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6. konferenca DAES

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Orodja za podporo  
odločanju v kmetijstvu  
in razvoju podeželja

Krško, 2013

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kmetijstvu in razvoju podeželja

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# Ekonometrične analize in matematično modeliranje

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## **CONSIDERING THE EFFECT OF UNCERTAINTY IN AN INPUT-OUTPUT ANALYSIS OF WATER CONSUMPTION: A FUZZY APPROACH APPLIED TO MACEDONIA**

Jordan HRISTOV<sup>a</sup>, Yves SURRY<sup>a</sup>

### **ABSTRACT**

This paper deals with reconsidering the reliability of the results obtained in the input-output analysis of water consumption by Hristov *et al.* (2012) due to uncertainty integrated in the data. The imprecision and uncertainty is studied within a workable fuzzy environment introduced by Beynon *et al.* (2005) where rank reversal is plausible. In addition, the water intensive structure of the Macedonian economy is investigated by applying Dietzenbacher eigenvector methodology (1992) in terms of water consumption. Similar as in the analysis in the work by Hristov *et al.* (2012), the water intensive structure in Macedonia mainly focused around agriculture and several industrial sectors is confirmed, given the fact that rank reversal was absent. Consequently, necessity to introduce changes in the agricultural production technology and specialization in the production in this region or maybe reconsidering the existing water pricing policy ought to be carefully considered by the policy makers.

Key words: fuzzy input-output, triangular membership function, rank order, water consumption

## **UPOŠTEVANJE UČINKA NEGOTOVOSTI V INPUT-OUTPUT ANALIZI PORABE VODE: MEHKI PRISTOP NA PRIMERU MAKEDONIJE**

### **IZVLEČEK**

V prispevku je predstavljena ponovna obravnava zanesljivosti rezultatov input-output analize porabe vode izpeljana s strani Hristov in sod. (2012) zaradi vpliva negotovosti vezane na podatke. Nenatančnost in negotovost je analizirana v okviru mehkega okolja vpeljanega s strani Beynon in sod. (2005), kjer je možna popolna sprememba ranga. Dodatno je preučevana tudi intenzivnost strukture makedonskega vodnega gospodarstva z vidika porabe vode z uporabo Dietzenbacherjeve metodologije eigenvektorja (1992). Podobno kot izhaja iz analize Hristov in sod. (2012) je tudi tu na podlagi dejstva, da popolna sprememba ranga ni prispejna, potrjeno, da je intenzivnost vodnogospodarskih struktur koncentrirana v kmetijstvu in nekaterih industrijskih sektorjev. Posledično je potrebna s strani kreatorjev politik posebna pazljivost pri vpeljavi sprememb kmetijskih tehnologij in specializaciji v regiji, oz. je potrebno spremeniti režime zaračunavanja porabe vode.

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Ključne besede: mehki input-output, funkcija trikotne porazdelitve, rangiranje, poraba vode

## 1 Introduction

As we become more aware of certain issues and realize their complexity, the number of phenomena that we are uncertain increases. To reduce the uncertainty, we always tend to collect more information. But sometimes the required information is not available and consequently the uncertainty increases even more.

Uncertainty is a topic that always leaves room for improvement and more in-depth investigation. Since the introduction of the fuzzy logic and fuzzy set theory by Zadeh in 1965, this concept has been implemented in many economics fields (Morillas *et al.*, 2011). Recently, the topic of uncertainty in the area of input-output analysis has been thoroughly investigated by economists using fuzzy logic or fuzzy set theory such as: Buckley (1989), Beynon *et al.* (2005), Beynon and Munday (2006), Diaz *et al.* (2006), etc. In addition, some applications of input-output analysis to environmental and sustainability problems have become a growing field of interest (Morillas *et al.*, 2011). However, the topic of fuzzy input-output analysis considering limiting natural resources for some economic activities hasn't been given substantial attention.

In a paper by Hristov *et al.* (2012) an input-output methodology was undertaken to analyse the water consumption in Macedonia. It was concluded that the Macedonian economy was characterized with intensive water consumption mainly focused around agriculture and several industrial sectors. However, while comparing and combining the two data sets to find an appropriate water demand quantities for some sectors, we notice irregularities in the data which may cause misleading interpretation. In the State Statistical Yearbook, some of the information that refers to water abstraction by sector is the same as the water supply data in the EUROSTAT data base. Whereas the information on water supply in the State Statistical Yearbook is much higher. For example in the EUROSTAT data base, the total *manufacturing sector* water supply is 230.5 million m<sup>3</sup> which is the exact water abstraction for technical purposes in the State Statistical Yearbook 2006. On the other hand in the State Statistical Yearbook 2006, the total supply to water for the *manufacturing sector* is 477.95 million m<sup>3</sup>.

Moreover, aggregation of sectoral primary factor returns is among the numerous sources of uncertainty and imprecision in input-output analysis (Beynon & Munday, 2008). Due to the absence of data for some sectors regarding the water consumption, we were forced to aggregate the 60 sector symmetric input-output table to only 28 sectors. Hence, beside uncertainty of the data to construct the water accounts, another source of uncertainty may stem from aggregation bias.

Given the uncertainty of the used data regarding the direct total water use as well as the aggregation for some sectors, the identification of agriculture in Hristov *et al.* (2012) as a key water consuming sector during the analysis might have been jeopardized. Thus, necessity of ranking the sectors in a fuzzy environment is appropriate.

Therefore, the aim of this study is to investigate the water intensive structure of the Macedonian economy by developing fuzzy input-output analysis. With less stressed imprecision and uncertainty, the findings may contribute to raised

awareness of this natural resource necessary for sustainable water management. In addition, explanation of potential water pricing policies may be conveyed. For this purpose, additional measurement of sectoral interdependencies is based on Dietzenbacher (1992) framework or the eigenvector method. The main incentive is that some of the sectors identified in Hristov *et al.* (2012) as key water use sectors by Rasmussen methodology were not considered to a great extent during the intersectoral water relationship analysis and the derived indicators. It has been shown that Dietzenbacher method is superior and provides better indicator of interindustry linkages than the Rasmussen methodology applied in the Hristov *et al.* (2012).

The paper is structured as follows. In the next section we give an overview of the framework throughout imprecision and uncertainty may be investigated in an input-output analysis. The fundamentals of the analysis and discussion which are presented in the third section, also depends on the Dietzenbacher methodology outlined along with the fuzzy approach. After considering the intensive water consumption in Macedonia from different perspectives in terms of uncertainty and imprecision, brief conclusion is given at the end of the paper.

## 2 Methodology

### 2.1 Fuzzy input-output framework

The identification of key sectors in terms of their buyer and supplier linkages is the fundamental aspect in input-output analysis. Irrespective the buyer-seller relationship based on the inverse of Leontief's model to search for the key sector, the uncertainty and imprecision associated with the direct (input-output) technical coefficients affects the procedure of identification of key sectors. However, the fuzzy environment gives the opportunity to overcome this problem.

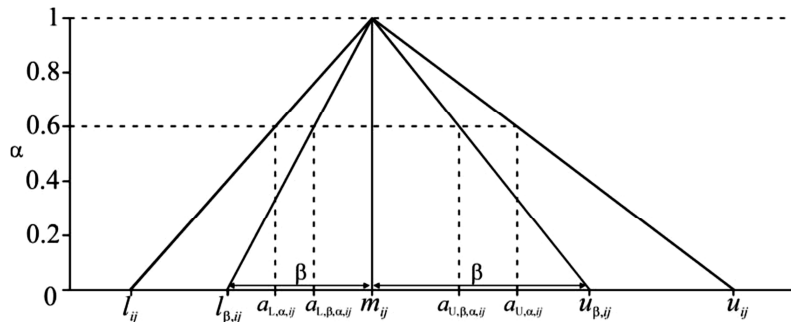


Figure 1: Triangular membership function based on  $(l, m, u)$  values and the  $\alpha$  and  $\beta$  parameters, *Source: Beynon et al. (2005)*

The main advantage of fuzzy logic and fuzzy set theory is to assign membership function to each object in the analysis. This means that there is no sharp boundary or value for the object and depends entirely on the defined membership function  $\bar{a}$ . In our work we incorporate fuzzy triangular membership function which depends on



three values: lower ( $l$ ), middle ( $m$ ) and upper ( $u$ ), i.e.  $\bar{a} = [l, m, u]$  (see figure 1). The  $\alpha$ -cut ranging from 0 to 1 has an effect over the membership function by closing the triangular number to an interval  $[a_{l,\alpha}, a_{u,\alpha}]$  whereby the  $\beta$  value also ranging from 0 to 1, represents the level of imprecision as proportional distance between the  $l$  and  $u$  values relative to  $m$  (Beynon & Munday, 2008). The larger the  $\beta$  value the larger the imprecision, i.e.  $\beta$  equals to 1 indicates the worst imprecision case (ibid).

The necessary overview on fuzzy set theory outlined in Zadeh (1965) will be omitted here, but what is of great importance is how the triangular membership functions of each fuzzy number are defined. A fuzzy number is a convex fuzzy subset of real number  $R$ , represented by its triangular membership function (Wang *et al*, 2006):

$$\bar{a}(x) = \begin{cases} \frac{x-l}{m-l} = \bar{a}_L & \text{if } l \leq x \leq m \\ \frac{x-u}{l-u} = \bar{a}_U & \text{if } m \leq x \leq u \end{cases} \quad (1)$$

where the triangular uncertain set  $\bar{a} = [l, m, u]$  has an inverse membership function:

$$\bar{a}^{-1}(\alpha) = \begin{cases} (1-\alpha)l + \alpha m = \bar{a}_L^{-1} & \text{if } 0 \leq \alpha \leq 1 \\ (1-\alpha)u + \alpha m = \bar{a}_U^{-1} & \text{if } 0 \leq \alpha \leq 1 \end{cases} \quad (2)$$

Although,  $\alpha$ -cut ensures that the membership function has a closed set, the  $\beta$  proportion value plays more important role in the fuzzy environment. With the introduction of the  $\beta$  parameter, the existence of solution as well as certainty of correctness to fuzzy input-output model are ensured (Buckley, 1989; Beynon and Munday, 2007). In other words, the constraint  $\sum_{i=1}^s u_{ij} < 1$  for  $s$  number of sectors should be satisfied. Concerning this constraint in fuzzy environment when we have most imprecision i.e.  $\alpha = 0$ , it becomes:

$$\sum_{i=1}^s m_{ij} + \beta(u_{ij} - m_{ij}) < 1 \quad (3)$$

Solving for  $\beta$ , it is noticeable that the parameter has a domain between 0 and 1, i.e.:

$$\beta < \frac{1 - \sum_{i=1}^s m_{ij}}{\sum_{i=1}^s (u_{ij} - m_{ij})} \quad (4)$$

Hence, the allowed upper bound on  $\beta$  that ensures solution to the fuzzy input-output matrix, defined as  $\beta_{max}$  is:

$$\beta_{\max} = \min \left( 1, \frac{1 - \sum_{i=1}^s m_{i1}}{\sum_{i=1}^s (u_{i1} - m_{i1})}, \frac{1 - \sum_{i=1}^s m_{i2}}{\sum_{i=1}^s (u_{i2} - m_{i2})}, \dots, \frac{1 - \sum_{i=1}^s m_{is}}{\sum_{i=1}^s (u_{is} - m_{is})} \right) \quad (5)$$

As consequence of the previously outlined definition of  $\beta$ , the upper bound of 1 is a general condition but sometimes may not be strictly required (Beynon *et al.* 2005). This will be noticed in our application to the Macedonian economy.

Similarly to most of the existing literature by Beynon and others, we adapt a general symmetric imprecision with  $l=0$  and  $u=2m$ . This choice is mainly based on computational simplicity. However, according to Diaz and Morrillas (2011) the proposed framework is strongly criticized because they consider this form of the membership function as unrealistic and arbitrary. In addition, the assigned upper value which is twice the observed ( $m$ ) gives the option that some of the direct technical coefficients may be higher than 1. However, the introduction of the  $\beta$  parameters corrects and ensures that this problem has a solution. Using the initial symmetric limits not necessarily means that the triangular membership functions will be symmetric (normal triangle) consistently as the level of imprecision ( $\beta$ ) increases. In other words, as the level of imprecision increases to the value that ensures solution of the fuzzy system, the lower and upper value may not increase at the same rate. The technical details of defining the lower and upper boundary matrices will not be described here in terms of  $l$ ,  $m$  and  $u$  values. For more interested readers please see Beynon *et al.* (2005). What is interesting is to emphasize that the technical coefficients for the symmetric case, obtains the following form:

$$a_{L,\beta,\alpha,ij} = (1 - (1 - \alpha)\beta)m_{ij} \quad \text{and} \quad a_{U,\beta,\alpha,ij} = (1 + (1 - \alpha)\beta)m_{ij} \quad (6)$$

When it comes to the ranking procedure, different methods display different rank order (Wang *et al.*, 2006). Beynon and his associates based their work on the formulas described by Chu and Tsao (2002). However, these ranking formulas are incorrect. Thus, the most appropriate method of ranking fuzzy numbers that will be consistent with the change of the fuzziness rate of the technical coefficient matrix is by centroids calculated by Wang *et al.* (2006, p.921) formulae. Moreover, the method based on Euclidean distances from the origin to the centroid points provide useful and simplified computational application in ranking fuzzy numbers.

The ranking of each fuzzy number is deployed over the  $\beta$  domin that ensures solution, i.e.  $\beta_{\max}$  where the general  $S(\cdot)$  area is obtained by:

$$S_{\beta} = \bar{a}_{\beta} * \bar{a}_{\beta}^{-1} \quad (7)$$

where:

$$\bar{a}_{\beta} = \frac{\int_l^m x \bar{a}_{\beta,L}(x) dx + \int_m^u x \bar{a}_{\beta,U}(x) dx}{\int_l^m \bar{a}_{\beta,L}(x) dx + \int_m^u \bar{a}_{\beta,U}(x) dx} \quad \bar{a}_{\beta}^{-1} = \frac{\int_0^1 \alpha \bar{a}^{-1}_{\beta,U}(\alpha) d\alpha - \int_0^1 \alpha \bar{a}^{-1}_{\beta,L}(\alpha) d\alpha}{\int_0^1 \bar{a}^{-1}_{\beta,U}(\alpha) d\alpha - \int_0^1 \bar{a}^{-1}_{\beta,L}(\alpha) d\alpha} \quad (8)$$

The main errors with Beynon *et al.* (2005) ranking formulae is in the second item  $\bar{a}_\beta^{-1}$ , i.e. "both numerator and denominator take a positive sign, which is fundamental error and makes the formulae wrong for any  $\alpha$  values" (Wang *et al.* (2006, p.921). Here, instead of summation we use subtraction in the second part of equation (8)!

Hence, equation (7) combined with (8) gives the necessary rank order for the considered sector, where rank reversal is plausible.

## 2.2 The eigenvector method

As indicated in the introduction, in the paper by Hristov *et al.* (2012) the indicators developed by Rasmussen (1956) are used in the linkage analysis. In what follows, is to give a brief overview of the method where both backward and forward linkages are linked to each other as the result of the properties of the eigenvalues and eigenvectors of nonnegative square matrices, i.e. the Perron-Frobenius theorem (Galanopoulos *et al.*, 2007).

Before deriving the backward and forward linkages in terms of eigenvector method, it is essential to go back at the basic input-output matrix algebra in order to better understand the associated relationship. Consider a nonnegative square matrix with  $n \times n$  flows among sectors ( $X$ ) where  $x$  is the row vector of total inputs (Miller and Blair, 2009). In, addition let  $A$  be the matrix of technical coefficients given by:

$$A = X\hat{x}^{-1} \quad (9)$$

where  $\hat{x}$  denotes a diagonal matrix with the elements of  $x$  on the leading diagonal.

If considering  $x$  to be a column vector of total outputs, a  $B$  matrix of input-output coefficients may be obtained, i.e:

$$B = \hat{x}^{-1} X \quad (10)$$

Hence, from equation (9) and (10) it is noticeable that  $X$  is:

$$X = A\hat{x} \text{ or } X = \hat{x}B \quad (11)$$

Therefore, if we assume that  $\lambda$  is dominant eigenvalue for the input - output matrix  $A$ , it is noticeable from the relationship in (11) that matrix  $B$  has the same dominant eigenvalue. This implies that:

$$Ay = \lambda y \quad (12)$$

such that  $y$  is the non-zero column eigenvector of a matrix  $A$  (Galanopoulos *et al.*, 2007).

### 2.2.1 Linkages

Given the Perron-Frobenius prepositions cited in Galanopoulos *et al.* (2007) the eigenvector may be interpreted as quantity vector defined either as "left hand" or "right hand" Perron vector. Backward linkages are associated with the "left-hand" Perron vector, whereas forward linkages are measured by the "right hand" Perron vector (Dietzenbacher, 1992). It can be shown that both backward and forward Perron vectors are indeed associated with the dominant eigenvalue of the input and output coefficient matrices.

Thus, given that there is a "left-hand" Perron vector  $q$  it implies that:

$$q' A = \lambda q' \quad (13)$$

where prime indicates transposition. Or:

$$A' q = \lambda q' \text{ since } q' A = A' q \quad (14)$$

The detailed derivation of the backward linkage reflected by the eigenvector is presented in Dietzenbacher (1992). However, what is important to know is that the below expression for the backward linkage indicator (BLI) is derived from equation (14).

$$BLI = nq' / q' e \quad (15)$$

whit  $e$  representing the column summation vector, i.e.  $e=1$  for each sector in the  $n \times n$  matrix (ibid).

Consireding forward linkage there exists a "right-hand" Perron vector  $z$  associated with the matrix of the output coefficients  $B$ . Hence:

$$\lambda z = Bz \quad (16)$$

Considering equation (10) and replacing  $X$  with equation (11) the following expression is obtained:

$$B = \hat{x}^{-1} A \hat{x} \quad (17)$$

Which may be substituted in equation (16), i.e:

$$\lambda z = \hat{x}^{-1} A \hat{x} z \quad (18)$$

Premultiplying expression (18) on both sides with  $\hat{x}$ , we get:

$$\hat{x}\lambda z = \hat{x}\hat{x}^{-1}A\hat{x}z \quad \text{or} \quad \lambda\hat{x}z = A\hat{x}z \quad (19)$$

If we set  $y = \hat{x}z$  then, equation (19) becomes:

$$\lambda y = Ay \quad (20)$$

which is equivalent to equation (12).

Since we proved that both  $A$  and  $B$  have the same dominant eigenvalue  $\lambda$  and using the relationship  $y = \hat{x}z$ , it is meaningful to say that the forward linkage indicator (FLI) obtains similar form as BLI, i.e:

$$FLI = nz / e'z \quad (21)$$

Each element of the two Perron vectors is normalized with respect to their means and hence allows for the assessment of essential or key sectors similar as in Hristov *et al* (2012) meaning, sector is considered as a *key* when the estimates of its indicators are above average, i.e. greater than 1.

An issue that we should be aware in our analysis is that the Macedonian symmetric input-output matrix is not diagonal dominant but the Dietzenbacher framework works well beside this problem.

### 3 Results and discussion:

This section provides an analogous analysis of the input-output relationship in Macedonia in terms of water consumption. Again, we deal with 28 sector symmetric input-output table for 2005 published by the State Statistical Office (2008). The list of the sectors is provided in appendix 1. These 28 sectors were initially 60, which we had to aggregate due to absence of data regarding water consumption. Hence, beside the uncertainty of the used data in terms of water use, aggregation of sectoral primary factor returns is additional source of uncertainty and imprecision in our input-output analysis. Hence, what we did is apply a fuzzy approach to the technical coefficients of the symmetric 28 input-output matrix to investigate if there is changes in the level of fuzziness given the imposed aggregation and inappropriate survey representation. Based the above provided technical details, the domain of  $\beta$  that ensures solution to the fuzzy input-output models is:

$$\beta_{max} = \min(1, 1.231, 7.977, 1.488, 1.213, 0.598, 8.923, 0.473, 1.315, 3.16, 1.637, 4.556, 0.412, 5.342, 2.323, 1.809, 1.086, 1.44, 7.854, 3.658, 13.272, 7.078, 4.023, 2.743, 1.038, 1.197, 2.511, 0.597, 1.522) = 0.412$$

Hence a value of 0.412 limits the analysis for the 28 sector input-output model. As indicated before an upper bound of 1 is not a strict general condition in fuzzy environment.

Figure 2 displays the fuzzification structure of the output multipliers concerning the key water consuming sectors found in Hristov *et al.* (2012). The results of the adopted fuzzification of the input-output matrix of technical coefficients, indeed displays change of the rate of fuzziness for some output multipliers as  $\beta$  goes from 0 to the level that ensures a solution. Considering the sectors coke and refined petroleum (12) as well as basic metal (16), when the level of uncertainty is increased change in their upper rate appears when  $\beta$  is around 0.2. In addition, electrical energy (25) sector changes the level of fuzzification with the other mining (3) sector in their lower bounds when  $\beta$  is around 0.3.

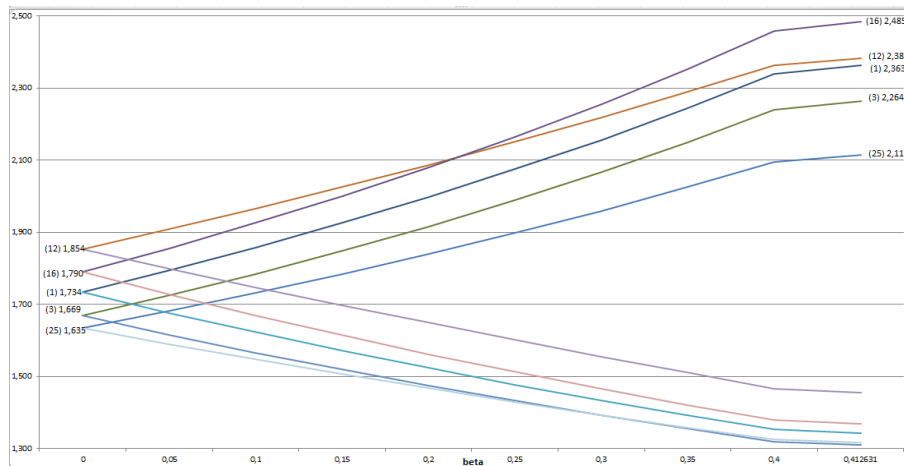


Figure 2: Bounds on the fuzzy triangular output multipliers for the key water consuming sectors found in Hristov *et al.* (2012) for  $0 < \beta < 0.412$

Appendix 2 provides together the lower and the upper bounds of the fuzzy triangular output multipliers for all 28 sectors. Due to the disorderliness of the series in the figure it is hard to notice that some of the changes for some sectors appear at an early stage of uncertainty ( $\beta$ ) and some later. Moreover, what we found at this stage is that for almost all output multipliers the rate of change for the upper and the lower values is not stable, i.e. the triangle is not symmetric though the general symmetry approach is applied. Consequently, this means that when the sectors are ranked in a fuzzy environment there will be changes in the ranking order given the proportional imprecision, i.e. rank reversal. However, since we are more interested to investigate the water consumption in terms of the indicator of total water consumption described in the Hristov *et al.* (2012) work, we will continue our analysis by skipping the ranking methodology of the output multipliers.

The reasons for hesitancy that were found in Hristov *et al.* (2012) in terms of identification of key water consumption sectors is confirmed in the fuzzy environment of the technical coefficients. The changes in the rate of the fuzzification affects the Rasmussen linkage analysis, since the direct (input-output) technical coefficient matrix is linked to the identification methodology.

Therefore, what we did next in our study is we apply the Dietzenbacher eigenvector method to the non-fuzzy input-output matrix in terms of water consumption. Table 1 provides the results from this analysis.

Table 1: Dietzenbacher and Rasmussen backward linkages (BLI) and forward linkages (FLI) indices in terms of water consumption

<i>Nr.</i>	<i>Sector</i>	<i>Dietzenbacher</i>		<i>Rasmussen</i>	
		<i>BLI</i>	<i>FLI</i>	<i>BLI</i>	<i>FLI</i>
1	Agriculture, forestry and fisheries	5.31	1.75	1.97	5.59
2	Mining and quarrying	4.98	12.30	0.06	0.52
3	Other mining and quarrying products	8.12	9.35	2.16	3.89
4	Food products and beverages	2.86	0.31	2.40	0.86
5	Tobacco products	0.07	0.00	3.15	0.02
6	Textiles	0.02	0.00	0.02	0.06
7	Wearing apparel; furs	0.04	0.00	0.04	0.01
8	Leather and leather products	0.01	0.00	0.03	0.01
9	Wood and products of wood and cork (except furniture); articles of straw and plaiting materials	0.57	0.23	0.85	0.25
10	Pulp, paper and paper products	0.01	0.03	0.04	0.12
11	Printed matter and recorded media	0.01	0.10	0.04	0.07
12	Coke, refined petroleum products and nuclear fuels	0.01	0.01	7.56	2.24
13	Chemicals, chemical products and man-made fibres	0.20	0.25	0.19	1.30
14	Rubber and plastic products	0.15	0.01	0.09	0.09
15	Other non-metallic mineral products	0.17	0.06	1.00	0.09
16	Basic metals	1.88	1.12	1.63	2.60
17	Fabricated metal products, except machinery and equipment	0.02	0.01	0.93	0.04
18	Machinery and equipment	0.33	0.15	0.23	0.30
19	Office, computers; Electrical machinery and apparatus; Radio, TV, communication	2.90	2.14	0.43	0.19
20	Medical, precision and optical instruments, watches and clocks	0.00	0.00	0.01	0.00
21	Motor vehicles, trailers and semi-trailers	0.02	0.00	0.28	0.01
22	Other transport equipment	0.08	0.00	0.45	0.00
23	Furniture; other manufactured goods	0.07	0.06	0.06	0.05
24	Secondary raw materials	0.00	0.01	0.75	0.07
25	Electrical energy, gas, steam and hot water	0.07	0.06	2.41	1.56
26	Collected and purified water, distribution services of water	0.00	0.00	0.42	0.04
27	Construction work	0.04	0.00	0.57	0.04
28	Services	0.04	0.01	0.24	7.97

Given the fact that the eigenvector method is superior and provides better indicator of interindustry linkages than the Rasmussen framework, the obtained results may be considered as robust. Table 1 displays that with the new linkage analysis, new sectors are considered as key in terms of water use. Agriculture (1), other mining (3) and basic metals (16) sectors kept their position as major water

users, but now the mining (2) and production of electrical machinery and related equipment (19) sectors are included in this group. These results are realistic since all these sectors in Hristov *et al.* (2012) obtained the largest indicators of direct water consumption per currency unit produced output. Meaning, the water consumption is high compared to their production and these sectors play a crucial influence on the Macedonian limited water resources. In addition, in the water transactoin matrix these sectors were the most considered in the intersectoral water relationship.

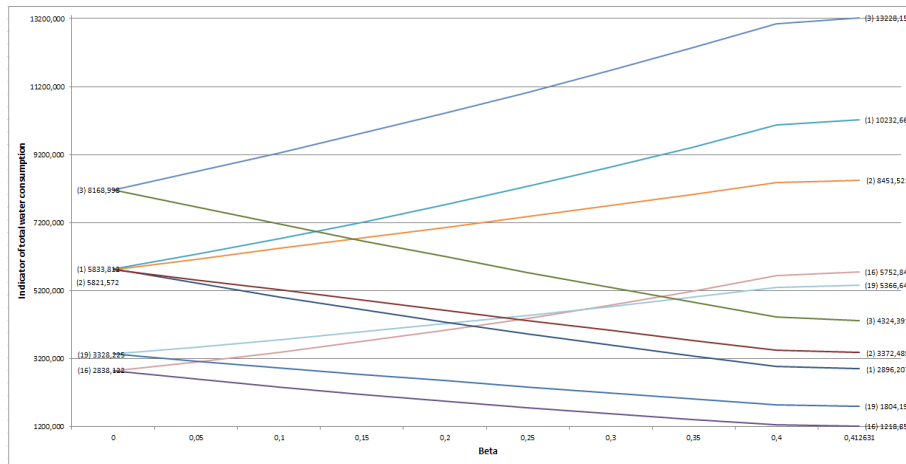


Figure 3: Bounds on the fuzzy triangular indicators of total water consumption for the key water consuming sectors determined with eigenvector method, for  $0 < \beta < 0.412$

Considering these new perspective in our analysis we are keener to investigate and apply the proposed fuzzy methodological framework to the water composition coefficients. As we indicated before, the main reason for this is that we are uncertain about the used data regarding the direct total water consumption. Consequently the uncertainty and imprecision will be reduced by analysing the intensive water consumption in Macedonia from fuzzy perspective.

Similar as before, we will focus our analysis only to the key water use sectors found by the eigenvector methodology, using the same domain that ensured solution to the fuzzy input-output model. Figure 3, displays the bounds on the fuzzy triangular indicators of total water consumption for the key water consuming sectors, whereby appendix 3 provides the bounds for all 28 sectors. Same as the Leontief output multipliers, the indicators of total water consumption is a row vector and determines the total amount of water that the economy will both directly and indirectly consume, if there is an increase by one unit of any given sector.



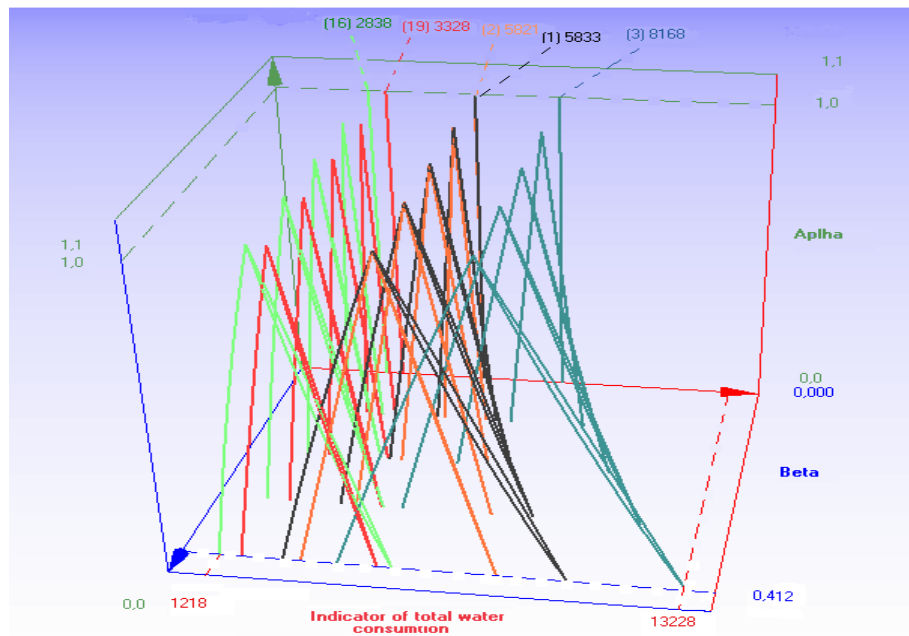


Figure 4: Fuzzy triangular membership functions of the indicators of total water consumption of the key sectors as function of  $\alpha$  and  $\beta$

What we may determine from figure 3 is that again as in the output multipliers there are changes in the rate of fuzziness for the new key water use sectors identified by the Dietzenbacher method. Agriculture (1) and other mining (3) have constant increase in the level of fuzziness. Contrary to this, the basic metals (16) and electrical energy (19) sectors have changes in their upper rate when the level of imprecision is around 0.3. When it comes into consideration the lower bound the associated rate of fuzziness for the mining sector (2) grows slower compare to agriculture (1) even at the very beginning when we have very little uncertainty. As the level of imprecision increases to the value that ensures solution of the fuzzy system, the lower and upper value do not increase at the same rate. This is visible in figure 4, where the fuzzy triangular membership functions of the respective sectors as functions of  $\alpha$  and  $\beta$  parameters, are displayed. Bigger the indicators are, the greater the uncertainty is. Therefore, in our last step of the analysis when the ranking of each fuzzy triangular water indicator is applied we would expect to experience rank reversal for the sectors that display changes in their level of fuzzification.

As indicated in the methodological framework section, we decided to implement the Wang *et al.* (2006) ranking formulas as the most appropriate ranking procedure of fuzzy numbers which are consistent with the change of the fuzziness rate. Examining figure 5, we may notice that indeed there is changes in the ranking of the basic metal (16) and the sector responsible for electrical equipment production (19). The rank reversal appears at fuzzy level of around 0.35 which is consistent

with the analysis from figure 3 and 4. However, this may be as consequence of the imposed general symmetry condition. Meaning that, larger the values of the indicators are, the larger level of variance is expected to be associated as the uncertainty and imprecision increases. Although throughout the analysis regarding changes in terms of rate of fuzzification, the mining sector (2) grew slower in the lower bound compare to agriculture (1), difference of rank order is not present here. Hence, what we may conclude from this analysis is that agriculture in a fuzzy environment remains the key water consuming with high indicator of direct water consumption. In addition, ranking the other key water use sectors identified by the Dietzenbacher method, it also conveys the idea that the heavy exploitation of the water resources in Macedonia by the industrial sectors is consistent in fuzzy environment. Therefore, due to the high indicator of direct water consumption, increase in the production of these sectors will impose a significant pressure on the natural freshwater resources and the environment.

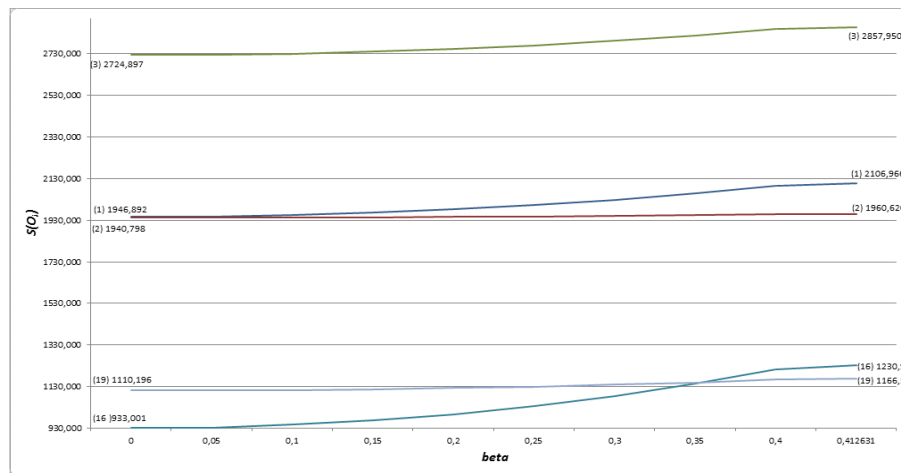


Figure 5: Ranking of the key water consuming sectors for  $0 < \beta < 0.412$

Clearly, the fuzzy environment combined with the Dietzenbacher method gave us the opportunity to reduce the uncertainty and imprecision in terms of input-output analysis of water consumption in Macedonia. In addition, the applied ranking method provide us the tool to consider whether or not policy options to be targeted at this natural resource. Similar as in Hristov *et al.* (2012) the water intensive structure mainly focused around agriculture and several industrial sectors is confirmed, given the fact that rank reversal was absent. Consequently, necessity to introduce changes in the agricultural production technology and specialization in the production in this region or maybe reconsidering the existing water pricing policy ought to be carefully considered by the policy makers.

In what follows is explanation of some aspects that might be considered in our analysis and the arguments behind the delimitation to these frameworks.

By defining a Beta probability distribution that best fits the direct technical coefficients (Diaz & Morillas, 2011), a more appropriate methodological framework would be to use stochastic analysis through Monte Carlo simulations. Although the Beta probability distribution allows for higher flexibility and defines the domain of the input-output coefficients with only two parameters, in our study it is impossible to use it because its implementation requires to have a wealth of prior data. Indeed, defining a Beta probability distribution requires prior information (primary data obtained from the firms' surveys used in constructing the symmetric input-output table) which are not at our disposals at the moment. Hence, due to this inconvenience we decide to choose the general symmetry approach even though it underestimates the true observed value and  $\nu$  tend to overestimate.

When it comes to the Dietzenbacher method, the reason for applying only to the non-fuzzy matrix in terms of water consumption is because the method that Buckley (1990) developed to find fuzzy eigenvalues for a given fuzzy matrix  $\bar{A}$  it is limited. It is limited in a sense, that the necessary requirement to ensure a solution in  $\bar{\lambda}\bar{y} = \bar{A}\bar{y}$  is  $\bar{\lambda} \geq 0$  and  $\bar{y} \geq 0$  should satisfy the same criteria as the positive elements in  $\bar{A} \geq 0$ . "When  $\bar{\lambda} < 0$  the equations to solve for the fuzzy eigenvalue are far more complicated and there is no guarantee that their solution will produce a bonafide fuzzy number for  $\bar{\lambda}$ " (Buckley, 1990, p.193). Given the fact that our symmetric input-output matrix is not diagonally dominant, we may argue that obtaining a negative eigenvalue is plausible in fuzzy analysis. Hence, the eigenvector method was omitted in the fuzzy environment.

#### 4 Conclusion

The acknowledgment of imprecision and uncertainty that exists in the data used for constructing the input-output tables or the data used to extend the traditional Leontief relationship in terms of environmental indicators, followed by aggregation of some sectors, was investigated in this paper. The fuzzy modeling allows to investigate the effects of uncertainty over the technical coefficients of the symmetric input-output table and the indicators of total water consumption in Macedonia. By imposing triangular membership functions and general symmetric imprecision, we were able to achieve greater transparency of the suspicious results found in Hristov *et al.* (2012) in terms of identification of the key water use sectors. We confirmed that Dietzenbacher is better methodology compare to Rasmussen in terms of identifying key sectors, irrespective of the considered aspect (environmental, monetary, ...). Meaning, the applied Dietzenbacher eigenvector method conveys the idea that that agriculture and several industrial sectors impose heavy exploitation on the limited water resources in Macedonia. In addition, ranking of each fuzzy triangular water indicator didn't display any rank reversal for most of these sectors. Although, for two sectors there was a presence of rank reversal at a level of high imprecision and uncertainty, this result may be argued to be related to the initially imposed general symmetric imprecision. Simply stated, larger the fuzzy number is the larger variation is expected to be associated.

Therefore, in general it may be concluded that the need to propose changes in the production technology or specialization in this region is necessary. Reconsidering the existing water pricing policy in Macedonia ought to be an option as well.

What is the most important contribution of this work is that this is the first attempt to apply such a methodology in the area of Western Balkan not just in terms of conventional monetary input-output analysis but also in terms of ecological footprints. Most of the existing literature is focused either on UK and Wales or the Andalusia region in Spain (Beynon *et al.* (2005), Beynon and Munday (2007), Diaz and Morillas (2011), Morillas *et al.* (2011)). Moreover, the applied correct centroid formulae described by Wang *et al.* (2006) in order to rank the fuzzy numbers is another significant difference compared to the Beynon work.

The reduced uncertainty from the obtained results brings to the forefront a new idea for further research that may raise the awareness of this natural resource even more. The approach to disaggregate the agriculture as major water consuming sectors could be of even greater importance for policy options for sustainable water management and potential water pricing policies. The end result of this extension will be a detailed and disaggregated input-output table of the Macedonian economy with a special emphasis on agriculture. Another idea that emerged from this fuzzy approach is to create a detailed water accounts similar as the one published by the Australian Bureau of Statistics (2010). In that sense, there won't be a necessity of a fuzzy approach since the exact relationship (supply and demand) between sectors will be known. Moreover, the detailed water accounts may serve to create a symmetric input-output table in terms of water consumption. This is something that the policy makers should really focus in the future!

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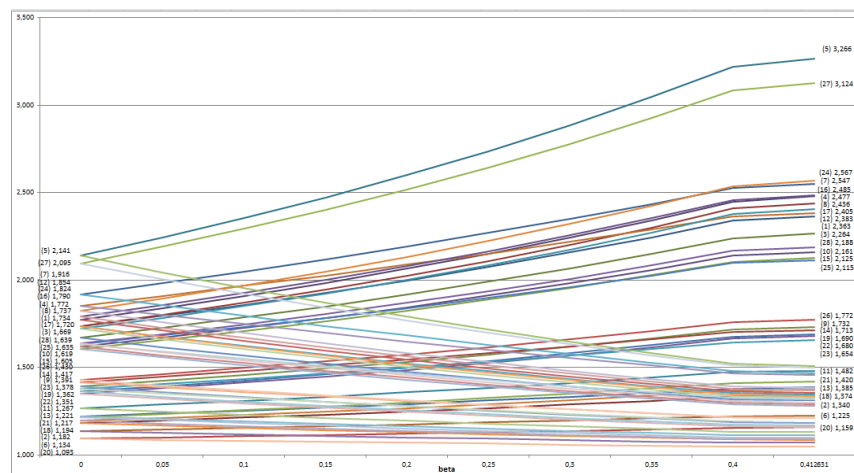
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**Annexes**

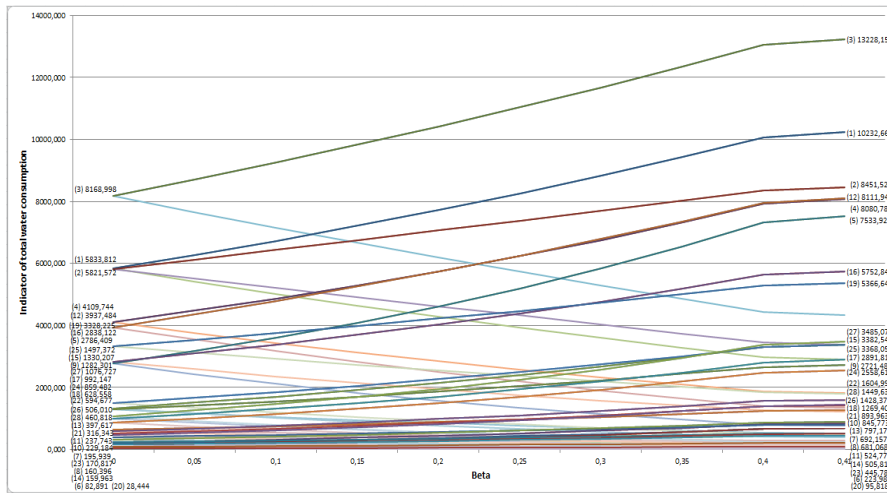
Appendix 1: List of sectors in the input-output table

Nr.	Sector	Nr.	Sector
1	Agriculture, forestry and fisheries	15	Other non-metallic mineral products
2	Mining and quarrying	16	Basic metals
3	Other mining and quarrying products	17	Fabricated metal products, except machinery and equipment
4	Food products and beverages	18	Machinery and equipment
5	Tobacco products	19	Office, computers; Electrical machinery and apparatus; Radio, TV, communication
6	Textiles	20	Medical, precision and optical instruments, watches and clocks
7	Wearing apparel; furs	21	Motor vehicles, trailers and semi-trailers
8	Leather and leather products	22	Other transport equipment
9	Wood and products of wood and cork (except furniture)	23	Furniture; other manufactured goods
10	Pulp, paper and paper products	24	Secondary raw materials
11	Printed matter and recorded media	25	Electrical energy, gas, steam and hot water
12	Coke, refined petroleum products and nuclear fuels	26	Collected and purified water, distribution services of water
13	Chemicals, chemical products and man-made fibres	27	Construction work
14	Rubber and plastic products	28	Services

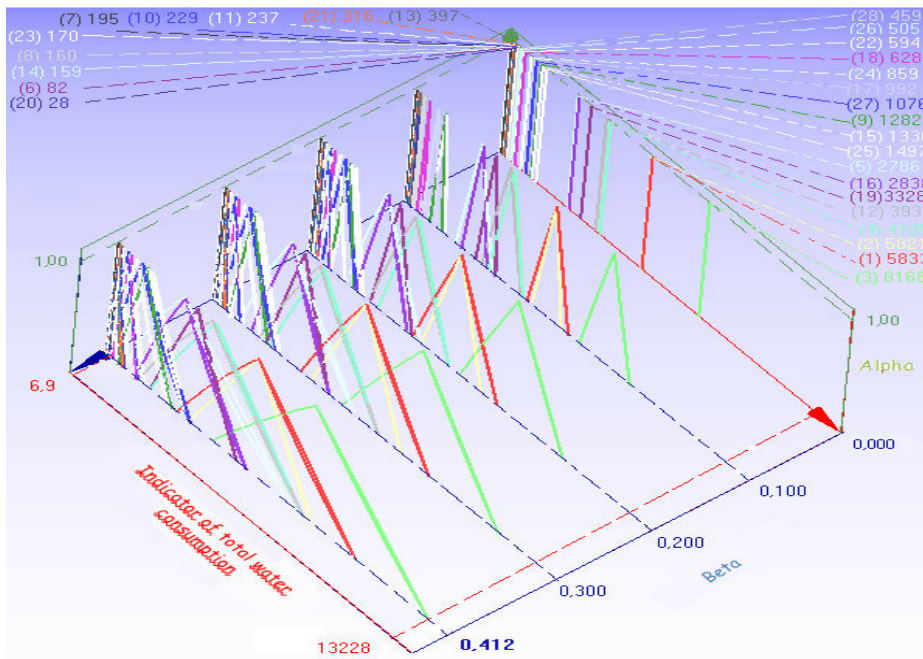
Appendix 2: Bounds on the fuzzy triangular output multipliers for the 28 sectors for  $0 < \beta < 0.412$



Appendix 3: Bounds on the fuzzy triangular indicators of total water consumption for the 28 sectors, for  $0 < \beta < 0.412$



Appendix 4: Fuzzy triangular membership functions of the indicators of total water consumption of all 28 sectors as function of  $\alpha$  and  $\beta$





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Študije potrošnih navad

Agrarna politika držav zahodnega Balkana

Ekonometrične analize in matematično modeliranje

Empirični modeli v podporo odločanju kmetijske politike

Modeli v podporo odločanju na ravni gospodarstva

Organizacije pridelovalcev, potrošne navade in poslovno odločanje

Pravo in razvoj podeželja

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