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Systemic Risk in U.S. Crop and Revenue Insurance Programs

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Abstract

The present study estimates the probability density function of the Federal Risk Management Agency's (RMA) net income from reinsuring crop insurance for corn, wheat, and soybeans. Based on 1997 data, it is estimated that there is a 5 percent probability that RMA will need to reimburse at least $1 billion to insurance companies, and that the fair value of RMA's reinsurance services to insurance firms equals $78.7 million. In addition, various hedging strategies are examined for their potential to reduce RMA's reinsurance risk. The risk reduction achievable by hedging is appreciable, but use of derivative contracts alone is clearly no panacea.

SYSTEMIC RISK IN U.S. CROP AND REVENUE INSURANCE PROGRAMS

Insurance companies have traditionally operated in markets where risk can be pooled or diversified. Futures and options markets have traditionally operated where risk is systemic. Yields and revenues obtained by crop producers have both systemic (drought and price drops) and poolable (localized yield shortfall) risks. Farmers cannot hedge the poolable or localized source of revenue risk on speculative markets, and insurance companies will not accept risk that has a systemic component (Gardner and Kramer; Kramer; Vaughan and Vaughan). As a result, a hybrid mechanism has evolved in U.S. crop insurance markets wherein the federal government agrees to accept the systemic risk so that private insurance companies will sell crop and revenue insurance to producers.[1]

To provide incentives for insurers to offer multi-peril crop insurance, the U.S. government has designed the Standard Reinsurance Agreement (SRA). Under the SRA, insurers can transfer to the Federal Risk Management Agency (RMA) a portion of losses that can occur with widespread yield shortfalls, in exchange for ceding their right to a portion of the gains when premiums are greater than indemnities. The SRA yields an increasing proportion of the firms' profits as positive returns increase and commits the RMA to taking responsibility for an increasing proportion of the losses as they increase. This leaves the RMA in its role as reinsurer with an uncertain level of total outlays. To date there has been no attempt to use speculative markets to hedge the systemic portion of this risk.

The purpose of this study is to break down the total risk absorbed by the U.S. crop insurance industry into poolable and systemic components. We then use option pricing theory to value the reinsurance that the federal government provides when it absorbs this systemic risk. Finally, we evaluate the possibility of using speculative markets in prices and yields to hedge the systemic risk accepted by the government. The analysis considers the insurance policies that most contribute to RMA's risk. These are Actual Production

History Buy-Up Coverage (BUP), Actual Production History Catastrophic Coverage (CAT), the Group Risk Plan (GRP), and Crop Revenue Coverage (CRC).[2] The crops studied are corn, soybeans, and wheat, as they represent the largest of the crops insured under these programs.

The research presented here is motivated in part by Miranda and Glauber, who argue that insurance companies could use derivatives markets as substitutes for government provision of reinsurance. Our work allows the individual insurance companies to pool their risk under the existing institutional structure and focuses on the systemic risk at a national level.

Insurance and Reinsurance Programs

Given the focus on RMA's reinsurance activities, attention is restricted to the major insurance programs available to farmers that are reinsurable by RMA, i.e., BUP, CAT, GRP, and CRC.

BUP pays indemnities when a farmer's yields (y) fall below ψ percent of the average of the individual's previous production (Y). In such instances, the farmer gets a payment per acre insured equal to the yield shortfall multiplied by π percent of the RMA expected price (P_{RMA}):

$$BUP(y; \pi, \psi) = \max[0, \pi \times P_{RMA} \times (\psi \times Y - y)]. \tag{1}$$

Farmers can choose price protection levels π from the set $\Pi_{BUP} \equiv \{0.6, 0.65, ..., 0.95, 1.0\}$ and yield coverage levels ψ from the set $\Psi_{BUP} \equiv \{0.5, 0.65, 0.75\}$. CAT is similar to BUP, except that the levels of price and yield protection are fixed at 60 percent and 50 percent, respectively:

$$CAT(y) = \max[0, 0.6 \times P_{RMA} \times (0.5 \times Y - y)]. \tag{2}$$

That is, $CAT(y) = BUP(y; \pi = 0.6, \psi = 0.5)$.

Unlike BUP and CAT policies (whose indemnities are based on the farmers' individual yields), GRP indemnities are based on county yields. Letting Y_i and y_i

represent county i's expected and realized yields, respectively, per-acre indemnities for GRP are calculated as

$$\mathrm{GRP}(y_i; \pi, \psi) = \max[0, \pi \times P_{RMA} \times (Y_i - y_i/\psi)]. \tag{3}$$

It is clear from (1) and (3) that GRP indemnities are computed differently from BUP and CAT indemnities, even after accounting for county- rather than farm-level yields. In particular, the yield coverage level (ψ) under GRP does not define the upper limit of indemnification, as it does under BUP and CAT. Farmers can select price protection levels $\pi \in \Pi_{GRP} \equiv \{0.9, 0.95, ..., 1.45, 1.5\}$ and yield coverage levels $\psi \in \Psi_{GRP} \equiv \{0.7, 0.75, 0.8, 0.85, 0.9\}$.

CRC is a revenue-protection product and is the most recent of the four programs analyzed. CRC provides a revenue guarantee equal to the revenue the farmer would get if his actual yields were ψ percent [$\psi \in \Psi_{CRC} \equiv \{0.7, 0.75, 0.8, 0.85\}$] of the historical average and the actual price equaled the expected price at planting time. More specifically, per-acre CRC indemnities can be stated as

$$\mathrm{CRC}(y, P_h; \psi) = \max[0, \max(P_p, P_h) \times \psi \times Y - P_h \times y], \tag{4}$$

where P_p and P_h are measures of the planting and harvest prices as defined by the CRC policy. Price P_p (P_h) is the average daily settlement price during the month prior to planting (harvest month) of the futures contract month immediately following harvest. From (4), it is clear that CRC indemnities depend crucially on market prices. This is in sharp contrast with BUP, CAT, and GRP indemnities, for which market prices are irrelevant.

RMA obligations on the policies written by insurers are determined from the indemnity levels described in the SRA. At the insurer's discretion, each of its written policies may be ceded to RMA outright (if the policy is sufficiently undesirable) or allocated to one of three different funds which determine the level of risk transferred to the RMA and the level maintained by the insurer. In this manner, the insurer may rid itself of its most undesirable policies and attenuate the risk of the portfolio of policies that it keeps.

In decreasing order of risk, the RMA funds are designated Assigned Risk Fund (ARF), Development Fund (DF), and Commercial Fund (CF). The high-risk policies that the insurance company elects to keep are placed in ARF, in which the overwhelming

majority of profits and losses are yielded to the RMA. Due to their high risk for RMA, for each state there is a limit on the total value of policies that can be placed in ARF. At the opposite end of the spectrum, insurers will designate as CF only the policies perceived to pose the least amount of risk to the firm and/or those that stand to yield the greatest profits.

Simulation Model

Estimating the probability density function (pdf) of RMA's net income from reinsurance is a complex modeling problem. Schematically, the procedure advocated here requires first completing the following tasks:

1. Estimating county-level yield pdfs and U.S. level price pdfs that reflect historical patterns.
2. Calibrating insurance policies and within-county yield pdfs, by using historical insurance data.
3. Assigning insurance policies to reinsurance funds, based on historical reinsurance data.

Having finished the three tasks above, Monte Carlo simulations are performed to draw a large number of simulated "annual" observations on crop yields and prices. In turn, these are used to compute simulated "annual" observations on RMA's net income from reinsurance. The histogram of the latter series provides an estimate of the unknown pdf of RMA's net income from reinsurance. A more detailed description of the whole process is provided next.

Simulation of County-Level Yields

Before employing historical yields for estimation purposes, it is imperative to correct them for the significant productivity advances that occurred over the period analyzed. The large positive trends in the yields of corn, soybeans, and wheat reported in Table 1 provide strong evidence of the need for such a correction. Table 1 contains estimates of the regression:

TABLE 1. Ordinary least-squares regression estimates of U.S. yields against time, 1972 through 1997

Regression Model: $\ln(y_{USt}) = a_0 + a_1 t + e_t$.

	Corn	Soybeans	Wheat
$\hat{a}_0$ [a]	4.5276**	3.4220**	3.4158**
	(0.0268)[b]	(0.0176)	(0.0153)
$\hat{a}_1$	0.01803**	0.01453**	0.00875**
	(0.00357)	(0.00235)	(0.00197)
R^2	0.430	0.614	0.442
Number of observations	26	26	26

Note: ** denotes significantly different from zero at the 1 percent level based on the two-tailed t-statistic.
[a]Variables with hats (ˆ) denote sample estimates.
[b]Numbers between parentheses below coefficient estimates are the respective standard deviations.

$$\ln(y_{USt}) = a_0 + a_1 t + e_t, \tag{5}$$

where y_{USt} represents U.S. yield in year t, e_t is an error term, a_0 is the intercept, and a_1 is the percentage annual change in yields (source: USDA-NASS). Following Miranda and Glauber, all of the historical yields used in the remainder of the study consist of 1997-equivalent yields, obtained by detrending the original yields by means of the percentage annual yield change estimates shown in Table 1.

Estimation of County-Level Yield pdf's. Skewness in crop yields has been identified (e.g., Gallagher; Ramirez) and is particularly important for this study in which insurance payments result from lower-than-average yields. Although Just and Weninger have raised concerns regarding the methods used to reject normality in the distribution of yields, the beta pdf has been used to model the behavior of yields in various studies as a means of capturing potential skewness (e.g., Nelson and Preckel; Babcock and Hennessy). Here we adopt the beta pdf (6) because it can reflect various levels of skewness and kurtosis, and its most appropriate shape can be estimated with historical data rather than imposed in an ad hoc manner.

$$B(y|\,\alpha, \beta, Y^L, Y^U) \equiv \frac{\Gamma(\alpha+\beta)}{\Gamma(\alpha)\,\Gamma(\beta)} \; \frac{(y - Y^L)^{\alpha-1}\,(Y^U - y)^{\beta-1}}{(Y^U - Y^L)^{\alpha+\beta-1}} \quad \text{for } Y^L \le y \le Y^U. \tag{6}$$

In (6), Y^L (Y^U) is the lower (upper) limit of the feasible range for random variable y, $\Gamma(\cdot)$ denotes the gamma function, and α and β are parameters which influence the shape of the pdf.

Using 1997-equivalent county yields from 1972 through 1997, a separate beta pdf was fitted for each individual county i, relying upon the method of moments. This was accomplished by setting the lower bound for yields equal to zero for all counties ($Y_i^L = 0$ $\forall$ i), estimating county i's mean yield (μ_i) and yield variance (σ_i^2), plugging in such values along with county i's maximum observed yield (y_i^U) in equations (7) through (9), and solving the latter for the numerical values of α_i, β_i, and Y_i^U.

$$\alpha_i = \left(\frac{\mu_i - Y_i^L}{Y_i^U - Y_i^L}\right)^2 \left(1 - \frac{\mu_i - Y_i^L}{Y_i^U - Y_i^L}\right) \frac{(Y_i^U - Y_i^L)^2}{\sigma_i^2} - \frac{\mu_i - Y_i^L}{Y_i^U - Y_i^L}, \tag{7}$$

$$\beta_i = \left(\frac{\mu_i - Y_i^L}{Y_i^U - Y_i^L}\right) \left(1 - \frac{\mu_i - Y_i^L}{Y_i^U - Y_i^L}\right) \frac{(Y_i^U - Y_i^L)^2}{\sigma_i^2} - 1 - \alpha_i, \tag{8}$$

$$\ln(y_i^U / Y_i^U) = -2.5426 - 0.9442 \ln(\alpha_i) - 0.9442 \ln(\beta_i). \tag{9}$$

Equations (7) and (8) are taken from Johnson and Kotz, whereas (9) is a regression estimate whose rationale is discussed in Appendix A. A total of 1,484 counties were analyzed for corn, 1,371 counties for soybeans, and 1,242 counties for wheat.

Modeling Systemic County-Level Yield Risk. One of the greatest challenges in running Monte Carlo simulations of county-level yields lies in imposing reasonable correlation levels across county observations. Cross-sectional correlations among county yields have dramatic effects on the pdf of RMA indemnities. As shown below, substantial changes in the variance of RMA indemnities result when low cross-sectional correlation levels are imposed, compared to scenarios with high correlation levels. If yields were uncorrelated across counties, by the law of large numbers the RMA would be able to eliminate almost all of its risk by holding a portfolio of policies well diversified across counties. In the opposite scenario of perfect correlation across county yields, geographic

diversification of the policy portfolio would provide no benefits to RMA in terms of risk reduction. Given that greater geographic correlation of yields reduces diversification benefits, it is essential for the simulations to maintain an appropriate degree of correlation among county yield draws.

For analytical and simulation purposes, each state was divided into a small number of divisions, and each division was divided further into three to five subdivisions containing a number of counties. *Rank* correlations (McClave and Benson) between successive levels (e.g., national-state, state-division, and so on) were estimated using 1997-equivalent yield data for 1972 through 1997.[3] In general, estimated rank correlations were very large in high-production areas, and positive but very disperse in other areas.[4] The estimated rank correlations between successive levels of geographic aggregation were used, along with Johnson and Tenenbein's method to obtain county-level yields with the desired cross-correlation structure.

To obtain a single "annual" draw of the set of county yields, first draw one realization x_{USt} for the United States from the uniform pdf with limits of zero and one [U(x)]. Second, draw one realization from U(x) for each state, and use Johnson and Tenenbein's approach to convert each of them into a uniform variable with the desired level of correlation with x_{US}. Third, draw one realization from U(x) for each division of each state, and use Johnson and Tenenbein's technique again to convert each of them into a uniform variable with the desired level of correlation to the state to which it belongs. This procedure is repeated at the subdivision and county levels so as to obtain a vector of county-level variables (x_{it})—each of them uniformly distributed between zero and one—that are correlated with each other through their correlations at the county-subdivision, subdivision-division, division-state, and state-national levels. Finally, convert each of the uniformly distributed county-level variables x_{it} into a yield realization y_{it} using the inverse of the corresponding beta pdf, i.e., $y_{it} = B^{-1}(x_{it}| \alpha_i, \beta_i, X_i^L, X_i^U)$. The results reported in the present study are based on 2,500 "annual" draws. That is, t = 1, ..., 2,500.

Simulation of Harvest Prices

Because the RMA reinsures CRC policies, which guarantee revenues rather than yields, it is necessary to simulate prices, as well. The price component of the CRC indemnity (4) is determined by using two price levels (P_p and P_h) in each year. Planting prices (P_p) are represented by constants equal to the 1997 CRC planting prices, because the model is calibrated for 1997 and P_p is known at the time of entering the insurance agreement. Therefore, P_p is set equal to \$2.73/bu for corn, \$6.97/bu for soybeans, and \$3.99/bu for wheat.

In contrast, harvest prices (P_h) are unknown at the time of signing the insurance contract. Hence, P_h is treated as a random variable. It is postulated that $\ln(P_h)$ is normally distributed, an assumption commonly made for commodity futures prices (recall that P_h is a monthly average of futures prices). Data on P_h for 1989 through 1998 were assembled from *RMA Manager's Bulletins* (USDA-RMA) to calculate sample means and variances of $\ln(P_h)$. The estimated parameters are used as parameters for the price pdfs.

The simulated price and yield series mimics the historical correlations between prices and yields. Correlation coefficients between harvest prices (P_{ht}) and 1997-equivalent U.S. yields (y_{USt}) were estimated using (10) with data for years t = 1989 through t = 1998:

$$\hat{\rho}_{P_h y_{US}} = \frac{\frac{1}{10}\sum_{t=1989}^{1998}[\ln(P_{ht}) - \ln(P_{pt})](y_{USt} - Y_{USt})}{\left\{\frac{1}{10}\sum_{t=1989}^{1998}[\ln(P_{ht}) - \ln(P_{pt})]^2\right\}^{0.5}\left[\frac{1}{10}\sum_{t=1989}^{1998}(y_{USt} - Y_{USt})^2\right]^{0.5}}. \qquad (10)$$

In (10), planting prices (P_{pt}) are defined as in (4), and Y_{USt} is year t's expected yields obtained by applying the regression equation for U.S. yields (5). As predicted by economic theory, correlation estimates are negative for the three crops, with $\hat{\rho}_{P_h y_{US}} = -0.788$ for corn, $\hat{\rho}_{P_h y_{US}} = -0.499$ for soybeans, and $\hat{\rho}_{P_h y_{US}} = -0.649$ for wheat.

The estimated correlations are imposed on the simulated series of harvest prices and national yields by means of Johnson and Tenenbein's method. For this particular case, however, application of Johnson and Tenenbein's technique requires a trial-and-error process. The reason for this is that, while simulated observations of (the logarithm of)

harvest prices are obtained directly from the normal pdf, there is no single pdf from which simulated observations of national yields can be drawn. Instead, each simulated observation of U.S. yields is obtained by getting one "annual" draw of the vector of county yields, calculating county-level production figures (by multiplying county yields by their respective acreages),[5] obtaining U.S. output by summing production across counties, and, finally, dividing U.S. output by total acres.

Calibration of Insurance Policies

Farmers are allowed to choose from a menu of price protection (π), yield coverage (ψ), and policy premium levels when contracting BUP and GRP, and from a pre-specified set of yield coverage (ψ) and premium levels for CRC policies. Since actual data on individual (π, ψ) allocations are not available, a workable method is needed to proceed with the analysis.[6]

The method advocated here computes *calibrated* values of π and ψ for each policy and each county, and then proceeds as if they were the only price protection and yield coverage levels available. With calibrated π and ψ values for county i and policy p (p = BUP, GRP, CRC), county i's total expected indemnities are equal to total premiums paid in county i under policy p. That is, calibration yields an expected loss ratio (i.e., total expected indemnities divided by total premiums paid) equal to one for each county-policy combination. Because CAT price protection and yield coverage levels are fixed (see [2]), achieving expected loss ratios equal to one requires a different kind of calibration. As explained below, CAT calibration is performed by means of the variance of acre-level yields. Next, we elaborate more fully on these issues.

Total indemnities for policy p in county i and "year" t (I_{pit}) are calculated using (11):

$$I_{pit} = A_{pit} \times I_p(y_{it}), \tag{11}$$

where A_{pit} is the number of acres insured and $I_p(y_{it})$ is the average indemnity paid per acre insured. The notation for $I_p(y_{it})$ stresses that in any given year t, county i's average per-acre indemnity paid under policy p crucially depends on the observed county-level yield (y_{it}).

Because the model is calibrated for 1997, A_{pit} values are taken to be county i's number of acres under the corresponding policies in 1997. Formulas for $I_p(y_{it})$ are policy specific, the simplest of them being the one corresponding to GRP:

$$I_{GRP}(y_{it}) \equiv GRP(y_{it}; \pi_{GRPi}, \psi_{GRPi}), \tag{12}$$

where $\pi_{GRPi} \in (0.9, 1.5)$ and $\psi_{GRPi} \in \Psi_{GRP}$ are county i's *calibrated* price protection and yield coverage levels for GRP, respectively. Calibration is achieved by selecting π and ψ so as to equate the expected value of county i's total GRP indemnities (i.e., $A_{GRPi1997}$ $II_{GRP}(y_{it})$ $B(y_{it}|\ \alpha_i, \beta_i, Y_i^L, Y_i^U)\ dy_{it}$) with county i's total GRP premiums.[7]

Formulas for $I_p(y_{it})$ corresponding to CAT, BUP, and CRC policies are more involved than (12), as they depend on farm-level yields. For CAT, the formula for $I_p(y_{it})$ is

$$I_{CAT}(y_{it}) \equiv \int CAT(y_{ait})\ d\Phi(y_{ait}|\ y_{it}, s_{it}^2), \tag{13}$$

where y_{ait} is the yield in the *a*th acre of county i in year t, and $\Phi(\cdot|\ y_{it}, s_{it}^2)$ is the cumulative normal pdf with mean y_{it} and variance s_{it}^2. For numerical tractability, the integral in (13) is approximated by adding up over ten partitions of the y_{ait} range, each with 10 percent mass probability. All of the y_{ait} values within the *d*th partition are set equal to $y_{dit} \equiv \Phi^{-1}(d/11|\ y_{it}, s_{it}^2)$, where $\Phi^{-1}(\cdot)$ is the inverse function of $\Phi(\cdot)$.

Acre-level yield means are the simulated state-level yields (y_{it}), because county yields are simple averages of acre-level yields. Acre-level yield variances (s_{it}^2) are calculated employing (14):

$$s_{it}^2 = \max\{0.001,\ \bar{s}_i\ \exp[\Phi^{-1}(z_{it}|\ b_{30} + b_{31}\ x_{it}, b_{20} + b_{21}\ x_{it})]\}, \tag{14}$$

where $\bar{s}_i$ is a county-specific constant obtained by *calibration* (see below), z_{it} is a random number drawn from the uniform pdf U(z), x_{it} is the uniformly distributed county-level variables used to calculate the state-level yield realization y_{it} (see sub-subsection "Modeling Systemic County-Level Yield Risk"), and b_{30}, b_{31}, b_{20}, and b_{21} are a set of constants whose values are reported in Table B.1. The rationale for the linear equations involving x_{it} in (14) is discussed in Appendix B.

CAT differs from the other insurance programs in that its price protection and yield coverage levels are fixed at 0.6 and 0.5, respectively. Hence, the acre-level yield variance (s_{it}^2) is the only term in (13) that can be calibrated so as to render county i's CAT expected loss ratio equal to one.[8] Given the variance formula in (14), calibration of s_{it}^2 is achieved by estimating the constant $\bar{s}_i$.

The expression to compute county i's average per-acre BUP indemnities in year t is

$$I_{BUP}(y_{it}) \equiv \int \; BUP(y_{ait}; \pi_{BUPi}, \psi_{BUPi})\, d\Phi(y_{ait} | \, y_{it}, \; s_{it}^2), \qquad (15)$$

where $\pi_{BUPi} \in (0.6, 1.0)$ and $\psi_{BUPi} \in \Psi_{BUP}$ are county i's *calibrated* BUP price protection and yield coverage levels.[9] Numerical estimation of (15) is performed following the same procedure and employing the same (y_{it}, s_{it}^2) values that are used to calculate (13).

Finally, average per-acre CRC indemnities are computed from formula (16):

$$I_{CRC}(y_{it}) \equiv w_{CRCi} \int [\int CRC(y_{ait}, P_{ht}; \psi_{CRCi})\, \theta(P_{ht} | \, y_{USt})\, dP_{ht}]\, d\Phi(y_{ait} | \, y_{it}, \; s_{it}^2)$$

$$+ (1 - w_{CRCi}) \int [\int CRC(y_{ait}, P_{ht}; \psi_{CRCi} - 0.05)\, \theta(P_{ht} | \, y_{USt})\, dP_{ht}]\, d\Phi(y_{ait} | \, y_{it}, \; s_{it}^2). \qquad (16)$$

In (16), $w_{CRCi} \in (0, 1)$ represents a *calibrated* CRC acreage share for county i, $\psi_{CRCi} \in \Psi_{CRC}$ is county i's *calibrated* CRC yield coverage, and $\theta(P_{ht} | \, y_{USt})$ is the conditional pdf of harvest prices. Analogous to the calibration criterion for all other policies, terms w_{CRCi} and ψ_{CRCi} are chosen so as to make county i's CRC expected loss ratio equal to one. An interpretation of the linear combination in (16) is that w_{CRCi} percent of acres in county i are covered at level ψ_{CRCi}, and the remaining ($100 - w_{CRCi}$) percent acres are covered at the immediately lower permitted level.

Allocation to Reinsurance Funds and Simulation of RMA's Net Income

The final requirement to simulate RMA's net income from reinsurance activities consists of allocating policies across reinsurance funds according to the risks they present to insurers. As pointed out earlier, calibration is achieved by equating expected loss ratios to one, which is equivalent to setting expected net income from insurance equal to zero.

Despite this, insurance firms can expect to profit from selling insurance because the SRA is structured so as to provide incentives for them. By judiciously allocating their insurance policies among RMA's reinsurance funds, insurance firms expect to extract a positive net income at the expense of RMA, which must bear the corresponding expected losses.[10]

The 1997 state-level allocations to each of ARF, DF, and CF in premium dollars are provided by RMA. In 1997, policies designated CF generally accounted for the largest dollar amount reinsured by the RMA in the main producing states. For other states, allocations varied substantially among the three funds. The 1997 state-level allocations are mimicked in the simulation. This is accomplished by first ranking all of the policies for a particular state according to their standard deviation of loss ratios (which are obtained at the calibration stage).[11] Then, beginning with the policies with the largest (smallest) standard deviation, policies are designated ARF (CF) until their share of premiums reaches the state's share of premiums designated ARF (CF) in 1997. All policies yet to be designated are then assigned to DF.[12] The process is then repeated for each of the remaining states.

Having allocated all policies into reinsurance funds, Monte Carlo simulations of RMA's net income can finally be performed. Each "annual" observation of RMA's net income is obtained by (*i*) computing total indemnities for each policy and county by means of (11), (*ii*) calculating RMA's portion of such indemnities and their premiums according to the reinsurance funds they belong to, and (*iii*) adding the figures obtained in step (2) across counties and policies.

RMA's Net Income pdfs without Hedging: Results and Discussion

Table 2 contains summary statistics of the RMA's estimated net income pdfs in the absence of hedging. For illustrative purposes, the RMA's estimated net income cumulative pdfs from reinsuring corn are depicted in Figure 1. To save space, analogous figures for soybeans and wheat are not included, as they display similar patterns.

Expected net income from reinsuring corn is a loss of $36.3 million, with losses occurring 40.5 percent of the time. Standard deviation of RMA's net income for corn is $295.5 million. Value at risk at the 5 percent and 10 percent levels [VAR(5%) and

TABLE 2. Statistics of the simulated pdf of RMA's net income without hedging, expressed in millions of dollars

				Aggregate	
	Corn	**Soybeans**	**Wheat**	**Hist. Corr.[a]**	**Perf. Corr.[b]**
Mean	–36.3	–24.6	–17.7	–78.7	–78.7
Standard Deviation	295.5	227.7	111.8	473.7	635.0
VAR(5%)	–633.8	–498.6	–226.2	–1,006.8	–1,358.5
VAR(10%)	–487.2	–344.3	–171.5	–779.0	–1,003.0
Minimum	–1,383.8	–1,078.8	–524.9	–2,012.6	–2,987.5
Maximum	277.7	221.1	190.8	641.9	689.6

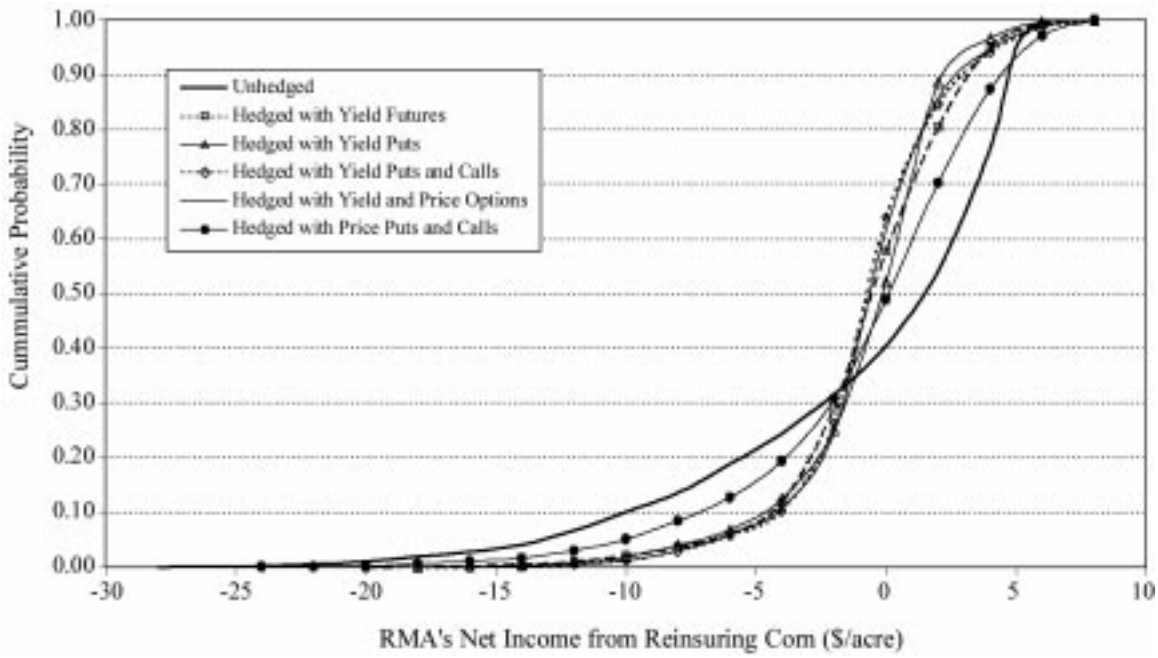
Note: Results are based on 2,500 simulated "annual" observations.
[a]"Hist. Corr." means simulation results corresponding to historical correlation levels across national crop yields.
[b]"Perf. Corr." denotes simulation results assuming that yields and prices are perfectly correlated across the three crops.

VAR(10%), respectively] are losses of $633.8 million and $487.2 million, respectively, and the minimum (maximum) "annual" net income obtained is a $1,383.8 million loss ($277.7 million gain).[13]

Results for soybeans are similar, albeit on a somewhat smaller scale. Expected net income is a loss of $24.6 million for RMA in its capacity as reinsurer of soybean policies. Net income ranges from a loss of $1,078.8 million to a gain of $221.1 million, with losses observed 42.2 percent of the time. Standard deviation of net income equals $227.7 million. In this instance, VAR(5%) and VAR(10%) consist of losses of $498.6 million and $344.3 million, respectively.

The scale is smaller still for RMA net income from reinsuring wheat, with an expected loss of $17.7 million and a standard deviation equal to $111.8 million. The greatest loss ($524.9 million) as well as the largest gain ($190.8 million) for wheat is smaller than the respective figures for corn and soybeans. In the case of wheat reinsurance, median net income is a loss, as expenses exceed revenues 50.6 percent of the time. VAR(5%) and VAR(10%) amount to losses of $226.2 million and $171.5 million, respectively.

For at least two reasons, it is important to estimate the pdf of RMA's net income aggregated across crops, as well. First, it may be argued that the ultimate risk faced by RMA stems from its total potential losses, rather than its potential losses from each individual crop. Second, it is of interest to assess to what extent diversifying among crops ameliorates the risk of RMA's total net income.



Note: Results are based on 2,500 simulated "annual" observations. Hedged positions are chosen so as to minimize the variance of RMA's net income from reinsuring corn. Yield (price)options are assumed to have strike yields (prices) of 120 bu/acre ($2.75/bu). All options and futures are also assumed to be bought and sold at their fair values.

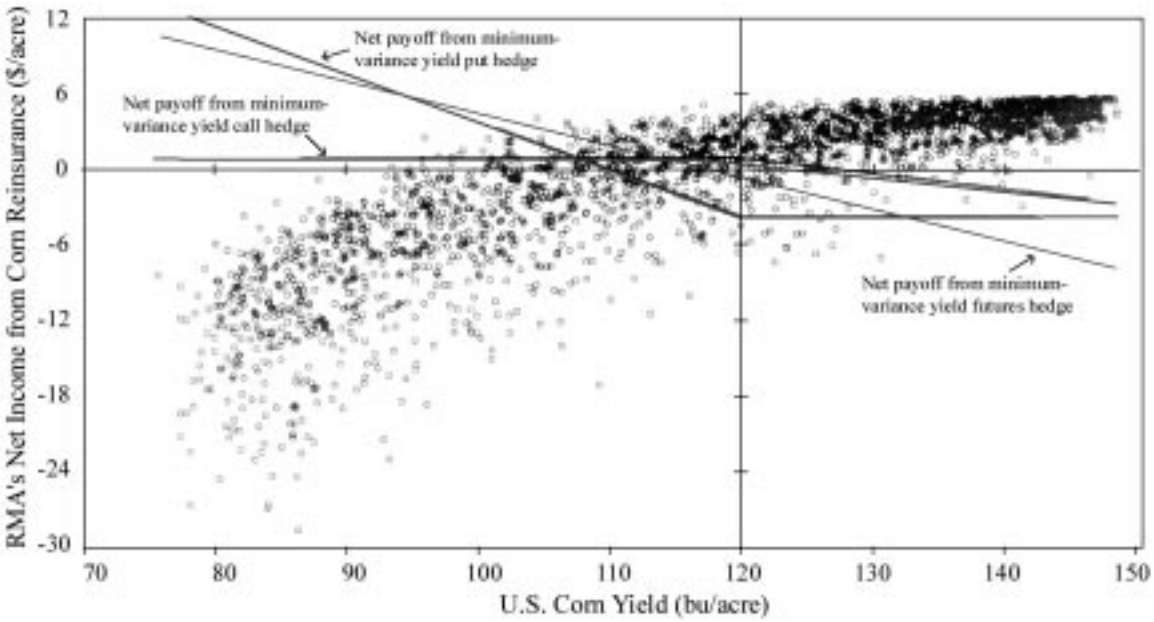
FIGURE 1. Estimated cumulative pdfs or RMA's net income from reinsuring corn

To simulate the pdf of RMA's aggregate net income, historical correlation levels among national yields for the three crops are imposed.[14] Summary pdf statistics are reported in the last column of Table 2. The mean of RMA's aggregate net income is a loss of $78.7 million, with values ranging from a maximum loss of $2,012.6 million and a maximum gain of $641.9 million. The standard deviation is $473.7 million, and VAR(5%) and VAR(10%) are losses of $1,006.8 million and $779.0 million, respectively. If yields and prices were perfectly correlated across the three crops, RMA's aggregate net income would be characterized by a standard deviation of $635.0 million, and VAR(5%) and VAR(10%) would be losses of $1,358.5 million and $1,003.0 million, respectively (see last column of Table 2). Therefore, RMA seems to benefit somewhat by diversifying across crops.

RMA's Net Income pdfs in the Presence of Hedging

The analysis in this section is performed on a per-acre (insured only) basis, to make it more intuitive. Given the substantial risks stemming from RMA's reinsurance activities, it is of interest to investigate the potential to attenuate such risks by hedging with yield and/or price derivatives. Because the relationship between reinsurance costs and yields crucially affects the potential for yield derivatives to decrease risks, it is essential to have simulated observations on national yields in addition to simulated harvest prices and RMA's net income. Simulated national yield observations are obtained as described in the subsection "Simulation of Harvest Prices."

Figure 2 displays the scatter diagram of RMA's net income per acre of corn against U.S. corn yields. Per-acre net income from reinsuring corn increases with corn yields, but at a decreasing rate. Yields of 75 bu/acre are associated with expected losses of around $30/acre, whereas for yields of 150 bu/acre there are expected gains of about $5/acre.



Note: Results are based on 2,500 simulated "annual" observations. Yield options are assumed to have strike yields of 120 bu/acre. Options and futures are also assumed to be bought and sold at their fair values.

FIGURE 2. Relationship between RMA's net income from reinsuring corn and U.S. corn yields

Notwithstanding the strong positive effect of yields on RMA's net income, there is substantial variation in net income for any given national yield. RMA may incur losses at all yield levels, but it may also reap gains, even for yields as low as 90 bu/acre.

The effect of hedging with corn yield futures on RMA's net income is investigated first.[15] It is assumed that futures contracts are based on planted acres and that RMA opens its futures position far enough in advance so that yield futures contracts are priced at their unconditional expected value of 117.6 bu/acre (i.e., the average national corn yield). It is further assumed that RMA's hedging objective is to minimize risk, in which case the optimal hedge is the so-called minimum-variance hedge (e.g., Ederington).

Here, the minimum-variance hedge is a short futures position proportional to the slope in a regression of RMA's net income per acre against national yields. The slope estimate is 0.26, which means that, on average, a one-bushel increase in national yields increases expected RMA's net income by $0.26/acre (see Figure 2). Hence, the expected income of the offsetting yield futures position (i.e., the minimum variance hedge) must be –$0.26/acre for each unit increment in U.S. corn yields. Adding the income from shorting the corresponding number of corn yield futures to RMA's unhedged net income gives RMA's net income after hedging.

Simulation results are summarized in Table 3 and illustrated graphically in Figure 1. Compared to the unhedged scenario, hedging with yield futures leaves RMA's expected net income unchanged at –$0.75/acre but substantially reduces the standard deviation of net income (from $6.11/acre to $3.34/acre). Hedging also reduces the maximum observed loss from $28.62/acre to $20.64/acre, and increases the maximum observed gain from $5.74/acre to $8.14/acre. Hedging improves VAR(5%) from a loss of $13.11/acre to a loss of $6.58/acre, and VAR(10%) from a loss of $10.08/acre to a loss of $4.44/acre.

Given the nonlinear kind of relationship between RMA's net income and U.S. yields depicted in Figure 2, holding an appropriate position in corn yield put options should provide a better hedge than holding yield futures. Yield put payoffs are linear in yields when yields fall below the puts' strike yield, so that a put holder's income increases by a constant amount per bushel drop in yields under the strike yield. Should final yields be greater than the yield strike at settlement, the put option expires worthless, limiting the loss to the amount initially paid to purchase the put.

TABLE 3. Statistics of the simulated pdf of RMA's net income, expressed in $/acre

		Hedged with:			
	Unhedged	**Yield Futures**	**Yield Puts and Calls**	**Yield and Price Options**	**Price Puts and Calls**
Corn:					
Mean	–0.75	–0.75	–0.75	–0.75	–0.75
Std. Dev.	6.11	3.34	3.12	3.07	4.74
VAR(5%)	–13.11	–6.58	–6.63	–6.58	–10.07
VAR(10%)	–10.08	–4.44	–4.13	–4.17	–7.14
Soybeans:					
Mean	–0.58	–0.58	–0.58	–0.58	–0.58
Std. Dev.	5.39	3.37	3.29	3.28	5.01
VAR(5%)	–11.81	–6.95	–6.60	–6.67	–10.90
VAR(10%)	–8.16	–4.15	–4.33	–4.33	–7.52
Wheat:					
Mean	–0.35	–0.35	–0.35	–0.35	–0.35
Std. Dev.	2.21	1.61	1.60	1.60	1.96
VAR(5%)	–4.47	–3.34	–3.40	–3.37	–4.07
VAR(10%)	–3.39	–2.47	–2.44	–2.49	–3.00

Note: Results are based on 2,500 simulated "annual" observations. Hedged positions are chosen so as to minimize the variance of RMA's net income from reinsuring the respective crops. Yield (price) options are assumed to have strike yields (prices) of 120 bu/acre ($2.75/bu) for corn, 40 bu/acre ($6.95/bu) for soybeans, and 34 bu/acre ($4.00/bu) for wheat. All options and futures also are assumed to be bought and sold at their fair values.

To assess the effectiveness of hedging with yield puts, a strike yield of 120 bu/acre is selected, as this is near the mean yield. Holding puts with this strike yield will attenuate losses only when yields fall below 120 bu/acre. Similar to futures, the put position that minimizes the variance of RMA's net income (below 120 bu/acre) is determined by regressing net income on yields. In this instance, however, the regression should only employ data corresponding to yields below 120 bu/acre. The estimated slope of $0.38/acre implies that, on average, RMA's unhedged net income is expected to decrease by $0.38/acre for every bushel drop in yields under 120 bu/acre. Therefore, the offsetting put position must provide $0.38/acre of income per bushel fall in yields. The cost of buying the appropriate put position is taken to be equal to its fair value of $3.75/acre.[16] Because the net income from the put position cannot be positive until the $3.75/acre cost is recovered, the put position results in losses unless yields are sufficiently low (110.25 bu/acre or less).

Compared to the no-hedging scenario, hedging with yield puts leaves the mean unchanged[17] (at a loss of $0.75/acre) but reduces risk substantially, cutting by almost half both the standard deviation (from $6.11/acre to $3.17/acre) and VAR(5%) (from a loss of $13.11/acre to a loss of $7.20/acre). Somewhat unexpectedly, however, the standard deviation achieved by hedging with yield puts ($3.17/acre) is negligibly smaller than the standard deviation obtained by hedging with yield futures ($3.34/acre). Even more perplexing is that VAR(5%) is worse when hedging with puts than when hedging with futures (losses of $7.20/acre and $6.58/acre, respectively). Further analysis reveals that the culprit responsible for VAR(5%) being worse for puts than for futures is the high cost of purchasing the former ($3.75/acre) compared to buying the latter ($0/acre).

From the scatter diagram in Figure 2 it is clear that RMA's net income is positively related to yields, for yields above 120 bu/acre. The estimated slope from the corresponding regression is 0.13. This implies that a combination of yield puts and calls should be more effective in reducing risks than yield puts alone. Focusing on yield puts and calls with a strike yield of 120 bu/acre, the optimal put position is the same as obtained before; whereas the optimal call position consists of selling calls so that RMA's net income is expected to decrease by $0.13/acre (i.e., the slope estimate) per bushel increase in yields above 120 bu/acre. The ensuing sale of calls at fair values generates $0.93/acre in revenues, which, combined with the $3.75/acre cost of the puts, nets a cost of $2.82/acre for the portfolio of puts and calls.

As expected, hedging with yield puts and calls is more effective in reducing risks than hedging with puts only. By hedging with both puts and calls instead of puts alone, the standard deviation of RMA's net income is reduced from $3.17/acre to $3.12/acre, and VAR(5%) is improved from a loss of $7.20/acre to a loss of $6.63/acre. It is important to stress, however, that the risk reduction performance of the put-call portfolio vis-à-vis hedging with futures seems negligible from a practical standpoint.

It is also relevant to analyze whether price derivatives can be added to the portfolio of yield derivatives to further mitigate RMA's net income risk. For example, it is clear from (4) that CRC indemnities depend directly and in a nonlinear fashion on harvest prices (P_h). When harvest prices are above planting prices (P_p), CRC indemnities are paid

if $(\psi_{CRC}\ Y - y)\ P_h > 0$. In contrast, when harvest prices are below planting prices, CRC indemnities are paid if $(\psi_{CRC}\ P_p\ Y - P_h\ y) > 0$. Therefore, holding yields constant, indemnities *increase* (*decrease*) by the amount $\psi_{CRC}\ Y - y > 0$ $(y > 0)$ per unit increase in harvest price if $P_h > (<)\ P_p$. In other words, holding yields constant, RMA's net income is expected to decline as harvest prices deviate from expected prices.

The preceding analysis suggests that a long position in both price puts and price calls with a strike price near the planting price (i.e., a bottom straddle or straddle purchase (Hull, pp. 187–188) would be the most appropriate addition to the portfolio of yield derivatives. To examine this, RMA's net income after hedging with yield options is regressed against futures prices. The hypothesized change in slope sign at the expected price level is taken care of by running separate regressions for data corresponding to prices above and below $2.75/bu (the 1997 corn CRC planting price is $2.73). As expected, the slope is negative (–2.14) and positive (2.41) for prices below and above $2.75/bu, respectively. Thus, the offsetting position in price options must have a payoff that decreases by –$2.14/acre (increases by $2.41/acre) for every $1/bu increase in the harvest price of corn below (above) $2.75/bu.

The fair cost of the offsetting position in price options is $0.78/acre. Adding this bottom straddle to the portfolio of yield options has very little effect on the pdf of RMA's net income (see Table 3). With its inclusion, standard deviation is reduced only slightly to $3.07/acre, compared to $3.12/acre when using yield options only. VAR(5%) falls from a loss of $6.63/acre to a loss of $6.58/acre, but VAR(10%) moves from a loss of $4.13/acre to a loss of $4.17/acre. Looking at the range of outcomes, the maximum loss is reduced from $18.61/acre to $18.08/acre, whereas the maximum gain is also reduced from $8.96/acre to $8.65/acre.

Unfortunately, in reality it may be problematic for RMA to use yield derivatives. This is true because currently there are no yield derivatives for soybeans and wheat. Further, yield derivatives for corn have languished since their introduction in 1995, and their market seems too thin for RMA's hedging needs. For these reasons, and given the strong correlation between prices and yields (e.g., see subsection "Simulation of Harvest Prices"), it is of interest to explore the potential of using only price derivatives to hedge RMA's net income.

To assess the hedging potential of price derivatives in isolation, it is assumed that RMA only hedges by means of corn price options with a strike price of $2.75/bu (i.e., the same strike price used to analyze the portfolio of yield and price derivatives).[18] The portfolio of price options that minimizes the variance of RMA's net income from reinsuring corn is obtained by running separate regressions of net income against harvest prices, for data corresponding to prices above and below $2.75/bu. The estimated slope for prices above (below) $2.75/bu is –12.03 (–9.62), which implies that the optimum portfolio involves purchasing calls and selling puts in a ratio of 12.03:9.62.[19]

Not surprisingly, Table 3 shows that the position in price options is not as effective at reducing RMA's net income risk as is the position in yield options. With price options, the standard deviation is $4.74/acre, compared to $6.11/acre with the naked position and $3.12/acre with the position in yield options. VAR(5%) is a loss of $10.07/acre, falling between the unhedged and yield-option-hedged VAR(5%) losses of $13.11/acre and $6.63/acre. Likewise, VAR(10%) is $7.14/acre for price option hedging, which is better than the no-hedge VAR(10%) ($10.08/acre) but not as good as VAR(10%) when using yield options ($4.13/acre).

The effectiveness of hedging RMA's net income from reinsuring soybeans and wheat is analyzed using the same methods that are used for corn. It is apparent from the summary figures reported in Table 3 that results for soybeans and wheat are quite similar to the results for corn, except for some differences in scale. Therefore, in the interest of space, discussion of the findings from hedging soybeans and wheat is omitted.

Two major observations can be made after examining the net income from reinsuring individual crops with and without hedging instruments. The first is that net income without hedging exhibits wide variability, with small probabilities of very large losses. This shape of the pdf is altered considerably by hedging with yield and/or price derivatives. Whereas all pdfs with and without hedging are noticeably skewed to the left, the no-hedge pdfs exhibit far less kurtosis. The second observation is that, despite the noticeable risk-reduction effect of holding appropriate positions in derivatives, there remains a substantial amount of variability in net income after hedging. While the likelihood of the worst results is much reduced, the range of potential outcomes after hedging is about the same as before hedging.

For the sake of completeness, the pdf of RMA's aggregate net income after hedging is also estimated, imposing historical correlation levels across national crop yields. It is found that while hedging leaves the mean of aggregate net income unchanged at a loss of $0.56/acre (due to the assumption that derivatives are fairly priced), it attenuates variability substantially. For example, hedging with both yield and price options reduces the standard deviation by more than half (from $3.36/acre to $1.56/acre), and improves VAR(5%) [VAR(10%)] from a loss of $7.13/acre ($5.52/acre) to a loss of $3.31/acre ($2.44/acre).

Summary and Conclusions

While crop insurance has received significant attention from economists, there has been little attempt at quantifying the level of risk accepted by the government in its role as reinsurer. The present study aims at filling this gap in the literature by estimating the probability density function (pdf) of the Federal Risk Management Agency's (RMA) net income from reinsuring corn, wheat, and soybeans. This is accomplished by means of Monte Carlo simulations in which correlated yields and prices are drawn and indemnities are then computed. Using the reinsurance obligations of the RMA, as described in the 1997 Standard Reinsurance Agreement (SRA), payments to and from the RMA are calculated from indemnity levels.

It is estimated that RMA's VAR (5%) is $1 billion. That is, there is a 5 percent probability that RMA will need to reimburse at least $1 billion to insurance companies. This is a number greater than the reimbursements made by the RMA in its worst reinsurance year (i.e., 1993), in which claims against the RMA exceeded premiums by $822 million (U.S. GAO). Another important result is the estimated fair value of RMA's reinsurance services to insurance firms. The expected net transfer from RMA to the insurance industry due to the SRA specifications is estimated at $36.3 million for corn, $24.6 million for soybeans, and $17.6 million for wheat, for a total of $78.7 million.[20] This figure is small compared to potential losses under worst-case scenarios, but it is a real cost to the government and a real benefit to the insurers.

The question of risk reduction for the reinsurer is also investigated. Assuming the existence of national yield and price derivatives markets, various hedging strategies are examined for their potential to reduce RMA's risks from reinsurance. The level of risk reduction achievable by hedging is found to be appreciable. For example, hedging with yield and price options may reduce RMA's VAR(5%) by more than half, from $1 billion to $467 million. However, use of these derivative contracts alone is clearly no panacea.

Endnotes

1. In the crop insurance market, diversification does little to mitigate the level of risk (Quiggin). For example, Miranda and Glauber computed the coefficient of variation (CV) of indemnity portfolios owned by the ten largest crop insurance firms, and found that CVs based on historical yield correlations were between 22 and 49 times larger than CVs based on zero yield correlations. Further, such high CVs did not seem to be due to poor diversification practices. This level of variability is much higher than that seen in other lines of insurance where government participation has not been required.

2. Actual Production History Insurance is also known as Multiple-Peril Crop Insurance.

3. Rank correlations are not affected by monotonic transformations of the underlying variables. This becomes important at the simulation stage, where uniformly distributed random variables are transformed into beta-distributed random yields (see next paragraph).

4. In a few low-production counties with small acreages and some missing yield observations (because land goes in and out of production over the years) the estimated correlation is negative. These cases are assigned a correlation level of zero in the simulations, because it seems unreasonable that the true correlation would be negative.

5. Note that county acreages must include not only insured acres but also uninsured acres.

6. Even if actual data on individual (π, ψ) allocations were available, modeling all of the permitted (π, ψ) combinations for each policy would not be tractable. For example, there are 27 (π, ψ) combinations for BUP alone.

7. Calibration begins by fixing ψ at its maximum level permitted ($\psi = 0.9$) and searching for the value of $\pi \in (0.9, 1.5)$ that equates the expected loss ratio to one. If this search fails, the process is repeated by successively fixing ψ at 0.85, 0.8, 0.75, and 0.7 until a $\pi \in (0.9, 1.5)$ is found that makes expected loss ratios equal to one. When this search also fails, average per-acre GRP indemnities are defined not as (12) but as $I_{GRP}(y_{it}) \equiv w_{GRPi} \times GRP(y_{it}; 1, \psi_{GRPi}) + (1 - w_{GRPi}) \times GRP(y_{it}; 1, \underline{\psi}_{GRPi})$ instead, where $w_{GRPi} \in (0, 1)$, $\psi_{GRPi} \in \Psi_{GRP}$, $\underline{\psi}_{GRPi} \in \Psi_{GRP}$, $\psi_{GRPi} \neq \underline{\psi}_{GRPi}$, and w_{GRPi}, ψ_{GRPi}, and $\underline{\psi}_{GRPi}$ are selected so that county i's GRP expected loss ratio equals one. The implicit assumption in using this linear combination is that a share w_{GRPi} of the GRP acres in county i are insured at coverage level ψ_{GRPi}, and the rest are insured at coverage level $\underline{\psi}_{GRPi}$.

8. There are very few counties for which no CAT policies were sold in 1997, so that their CAT premiums are zero. In such instances, $\bar{s}_i$ is calibrated by using BUP premiums, fixing BUP price protection and yield coverage levels at $\pi = 0.75$ and $\psi = 0.65$, respectively, and then proceeding as if they were CAT policies.

9. For calibration, an attempt is first made to equate expected indemnities to premiums by fixing $\psi = 0.65$ and adjusting π. If there is no $\pi \in (0.6, 1.0)$ which accomplishes this, the same process is repeated for $\psi = 0.5$ and, if necessary, for $\psi = 0.75$. As with GRP, it is sometimes necessary to use a linear combination of indemnities at two different yield coverage levels because the previous search procedure fails to produce expected loss ratios equal to one. In this instance, average per-acre BUP indemnities are defined as $I_{BUP}(y_{it}) \equiv w_{BUPi} [\int BUP(y_{it}; 1, \psi_{BUPi})\, d\Phi(y_{ait} | y_{it}, s_{it}^2)] + (1 - w_{BUPi}) [\int BUP(y_{it}; 1, \underline{\psi}_{BUPi})\, d\Phi(y_{ait} | y_{it}, s_{it}^2)]$ instead of (15), where $w_{BUPi} \in (0, 1)$, $\psi_{BUP} \in \Psi_{BUP}$, $\underline{\psi}_{BUPi} \in \Psi_{BUP}$, $\psi_{BUPi} \neq \underline{\psi}_{BUPi}$, and w_{BUPi}, ψ_{BUPi}, and $\underline{\psi}_{BUPi}$ render county i's BUP expected loss ratio equal to one.

10. This implies that RMA's expected net income from reinsurance (without hedging) can also be interpreted as the negative of insurance firms' expected net income.

11. Due to the way policies are calibrated, the standard deviation of loss ratios is identical to the CV of indemnities.

12. A small number of policies cannot be calibrated so as to yield expected loss ratios equal to one, because their expected loss ratios exceed one for all of the permitted coverage levels. Such policies are calibrated so as to minimize their expected loss ratios within the coverage levels allowed and are assumed to be ceded to the RMA by the insurers. Such instances are confined to counties with very few insured acres.

13. VAR(z percent) is merely the dollar value \$v such that Probability(net income ≤ \$v) = z percent (Jorion).

14. More specifically, in each "annual" simulation the historical correlation structure is imposed to draw the vector of national-level uniformly distributed variables $\mathbf{x}_{US} \equiv [x_{US,corn}, x_{US,soybeans}, x_{US,wheat}]$ (see sub-subsection "Modeling Systemic County-Level Yield Risk"). This is done using the correlation matrix, which the @Risk software makes available to the user.

15. Yield futures do currently exist for corn and are traded on the Chicago Board of Trade. Traded contracts are based on corn yield estimates of harvested acres made by the National Agricultural Statistics Service.

16. The \$3.75/acre figure would be the put position's value in the presence of efficient markets (so that put prices are equal to their expected payoffs). It is obtained by computing the average of the put position's payoffs [$= 0.38 \times \max(0, 120 - y_{USt})$] over 2,500 simulated yield draws.

17. The unchanged mean is an implication that the put position's cost equals its fair value.

18. To save space, hedging with price futures is not discussed, because put-call parity (Hull, p. 167) implies that any futures position can be replicated by an appropriate combination of puts and calls with the same strike price. Hence, an optimal portfolio of puts and calls will always perform at least as well as an optimal futures position.

19. Note the striking contrast with the optimal portfolio of yield options, which involved selling calls and buying puts in a ratio of 0.13:0.38. The reason for such a difference is the negative correlation between prices and yields.

20. This is in addition to the administrative and operations expense subsidies that RMA pays to insurance companies.

21. The value of N = 26 is the number of annual yield observations per county. Parameter Y^U is standardized to 1 to facilitate numerical calculations, as $B(y|\ \alpha, \beta, Y^L = 0, Y^U) = B(z|\ \alpha, \beta, Y^L = 0, Y^U = 1)$ for $z \equiv y/Y^U$. This also implies that the ratio y^U/Y^U is independent of Y^U, explaining why Y^U is not included in the right-hand side of (A.1).

22. For example, the variance of farm-level yields may be greater than usual in drought years, if some production units experience very low yields while others are not severely affected by dry weather.

23. For instance, without rescaling, a negative relationship between CVR and county yield would be found if counties with low (high) expected yields have relatively large (small) CVRs.

24. In essence, rescaling is the reverse of the process by which county yields are simulated in the model.

Appendix A: Derivation of Expression (9)

The motivation for (9) is that the maximum value from a sample of N observations from the beta pdf $B(y|\ \alpha, \beta, Y^L, Y^U)$, $y^U(N, \alpha, \beta, Y^L, Y^U)$ is a random variable that is almost surely smaller than Y^U. Because $y^U(N, \alpha, \beta, Y^L, Y^U)$ is a negatively biased estimate of Y^U, a feasible method is needed so as to obtain a better estimate of Y^U. To assess the magnitude of the likely bias in the historical data set, preliminary estimates of α_i ($\hat{\alpha}_i$) and β_i ($\hat{\beta}_i$) were obtained for each county i by setting $Y_i^L = 0$ and letting Y_i^U range from y_i^U through 1.5 y_i^U. It was found that in all instances $2 < \hat{\alpha}_i < 15$, $2 < \hat{\beta}_i < 15$, and $0 < \hat{\alpha}_i - \hat{\beta}_i < 7$.

The $\hat{\alpha}_i$ and $\hat{\beta}_i$ estimates were used to construct 56 (α, β) pairs spanning the likely range of unknown α_is and β_is in the available data. For each (α, β) pair, 1,000 observations on $y^U(N = 26, \alpha, \beta, Y^L = 0, Y^U = 1)$ were obtained by (1) drawing 26 observations from the corresponding beta pdf,[21] (2) obtaining their maximum value (y^U), (3) repeating steps (1) and (2) 1,000 times, and (4) calculating the average from the 1,000 observations about y^U. Letting $\bar{y}^U$ denote such averages, regression (A.1) was run using ordinary least squares on the resulting 56 observations:

$$\ln(\bar{y}^U / Y^U) = -2.5426^{**} - 0.9442^{**} \ln(\alpha) + 1.7203^{**} \ln(\beta) + \text{error}, \quad R^2 = 0. \qquad \text{(A.1)}$$

$$\qquad (0.0449) \qquad (0.0259) \qquad (0.0207)$$

In (A.1), numbers between parentheses below coefficient estimates are the respective standard deviations, and "**" denotes significantly different from zero at the 1 percent level based on the two-tailed t-statistic. The log-log specification in (A.1) was adopted because it had better fit than alternative specifications such as linear and log-linear.

Appendix B: Variability of Within-County Yield PDFs

It seems reasonable for the shape of the pdf of within-county yields to vary with county-level yields.[22] To examine this hypothesis, the CV ratio $CVR_{i,t} \equiv CVF_{it}/CVF_i$ is constructed for each county i with data for at least 100 producers, for years t = 1983 through 1994. Variable CVF_{it} is an estimate of county i's CV of farm-level yields in year t, obtained by using RMA records on BUP and CAT participants. Variable CVF_i is the average CVF_{it} for county i across time.

The short time series prevents us from estimating the relationship between $CVR_{i,t}$ and county yields county by county. However, a general relationship between CVRs and county yields can be estimated by combining series across counties. Such a combination requires rescaling county yields; otherwise, the estimated relationship could be mostly driven by county characteristics.[23] Rescaling is achieved by using the beta function (6) estimated for each county, along with 1997-equivalent county yields from 1983 through 1994. The rescaled yield for the *i*th county in year t, $x_{it} = B(y_{it}| \alpha_i, \beta_i, Y_i^L, Y_i^U)$, is a number between 0 and 1 denoting the probability of county i's yields being less or equal than the yield realized in year t (y_{it}).[24]

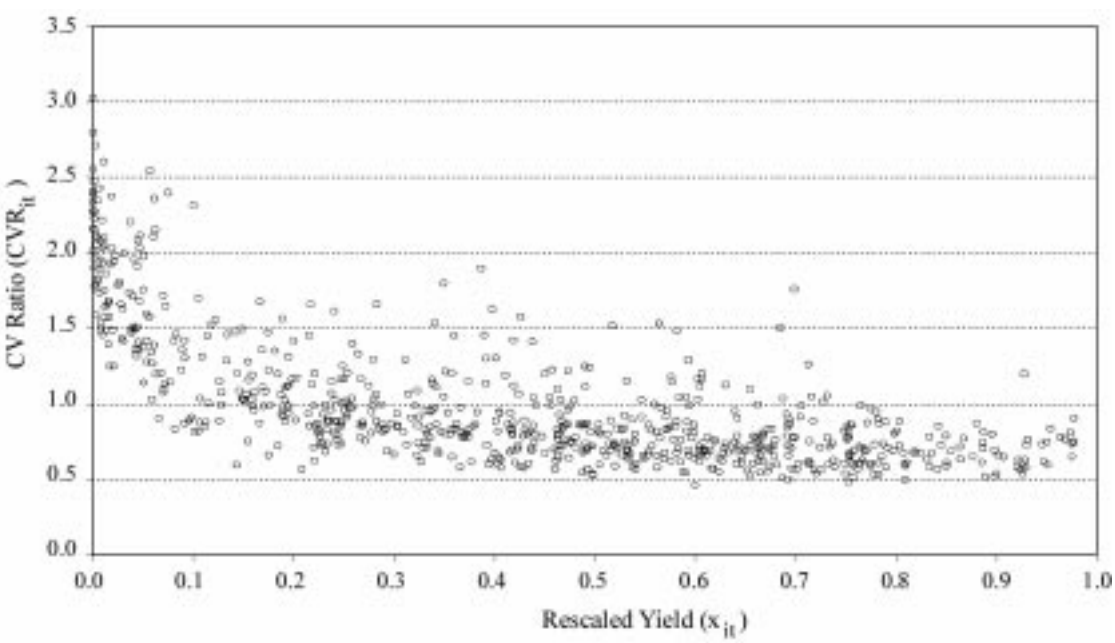


FIGURE B.1. Relationship between CV ratio (CVR_{it}) and rescaled yield (x_{it}) for corn

Figure B.1 displays a scatter plot of CVRs versus x's for corn. Analogous scatter plots for soybeans and wheat display very similar patterns and are omitted in the benefit of space. From Figure B.1, it can be observed that CVR tends to decrease at a decreasing rate as x rises, and that the dispersion of CVR data points varies inversely with the level of x. To model such patterns, regressions of the logarithm of CVR against x were run by weighted least squares, using as weights the predicted squared errors from ordinary least squares regressions of the logarithm of CVR versus x. Regression results are reported in Table B.1 for all three crops, where the weighted least squares regression is shown as model 3. The visual patterns observed in the CVR-x scatter plots are strongly significant. The estimated coefficients for models 2 and 3 in Table B.1 are plugged into equation (14) to calculate the variances of acre-level yields.

TABLE B.1. Regression estimates of the relationship between CV ratio (CVR_{it}) and rescaled yield (x_{it})

Regression Models:[a]

1. $\ln(CVR_{it}) = b_{10} + b_{11}\, x_{it} + e_{1it}$.
2. $\hat{e}^2_{1it} = b_{20} + b_{21}\, x_{it} + e_{2it}$, where $\hat{e}_{1it} \equiv \ln(CVR_{it}) - \hat{b}_{10} - \hat{b}_{11}\, x_{it}$.
3. $\ln(CVR_{it})/(\hat{b}_{20} + \hat{b}_{21}\, x_{it})^{0.5} = b_{30} + b_{31}\, x_{it} + e_{3it}$.

		Corn	Soybeans	Wheat
Model 1:	$\hat{b}_{10}$	0.3983**	0.3473**	0.3127**
		(0.0169)[b]	(0.0183)	(0.0173)
	$\hat{b}_{11}$	–0.8280**	–0.9938**	–0.8279**
		(0.0261)	(0.0363)	(0.0318)
	R^2	0.510	0.508	0.410
	No. observations	971	730	544
Model 2:	$\hat{b}_{20}$	0.1087**	0.0954**	0.1048**
		(0.0084)	(0.0065)	(0.0070)
	$\hat{b}_{21}$	–0.0792**	–0.0619**	–0.0597**
		(0.0131)	(0.0128)	(0.0128)
	R^2	0.037	0.031	0.039
	W[c]	35.51**	22.66**	21.37**
	No. observations	971	730	544
Model 3:	$\hat{b}_{30}$	1.3907**	1.2520**	1.0745**
		(0.0619)	(0.0660)	(0.0607)
	$\hat{b}_{31}$	–3.3074**	–3.9242**	–3.0749**
		(0.0956)	(0.1309)	(0.1115)
	R^2	0.553	0.553	0.438
	No. observations	971	730	544

Note: ** denotes significantly different from zero at the 1% level based on the two-tailed t-statistic.

[a]Variables with hats (^) denote sample estimates. Variables e_{1it}, e_{2it}, and e_{i3t} are errors.

[b]Numbers between parentheses below coefficient estimates are the respective standard deviations.

[c]W is the critical value for the heteroskedasticity test developed by White, which is distributed as χ^2_1.

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