Production Risk and the Estimation of Ex Ante Cost Functions

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Abstract

Cost function estimation under production uncertainty is problematic because the relevant cost is conditional on unobservable expected output. If input demand functions are also stochastic, then a nonlinear errors-in-variables model is obtained and standard estimation procedures typically fail to attain consistency. But by exploiting the full implications of the expected profit maximization hypothesis that gives rise to ex ante cost functions, it is shown that the errors-in-variables problem can be effectively removed, and consistent estimation of the parameters of interest can be achieved. A Monte Carlo experiment illustrates the advantages of the proposed procedure as well as the pitfalls of other existing estimators.

Key words: cost function, duality, expected profit maximization, nonlinear errors-in-variables, stochastic production

PRODUCTION RISK AND THE ESTIMATION OF EX ANTE COST FUNCTIONS

1. Introduction

Following the pioneering work of Shephard (1953), Diewert (1971), and McFadden (1978), the cost function approach has proven very useful and popular in applied production studies. Insofar as the hypothesis of cost minimization is correct, estimating a cost function is usually deemed preferable to estimating a primal specification of the technology because, by using input prices instead of input quantities on the right-hand side of estimating equations, one removes a potential source of simultaneous equation bias. Specifically, in the cost function framework input choices are modeled as a function of input prices and the output level. But, as emphasized in the recent article by Pope and Just (1996), a problem then arises when the production technology is inherently stochastic. Such a case is very important in agricultural and environmental production models, where climatic and pest factors outside of the producer's control affect realized output in a nontrivial fashion. When producers make their input choices prior to the resolution of this production uncertainty, the standard cost function specification (which is conditional on the realized output level) is not relevant. In this setting one should instead study input choices conditional on the expected output level, i.e., estimate the structure of an "ex ante" cost function.

Estimating ex ante cost functions turns out to be problematic because the expected output level that is relevant for the cost-minimization problem is not observable. Pope and Just (1996) propose a solution that estimates the expected output level jointly with the cost function model, and they argue that their procedure yields consistent estimation of the parameters of the cost function. This interesting approach exploits duality to recover the form of the production function that is implied by the cost function being estimated, and then uses this production function, together with observed input quantities, to estimate the (unobserved) expected output level. But this representing unobserved expected output as a function of inputs introduces simultaneity in the specified model. This simultaneity is most apparent when the cost function is equivalently represented in terms of cost-minimizing input demands, such that input quantities appear as both left-hand-side variables (the dependent variables of input demand equations) and right-hand-side variables (as variables "estimating" expected output). Because of this simultaneity, Pope and Just's (1996) ex ante procedure needs to assume that expected output is a deterministic function of observed input quantities. Consequently, the proposed ex ante estimation procedure achieves consistency if input choices hold deterministically. But when input demands are stochastic (at least as far as the econometrician is concerned), as one would expect in any empirical application, the consistency property of estimates obtained from the ex ante procedure is called into question.

The crux of the matter is that, in general, in empirical applications of the ex ante cost model one should really allow for two distinct sources of errors: the primal error due to the stochastic production function, and input demand errors. The joint presence of these sources of errors is crucial. As shown in this paper, the presence of these two types of errors typically implies that the ex ante cost model that one obtains belongs to the class of nonlinear errors-in-variables models (Y. Amemiya 1985; Hsiao 1989). Unlike in simultaneous equations models, where the relation of interest is specified to hold between observable variables, in an errors-in-variables model one has a relation between unobservable variables. If the errors-in-variables model were linear, then one could exploit a useful equivalence between linear errors-in-variables models and linear simultaneous equations models and obtain consistent estimation procedures. Fuller (1987) provides an extensive analysis of linear errors-in-variables models. But in fact the ex ante cost function model is inherently nonlinear. As noted by Y. Amemiya (1985), a nonlinear errors-in-variables model is not isomorphic to a simultaneous equations model, and for such nonlinear errors-in-variables models it is notoriously difficult to obtain estimators that are consistent in the usual sense.

In this paper I provide an explicit characterization of the ex ante cost function problem and detail the conditions that give rise to a nonlinear errors-in-variables problem.

In such a setting, the ex ante procedure leads to inconsistent estimates. Appeals to procedures that work in a simultaneous equations setting, such as three-stage least squares using instrumental variables, are also unlikely to produce consistent estimates. But for the stochastic production setting of interest here, however, I am able to derive a procedure that in fact yields consistent estimators. The procedure exploits the economic context that makes it interesting to estimate the ex ante cost function, namely, expected profit maximization. By appealing to behavioral implications of expected profit maximization, I am able to effectively remove the errors-in-variables problem from the model. Because of its simplicity, I believe that this approach is of considerable interest for a number of applications. My claims about the inconsistency of existing estimators of the ex ante cost function, and the consistency of my proposed procedure that exploits the implications of expected profit maximization, are illustrated by means of a Monte Carlo experiment. Related implications for modeling the dual structure of stochastic production are discussed.

2. The Problem

The problem is that of estimating the parameters of the cost function corresponding to a stochastic production function. Under production uncertainty it may not be obvious that there exists a cost function that is "dual" to the production function, but Chambers and Quiggin (2000) provide an appealing derivation of such duality under uncertainty in the context of the state-contingent framework. To briefly characterize this approach in a notation that is suitable for my later analysis, suppose that there are state-contingent production functions $G(x,e;q_0)$, where $x \in \mathbb{R}_+^n$ is the vector of inputs, ε is a variable indexing the state of nature, and q_0 is the vector of all parameters appearing in the (state-contingent) production function. The functions $G(x,e;q_0)$ are assumed non-decreasing and quasi-concave in x. For notational simplicity, consider the discrete case such that there are S states of nature, i.e., $e \in e_1, e_2, \ldots, e_S$. A typical (and general) production objective for competitive producers is expected utility maximization, which here can then be written as

$$\max_{x} \sum_{i=1}^{S} \ell_i U \left(pG(x,e_i;q_0) - w \cdot x \right) \tag{1}$$

where $p \in \mathbb{R}_{++}$ is the output price, $w \in \mathbb{R}_{++}^{n}$ denotes the input price vector, and $\ell_i \in (0,1)$ represents the probability of the ith state of nature (such that $\sum_{i=1}^{S} \ell_i = 1$). The utility function $U(.)$ is assumed to be strictly increasing and concave, thus allowing for the possibility that producers may be risk averse.

Following Chambers and Quiggin (1998), a reformulation of this general production problem that exploits the notion of cost minimization takes the form

$$\max_{y_1, y_2,..., y_S} \left\{ \sum_{i=1}^{S} \ell_i U \left(py_i - c(y_1, y_2,..., y_S, w; q_0) \right) \right\} \tag{2}$$

where $y_i \in \mathbb{R}_+$ ($i = 1,2,\ldots,S$) denotes state-contingent output levels, and the cost function $c(y_1, y_2,..., y_S, w; q_0)$ is defined as

$$c(y_1, y_2,..., y_S, w; q_0) \equiv \min_{x} \left\{ w \cdot x \,\middle|\, G(x,e_i;q_0) \geq y_i \;, i = 1,2,\ldots,S \right\} \;, \tag{3}$$

assuming that the state-contingent output vector $(y_1, y_2,..., y_S)$ can be produced (i.e., $\exists x \in \mathbb{R}_+^n$ such that $G(x,e_i;q_0) \geq y_i, \forall i$). This is what Chambers and Quiggin (2000) call the "effort cost function." Whereas $c(y_1, y_2,..., y_S, w; q_0)$ is conceptually attractive, its empirical implementation is problematic.[1] But a useful simplification is possible under the additional assumption of risk neutrality (i.e., $U(.)$ is linear), such that the producer problem in (2) reduces to expected profit maximization and can be written as

$$\max_{\bar{y}} \left\{ p\bar{y} - C(\bar{y}, w; q) \right\} \tag{4}$$

where $\bar{y} \in \mathbb{R}_+$ denotes a given expected output level, q is the vector of all relevant parameters (which here include the parameters that describe the states of nature), and the cost function $C(\bar{y}, w; q)$ satisfies

$$C(\bar{y}, w; q) \equiv \min_{y_1, y_2,..., y_S} \left\{ c(y_1, y_2,..., y_S, w; q_0) \,\middle|\, \sum_{i=1}^{S} \ell_i y_i \geq \bar{y} \right\}. \tag{5}$$

The cost function $C(\bar{y}, w; q)$ is what Pope and Just (1996) call the "ex ante cost function." In their derivation expected profit-maximization is postulated outright, such that the producer problem is written as

$$\max_{x} \left\{ E\left[pG(x, e; q_0) - w \cdot x \right] \right\} \tag{6}$$

where E is the mathematical expectation operator (which is defined over the distribution of the random variable e). By defining an "expected output" function as $g(x;q) \equiv E\left[G(x,e;q_0)\right]$, where q is the vector of all relevant parameters (which here include parameters of the distribution of the random variable e), this expected-profit-maximization problem can be equivalently expressed in terms of two distinct problems. First, the producer chooses the optimal input vector to produce a given level of expected output, that is, he or she solves

$$\min_{x} \left\{ w \cdot x \,\middle|\, \bar{y} \leq g(x; q) \right\}. \tag{7}$$

Let $x^* = h(\bar{y}, w; q)$ denote the solution to problem (7). Then the ex ante cost function is defined as $C(\bar{y}, w; q) \equiv w \cdot h(\bar{y}, w; q)$. Given the optimal input choices summarized by $C(\bar{y}, w; q)$, the second step is for the producer to choose the optimal level of expected output that maximizes expected profit, that is, to solve the program in equation (4).[2]

Note that the ex ante cost function $C(\bar{y}, w; q)$, by construction, reflects the producers' expectations in addition to the technological properties of the stochastic production function. For example, changes in the producers' beliefs about the distribution of the random variable e would affect the structure of this ex ante cost function. Hence, any empirical specification of the ex ante cost function is bound to represent, in some sense, a reduced-form function whose meaning is somewhat different from what one ascribes, from duality, to standard cost function. But $C(\bar{y}, w; q)$ here does describe a relevant cost-minimization behavior in a parsimonious way, and therefore it is often of considerable interest to estimate its parameters. Unfortunately, $C(\bar{y}, w; q)$ is conditional on expected (or planned) output $\bar{y}$, which is not observable, and hence direct estimation of the ex ante cost function is not feasible.

3. Ex Ante Cost Function Estimation

As Pope and Just (1996) correctly note, previous applications with data that likely were generated by a stochastic process (such as agricultural production data) have simply ignored the problem. That is, researchers have routinely estimated $C(y,w;q)$, where y is the observed (ex post or realized) output, when in fact they should have been estimating $C(\bar{y},w;q)$. This approach, which is here labeled as the "standard" approach, essentially uses observed output y as the proxy for the unobserved expected output $\bar{y}$. But because y "measures" the true variable $\bar{y}$ only with error, naïve (least-square) type estimators that ignore this problem lead to inconsistent estimates.

To overcome the inconsistency of the standard cost function approach when production is stochastic, Pope and Just (1996) propose an alternative and original estimation procedure that entails estimating $\bar{y}$ simultaneously with the ex ante cost function. First, recall that if $\bar{y}$ were observable the parameters q could be estimated efficiently by fitting the system of n input demand functions $h\ \bar{y},w;q$, which, by Shephard's lemma, are related to the ex ante cost function by $h\ \bar{y},w;q \equiv \nabla_w C\ \bar{y},w;q$. But because $\bar{y}$ is not observable, Pope and Just (1996) propose to replace it by the output level which solves

$$\max_{\bar{y}} \left\{ \bar{y} \,\middle|\, \min_{w} \left[1 - C(\bar{y},w;q) + w \cdot x \right] \geq 1 \right\}. \tag{8}$$

Denote such a solution by $\bar{y}^0$. Under standard regularity conditions, by duality theory it must be then that $\bar{y}^0 \equiv g(x;q)$.[3] Hence, this method reduces to estimating the set of input demand equations with $\bar{y}$ replaced by the expected output function $g(x;q)$. Although this point was perhaps not emphasized enough, it was certainly articulated explicitly by Pope and Just (1996) (e.g., in the first unnumbered equation on page 240). With such a substitution, to allow input demands to be stochastic one would need to write the system of input demand equations as

$$x = h\ g(x;q),w;q\ + e \tag{9}$$

where e is the error vector of input demands.

In estimating the ex ante cost model, Pope and Just (1996) recognize and address two problems. First, as they emphasize, popular functional forms for the ex ante cost function $C(\bar{y}, w; q)$ (such as the translog) do not admit a closed-form solution for the underlying production function (i.e., an explicit solution for the problem in [8]). In such a case, the method that they propose can be useful because it provides a procedure that constructs $g(x;q)$ numerically as part of the estimation algorithm. Of course, this observation should not obscure the basic point that, in this approach, $g(x;q)$ (whether analytically or numerically) is being used for the unobserved expected output level $\bar{y}$. A second problem is that not all parameters are estimable by using the input demand equations in (9). Intuitively, this is due to the fact that with (9) one is trying to estimate a cost function without observing output, which means that equations (9) define a simultaneous equation system that is not identified. To overcome this problem Pope and Just (1996, p. 240) suggest adding an equation to the estimating system. In my notation, I would then estimate a system of $n + 1$ equations given by the n input demand equations in (9) plus the production function equation, that is[4]

$$y = g(x;q) + u \tag{10}$$

where u is an error term induced by the random variable e (i.e., $u \equiv y - E[G(x,e;q)]$).

If the functional specification is such that the parameter vector q is now identified, then the system of equations (9)-(10) can be used to estimate this parameter vector. But although joint estimation of equations (9)-(10) is, in principle, possible, it is now apparent that there is still a major unresolved issue in this setting. Specifically, the system of $n + 1$ equations in (9)-(10) entails that the (possibly stochastic) vector of input quantities x appears on the right-hand side of all equations. This simultaneity feature was not explicitly discussed in Pope and Just (1996). Clearly, if input choices hold deterministically (such that $e \equiv 0$ in equations [9]), then their proposed estimation procedure will produce consistent estimates of the underlying parameters. But if one were to allow for the realistic feature of errors in input demands, the ex ante procedure is unlikely to yield consistent estimates.

Recognizing that simultaneous equation bias might be a problem if input demands are allowed to be stochastic has led Pope and Just (1998) to implement, in a related setting, a three-stage least squares estimation procedure that uses instrumental variables (IV). But whether or not such an "IV ex ante" approach leads to consistent estimates is an open question because, for reasonable specifications of the stochastic nature of input demands, the simultaneous equations representation of (9)-(10) is not the appropriate one. Rather, when both production and input demands are stochastic, the model that is obtained is likely to give rise to an errors-in-variables problem. Because the model is also inherently nonlinear, estimation techniques that yield consistent estimators for simultaneous equation models do not typically work here (Y. Amemiya 1985; Hsiao 1989).

4. Stochastic Input Demands and the Errors-in-Variables Problem

It is clear at this point that the stochastic nature of input demands plays a crucial role in the properties of the ex ante estimators discussed in section 3. To gain more insights into this problem, it is necessary to be precise about the source of these error terms. Here I analyze in detail what McElroy (1987) has called the "additive generalized error model" (AGEM). This rationalization provides an attractive and coherent explanation for stochastic input demands and for this reason was advocated explicitly in Pope and Just's (1996, 1998) empirical applications. Specifically, producers are assumed to minimize cost conditional on a production function which, in our setting, can be written as $g(x-e;q)$, where the vector e is parametrically known to producers. Hence, optimal input choices are written as

$$x = h(\bar{y}, w; q) + e \tag{11}$$

with total production costs $C \equiv w \cdot x$ given by

$$C = C(\bar{y}, w; q) + w \cdot e \quad . \tag{12}$$

By assuming that the vector e, while parametrically known to producers, is unobservable to the econometrician, the deterministic input demand setting at the producer level translates naturally into an internally consistent stochastic input demand setting for the purpose of estimation (McElroy 1987).

Although clearly appealing from an economic point of view, the AGEM rationalization for stochastic input demands, in conjunction with the assumed stochastic production structure, turns out to create a problem for the ex ante estimation procedure. Specifically, although one can find the expected output function $g(.;q)$ dual to the cost function being used (by solving [8], say), the argument of this function that is relevant for the purpose of computing expected output $\bar{y}$ cannot be observed. In other words, if one defines $\bar{x} \equiv x - e$, then the $(n+1)$ equation system of input demands and production function implied by the AGEM model is

$$x = h\left(g(\bar{x};q), w; q\right) + e \tag{13}$$

$$y = g(\bar{x}, q) + u \tag{14}$$

where $g(\bar{x}, q) \equiv \bar{y}$. Clearly, the system of equations (13) and (14) cannot be estimated directly because $\bar{x}$ is not observed. Indeed, the problem here is completely analogous to the one that I have set out to solve (i.e., estimating $C\left(\bar{y}, w; q\right)$ when $\bar{y}$ is not observed). Thus, with stochastic input demands and stochastic production, the estimating equations for the ex ante cost model belong to the class of nonlinear errors-in-variables models. As mentioned earlier, such models are conceptually distinct from simultaneous equation models, and the estimators that apply to the latter do not typically work for the former (Y. Amemiya 1985).

Whereas the AGEM specification is useful for an explicit characterization of our problem, it should be clear that AGEM per se is not crucial to obtain an errors-in-variables model. Other internally consistent rationalizations for the stochastic terms of input demands can yield an errors-in-variables problem when stochastic input demands are combined with a stochastic output. Consider, for example, the following alternative rationalization for stochastic input demands: agents make decision errors. To steer clear of making inconsistent assumptions, one needs to be explicit about the decision framework. In particular, the assumption here is that there are "input errors" that cannot be avoided, but producers are aware that such errors will be committed and they know the distribution of these errors. This is equivalent to saying that producers choose $\bar{x}$, say, but the choice $x = \bar{x} + e$ is implemented, where e denotes a vector of input demand errors satisfying $E[e]=0$. Of course, $\bar{x}$ is not observable whereas x is observed. But once x is implemented, it is x which enters the production function (in other words, input errors here are "productive").

Specifically, the production function is written as $G(x,e;q_0)\equiv G(\bar{x}+e,e;q_0)$, and the expected-profit-maximization problem can be written as

$$\max_{\bar{x}} \left\{E\left[\, pG(\bar{x}+e,e;q_0)-w\cdot(\bar{x}+e)\right]\right\} \tag{15}$$

where the expectation operator E here is defined over the distribution of the random variables e and e. In this setting the relevant expected output function is $g(\bar{x};q)\equiv E\left[G(\bar{x}+e,e;q_0)\right]$, where again the expectation operator E is defined over the distribution of e,e, and q is the vector of all relevant parameters (which here include the parameters of the distributions of e,e). The ex ante cost function dual to the expected output function is therefore defined as

$$C(\bar{y},w;q)\equiv \min_{\bar{x}}\left\{w\cdot\bar{x}\;\middle|\;\bar{y}\le g(\bar{x};q)\right\} \tag{16}$$

and the expected-profit-maximization problem in (15) can then be stated as the program in (4). In this setting the stochastic input demands equations can be written as $x=h\ \bar{y},w;q\ +e$, where, by Shephard's lemma, $h\ \bar{y},w;q\ \equiv\nabla_w C\ \bar{y},w;q$. As before, these demand functions cannot be estimated directly (because $\bar{y}$ is not observable). Furthermore, trying to estimate the expected output $\bar{y}$ simultaneously with input demands leads to a system with the structure of equations (13)-(14). Hence, the estimating system entailed by this decision errors framework is isomorphic to the model implied by the AGEM rationalization discussed earlier (the true choices $\bar{x}$ are not observed).[5]

Based on the foregoing, it is apparent that allowing for stochastic input demands introduces subtle issues for the interpretation and estimation of the ex ante cost function. Recall that the hallmark of this approach is to exploit duality to recover the expected output function dual to the adopted specification of the ex ante cost function. But duality relies crucially on the assumed optimizing behavior of producers, and the dual form that one recovers can only reflect the optimizing choices of producers. If the identity between observed input quantities and optimal producer choices is broken, by allowing stochastic terms in input demands, the internal consistency of the proposed ex ante procedure is affected. The preceding structural explanations of input demands make it clear that the ex ante procedure does apply in a special case: that of nonstochastic input demands. If input demands do not have error terms ($e \equiv 0$), then $x = \bar{x}$ and the ex ante procedure effectively removes the errors-in-variables problem (while still allowing for stochastic production). Similarly, the current discussion also identifies the other special case that arises when production is not stochastic ($u \equiv 0$). In this case, which is implicitly assumed in most existing empirical applications, one has $y = \bar{y}$ and the errors-in-variables problem disappears from the cost model (while still allowing for stochastic input demands).[6] But with the joint presence of error terms in input demand equations and in the production equation, exploiting duality does not eliminate "unobserved" variables and the ex ante cost model is still affected by an errors-in-variables problem.

5. A "Full Information" Solution

Existing econometric results on the consistency of estimators for the nonlinear errors-in-variables problem are rather discouraging for the purpose of estimating the parameters of the ex ante cost function. The standard instrumental variable approach that applies to nonlinear simultaneous equation models fails to achieve consistency in the usual sense. Y. Amemiya (1985) has investigated the use of an alternative notion of asymptotic convergence that applies when error variances (of the unobservable variable) are small and sample sizes are large. But such an asymptotic theory may not apply to typical econometric problems, where one cannot expect replicated experiments as the sample size increases. Hausman et al. (1991) and Hausman, Newey, and Powell (1996) also obtain a consistent estimator for a class of nonlinear errors-in-variables models when there is a single repeated observation on the unobserved regressor. But for the purpose of estimating ex ante cost functions, such repeated observations on expected output are usually not available (especially when estimation relies on time-series data).

Fortunately, an alternative procedure to estimate the ex ante cost function suggests itself in the context of the economic problem where the ex ante cost function is relevant. Specifically, recall that interest in the ex ante cost function $C(\bar{y}, w; q)$ is motivated here by the assumption that producers solve the expected-profit-maximization problem in equation (6). Because this expected-profit-maximization problem equivalently can be written as (4), then from the optimality condition of problem (4) one finds the solution $\bar{y}^* = s(p, w; q)$, where the parametric structure of the ex ante supply function $s(p, w; q)$ is implied by the structure of the ex ante cost function $C(\bar{y}, w; q)$. This optimal expected production level depends on the (exogenously given) output price p. If such an output price is observable (as is usually the case) then p provides the obvious "instrument" for the unobserved expected output, and the function $s(p, w; q)$ provides the correct nonlinear mapping for this instrument. Thus in this setting one can estimate the parameters of the cost function by fitting the system of n input demand equations:

$$x = h\ s(p, w; q), w; q\ + e \quad . \tag{17}$$

If so desired, the system of input demand functions in (17) can be supplemented by the expected output function equation, that is

$$y = s(p, w; q) + u \quad . \tag{18}$$

Note, however, that here equation (18) is not necessary in order to identify all the parameters of the model. Unlike the ex ante input demand system in (9), the system in (17) typically allows for the estimation of all cost parameters (again, this is made possible by the presence of the output price p).

The approach that I have suggested, based on the expected-profit-maximization problem actually solved by the producer, will yield consistent estimates of the parameters of the underlying technology because it effectively removes the errors-in-variables problem. It bears repeating that my proposed approach does not require additional assumptions relative to those inherent in the setting being analyzed. Specifically, the hypothesis of expected profit maximization is already made to motivate interest in the ex ante cost function; and, given that, the shape of the ex ante supply function $s(p, w; q)$ is fully determined by the cost function $C(\bar{y}, w; q)$ via the optimality conditions for problem (4). Although this alternative route to estimate the ex ante cost function is reasonably straightforward, for many functional forms specifications of $C(\bar{y}, w; q)$ one will not be able to solve explicitly for the ex ante supply function $s(p, w, q)$. In such a case one could retrieve numerically $s(p,w,q)$, from a given specification for $C(\bar{y},w;q)$, as part of the estimation routine (in a manner similar to that implemented by Pope and Just [1996] for their procedure).

6. A Monte Carlo Illustration: The Generalized CES Model

To illustrate the properties of the alternative estimators for the ex ante cost function, I have constructed a Monte Carlo experiment that carefully represents all the features of the problem being analyzed. For this purpose, I work with a cost function that admits a closed-form solution for the dual production function. Hence, I can avoid the complications of retrieving this function numerically as part of the estimation routine, a computational task that featured prominently in Pope and Just (1996) but which is peripheral to the main issue analyzed here. Specifically, I consider a generalized constant elasticity of substitution (CES) cost function that allows for decreasing returns to scale (such that it can be consistent with the expected-profit-maximization problem that has been used to motivate the ex ante cost function).

6.1 Experiment Design

The AGEM specification of this CES cost function is written as

$$C = \bar{y}^{b} \left(\sum_{i=1}^{n} a_i w_i^{1-s} \right)^{\frac{1}{1-s}} + \sum_{i=1}^{n} w_i e_i \qquad (19)$$

where $a_i > 0 \quad \forall i$, $\sum_{i=1}^{n} a_i = 1$, $s > 0$, $s \neq 1$ and $b > 1$. The parameter s is the constant Allen-Uzawa elasticity of substitution between inputs. The parameter b controls the curvature of the cost function in $\bar{y}$, and the condition $b > 1$ ensures that the cost function is (strictly) convex in $\bar{y}$. From Shephard's lemma, input demands consistent with this cost function are

$$x_i = a_i \bar{y}^{b} w_i^{-s} \left(\sum_{k=1}^{n} a_k w_k^{1-s} \right)^{\frac{s}{1-s}} + e_i \qquad i = 1, \ldots, n \quad . \qquad (20)$$

Consistent with the AGEM specification, the terms e_i are parametrically known to the producers but are treated as random variables by the econometrician. Hence, the parameter vector to be estimated is $q \equiv (a, b, s)$. For this particular cost function it is verified that the (expected) production function (i.e., the solution to problem [8]) can be derived explicitly as

$$g(x-e;q) = \left[\sum_{i=1}^{n} a_i^{\frac{1}{s}} \cdot (x_i - e_i)^{\frac{s-1}{s}}\right]^{\frac{s}{b(s-1)}} . \qquad (21)$$

Hence, equation (21) here can be used to implement the ex ante methods discussed earlier.

If producers maximize expected profit, then they will choose the level of expected output such that the ex ante marginal cost equals output price, i.e., they will choose the level of expected output

$$s(p,w;q) = \left(\frac{p}{b}\right)^{\frac{1}{b-1}} \left[\sum_{i=1}^{n} a_i w_i^{1-s}\right]^{\frac{1}{(s-1)(b-1)}} . \qquad (22)$$

Hence, the supply function in (22) here can be used to implement the proposed method based on expected profit maximization.

Now the Monte Carlo experiment proceeds as follows.

A. First, I set the number of inputs at four (i.e., $n = 4$) and the true values of the parameters as follows:

   $a_1 = 0.1$, $a_2 = 0.2$, $a_3 = 0.3$, $a_4 = 0.4$, $b = 1.2$, $s = 0.5$.

B. Next, I choose the design matrix of exogenous variables (the vectors of expected output $\bar{y}$ and of input prices *w*), which is then held fixed throughout. Here I use an initial sample of 25 observations taken from a recent application using agricultural data (see the Appendix for more details). All variables are normalized to equal unity at their sample mean.

C. For each replication $j = 1, \ldots, J$, I construct a pseudo sample of optimal input quantities by using equations (20), with the vector e generated as $N(0, \Omega)$. Similarly, for each replication I construct a pseudo sample of stochastic output as $y = \bar{y} + u$, where the random term u is generated as $N(0, f^2)$. The standard deviation of each random variable is set to 10 percent of the corresponding mean.[7] Thus, the output stochastic term is set at $f = 0.1$. For the covariance matrix of the terms e_i I consider three cases: one with independent input demand errors Ω_0, one with such errors being negatively correlated Ω_1 and one with these input errors being positively correlated Ω_2. Specifically, the three covariance matrices for the vector e that I consider are

$$\Omega_i = \begin{matrix} w_{11}^2 & r_i w_{11} w_{22} & r_i w_{11} w_{33} & r_i w_{11} w_{44} \\ r_i w_{11} w_{22} & w_{22}^2 & r_i w_{22} w_{33} & r_i w_{22} w_{44} \\ r_i w_{11} w_{33} & r_i w_{22} w_{33} & w_{33}^2 & r_i w_{33} w_{44} \\ r_i w_{11} w_{44} & r_i w_{22} w_{44} & r_i w_{33} w_{44} & w_{44}^2 \end{matrix} , \qquad i = 0,1,2 \quad .$$

For all cases I set $w_{11} = 0.01$, $w_{22} = 0.02$, $w_{33} = 0.03$ and $w_{44} = 0.04$. For Ω_0 I set $r_0 = 0$, for Ω_1 I set $r_1 = -0.3$, and for Ω_2 I set $r_2 = 0.3$.

D. For each covariance structure, I generate 2,000 pseudo-random samples of observations (i.e., $J = 2{,}000$) using 1,000 random draws and their 1,000 antithetic counterparts.[8] For each sample, five models are estimated:

(i) The true model consisting of four input equations in (20). The results from this model provide a useful benchmark for evaluating the feasible estimators.

(ii) The standard model, which is the same as the true model but with *y* replacing $\bar{y}$.

(iii) The ex ante procedure suggested by Pope and Just (1996), consisting of five equations (four input equations and the output equation with the structure of equations [9]-[10]), that is

$$x_i = a_i w_i^{-s} \left[\sum_{k=1}^{n} a_k^{\frac{1}{s}} x_k^{\frac{s-1}{s}} \right]^{\frac{s}{(s-1)}} \left[\sum_{k=1}^{n} a_k w_k^{1-s} \right]^{\frac{s}{1-s}} + e_i \qquad i = 1, \ldots, n \tag{23}$$

$$y = \left[\sum_{k=1}^{n} a_k^{\frac{1}{s}} \cdot x_k^{\frac{s-1}{s}} \right]^{\frac{s}{b(s-1)}} + u \quad . \tag{24}$$

(iv) The "IV ex ante" procedure suggested by Pope and Just (1998), which estimates equations (23) and (24) by nonlinear three-stage least squares using a set of instrumental variables (which includes output price *p*).[9]

(v) The new approach proposed in this paper, which uses the ex ante supply function $s(p,w;q)$ in lieu of the unobserved expected output $\bar{y}$.[10] Because this approach relies on the implications of expected profit maximization, it is labeled "max $E[\Pi]$." Hence, here I fit the following system of four input demand equations plus the output equation:

$$x_i = a_i w_i^{-s}\left(\frac{p}{b}\right)^{\frac{b}{b-1}}\left(\sum_{k=1}^{n} a_k w_k^{1-s}\right)^{\frac{1}{(1-s)}\left(s+\frac{b}{(1-b)}\right)} + e_i \qquad i=1,\ldots,4 \qquad (25)$$

$$y = \left(\frac{p}{b}\right)^{\frac{1}{b-1}}\left(\sum_{k=1}^{n} a_k w_k^{1-s}\right)^{\frac{1}{(s-1)(b-1)}} + u \quad . \qquad (26)$$

6.2 Estimation

Each of the alternatives entails estimating a system of M equations using T observations.[11] Thus, for each alternative the model can be written as $Y = f(Z,q) + v$, where Y is the $TM \times 1$ stacked vector of the left-hand-side variables, $f(.)$ is a nonlinear (vector-valued) function, Z is the (stacked) $TM \times K$ matrix of all right-hand-side variables, q is the vector of all parameters to be estimated, and v is the $TM \times 1$ stacked residual vector. The error terms are assumed to be contemporaneously correlated but serially independent, that is, $E[vv'] = \Psi \otimes I_T$, where Ψ is the $M \times M$ contemporaneous covariance matrix and I_T is the identity matrix of order T. For four of the models considered (true, standard, naïve ex ante, and our new procedure) the system of interest is treated as a standard nonlinear seemingly unrelated regression model. Iterated minimum distance estimation is used (which converges to the maximum likelihood estimator). Specifically, at each iteration stage the vector of parameters is found by minimizing

$$\left[Y - f(Z,q)\right]'\left[\Psi^{-1} \otimes I_T\right]\left[Y - f(Z,q)\right]$$

where Ψ is the current estimate of the contemporaneous covariance matrix, which is updated at each iteration step until convergence. For the IV estimator, on the other hand, at each iteration the vector of parameters is found by minimizing

$$Y - f(Z,q)\,'\;\Psi^{-1} \otimes W\,W'W^{-1}W'\;\;Y - f(Z,q)$$

where W is the $T \times q$ matrix of all instrumental variables, and again the estimate of the contemporaneous covariance matrix Ψ is updated at each iteration step until convergence.[12]

6.3 Results

The results are summarized in Tables 1 to 4. Table 1 reports the average percentage bias for each parameter, for each estimation method and for all three covariance structures considered.[13] Average percentage bias is computed as

$$\frac{1}{N}\sum_{j=1}^{N}\frac{\hat{q}_i^{\,j} - q_i}{q_i}\times 100\,,$$

where $\hat{q}_i^{\,j}$ is the estimated *i*th parameter in the *j*th replication. All five methods do a reasonably good job at estimating the mean parameters a_i. Also, the proposed new model, based on expected profit maximization, is essentially unbiased and performs as well as the (unfeasible) true model. It is clear, on the other hand, that both the standard and the ex ante procedures yield estimates that are affected by considerable bias. Specifically, the standard model gives very poor estimates of the scale parameter b (as expected, because this is the parameter attached to the unobserved output level). The ex ante procedure does a better job than the standard model at estimating this scale parameter, although the estimated b is affected by considerable bias in this case as well. Furthermore, this ex ante model provides a much more biased estimator for the elasticity of substitution s (for example, for the case of uncorrelated e_i, the ex ante estimate of s has an average bias of 31 percent, whereas the standard model’s bias is less than 1 percent). The IV ex ante procedure performs better than the ex ante approach, although estimates are still affected by considerable bias.[14] As expected, changing the correlation structure of the e_i does not affect the performance of the true model nor that of our proposed model. It does not affect the performance of the standard model either, which is intuitively sensible (because for the standard model it is the random term u embodied in y, not the random vector e, that leads to inconsistency). But changing the correlation structure of the e_i does affect the performance of the ex ante procedure; with positively correlated e_i the bias in the scale parameter gets larger and the bias in the elasticity of substitution gets smaller, whereas the opposite holds true for negatively correlated e_i. The conclusions based on the average percentage bias of Table 1 are supported by the average percentage root mean square errors (RMSE) reported in Table 2. The entries of this table are computed as

$$\sqrt{\frac{1}{N}\sum_{j=1}^{N}\left(\frac{\hat{q}_i^j - q_i}{q_i}\times 100\right)^2},$$

and thus account for the sampling variance of each estimator (in addition to the bias). From Table 2 it is clear that the performance of the proposed model is comparable to that of the true model, whereas both the standard model and the ex ante procedure yield estimates that are far less precise.

Table 3 reports the average R^2, over all replications, for each equation in each estimation method. Specifically, the R^2 for each equation is defined as the square of the correlation coefficient between observed and fitted left-hand-side variable. This table provides an ex post check on the signal-to-noise ratio that we have implemented in our Monte Carlo experiment. Note that the "fit" of the various models is similar to that of many empirical applications. Indeed, in some sense the experiment has been conservative in that the magnitude of the production error that I have used is relatively large compared with the magnitude of the input demand errors (thus, my setup is somewhat slanted in favor of both ex ante procedures relative to the standard procedure).

Finally, Table 4 illustrates the finite-sample properties of the five estimators considered as the sample size increases. Specifically, to get an idea of the asymptotic convergence of the various estimators I allow the sample size to increase from 25 to 400 (each time I double the design matrix, such that the exogenous variables are multiple repeats of those reported in the Appendix). For the true model and our proposed model it is clear that the small-sample bias converges to zero as the sample size is increased. On the other hand, for the standard model, for the ex ante procedure, and for the IV ex ante method, the bias does not seem to be influenced by the increasing sample size. In particular, it is clear that the ex ante procedure leads to inconsistent parameter estimates. Indeed, the ex ante procedure arguably produces worse results than the standard approach. Of course, the ranking of these two inconsistent estimators likely depends on the magnitude of the randomness of the production function relative to the randomness of the input demand functions (recall that the errors-in-variables problem is due to u in the standard model, whereas it is due to e in the ex ante procedure).

TABLE 1. Average percentage bias in estimated parameters (T = 25)

	Covariance structure for the e_i's		
	Zero Correlation	**Negative Correlation**	**Positive Correlation**
True model			
a_1	-0.0099	-0.0053	-0.0076
a_2	-0.0059	-0.0022	-0.0038
a_3	0.0026	0.0023	0.0014
b	-0.0044	-0.0008	-0.0106
s	0.0217	0.0152	0.0133
Standard			
a_1	-0.0707	-0.0333	-0.0967
a_2	0.3036	0.3294	0.2829
a_3	-0.0900	-0.0944	-0.0892
b	-28.1709	-28.0437	-28.2886
s	-0.2483	-0.2961	-0.1942
Ex ante			
a_1	-0.9651	-1.2241	-0.6876
a_2	-0.3532	-0.4397	-0.2513
a_3	0.9321	1.2031	0.6587
b	10.0514	3.9848	15.8727
s	31.5560	39.5238	22.9752
IV ex ante			
a_1	-0.6934	-0.7120	-0.6318
a_2	-0.1624	-0.2754	-0.0575
a_3	0.2658	0.3369	0.1881
b	6.3656	2.9518	9.7748
s	18.8045	23.6668	13.5399
$\max E[\Pi]$			
a_1	0.0001	0.0054	-0.0011
a_2	0.0004	0.0019	-0.0004
a_3	0.0039	-0.0003	0.0030
b	0.0023	0.0005	0.0036
s	-0.0932	-0.0273	-0.0714

TABLE 2. Percentage RMSE in estimated parameters (T = 25)

	Covariance structure for the e_i's		
	Zero Correlation	**Negative Correlation**	**Positive Correlation**
True model			
a_1	1.9789	1.6518	1.7121
a_2	1.8872	2.0389	1.5723
a_3	1.6508	1.8927	1.3787
b	5.7451	1.8892	7.8247
s	12.5260	8.8782	11.0176
Standard			
a_1	2.0570	2.3233	1.7462
a_2	1.9383	2.2543	1.6151
a_3	1.6710	1.8842	1.4034
b	29.9088	29.3855	30.4255
s	13.7342	15.3372	11.7556
Ex ante			
a_1	2.5069	2.9191	2.0513
a_2	2.2654	2.6728	1.8754
a_3	2.2115	2.5664	1.7860
b	21.3586	16.5466	26.4365
s	34.3807	42.1902	25.8993
IV ex ante			
a_1	2.5643	2.8354	2.2131
a_2	2.1008	2.4242	1.7613
a_3	1.8463	2.0761	1.5461
b	18.2406	15.4823	21.0803
s	24.3597	28.9177	19.4645
max $E[\Pi]$			
a_1	1.9967	1.9478	1.6973
a_2	1.6915	1.2049	1.4824
a_3	1.5558	1.2857	1.3435
b	0.1681	0.0676	0.2095
s	11.7455	7.8556	10.8120

TABLE 3. Average R^2 of estimated equations (T = 25)

	Covariance structure for the e_i's		
	Zero Correlation	**Negative Correlation**	**Positive Correlation**
True model			
x_1 eqn	0.81	0.81	0.81
x_2 eqn	0.87	0.87	0.87
x_3 eqn	0.77	0.77	0.77
x_4 eqn	0.81	0.81	0.81
Standard			
x_1 eqn	0.61	0.61	0.61
x_2 eqn	0.74	0.74	0.74
x_3 eqn	0.53	0.53	0.53
x_4 eqn	0.60	0.60	0.60
Ex ante			
x_1 eqn	0.77	0.71	0.84
x_2 eqn	0.88	0.85	0.92
x_3 eqn	0.83	0.79	0.88
x_4 eqn	0.89	0.86	0.92
y eqn	0.68	0.71	0.64
IV ex ante			
x_1 eqn	0.78	0.72	0.85
x_2 eqn	0.88	0.85	0.92
x_3 eqn	0.84	0.79	0.88
x_4 eqn	0.90	0.87	0.93
y eqn	0.68	0.71	0.65
$\max E[\Pi]$			
x_1 eqn	0.81	0.81	0.81
x_2 eqn	0.87	0.87	0.87
x_3 eqn	0.77	0.77	0.77
x_4 eqn	0.81	0.81	0.81
y eqn	0.73	0.73	0.73

TABLE 4. Average percentage bias in estimated parameters and sample size

	***T* = 25**	***T* = 50**	***T* = 100**	***T* = 200**	***T* = 400**
True Model					
a_1	-0.0099	-0.0053	-0.0023	-0.0011	-0.0005
a_2	-0.0059	-0.0016	-0.0008	-0.0004	-0.0002
a_3	0.0026	0.0012	0.0006	0.0003	0.0001
b	-0.0044	-0.0022	-0.0008	0.0001	-0.0004
s	0.0217	0.0111	0.0033	0.0030	0.0005
Standard					
a_1	-0.0707	-0.0703	-0.0707	-0.0660	-0.0705
a_2	0.3036	0.3069	0.3083	0.3093	0.3110
a_3	-0.0900	-0.0945	-0.0933	-0.0929	-0.0990
b	-28.1709	-28.9322	-29.1527	-29.3885	-29.3740
s	-0.2483	-0.4271	-0.4931	-0.5191	-0.5075
Ex ante					
a_1	-0.9651	-0.9892	-0.9677	-0.9684	-0.9620
a_2	-0.3532	-0.3887	-0.4137	-0.4115	-0.4098
a_3	0.9321	0.9380	0.9489	0.9363	0.9295
b	10.0514	8.8001	8.2177	8.0898	7.9065
s	31.5560	32.5978	33.0348	33.3495	33.3922
IV Ex ante					
a_1	-0.6934	-0.5602	-0.5107	-0.4869	-0.4622
a_2	-0.1624	-0.0887	-0.0673	-0.0515	-0.0400
a_3	0.2658	0.2351	0.2427	0.2314	0.2196
b	6.3656	3.3162	1.8069	1.0691	0.6704
s	18.8045	18.5404	17.8961	17.5097	17.2222
$\max E[\Pi]$					
a_1	0.0001	-0.0005	0.0001	0.0003	0.0000
a_2	0.0004	0.0008	0.0005	0.0005	0.0000
a_3	0.0039	0.0009	0.0003	0.0001	0.0001
b	0.0023	0.0012	0.0005	0.0003	0.0001
s	-0.0932	-0.0276	-0.0172	-0.0088	-0.0041

7. Further Discussion

The results of the Monte Carlo experiment provide a compelling example of the deleterious consequences of ignoring production risk when estimating a cost function. Indeed, these results are a bit more general in that it is not even necessary to postulate production risk (in addition to input demand errors) in order to obtain an errors-in-variables cost function model. The above setting would in fact be unchanged if no production risk were present, but the error term u arose in a manner similar to the e_i, that is, from an AGEM rationalization. In other words, one could postulate that the profit-maximizing agents have a production function written as $y = g(x-e;q)+u$, where the terms e and u are known to the producer but are unobservable to the econometrician. Defining $\bar{y} = y - u$, the relevant cost function for this case is also written as $C(\bar{y}, w; q)$, where $\bar{y}$ is not observed by the econometrician. Hence, estimation of a standard cost function, conditional on observed output, is a problematic task for a wider (and realistic) class of problems than that of production uncertainty. But regardless of the source of the production error u, the approach that I have suggested, based on the expected-profit-maximization problem actually solved by producers, yields consistent estimates of the parameters of the underlying technology.

As mentioned earlier, a practical problem is that for many flexible specifications of $C(\bar{y}, w; q)$ one cannot solve explicitly for the ex ante supply function $s(p, w, q)$. In such a case one could numerically retrieve $s(p, w, q)$ as part of the estimation routine. Alternatively one can recognize that, in this context, it is better to specify and estimate an expected profit function rather than an ex ante cost function. Specifically, if the value function of problem (6) is written as $\Pi\left(p, w; q\right)$, then under standard assumptions this expected profit function exists and is continuous, linearly homogeneous, and convex in (p, w). This expected profit function is completely analogous to the standard profit function that obtains under conditions of certainty (as analyzed, for example, by Lau 1976). Thus, instead of specifying an ex ante cost function $C(\bar{y}, w; q)$, under production uncertainty the analysis can proceed by specifying the parametric structure of the expected profit function $\Pi\left(p, w; q\right)$. By Hotelling's lemma, this implies a coherent structure for the output supply function $s(p,w;q) = \Pi_p(p,w,;q)$ and the vector of input demand functions $x(p,w;q) = -\nabla_w \Pi(p,w;q)$, where $x(p,w;q) \equiv h\big(s(p,w;q), w;q\big)$. Hence, from a proper parametric specification of $\Pi \;\; p, w; q$ (say, Lau's [1974] normalized quadratic model), one can derive a coherent set of output supply and input demand equations that can be used in estimation. Because this route essentially removes the errors-in-variables problem, estimation of this set of equations produces consistent estimates of all the underlying parameters that are identified. If interest centers explicitly on the properties of the ex ante cost function, then one can exploit duality to retrieve the latter (numerically or analytically) from the expected profit function, i.e., by solving

$$C(\bar{y}, w; q) = \max_{p} \left\{ p\bar{y} - \Pi(p, w; q) \right\}. \tag{27}$$

The method that I have proposed to estimate the ex ante cost function model crucially depends on the hypothesis that the expected-profit-maximization problem in (6) applies. But how should one estimate the parameters of the ex ante cost function $C(\bar{y}, w; q)$ if such an expected-profit-maximization problem does not apply? It is important to re-emphasize, at this juncture, that the cost function $C(\bar{y}, w, q)$ is of interest precisely because of the expected-profit-maximization problem that producers are assumed to face. This framework could in fact be extended somewhat and still allow for the ex ante cost function $C(\bar{y}, w; q)$ to play a meaningful role. For example, both output price and production could be allowed to be stochastic, under some suitable restrictions, and the method that I have proposed to estimate $C(\bar{y}, w; q)$ could be adapted to this broader setting.[15] But more generally, when price and production risks are unrestricted and/or decision makers are risk averse, the cost function $C(\bar{y}, w; q)$ may not be of much interest anyway and, as discussed earlier in section 2, one may need to revert to more general cost function concepts.

8. Conclusion

Under production risk, a likely object of interest in production studies is the ex ante cost function, as noted by Pope and Just (1996). But when input demand equations (in addition to the production function) are also genuinely stochastic, the ex ante procedure is unlikely to improve over the standard estimation procedure because it does not solve the fundamental problem that arises in this context, that is, that the ex ante cost model inevitably leads to a nonlinear errors-in-variables problem. It is notoriously difficult to obtain consistent estimators for this class of models. For the particular case of an ex ante cost function that naturally arises in the context of the expected-profit-maximization hypothesis, however, I have shown that it is possible to achieve consistent estimation for the parameters of the ex ante cost function. Specifically, by exploiting the full implications of the expected-profit-maximization hypothesis one can effectively remove the errors-in-variables problem. The results of a carefully structured Monte Carlo experiment provide support for my claim about the properties of various estimation procedures. In particular, the proposed procedure to estimate the ex ante cost function yields estimates of the underlying technological parameters that are equivalent to those of the (unfeasible) true model.

Endnotes

1. For example, only one of the many possible state-contingent outputs y_i is realized (and therefore observed) for any one resolution of uncertainty.

2. It is assumed that $C(\bar{y}, w; q)$ is strictly convex in $\bar{y}$, which in turns requires the expected output function $g(x;q)$ to be strictly concave in x. This guarantees that the solution to problem (4) is unique, if one exists.

3. Regularity conditions include that $g(x;q)$ be quasi-concave in x, which is guaranteed by the assumed curvature conditions for expected profit maximization [i.e., $g(x;q)$ is concave].

4. Again, if the form of $g(.)$ that is consistent with the parameterization of $C(.)$ is not known, then $g(.)$ can be retrieved numerically.

5. Of course, here realized output can be written as a function of observed inputs, because in this case input errors are productive. Hence, $y = \tilde{g}(x, \tilde{q}) + u$, where $\tilde{g}(x;\tilde{q}) \equiv E\left[G(x, e; q_0)\right]$ (this expectation operator is defined only over the random variable e, and hence the vector $\tilde{q}$ differs from q because it includes parameters of the distribution of e but not of e). But writing $y = \tilde{g}(x, \tilde{q}) + u$ is not very useful in estimating the ex ante cost function because it is $g(\bar{x};q)$, and not $\tilde{g}(x;\tilde{q})$, which in this setting is dual to $C(\bar{y}, w; q)$.

6. Of course, if $e \equiv 0$, the system of n input demands would have to hold deterministically, whereas if $u \equiv 0$ then the output equation would need to hold deterministically. Hence such cases are somewhat uninteresting from an empirical point of view.

7. Given the normalizations chosen for the exogenous variables, the mean of x_i is approximately equal to a_i and the mean of y is equal to one.

8. For each draw I checked the regularity conditions $(x_i - e_i) > 0$, which turned out to be always satisfied.

9. I rely on four primitive instrumental variables: three input prices (deflated by the fourth input price) and the output price (deflated by the fourth input price). I use the four primitive variables plus their squares and cross products that, together with a constant, give a total of 15 instruments that are used in the IV procedure.

10. Consistent with the assumption of expected profit maximization under competition and stochastic production, the price series used in the Monte Carlo experiment was generated as $p = C_{\bar{y}}(\bar{y}, w; q)$, where $C_{\bar{y}}(\bar{y}, w; q)$ is readily obtained from the CES cost function specification in the text. Note that this output price series is used by both the IV ex ante approach and by the procedure proposed here.

11. Note that the first two methods entail $M = 4$, whereas for the last three methods $M = 5$.

12. Thus, this yields what is usually referred to as the nonlinear three-stage least squares estimator (e.g., T. Amemiya 1985). As mentioned earlier, here $q = 15$.

13. Because $\sum_{i=1}^{n} a_i = 1$, only three α_i parameters need to be estimated.

14. The performance of the IV estimator could be improved by the bias adjustment method proposed by Y. Amemiya (1990). But such a computationally intensive method still does not lead to consistency and in my context is bound to be inferior to the procedure I am proposing.

15. For example, as noted by Pope and Just (1996), in our setting output price can also be allowed to be a random variable provided p and ε are independently distributed. But then the relevant output price for producers' decision is the expected price $\bar{p} \equiv E[p]$. If $\bar{p}$ is observed (say, a futures price), the analysis of this paper carries through directly. If $\bar{p}$ is not observed, on the other hand, then the procedure proposed here needs to be augmented by a model specifying how $\bar{p}$ is formed, say, by postulating "rational expectations" (see Pesaran 1987 for a comprehensive introduction).

Appendix

Description of the Data

Ball et al. (1997) report a detailed data set pertaining to the U.S. agricultural sector for the period 1949–1994. To implement our Monte Carlo experiment we take four input price series from their data: labor (w_1), materials (w_2), energy (w_3), and capital (w_4). The aggregation of input prices in these four categories has been very common in the applied literature (leading to the so-called "KLEM" models; see Berndt and Wood 1975 for an early example). Variables w_1, w_3 and w_4 are reported directly by Ball et al. (1997), whereas w_2 had to be computed from the three non-energy intermediate input price series that they report. We did so by using Fisher's ideal index formula (with mean values over the entire period as the base). The expected output series $\bar{y}$ was generated as the fitted series of a linear regression of the quantity index for crop outputs, as reported by Ball et al. (1997), on the following variables: price of crops (lagged one period), price of livestock (lagged one period), price of the four inputs as described above (labor, materials, energy and capital), and a time trend. Whereas the computations just described were carried out for the entire period reported, for the purpose of our Monte Carlo experiment we utilize only the last 25 observations. Finally, the five data series that we utilize were scaled to equal one at the mean of the period that we use (i.e., for the period 1970–1994). The data so obtained, and used in the Monte Carlo experiment, are reported in Table A1.

TABLE A1. Data used in the Monte Carlo experiment

$\bar{y}$	w_1	w_2	w_3	w_4
0.7587	0.4076	0.4549	0.3260	0.4446
0.7656	0.4137	0.4769	0.3397	0.4093
0.7739	0.4291	0.4985	0.3410	0.4176
0.7450	0.4987	0.6769	0.3709	0.4384
0.8026	0.5475	0.8178	0.5463	0.4042
0.8779	0.5838	0.8364	0.5755	0.3460
0.8660	0.6495	0.8506	0.6225	0.5161
0.8997	0.7116	0.8493	0.6764	0.6261
0.9094	0.7715	0.8532	0.7145	0.6713
0.9349	0.8364	0.9401	0.9270	0.8419
0.9584	0.8820	1.0281	1.2636	1.1237
0.9854	0.8903	1.0919	1.4455	1.4533
1.0766	1.0416	1.0791	1.4508	1.6860
1.0014	0.9196	1.1315	1.4095	1.6439
1.0863	0.9969	1.1519	1.3832	1.8139
1.1295	1.1571	1.0834	1.3698	1.4502
1.1026	1.1394	1.0380	1.2592	1.2138
1.1055	1.0964	1.0693	1.1339	1.3617
1.0135	1.0547	1.2200	1.1423	1.2031
1.1447	1.3850	1.2874	1.1936	1.1098
1.1899	1.6175	1.2802	1.3550	1.1115
1.1932	1.6027	1.2831	1.3232	1.1237
1.1965	1.6334	1.2936	1.2816	1.2083
1.2147	1.8553	1.3301	1.2958	1.0642
1.2684	1.8786	1.3777	1.2534	1.3174

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