases with α in some range around $\alpha = 0.5$. Note that conditions $V_{Ai}'(0.5) > 0$ and $V_{Ai}(0.5) = 0$ imply that $V_{Ai}(\alpha) > 0$ for all $\alpha \in (0.5, \hat{\alpha})$, where $\hat{\alpha} = \inf\{\alpha : V_{Ai}(\alpha) = 0,\ \alpha > 0.5\} < 1$ because, by Lemma 3, $V_{Ai}(1) < 0$ and function $V_{Ai}(\alpha)$ is continuous (as it is differentiable). This implies that there is an $\hat{\alpha} \neq 0.5$ with $U_{ri}(\hat{\alpha}) = U_{ri}(1-\hat{\alpha}) > U_{ri}(0.5)$ that is also in equilibrium since $V_{Ai}(\hat{\alpha}) = -V_{Ai}(1-\hat{\alpha}) = 0$. Summarizing, we have the following.

COROLLARY. Suppose that Assumptions 1 through 3 and 5 hold, and $V'_{Ai}(0.5) > 0$. Let $\mathrm{E} = \{\alpha : V_{ri}(\alpha) = 0,\ \alpha \neq 0.5\}$. Then $\sup_{\alpha \in \mathrm{E}} U_{ri}(\alpha) > U_{ri}(0.5)$.

The following example illustrates some of the equilibrium properties and welfare implications in the case of mean-variance utility.

EXAMPLE 2. As in Example 1, let $\Pr\{y^o_{ri} = y^o_r\} = 1$ so that all yield risk is systematic within a region, and there is no farm-specific risk. Under Assumption 5 yields for different crops y^c_r and y^s_r are independent within a region. The inverse demand functions are $P_c(y) = P_s(y) = 1/y$, and production costs are zero, $c^s = c^c = 0$. Furthermore, let y^o_r follow a two-point distribution: $y^o_r = \underline{y}$ and $y^o_r = \bar{y}$ with equal probability. The crop revenues are $R^c_{Ai}(\alpha) = R(1-\alpha; y^c)$ and $R^s_{Ai}(\alpha) = R(\alpha; y^s)$, where $R(\alpha; y^o) = (n(\alpha + (1-\alpha)y^o))^{-1}$, and the random variable $y^o = y^o_B / y^o_A$ follows a three-point distribution ($\bar{y}/\underline{y}$,0.25;1,0.5;$\underline{y}/\bar{y}$, 0.25) because y^o_A and y^o_B are independent, $o = c, s$. Hence, the first and second moments of the crop revenues are calculated as follows: $ER^c_{Ai}(\alpha) = ER(1-\alpha) = 1/n(0.5 + 0.25/(1-\alpha + \alpha\underline{y}/\bar{y}) + 0.25/(1-\alpha+\alpha\bar{y}/\underline{y}))$, $ER^c_{Ai}(\alpha)^2 = ER(1-\alpha)^2 = 1/n^2(0.5 + 0.25/(1-\alpha + \alpha\underline{y}/\bar{y})^2 + 0.25/(1-\alpha+\alpha\bar{y}/\underline{y})^2)$, $ER^s_{Ai}(\alpha) = ER(\alpha)$, and $ER^s_{Ai}(\alpha)^2 = ER(\alpha)^2$.

In this case, the expectation of the returns to farming is invariant to the acreage allocation because $Ew(\alpha) = (1-\alpha)ER^c_{Ai}(\alpha) + \alpha ER^s_{Ai}(\alpha) = 1/n$ for all $\alpha \in [0,1]$. And so, acreage allocation, α, affects the expected utility only through the riskiness of the grower's position. It is straightforward to show that the variance of the crop revenue mix, $Var[w(\alpha)]$, is U-shaped with the peak at $\alpha = 0.5$. The variances of revenues for crop c and s in region A respectively increase and decrease as α increases (the converse is true for region B). The returns to farming are "risk free" (and hence maximize producer welfare) if each region specializes in one crop, say, region A is the exclusive producer of crop c and region B is the exclusive producer of crop s. Then under any resolution of yield uncertainty we have $R^c_{Ai}(0) = R^s_{Bj}(0) = 1/n$ for all producers. In this case, the expected utility of wealth derived

from farming is $E[u(R^c_{Ai}(0))] = E[u(R^s_{Bj}(0))] = 1/n$ for each producer in both regions. From the producer welfare point of view, the expected utility is maximized under complete specialization in one crop in each region: $\alpha^e = 0$ or $\alpha^e = 1$, that is, $U_{ri}(0) = U_{ri}(1) > U_{ri}(\alpha)$ for all $\alpha \in (0,1)$. This is because an individual grower does not take into account the effect of acreage allocation on systematic yield risk at the regional level.

The trade-offs between risk and return for the two crops are depicted in Figure 2, where $\sigma(R^o_{ri})$ is the standard deviation of crop revenue R^o_{ri}. Note that the expected gross revenues are non-monotone in the share of land planted to crop s. This is because the inverse demand function is convex, which implies that the effect of a decreased volatility and negative price-yield correlation go in the opposite directions. For example, when plantings of crop c in region A are large, $1-\alpha > 0.75$, the volatility effect dominates and the revenue for crop c in region A, $E[R^c_{Ai}]$, falls as α increases. However, for $\alpha \geq 0.25$ the effect due to a weaker negative price-yield correlation dominates, and the revenue increases with the share of crop c produced in region B. Furthermore, for $\alpha > 0.5$, the crop c output variability increases with α because more of the systemic risk is now coming from region B where most of the crop is grown. This has a positive effect on the expected price for crop c,

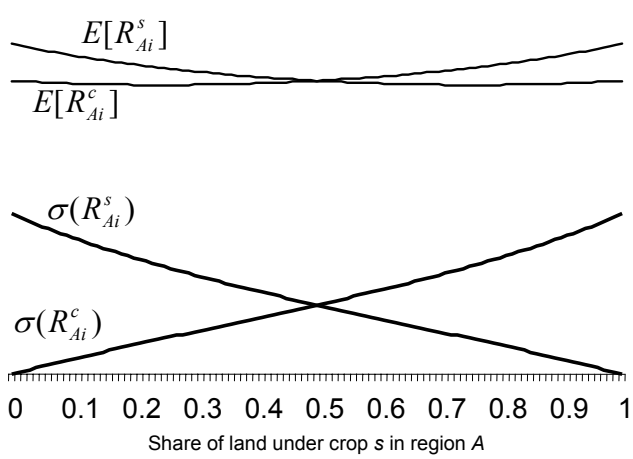

FIGURE 2. Crop revenues' risks and returns

$EP_c((1-\alpha)ny_A^c + \alpha ny_B^c)$, so that the expected revenue for crop c in region A increases at an increasing rate.

To determine the lower bound on the disutility of risk such that equilibria with incomplete specialization exist, we also need the following: $E[R_A^s] = 1/n$, $E[(R_A^s(0.5))']$ $= -(1-\bar{y}/\underline{y})^2 \ (1+\bar{y}/\underline{y})^{-2}/n$, and $E[(R_A^s)'R_A^s] = 2E[(R_A^s)']/n$. A little algebra shows that $Cov[R_A^s,(R_A^s)'] = E[(R_A^s)'R_A^s] - E[(R_A^s)']E[R_A^s] = E(R_A^s)'/n$ and $Var[R_A^s]$ $= 1/n^2((1+(\bar{y}/\underline{y})^2)\ (1+\bar{y}/\underline{y})^{-2} - 0.5)$ Now it is easy to check that $Var[R_A^s] <$ $-Cov[R_A^s,(R_A^s)']$, and so Result 4 can be applied. After a bit of manipulation, the minimum level of risk disutility necessary for multiple equilibria is $\hat{\lambda} = n$.

For concreteness, consider the following numerical values: $n = 10$, $\bar{y}/\underline{y} = 2$, $\lambda = 11$. Substituting these values in the optimality condition $V_{ri}(\alpha^*) = 0$ obtains three equilibrium inter-regional acreage allocations: $\alpha^* = 0.33$, $\alpha^* = 0.5$, and, by symmetry, $\alpha^* = 0.67$. The producer expected utility evaluated at these equilibria is $U_{ri}(0.33) = U_{ri}(0.67) = 0.098$ $> U_{ri}(0.5) = 0.97$, $r = A, B$ (see the corollary). Note that if the level of risk-disutility is less than $\hat{\lambda} = 10$, the only possible equilibrium is $\alpha^* = 0.5$. In this case, $\lambda > \hat{\lambda} = n$ is also a necessary condition for the existence of equilibria with incomplete specialization (see Example 3).

A Necessary and Sufficient Condition for Incomplete Specialization

A somewhat different approach to establishing conditions for the existence of equilibria with incomplete diversification relies on the convex cone representation of a concave utility function. Specifically, any utility function can be written as a linear (convex) combination of very simple concave functions $\min[w,z]$ as follows

$$u(w) = \int \min[w,z]dH(z), \tag{5}$$

where $H'(z) = -u''(z) \geq 0$ if u is twice differentiable. Note that Lemmas 2 and 3 imply the following. Condition $V_{Ai}(\alpha) < 0$ for $\alpha \in (0,0.5)$ guarantees the existence of an $\alpha' \in (0,\alpha)$

such that $V_{Ai}(\alpha') = V_{Ai}(1-\alpha') = 0$, and α' constitutes a partial specialization equilibrium. This is because, by Lemma 2, $V_{Ai}(0.5) = V_{Bi}(0.5) = 0$, and, by Lemma 3, $V_{Ai}(0) > 0 > V_{Ai}(1)$, and function $V_{Ai}(\alpha)$ is continuous.

Using the convex cone representation of the utility function, we have $E[u'(w)]$ $= E[\int 1_{w \le z} dH(z)]$ where the expectation is with respect to the random wealth w. Then the necessary condition for $V_{Ai}(\alpha) < 0$ is that there exists a wealth level z such that $H'(z) > 0$ and $E[1_{(1-\alpha)R^c_{Ai}+\alpha R^s_{Ai} \le z}(R^s_{Ai}(\alpha) - R^c_{Ai}(1-\alpha))] < 0$. Because one of the terms in the last expression is an indicator function, and the strict inequality assures that $\Pr\{(1-\alpha)R^c_{Ai} + \alpha R^s_{Ai} \le z\} > 0$, this inequality can be rewritten as

$$E[R^s_{Ai}(\alpha) - R^c_{Ai}(1-\alpha) \mid (1-\alpha)R^c_{Ai}(1-\alpha) + \alpha R^s_{Ai}(\alpha) \le z] < 0. \tag{6}$$

Condition (6) has an intuitive interpretation. First, note that, using Result 2 under Assumptions 1 through 3, we have $E[R^s_{Ai}(\alpha) - R^c_{Ai}(1-\alpha)] > 0$ for $\alpha < 0.5$ (see note 14). And so, the inequality in equation (6) is solely attributed to the constraint that the aggregate crop revenue, $(1-\alpha)R^c_{Ai} + \alpha R^s_{Ai}$, is below a certain level z. In this light, inequality (6) indicates that the "conditional dispersion" associated with crop s revenue is greater than that for crop c when the regional acreage under crop s is small, $\alpha \in (0,0.5)$. The knowledge that the combined generated crop revenue will not exceed a certain level implies that the expectation of crop s revenue conditioned on this information falls more than does the conditional expectation of crop c revenue. This is in spite of the fact that the weight of the crop s revenue, α, in the total farm revenue is smaller than that for crop c, $\alpha < 0.5 < 1-\alpha$.

Condition (6) is necessary for the marginal utility of allocating more farm acreage to crop c to be positive, even though the share of acreage under crop c exceeds that under crop s in region *A*. When condition (6) holds on a set of $z \in I$ with a positive Lebesgue measure, that is, $\int_I dz > 0$, there must exist a differentiable utility function $u(.)$ such that $V_{Ai}(\alpha) < 0$ for $\alpha < 0.5$. Summarizing, we have the following.

RESULT 5. Suppose that Assumptions 1 through 3 and 5 hold. There exists a differentiable utility function $u(w) = \int \min[w, z] dH(z)$ such that there are equilibria with incomplete specialization $\alpha^* \in (0,0.5) \cup (0.5,1)$ if and only if condition (6) is satisfied for $z \in I$, where set I has a positive measure with respect to $H(z)$, that is, $\int_I dH(z) > 0$.

Note that Result 5 conveys little in terms of the properties of the utility function and attitudes toward risk; it only states the conditions for the existence of a utility function such that there are multiple equilibria. For example, any utility function (5) with the property that there exists a small number $\gamma > 0$ such that $\int_{z \notin I} dH(z) \,/ \int_{z \in I} dH(z) \le \gamma$ is a viable candidate. The following example is instructive and highlights a more elegant approach for obtaining the same characterization as in Example 2.

EXAMPLE 3. Consider the same environment as in Example 2 except there is no restriction on the utility function. Pick a sufficiently small wealth level $z < 1/n$. Then it is easy to verify that the condition $(1-\alpha)R^c_{Ai}(1-\alpha) + \alpha R^s_{Ai}(\alpha) = (1/n)[(1-\alpha)/(1-\alpha+\alpha y^c) + \alpha/(\alpha+(1-\alpha)y^s) \le z$ can only be satisfied when $y^c, y^s > 1$. Because random variables $y^o = y^o_B / y^o_A$ follow a three-point distribution $(\bar{y}/\underline{y}, 1/4; 1, 1/2; \underline{y}/\bar{y}, 1/4)$, $o = c, s$, condition $(1-\alpha)R^c_{Ai}(1-\alpha; y^c) + \alpha R^s_{Ai}(\alpha; y^s) \le z$ may only hold when the realizations of both y^c and y^s are high: $\Pr\{y^c = y^s = \bar{y}/\underline{y}\} = 1$, where $\bar{y}/\underline{y} > 1$. In particular, it is satisfied when $\bar{y}/\underline{y} \ge (D + \sqrt{D^2 + 4zn(2-zn)})/(2zn)$, where $D = (1-zn)(\alpha^2 + (1-\alpha)^2)/(\alpha(1-\alpha))$, which is always satisfied when $zn \le 1$. But in this case, we have $E[R^s_{Ai} - R^c_{Ai} \mid (1-\alpha)R^c_{Ai} + \alpha R^s_{Ai} \le z] = E[R^s_{Ai} - R^c_{Ai} \mid y^c = y^s = \bar{y}/\underline{y}] = (1/n)[1/(\alpha + (1-\alpha)\bar{y}/\underline{y}) - 1/(1-\alpha+\alpha\bar{y}/\underline{y})]$ $= (1/n)(1-2\alpha)(1-\bar{y}/\underline{y}) \,/[(1-\alpha+\alpha\bar{y}/\underline{y})\,(\alpha+(1-\alpha)\bar{y}/\underline{y})] < 0$ because $\alpha < 0.5$ and $\bar{y}/\underline{y} > 1$. And so, if there is sufficient dispersion in the yield distributions (i.e., $\bar{y}/\underline{y}$ is large) there always exists a utility function (possibly exhibiting a large degree of risk aversion at low wealth levels) such that there are equilibria with partial regional specialization.

Alternatively, let us fix the yield dispersion parameter, $\bar{y}/\underline{y}$, and look for the value of the wealth level z such that condition (6) is satisfied. For any $z \in (0,1/n)$ condition (6) is operative because $(1-\alpha)R^c_{Ai}(1-\alpha; y^c) + \alpha R^s_{Ai}(\alpha; y^s) < 1/n$ when $y^c = y^s = \bar{y}/\underline{y} > 1$, and we showed that $(1-\alpha)R^c_{Ai}(1-\alpha; \bar{y}/\underline{y}) + \alpha R^s_{Ai}(\alpha; \bar{y}/\underline{y}) \le z$ implies that condition (6) holds. To construct a candidate utility function (5), consider the simplest possible measure $H(z)$ $= \mu z$ for $z \in [0, \le 1/\mu]$ and $H(z) = 1$ for $z > 1/\mu$, where $\mu > 0$. Hence, if $1/\mu < 1/n$ or $\mu > n$ there must exist incomplete specialization equilibria with $\alpha^* \in (0,0.5) \cup (0.5,1)$. Substituting this weighting measure in (5) and integrating over the intervals $z \le w$ and $z > w$ yields: $u(w) = w - \mu w^2/2$, where $\mu > n$. Compare this with the finding in Example 2 that the level of risk disutility must be $\lambda > n$ for the equilibria with partial specialization to exist in the case of mean-variance preferences.

Discussion

At the farm level, the problem of acreage allocation by a risk-averse producer is the standard portfolio selection problem in which the total wealth (land) needs to be allocated among risky assets (crops). What makes the problem studied in this paper distinct is the nature of the endogeneity of asset returns (net crop revenues) as a result of the investment decisions (land allocation) made by growers. While crop revenues are taken as exogenous by the individual producer, they depend on the distribution of crop acreage between the two regions because of the spatial yield correlation structure. Because crop yields within the region are likely more correlated than across the regions, the degree of systematic yield risk is affected by the inter-regional land allocation. If each region completely specializes in one crop, crop outputs are possessed of a relatively high degree of systematic risk compared with the situation when both regions produce both crops.

The way in which the probability distributions of crop revenues and the expected incremental returns from switching crops are affected by the redistribution of crop acreages across regions is, in general, ambiguous. This is because a crop revenue gross of production costs is the product of the producer-specific yield and the price that is common to all producers. On the one hand, the expected revenue decreases as the individual yield becomes more corre-

lated with the total output. This effect is operative because the individual yields in the region where most of the crop is produced are more correlated with the total output than are the individual yields in the region where most of the crop is produced elsewhere. Hence, as regions become more specialized in one crop, the "price-yield correlation" effect on the expected crop revenues is unambiguous because of the monotonicity of the price function. Now consider the crop revenue's second component, the crop price that depends on the aggregate output. An increase in the systematic risk present in the crop output may have a positive or negative impact on the expected crop price depending on the curvature properties of the inverse demand function. This is much the same as how the effect of uncertainty on the expected value of a function depends on whether the function is concave or convex.[16] As a result, the effect of the inter-regional land allocation on the expected crop revenues is sensitive to the specifics of the environment under scrutiny.

In general, the motives for the regional specialization in a small number of crops may stem from the non-monotone expected crop revenue differential, quite apart from the risk management aspects of the acreage allocation decisions. The problem is that the existence of multiple local optima may be due to the difficult-to-discern features of the production and demand environment, such as the curvature of the demand functions and the differences in spatial yield correlations across regions and within a region. Because inter-regional land allocation has overall ambiguous effects on the expected incremental return from switching crops, we create a specialized environment to better focus on the effects that land allocation has on the risks associated with crop revenues.

To this end, the paper exploits the symmetry in growing conditions both within and across regions that simplifies the analysis and isolates the role of risk aversion on production decisions. In this way, the number of dimensions is reduced because it is natural to focus on equilibria in which regions "mirror" each other in acreage allocation given the symmetry in all other aspects. Note that the symmetry across farms and probability distributions of crop revenues, conditional on the corresponding acreage allocation, eliminates any reasons for asymmetries in land allocation decisions stemming from the variations in production or marketing opportunities across farms or regions.

This approach is also a convenient calibration device since, under any level of risk aversion, the full diversification in production such that growers in each region allocate a half of

their acreage to each crop is equilibrium. While it is the only equilibrium under risk neutrality, equilibria with incomplete specialization in which growers allocate a greater share or even all of their acreage to one of the crops may exist under risk aversion. Because of the symmetry among growers across both regions, the welfare estimates are straightforward since a representative agent exists. In particular, we find that producer welfare is affected by inter-regional crop acreage allocation in a non-monotone manner and may be higher under incomplete specialization relative to the full diversification outcome.

Conclusions

This study inquiries into the acreage allocation decisions at farm and regional levels under risk aversion, yield uncertainty, and endogenous crop prices in a two-crop, two-region setting. The main insight is that a partial specialization in one crop at a regional level may be an equilibrium dominant strategy relative to the more diversified crop mix produced on farm. This applies to the environments where one can identify growing regions that are possessed of two features. The first feature is that growing conditions are relatively homogenous and there is higher farm-level yield co-variability within each region as compared with that across the regions. The second feature is that the sizes of the regions are large enough to have a substantial impact on output prices. Also, complete specialization where each region grows only one crop is found to be impossible when yields for different crops within a region are uncorrelated unless there are comparative advantages in production (or marketing) across producers. This is because the expected revenue for the crop that is grown elsewhere will be rather high because of the lack of the negative price-yield correlation.

In general, production of multiple crops on each farm leads to a less effective "natural hedge" and may increase the individual crop's riskiness. This, of course, needs to be balanced with the reduction of whole-farm revenue risk through crop enterprise diversification. Unless the disparity in expected crop revenues is small, risk-averse producers will not totally avoid allocating some of their land to a riskier crop. In particular, complete specialization is a plausible equilibrium outcome under the following conditions: a high level of risk aversion, limited benefits derived by crop enterprise diversification because of high correlation of farm-level yields for different crops, and low crop price elasticities that provide a cap on the gap between the expected crop revenues.

The analysis in this paper relies on a somewhat crude spatial yield dependence structure. Namely, it is held that the spatial yield correlation is invariant for all farms within a region. And so, the only way the "spatial" aspect of the environment is captured in the model is through the decline in the correlation among yields on farms in different regions relative to that in the same region. In reality, spatial yield correlation likely declines "slowly" as the distance between the land plots increases within a certain range and differs across farms in each region as well as across regions (Wang and Zhang 2003).[17] An effort to consider this more realistic spatial yield dependence characterized by a continuous, and possibly heterogeneous, decline in correlation is left for future research. The environments with the discrete jump in yield dependence across regions are plausible if regions are spatially, or otherwise, separated. This can happen if the "distance" between the regions is significant as compared with the size of the regions.

To inspect the circumstances under which (partial) regional specialization constitutes equilibrium, a number of factors were omitted that are controlled by producers and are important in any analysis of acreage allocation decisions. In particular, through the choice of inputs, farming methods, and pest management, producers can affect the probability distribution of farm-level yields (endogeneity of yield risk at the farm level). Also, the price is taken to be a deterministic function of the total supply, which assumes away any price risk stemming from the demand side of the market. Furthermore, in a sense, the price and yield risks are treated as perfect substitutes, while production and marketing contracts, storage following harvest, and pre-harvest pricing are likely to provide more opportunities to "hedge" price risk as opposed to yield risk. Finally, the interaction of acreage allocation decisions with crop insurance schemes is left unattended.

The implications of this research warrant an empirical inquiry into the extent to which risk management is a possible determinant of the regional acreage allocation patterns observed in real-world agricultural landscapes. Empirical data for estimating the appropriate model for the spatial yield dependence structure is likely readily available because of the recent progress made in the area of geographic information systems and the statistical analysis of spatial data. While the primary intent of this analysis is to characterize conditions for regional specialization, there is a normative implication in terms of the redistributive effects of the inter-regional land allocation. One could conjecture that a suitable reallocation of crop production among

areas that share similar growing conditions because of common soil, climate, and other environmental characteristics may obviate or diminish the need for agricultural revenue insurance markets beleaguered by moral hazard and adverse selection problems. This is, of course, given that the standard caveats due to the wide-ranging environmental impacts of crop production on soil, climate, and yield productivity are taken into account.

Endnotes

1. For example, the estimates of the farm price–yield correlation for corn in the United States at the county level indicate that the negative correlation is more pronounced in the Corn Belt area than it is outside of that area (Harwood et al. 1999).

2. Some entropy-based measures of enterprise diversification across different types of agricultural operations are provided in Jinkins (1992). There is a large literature analyzing benefits and costs of enterprise diversification on farm operations such as soil effects of crop rotation, labor management during the planting time, a broader managerial expertise, and farm machinery and equipment requirements. In addition to identifying complementarities and substitutabilities in multiple output farming operations and a number of technological and marketing constraints for diversification, most of the papers in this area are concerned with the effect of producer heterogeneities such as farm size and wealth on incentives to diversify (see Pope and Prescott 1980; Beneke 1998; Dodson 1993; Schoney, Taylor, and Hayward 1994; Held and Zink 1982; and Sonka and Patrick 1984). Nartea and Barry (1994) inquired into the risk reduction effects of geographical diversification involving farming several noncontiguous locations.

3. Very informally, the observation that the effective negative price-yield correlation provides a "natural hedge" for crop revenues is probably best summarized by Neil Harl: "... the only thing worse for a farmer than bad weather is good weather" (quoted in Goodwin 2000, p. 76). And so, we are interested in the (collective) choice between the "natural hedge" common to all producers in a "large" region where a small number of crops are produced and the "individual hedge" derived through production of a plural number of crops on each farm.

4. In a related line of inquiry, Chavas (1993) and Hennessy (1997) analyze the effects of exogenous price uncertainty on equilibrium land allocation and the Ricardian rent.

5. In a recent study, Wang and Zhang (2003) find that the positive dependence among crop yields varies inversely with distance in a somewhat discrete manner: it exists when the distance between the land plots is within a certain range but rapidly approaches zero when the land plots are sufficiently far apart. Using county-level data, Wang and Zhang estimate that the range for the positive correlation among yields for the three major U.S. field crops—corn, soybeans, and wheat—lies within a 570-mile radius, or even smaller. Nartea and Barry (1994) find a significant negative relationship between yield correlation and distance for corn and soybeans in central Illinois once the distance between land parcels exceeds 30 miles (the geographic dispersion of fields in their study was limited to 125 miles).

6. Hennessy and Lapan (2003) provide a fundamental treatment of the concept of "more systematic risk" and its formalizations in a broad economic setting with some applications. The supermodular stochastic order is widely used in insurance and financial management literatures.

7. If function ϕ is twice differentiable this is equivalent to $\partial^2\phi / \partial x_i \partial x_j \geq 0$ for all $i \neq j$. This property of supermodular functions will be used in proofs.

8. Parts (b) and (c) can be jointly stated as $F_{r1}^c \cdot \ldots \cdot F_{rn}^c \cdot F_{r1}^s \cdot \ldots \cdot F_{rn}^s \prec_{sm} F_r$. Stating the positive dependence among the farm-level yields in each region separately in the manner of Assumption 1 facilitates the subsequent exposition (see part [b] of Assumptions 4 and 5.)

9. The assumption that yields are independent across regions (part [a] of Assumption 1) can be weakened to allow for the negative dependence, i.e., $F \prec_{sm} F_A F_B$. The independence condition is easier to work with and does not change the qualitative nature of the results.

10. Note that we assume that the two crops are neither substitutes nor complements in consumption. This assumption simplifies the subsequent analysis but should not affect the qualitative nature of the results unless the substitution effects are significant.

11. Unless stated otherwise, primes denote differentiation.

12. Note that under symmetry, $\beta_{ri}^{o} = 1$, the dependence structure used in Result 3 can be obtained without part (a) of Assumption 4 by assuming that $F_r(y_{r1}^c,...,y_{rn}^c, y_{r1}^c,...,y_{rn}^s)$ = $\min[F_{r1}^c(y_{r1}^c),...,F_{r1}^c(y_{rn}^c),\ F_{r1}^s(y_{r1}^c),...,F_{r1}^s(y_{rn}^s)]$ since by part (b) of Assumption 3 farms are symmetric, $F_{ri}^{o} = F_{r1}^{o}$ for all *i*.

13. For concreteness, we use the optimality condition for region *A*. The optimality condition for the other region is completely analogous and is implied by (3).

14. The proof of Result 2 ascertains that the expected revenue differential in each region is monotone in α, and the evaluations of the differential at $\alpha = 0$ and $\alpha = 1$ have the opposite signs.

15. Condition $Var[R_A^s] < -Cov[R_A^s,(R_A^s)']$ implies that $\partial Var[R_{Ai}^s(0.5)]/\partial\alpha = 2Cov[R_A^s,(R_A^s)']$ < 0.

16. An increase in systemic risk implies an increase in the aggregate yield volatility because the yield diversification across areas is less effective. Formally, we have $\int H(y_1+...+y_n)dG \ \leq \int H(y_1+...+y_n)dG'$ if $G \prec_{sm} G'$ and $H'' \geq 0$ because $H(y_1+...+y_n)$ is supermodular when *H* is convex (take $H(y) = y^2$).

17. Also, it is likely that the spatial yield dependence structure is affected by cropland allocation itself when the risks of a spreading epidemic or infestation as well as other environmental impacts are affected by the density of plant population.

Appendix

Proofs of Results and Lemmas

Proof of Result 1

Suppose to the contrary that in equilibrium $\alpha^*_{Ai} = 0$ and $\alpha^*_{Bk} = 1$ for all $i = 1,..,n$ and $k = 1,...,m$. Then the optimality conditions are $E[u'_{Ai}(R^c_{Ai})R^s_{Ai}] \le E[u'_{Ai}(R^c_{Ai})\ R^c_{Ai}]$ and $E[u'_{Bk}(R^s_{Bk})R^s_{Bk}] \ge E[u'_{Bk}(R^s_{Bk})\ R^c_{Bk}]$, where $R^c_{ri} = P_c(\sum_{j=1}^{n} y^c_{Aj})y^c_{ri} - c^c_{ri}$ and R^s_{ri} $= P_s(\sum_{j=1}^{n} y^s_{Bj})y^s_{ri} - c^s_{ri}$. Consider the left-hand side of the first inequality. Differentiation establishes that the function $h(y^c_{A1},...,y^c_{An},\ y^s_{Aj}) = u'_{Ai}(R^c_{Ai})R^s_{Ai}$ is supermodular in (y^c_{Aj}, y^s_{Ai}) for all j if condition (i) holds. Because the yields are independent across regions by part (a) of Assumption 1 and are positively dependent across crops within a region by part (c) of Assumption 1, we have $E[u'_{Ai}(R^c_{Ai})R^s_{Ai}] = E[u'_{Ai}(P_c(\sum_{j=1}^{n} y^c_{Aj})]y^c_{Ai} - c^c_{Ai})E[P_s(\sum_{j=1}^{m} y^s_{Bj})]y^s_{Ai}$ $- c^s_{Ai}] \ge E[u'_{Ai}(R^c_{Ai})]E[R^s_{Ai}]$. Alternatively, the last inequality always holds under condition (ii). On the other hand, risk aversion ($u'' < 0$) implies that $E[u'_{Ai}(R^c_{Ai})R^c_{Ai}] < E[u'_{Ai}(R^c_{Ai})]$ $E[R^c_{Ai}]$. Hence, from the optimality condition for farms in region A, we obtain that $ER^s_{Ai} < ER^c_{Ai}$ for all i. Similarly, the optimality condition in region B implies that $ER^s_{Bk} > ER^c_{Bk}$ for all k. Because the inverse demands are decreasing functions; y^s_{Bj} and y^s_{Ai}, and y^c_{Aj} and y^c_{Bi} are pair-wise independent by part (a) of Assumption 1; $\{y^o_{ri}\}$ are positively dependent by part (b) of Assumption 1; and condition (iii) holds; these last two inequalities are mutually exclusive. The proof in the case $\alpha^*_{Ai} = 1$ and $\alpha^*_{Bk} = 0$ is analogous.

Proof of Lemma 1

For concreteness, take $\alpha_A = 0$ and $\alpha_B = 1 - \alpha_A$. Differentiating $ER^o_{Ai} =$ $P_o((1-\alpha_A)\sum_j y^o_{Aj} + \alpha_A \sum_j y^o_{Bj})y^o_{ri} - c^o_r$ at $\alpha_A = 0$ yields $E[(R^o_{Ai}(\alpha_A = 0))'] =$ $E[P'_o(\sum_j y^o_{Aj})(\sum_j y^o_{Bj} - \sum_j y^o_{Aj})y^o_{Ai}]$. Because, by Assumption 2, all farms in region A are homogenous, we can write $E[(R^o_{Ai}(0))'] = (1/n)E[P'_o(\sum_j y^o_{Aj})(\sum_j y^o_{Bj} - \sum_j y^o_{Aj}) \sum_j y^o_{Aj}]$. And so, the sign of $E[(R^o_{Ai}(0))']$ depends on the direction of the inequality $E[P'_o(\sum_j y^o_{Aj})\sum_j y^o_{Bj} \sum_j y^o_{Aj}] \geq (<) E[P'_o(\sum_j y^o_{Aj})(\sum_j y^o_{Aj})^2]$. Rewrite it as follows: $Ef(x,y) \geq (<) Ef(x,x)$, where $x = \sum_j y^o_{Aj}, y = \sum_j y^o_{Bj}$, and $f(x,y) = P'_o(x)xy$. Note that the sums of crop yields in regions A and B are independent (part [a] of Assumption 1) and are possessed of the same marginals. Therefore, the direction of the inequality depends on whether the function $f(x,y) = P'_o(x)xy$ is supermodular or submodular: the expected value of a supermodular (submodular) function increases (decreases) as the dependence (in the sense of the supermodular stochastic dominance) increases. Differentiating twice $f(x,y)$ gives the condition required in the result.

Proof of Lemma 2

We need to show that $U_{Ai}(\alpha) = Eu((1-\alpha)R^s_{Ai}(1-\alpha) + \alpha R^c_{Ai}(\alpha)) = U_{Ai}(1-\alpha)$ $= Eu((1-\alpha)R^s_{Bj}(\alpha) + \alpha R^c_{Bj}(1-\alpha)) = U_{Bj}(1-\alpha)$. The second equality is due to Assumption 2, part (a), that the crop revenues are exchangeable random variables given that the shares of acres allocated to crops in each region are the same. The third equality is due to Assumption 2, part (a) and Assumption 3, part (b), that the probability distributions of yields within the region and across the regions are exchangeable random variables. Also, observe that $V_{ri}(\alpha)$ $= -V_{ri}(1-\alpha)$, which implies that $V_{Ai}(0.5) = V_{Bj}(0.5) = 0$, and $\alpha^*_A = \alpha^*_B = 0.5$ is equilibrium.

Proof of Result 2

Equilibrium without specialization is given by $EH(\alpha^*, i) = E[R^c_{Ai}(1-\alpha^*) - R^s_{Ai}(\alpha^*)] = 0$ for all $i = 1,...,n$, with the similar condition for region B satisfied by symmetry. We can rewrite

the second term as $ER^s_{Ai}(\alpha) = E[P_s((1-\alpha)\sum_{j=1}^n y^s_{Aj} + \alpha\sum_{j=1}^n y^s_{Bj}))y^s_{Bi} - c^s] =$ $E[P_c((1-\alpha)\sum_{j=1}^n y^c_{Aj} + \alpha\sum_{j=1}^n y^c_{Bj}))\ y^c_{Bi} - c^c]$, where part (b) of Assumption 3 is used to get the first equality, and part (a) of Assumption 3 is used to get the second. Substitution yields $EH(\alpha,i) = E[P_c((1-\alpha)\sum_{j=1}^n y^c_{Aj} + \alpha\sum_{j=1}^n y^c_{Bj}))(y^c_{Ai} - y^c_{Bi})]$. Because all farms are homogenous with identical yield probability distributions we have $E[H'(\alpha,i)] =$ $(1/n)E[\sum_{j=1}^n H'(\alpha,j)] = -(1/n)E[P'_c((1-\alpha)\sum_{j=1}^n y^c_{Aj} + \alpha\sum_{j=1}^n y^c_{Bj}))\ (\sum_{j=1}^n y^c_{Bj} - \sum_{j=1}^n y^c_{Aj})^2] >$ 0. Next, evaluate the expected revenue differential $EH(\alpha,i)$ at $\alpha = 0, 0.5$, and 1: $EH(0,i) = E[P_c(\sum_{j=1}^n y^c_{Aj})y^c_{Ai})\ - P_c(\sum_{j=1}^n y^c_{Bj})y^c_{Ai}] < 0$, $EH(1,i)\ = -EH(0,i) > 0$, and $EH(0.5,i) = 0$. Therefore, $\alpha^* = 0.5$ is the unique equilibrium.

Proof of Result 3

At $\alpha^*_{Ai} = 0$ and $\alpha^*_{Bj} = 1$, the optimality conditions are $V_{Ai}(0) \le 0$ ($V_{Bi}(1) \ge 0$). Hence, upon using condition (i), we need to show that

$$E[u'_{Ai}(P_c(ny^c_A)\beta^c_{Ai}y^c_A - c^c_{Ai})(P_s(ny^s_B)\beta^s_{Ai}y^s_A - c^s_{Ai})] \le$$
$$E[u'_{Ai}(P_c(ny^c_A)\beta^c_{Ai}y^c_A - c^c_{Ai})(P_c(ny^c_A)\beta^c_{Ai}y^c_A - c^c_{Ai})]. \qquad \text{(A.1)}$$

The proof proceeds in two steps.

Step 1. Consider the left-hand side of (A.1). Observe that by part (a) of Assumption 3, we have $\Pr\{P^s(ny^s_B)\beta^s_{Ai}y^s_A - c^s_{Ai} \le R\} = \Pr\{P^c(ny^c_B)\beta^c_{Ai}y^c_A - c^c_{Ai} \le R\}$, and, hence, $E[u'_{Ai}(P_c(ny^c_A)\ \beta^c_{Ai}y^c_A - c^c_{Ai})\ (P_s(ny^s_B)\beta^s_{Ai}y^s_A - c^s_{Ai})] = E[u'_{Ai}(P_c(ny^c_A)\beta^c_{Ai}y^c_A - c^c_{Ai})$ $(P_c(ny^c_{r_1})\beta^c_{Ai}y^c_{r_2} - c^c_{Ai})]$ given that the dependence structure corresponding to the one among y^c_A, y^s_B, y^s_A is maintained among $y^c_A, y^c_{r_1}, y^c_{r_2}$. Because the "average" yield for crop c in region A, y^c_A, and the "average" yield for crop s in region B, y^s_B, as well as y^s_A and y^s_B, are pairwise independent (part [a] of Assumption 1), and part (b) in Assumption 4 holds, we have $r_1 = B$ and $r_2 = A$ for the identity to hold.

Step 2. Now we show that $E[u'_{Ai}(P_c(ny^c_A)\beta^c_{Ai}y^c_A - c^c_{Ai})\ (P_c(ny^c_B)\beta^c_{Ai}y^c_A - c^c_{Ai})]$

$\leq E[u'_{Ai}(P_c(ny^c_A)\ \beta^c_{Ai}y^c_A - c^c_{Ai})\ (P_c(ny^c_A)\beta^c_{Ai}y^c_A - c^c_{Ai})]$. Imagine that yields y^c_A and y^c_B are comonotonic random variables. Formally, if $\Pr\{y^c_A \leq x,\ y^c_B \leq y\} = \min[\Pr\{y^c_A \leq x\}$, $\Pr\{y^c_B \leq y\}]$ then, instead, we must have $r_1 = A$ and $r_2 = A$ in the last identity, and the inequality (A.1) must hold as equality. Because by assumption the yields are not perfectly correlated across regions (in fact, the yields are independent across regions), that is, $\Pr\{y^c_A \leq x, y^c_B \leq y\} = \Pr\{y^c_A \leq x\}\ \Pr\{y^c_B \leq y\} \prec_{sm} \min[\Pr\{y^c_A \leq x\},\ \Pr\{y^s_B \leq y\}]$, inequality in (A.1) holds if the function $f(x,y) = u'_{Ai}(P_c(ny)\beta^c_{Ai}y\ - c^c_{Ai})\ (P_s(nx)\beta^s_{Ai}y - c^s_{Ai})$ is supermodular. Differentiating twice $f(x,y)$ establishes that such a property adheres if condition (ii) is satisfied. The same analysis applies in region *B*.

Proof of Lemma 3

By definition, we have $V_{Ai}(0) = Eu'(P_c(\sum y^c_{Aj})y^c_{Ai} - c^c)(P_s(\sum y^s_{Bj})y^s_{Ai}\ - c^s$ $-(P_c(\sum y^c_{Aj})y^c_{Ai} - c^c))$. Observe that $Eu'(P_c(\sum y^c_{Aj})y^c_{Ai} - c^c)(P_s(\sum y^s_{Bj})y^s_{Ai} - c^s)$ $= Eu'(P_c(\sum y^c_{Aj})y^c_{Ai} - c^c)E[P_s(\sum y^s_{Bj})y^s_{Ai} - c^s] \geq Eu'(P_c(\sum y^c_{Aj})y^c_{Ai} - c^c)E[P_s(\sum y^s_{Bj})y^s_{Bi}$ $- c^s] = E[u'(P_c(\sum y^c_{Aj})y^c_{Ai} - c^c)]E[P_c(\sum y^c_{Aj})y^c_{Ai} - c^c] \geq E[u'(P_c(\sum y^c_{Aj})y^c_{Ai} - c^c) \times$ $(P_c(\sum y^c_{Aj})y^c_{Ai} - c^c)]$. The first equality is due to the yield independence for the two crops within each region (Assumption 5) as well as across regions (part [a] of Assumption 1). The first inequality is due to the inter-regional yield independence and positive intra-regional dependence (parts [a] and [b] of Assumption 1), and symmetry across regions (Assumption 3, part [b]). The second equality is due to Assumption 3 that the crop revenues are exchangeable random variables and regions are symmetric. The last inequality is due to risk aversion. Hence, we have $V_{Ai}(0) > 0$, and $V_{Ai}(1) = -V_{Ai}(0) < 0$.

Proof of Result 4

We need to show that condition $V'_{Ai}(0.5) > 0$ reduces to the condition stated in the result when $U_{Ai}(\alpha) = (1-\alpha)ER^c_{Ai} + \alpha ER^s_{Ai} - \lambda((1-\alpha)^2 Var[R^c_{Ai}] + \alpha^2 Var[R^s_{Ai}])$, where yield independence across different crops in a region (Assumption 5) is used to calculate the variance

of the whole-farm revenue. Differentiating (revenues are exogenous at the producer level) yields gives $V_{Ai}(\alpha) = E[R^s_{Ai} - R^c_{Ai}] + 2\lambda((1-\alpha)\ Var[R^c_{Ai}] - \alpha Var[R^s_{Ai}])$. Differentiating one more time (now crop revenues are endogenous) yields $V'_{Ai}(0.5) = 2(E[(R^s_A)'] - \lambda\ (Var[R^s_A] + Cov[R^s_A, (R^s_A)']))$, where $R^s_A = R^s_{Ai}(0.5)$, and the independence and symmetry among the crop revenues was used. The condition stated in the result is meaningful only if the sign of $E[(R^s_A)']$ is negative. Next, we check that this is indeed the case. Note that $E[(R^s_A)']$ $= E[P'_s(0.5\sum_{r,j} y^s_{rj})\ y^s_{Ai}(\sum_j y^s_{Aj} - \sum_j y^s_{Bj})] = (1/n)E[P'_s(0.5\sum_{r,j} y^s_{rj})\ \sum_i y^s_{Ai}(\sum_j y^s_{Aj} - \sum_j y^s_{Bj})] = (1/n)(E[G(x,y) \mid x > y]\ \Pr(x > y) + E[G(x,y) \mid x < y]\ \Pr(x < y))$ $= (1/n)(E[G(x,y) \mid x > y] + E[G(y,x) \mid x > y])\Pr(x > y)$, where $x = \sum_j y^s_{Aj}$, $y = \sum_j y^s_{Bj}$, and $G(x,y) = P'_s(0.5(x+y))x(x-y)$. Because the regions are symmetric, we have $\Pr(x > y) = \Pr(x < y)$, and $G(x,y) < -G(y,x)$ for $x > y$. Hence, we obtain $E[(R^s_A)'] < 0$. The analysis for region *B* is analogous.

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