AgEcon SEARCH
RESEARCH IN AGRICULTURAL & APPLIED ECONOMICS

The World's Largest Open Access Agricultural & Applied Economics Digital Library

This document is discoverable and free to researchers across the globe due to the work of AgEcon Search.

Help ensure our sustainability.

Give to AgEcon Search

AgEcon Search
http://ageconsearch.umn.edu
aesearch@umn.edu

*Papers downloaded from **AgEcon Search** may be used for non-commercial purposes and personal study only. No other use, including posting to another Internet site, is permitted without permission from the copyright owner (not AgEcon Search), or as allowed under the provisions of Fair Use, U.S. Copyright Act, Title 17 U.S.C.*

On the Premium for Revenue Insurance under Joint Price and Yield Risk

Alexander E. Saak

Working Paper 04-WP 368
July 2004

Center for Agricultural and Rural Development
Iowa State University
Ames, Iowa 50011-1070
www.card.iastate.edu

Alexander Saak is an assistant scientist at the Center for Agricultural and Rural Development and a U.S. farm policy analyst at the Food and Agricultural Policy Research Institute, Iowa State University.

This publication is available online on the CARD Web site: *www.card.iastate.edu*. Permission is granted to reproduce this information with appropriate attribution to the authors.

For questions or comments about the contents of this paper, please contact Alexander Saak, 565 Heady Hall, Iowa State University, Ames, IA 50011-1070; Ph: 515-294-0696; Fax: 515-294-6336; E-mail: asaak@iastate.edu.

Iowa State University does not discriminate on the basis of race, color, age, religion, national origin, sexual orientation, sex, marital status, disability, or status as a U.S. Vietnam Era Veteran. Any persons having inquiries concerning this may contact the Director of Equal Opportunity and Diversity, 1350 Beardshear Hall, 515-294-7612.

Abstract

This note provides two results pertaining to the pricing of agricultural revenue insurance contracts under joint price and yield risk. First, a weakening of the concordance ordering is used to sign the effect of greater dependence between the multiplicative risks (price and yield) on the expected indemnity payment. Second, sufficient conditions are found when the premium rate for revenue insurance is smaller (greater) than the premium for the corresponding single risk (price or yield) insurance.

Keywords*:* concordance order, revenue insurance.

ON THE PREMIUM FOR REVENUE INSURANCE UNDER JOINT PRICE AND YIELD RISK

1. Introduction

In recent years, several revenue insurance products have become popular among U.S. agricultural producers (USDA 2001a). Unlike other risk management instruments that insure separately against price or yield risk, a revenue insurance contract provides a joint coverage.[1] As a result, the determination of the actuarially fair premium requires pricing put options on a non-traded asset such as gross farm revenue (Yin and Turvey 2003). One of the theoretical and empirical problems with the pricing of revenue insurance contracts is accounting for the dependence between the price and yield risks (e.g., USDA 2001b; Miller, Kahl, and Rathwell 2000). The modeling of dependencies among risks is an area of active research in actuarial, finance and agricultural economics literatures (e.g., Denuit, Dhaene, and Ribas 2001 and references therein; Hennessy, Saak, and Babcock 2003). However, most of the previous literature focused on the case of multiple additive risks corresponding to a portfolio of insurance policies held by the insurer (or the insured).

The goal of this paper is to analyze how an increase in dependence between multiplicative risks affects the actuarially fair value of an individual insurance policy. The primary example is agricultural revenue insurance that insures gross crop revenue defined as the product of random price and yield, which are prone to be negatively correlated.[2] Also, premiums for revenue insurance and the corresponding yield insurance policy are compared. The indemnity payment under yield insurance policy is triggered by the yield loss that is multiplied by the predetermined (expected) price.[3]

Section 2 presents a weakening of the concordance ordering that localizes the domain of realizations where risks become more dependent in the sense of the ordering. Restricting the set of possible dependence structures in this manner assures that the expectation of the indemnity function is monotone with respect to the concordance ordering. In Section 3, the local concordance ordering is used to ascertain how an in-

crease in dependence between multiplicative risks affects the expected indemnity payment. If an increase in dependence involves the values of price and yield such that the realized revenue lies below the revenue guarantee, the premium decreases. The converse holds if the product of price and yield exceeds the revenue guarantee in the domain where the increase in the concordance order takes place.

Section 4 provides two conditions sufficient for an unambiguous comparison of the premiums for revenue and yield insurances. Because in practical terms price and yield can be viewed as either negatively dependent or independent, the proposed conditions are based on evaluating the revenue premium under the "boundary" dependence structures: extreme negative dependence (the Frechet lower bound) and independence. The yield insurance is always cheaper than the revenue counterpart if the negative price-yield dependence, as compared to price-yield independence, is "operative" in the domain of price-yield realizations that are not covered by the revenue insurance. On the other hand, under certain restrictions on the marginal distributions and the coverage level, the revenue insurance is cheaper. This is the case when the strength of the negative price-yield dependence is weaker relative to the Frechet lower bound in the portion of the distribution support in which there is no indemnity payment under revenue insurance.

2. Local Concordance Order

To model the strength of dependence between price and yield we use a weaker version of the concordance order (also known as the positive quadrant dependence order) for bivariate distributions with fixed univariate marginals (Joe 1997). Let $F_X(x) = F(x,\infty)$ and $F_Y(y) = F(\infty, y)$ denote marginal probability distributions with densities f_X and f_Y and $supp(F_X) = \{x : f_X(x) > 0\}$.

DEFINITION. A probability distribution $F(x,y)$ is smaller than probability distribution $F'(x,y)$ in the local concordance order on set $A \subseteq supp(F(x,\infty)) \times supp(F(\infty,y))$ (denoted $\prec_c^A$) if $F(x,y) \le F'(x,y)$ for all $(x,y) \in A$ and $F(x,y) = F'(x,y)$ for all $(x,y) \notin A$, where $F_X = F'_X$, $F_Y = F'_Y$.

Tchen (1980) and Epstein and Tanny (1980) showed that $F \prec_c^A F'$ is equivalent to the following two conditions.[4] First, there exists a sequence of probability distributions $F = F_1, ..., F_n = F'$ such that F_i is obtained from F_{i-1} by adding mass $\varepsilon_i > 0$ at some points (x, y) and (x', y') while subtracting mass ε_i at the points (x', y) and (x, y') where $x < x'$, $y < y'$, and (x, y), (x', y'), (x', y), $(x, y') \in A$. We will refer to a single transformation that redistributes the probability mass toward the outcomes in which the components are better aligned as an elementary increase in concordance (EIC), denoted by $e(x, x', y, y', \varepsilon)$.[5]

The other characterization relies on the properties of supermodular (also known as superadditive) functions. The local concordance ordering adheres if $E\varphi(X, Y) \leq E\varphi(X', Y')$ for any function φ that is supermodular on set A such that the expectations exist, where random variables X, Y and X', Y' have the joint probability distribution $F(x, y)$ and $F'(x, y)$, respectively. This implies that the corresponding conditional expectations are also ordered in the same manner: $E[\varphi(X, Y) \mid (X, Y) \in A] = \int_A \varphi(x, y) dF / (1 - \int_{A^c} dF) \leq \int_A \varphi(x, y) dF' / (1 - \int_{A^c} dF') = E[\varphi(X', Y') \mid (X', Y') \in A]$. The inequality follows because, by definition, $\int_A dF = \int_A dF'$. A function ϕ is called supermodular (submodular) if for any evaluations $x_1' > x_1''$ and $x_2' > x_2''$, we have $\phi(x_1', x_2') + \phi(x_1'', x_2'') \geq (\leq)\ \phi(x_1', x_2'') + \phi(x_1'', x_2')$. The supermodularity is equivalent to the "increasing differences" property: $\Delta_1^\tau \Delta_2^\delta \phi(x_1, x_2) \geq 0$ where $\tau > 0$, and $\delta > 0$, $\Delta_1^\tau \phi(x_1, x_2) = \phi(x_1 + \tau, x_2) - \phi(x_1, x_2)$. In other words, the value of a supermodular function increases more with x_i when other $x_j, j \neq i$ take on high values. One of the attractive features of the (local) concordance ordering is its immediate connection with a more familiar notion of correlation. One can easily show that $F \prec_c^A F'$ implies that $Cov[f(X), g(Y)] \leq Cov[f(X')\ g(Y')]$ for any functions f and g monotonic in the same direction given that the covariances exist.

Next, we use the notion of the local concordance ordering to determine the effect of an increase in price-yield dependence on the expected indemnity for a revenue insurance contract.

3. Premium for Revenue Insurance and Price-Yield Dependence

The revenue insurance contract has the standard indemnity stream of the form

$$\max[\beta E[PY]-PY,0], \tag{1}$$

where P and Y are non-negative random variables representing price and yield risks, respectively, with the joint probability distribution F and density f; $\beta \in (0,1]$ is the coverage level; and E is the mathematical expectation.[6]

PROPOSITION 1. *An actuarially fair premium for revenue insurance,* $\Pi_R(F)$ $=\int \max[\beta\int tsdF \ -py,0]dF$, *decreases (increases) under* $F \prec_c^A F'$ *(*$F \prec_c^B F'$*) where* $A = \{(p,y) \mid py \le \beta\int tsdF']\}$ *and* $B=\{(p,y) \mid py \ge \beta\int tsdF\}$.

Proof. First, we consider $F \prec_c^A F'$. By definition, we have $\Pi_R(F)=\int_{py\le\beta\int tsdF}(\beta\int tsdF$ $-py)dF \ge \int_{py\le\beta\int tsdF'}(\beta\int tsdF-py)dF = \int_{py\le\beta\int tsdF'}(\beta\int tsdF'-py)dF' \ -(\beta\int_{py\le\beta\int tsdF'}dF)$ $\int pyd(F'-F)+\int_{py\le\beta\int tsdF'}pyd(F'-F)=\Pi_R(F')+(1-\beta\int_{py\le\beta\int tsdF'}dF)\int pyd(F'-F)$ $\ge \Pi_R(F')$. The first inequality is because the function $\varphi(p,y)=py$ is supermodular and the expected value of a supermodular function is monotone in the (local) concordance order, $\int pydF \ \le \int pydF'$. Then, the fact that the increase in dependence is "local" and takes place on set A is used. Specifically, $F \prec_c^A F'$ implies that $\int_{py\le\beta\int tsdF'}dF$ $=\int_{py\le\beta\int tsdF'}dF'$ and $\int_{py>\beta\int tsdF'}pyd(F'-F) \ =0$. The last inequality follows because $\beta \le 1$.

The case with $F \prec_c^B F'$ is proved similarly. We have $\Pi_R(F)=\int_{py\le\beta\int tsdF}(\beta\int tsdF$ $-py)dF=\int_{py\le\beta\int tsdF}(\beta\int tsdF-py)dF' \le \int_{py\le\beta\int tsdF}(\beta\int tsdF'-py)dF' \le \int_{py\le\beta\int tsdF'}(\beta\int tsdF'$ $-py)dF'=\Pi_R(F')$. The second equality is because $F \prec_c^B F'$ implies that $F=F'$ for all (p,y) such that $py \le \beta\int tsdF$. Both inequalities are due to $\int tsdF \le \int tsdF'$, which is also implied by $F \prec_c^B F'$.

Note that conditions in Proposition 1 do not overlap in the sense that it is impossible to have $F \le F'$ for all (p, y) such that $\beta\int tsdF \le py \le \beta\int tsdF'$ (with strict inequality for some (p, y)), and $F = F'$ otherwise.[7] Of course, conditions provided in Proposition 1 are only sufficient and provide little guidance when set A includes the realizations of price and yield that are both above and below the guaranteed revenue.

Next, we study the effect on the premium when an increase in dependence is "small," although it is not restricted to any particular area of the distribution support. For simplicity, we consider discrete random variables with finitely many atoms: $p_1 < p_2 < ... < p_n$ and $y_1 < y_2 < ... < y_m$. We examine the effect of EIC $e(p_{i_1}, p_{i_2} y_{j_1} y_{j_2}, \varepsilon)$ of distribution F on the expected value of the indemnity for revenue insurance. Let (P^e, Y^e) denote random variables with the probability distribution F^e that is obtained from F through an EIC; that is, $f^e(p_i, y_j) - f(p_i, y_j) = \delta_{ij}$ where $\varepsilon \in (0, \min[f(p_{i_1}, y_{i_2}), f(p_{i_2}, y_{i_1})]]$, $i_1 < i_2$, $j_1 < j_2$, $\delta_{ij} = \varepsilon$ if $i = i_1, j = j_1$ or $i = i_2, j = j_2$, $\delta_{ij} = -\varepsilon$ if $i = i_2, j = j_1$ or $i = i_1, j = j_2$, and $\delta_{ij} = 0$ otherwise.

PROPOSITION 2. *An actuarially fair premium for revenue insurance decreases (increases) under an EIC of $F(p, y)$ depending on whether*

$$E[P^e Y^e] \ge p_{i_2} y_{j_2} \Pr\{PY \le \beta E[P^e Y^e]\} + (1/\beta - \Pr\{PY \le \beta E[P^e Y^e]\}) \tag{2}$$

$$(p_{i_2} y_{j_1} + p_{i_1} y_{j_2} - p_{i_1} y_{j_1}), \text{ or}$$

$$E[P^e Y^e] \le p_{i_2} y_{j_2} \Pr\{PY \le \beta E[PY]\} + (1/\beta - \Pr\{PY \le \beta E[PY]\}) \tag{3}$$

$$(p_{i_2} y_{j_1} + p_{i_1} y_{j_2} - p_{i_1} y_{j_1}).$$

Proof. Decompose the difference as follows: $\Pi(F^e) - \Pi(F) = M + N$, where $M = \sum_{i,j} ((\beta(E[PY] + \varepsilon \Delta p \Delta y) - p_i y_j)_+ - (\beta E[PY] - p_i y_j)_+) f(p_i, y_j)$ and

$N = \varepsilon((\beta E[P^e Y^e] - p_{i_2} y_{j_2})_+ + (\beta E[P^e Y^e] - p_{i_1} y_{j_1})_+ - (\beta E[P^e Y^e] - p_{i_2} y_{j_1})_+$

$-(\beta E[P^eY^e]-p_{i_1}y_{j_2})_+)$, where $E[PY]=\sum_{t,s}p_ty_sf(p_t,y_s)$, $\Delta p\Delta y = (p_{i_2}-p_{i_1})$ $(y_{j_2}-y_{j_1})$. Because $\Delta p\Delta y>0$, we obtain the following bounds for *M*: $\beta\varepsilon\Delta p\Delta y$ $\sum_{i,j}1_{p_iy_j\le E[P^eY^e]}f(p_i,y_j)\ge M\ge\beta\varepsilon\Delta p\Delta y\sum_{i,j}1_{p_iy_j\le E[PY]}f(p_i,y_j)$. Turning to N, we find that $N=\varepsilon(\min[p_{i_2}y_{j_1},p_{i_1}y_{j_2}]-p_{i_1}y_{j_1})>0$ if $\min[p_{i_2}y_{j_1},p_{i_1}y_{j_2}]\le\beta E[P^eY^e]$ $\le\max[p_{i_2}y_{j_1},p_{i_1}y_{j_2}]$, $N=-\varepsilon\Delta p\Delta y<0$ if $\beta E[P^eY^e]\ge p_{i_2}y_{j_2}$, and the sign of N is indeterminate if $\max[p_{i_2}y_{j_1},p_{i_1}y_{j_2}]<\beta E[P^eY^e]<p_{i_2}y_{j_2}$. Combining these conditions with the bounds for *M* completes the proof.

In general, the right-hand sides of inequalities (2) and (3) are non-monotone in the coverage level, β. As expected by Proposition 1, condition (2) is satisfied when an EIC involves the realizations of price and yield that lie below the guaranteed revenue, $p_{i_2}y_{j_2}$ $\le\beta E[P^eY^e]$, while condition (3) is satisfied when an EIC involves the realizations above the guaranteed revenue, $p_{i_1}y_{j_1}\ge\beta E[P^eY^e]$, when $\beta\le 1$. Proposition 1 can also be established by repeatedly applying Proposition 2.[8] In the manner of Proposition 2, it can be shown that the effect of an EIC on the premium rate, $\hat{\Pi}(F)=\Pi(F)/\beta\int pydF$, is negative (positive) depending on whether

$$E[P^eY^e]\ge p_{i_2}y_{j_2}L(\beta E[P^eY^e])+(1-L(\beta E[P^eY^e]))\ (p_{i_2}y_{j_1}+p_{i_1}y_{j_2}-p_{i_1}y_{j_1}),\text{ or} \tag{2a}$$

$$E[P^eY^e]\le p_{i_2}y_{j_2}L(\beta E[PY])\ +(1-L(\beta E[PY]))\ (p_{i_2}y_{j_1}+p_{i_1}y_{j_2}-p_{i_1}y_{j_1}) \tag{3a}$$

holds, where $L(R)=E[PY\mid PY\le R]\Pr\{PY\le R\}/E[PY]\le 1$. These conditions are analogous to conditions (2) and (3).

We can isolate two partial effects on $\Pi_R(F)$ induced by $F\prec_c^A F'$: (i) as previously noted, the expected revenue, and hence the guaranteed revenue coverage, $\beta E[PY]$, increases; and (ii) the transformation of the probability distribution of revenue, PY, has an effect on $\Pi_R(F;\bar{R})=E\max[\bar{R}-PY,0]$ as a result of $F\prec_c^A F'$, keeping the revenue guarantee, $\bar{R}=\beta E[PY]$, fixed. Effect (ii) is ambiguous and depends on the subset of the

domain in which the dependence between P and Y increases in the sense of the local concordance ordering. If the insurance never pays off in the range of price and yield realizations with greater dependence (set *B*), the expected indemnity increases because (obviously) effect (ii) has no impact on $\Pi_R(F;\overline{R})$.

On the other hand, if the insurance always pays off in the subset of price and yield realizations with greater dependence, A, then the expected indemnity decreases. This is because the probability mass is shifted toward outcomes in which revenue is more dispersed such as $p_{i_1}y_{j_1}$ and $p_{i_2}y_{j_2}$, and away from the outcomes in which revenue is more stable $p_{i_1}y_{j_2} \in (p_{i_1}y_{j_1}, p_{i_2}y_{j_2})$ and $p_{i_2}y_{j_1} \in (p_{i_1}y_{j_1}, p_{i_2}y_{j_2})$. Therefore, keeping the revenue guarantee, $\overline{R}$, fixed, the expected payout must fall because the probability weight is shifted toward the outcomes with a smaller average payout, $p_{i_1}y_{j_1} + p_{i_2}y_{j_2}$ $> p_{i_1}y_{j_2} + p_{i_2}y_{j_1}$. Furthermore, in this case effect (ii) dominates effect (i). The reason is that effect (ii) works through transforming the probabilities with which the indemnity payments occur, while effect (i) shifts the magnitude of all payouts, albeit conditional on the event that the revenue falls short of the revenue guarantee. Hence, the impact of effect (i) is scaled down by both the probability that there is an indemnity payment and the share of the expected revenue covered by the insurance: $\beta \Pr\{PY \le \beta E[P'Y']\}$.

As previously mentioned, Proposition 2 demonstrates that the effect of an increase in price-yield dependence on the expected indemnity depends on the level of coverage in a non-monotone manner. However, loosely speaking, the set of joint probability distributions exhibiting a greater degree of price-yield dependence that raises (lowers) the expected indemnity expands (contracts) when the level of insurance coverage, β, decreases. This is formalized in the following.

COROLLARY. Let $F \prec_c^A F'$ for some $A \subseteq supp(F(x,\infty)) \times supp(F(\infty,y))$. (a) Suppose that $\overline{\beta} = \sup_{(p,y)\in A}\{py\} \ / \int pydF' < 1$. Then the actuarially fair premium decreases, $\Pi_R(F,\beta) \ge \Pi_R(F',\beta)$, for any $\beta \ge \overline{\beta}$; (b) There exists $\underline{\beta} = \min[1, \inf_{(p,y)\in A} py / \int pydF]$ such that the premium increases, $\Pi_R(F,\beta) \le \Pi_R(F',\beta)$, for any $\beta \le \underline{\beta}$.

Note that conditions (a) and (b) provide a sense in which it is "more likely" that the expected indemnity increases as the strength of positive dependence between price and yield increases when the level of coverage is low. This is, of course, because ordinarily, agricultural revenue insurance covers less than 100 percent of the expected revenue.

EXAMPLE 1. Let price and yield take one of three values: low (1), medium (2), or high (3), with equal probability (see Figure 1). Price and yield are independent random variables if $\varepsilon = 0$ and negatively dependent in the sense of the concordance order for $\varepsilon \in [-1/9, 0)$. Consider EIC $e(1,2,1,2,\varepsilon)$ that involves realizations of price and yield such that revenues are low, $py \le 4 = E[P]E[Y]$ (see Figure 1a). The relationships between the premium rate and the strength of positive dependence for different levels of coverage are presented in Figure 2. In contrast, in the case of EIC $e(2,3,2,3,\varepsilon)$ (see Figure 1b), the premium rate increases for all $\beta \le 1$.

Before closing this section, consider the effect of an increase in price-yield dependence on the premium for a revenue insurance policy with the expected indemnity equal to[9]

$$\Pi_{CRC} = E\max[\beta \max[EP, P]EY - PY, 0]. \tag{4}$$

Using the fact that the indemnity function in equation (4) is submodular in (P, Y) when either $P \ge EP$ or $P \le EP$ and $PY \le \beta E[P]E[Y]$, we can easily obtain the following.

PROPOSITION 3. *Actuarially fair premium* (4) *decreases under* $F \prec_c^{A_1} F' \prec_c^{A_2} F''$ *or* $F \prec_c^{A_2} F' \prec_c^{A_1} F''$; *i.e.,* $\Pi_{CRC}(F) \ge \Pi_{CRC}(F') \ge \Pi_{CRC}(F'')$, *where* $A_1 = \{(p,y): py \le \beta E[P]E[Y], p \le E[P]\}$ *and* $A_2 = \{(p,y): p > E[P]\}$.[10]

a)

$Y \setminus P$	1	2	3
1	$1/9+\varepsilon$	$1/9-\varepsilon$	$1/9$
2	$1/9-\varepsilon$	$1/9+\varepsilon$	$1/9$
3	$1/9$	$1/9$	$1/9$

b)

$Y \setminus P$	1	2	3
1	$1/9$	$1/9$	$1/9$
2	$1/9$	$1/9+\varepsilon$	$1/9-\varepsilon$
3	$1/9$	$1/9-\varepsilon$	$1/9+\varepsilon$

FIGURE 1. Probability distribution of price and yield and elementary increase in concordance

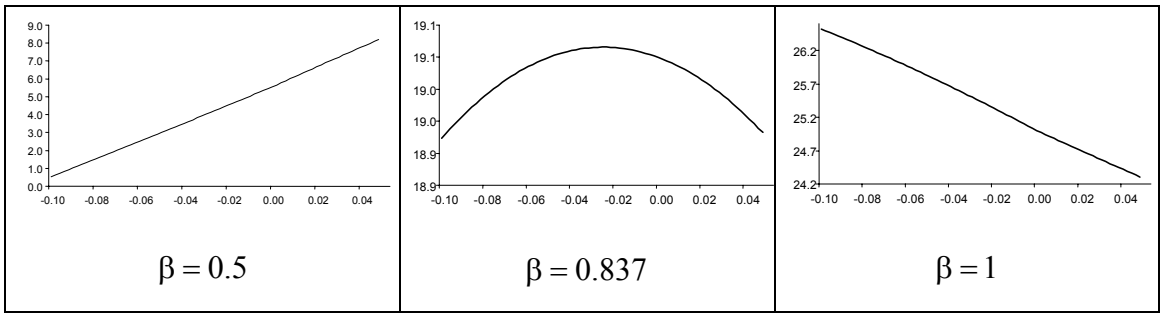

FIGURE 2. Premium rate for revenue insurance as a function of ε

Comparing conditions in Propositions 1 and 3 provides a sense in which the premium for insurance contract (4) is "more likely" to decrease compared with that for insurance contract (1) as the strength of (negative) price-yield dependence weakens.

4. Premiums for Revenue and Yield Insurance

The expected indemnity payment under a yield insurance contract is

$$\Pi_Y = EP \max[\beta EY - y, 0]\,. \tag{5}$$

We will need the following notation. Let $F_P(p) = F(p,\infty)$ and $F_Y(y) = F(\infty, y)$ denote marginal probability distributions for price and yield, respectively, with densities f_P and f_Y. Let $G^{-1}(v) = \inf\{x : G(x) \geq v\,,\ 0 < v < 1\}$ denote a left-continuous inverse of the univariate probability distribution G. Also, let $Q(y) = F_P^{-1}(1 - F_Y(y))$, which can be regarded as an inverse crop demand function. When can we assuredly say that the expected indemnity for revenue insurance contract, $\Pi_R(F,\beta)$, is greater (smaller) than that for the corresponding yield insurance contract, $\Pi_Y(F_P, F_Y, \beta)$?

PROPOSITION 4. (a) *An actuarially fair premium for revenue insurance is greater than that for yield insurance when* $F \prec_c^A F_P F_Y$ *where* $A = \{(p,y) : py \leq \beta E[P]E[Y]\}$.

(b) *Suppose that (i)* $f_P(Q(y))Q(y) \geq f_Y(y)y$ *for all* y*, and (ii)* $\hat{y} \leq \min[\beta EY, Q^{-1}(EP)]$ *where* $\hat{y} = \sup\{y : Q(\hat{y})\hat{y} = \beta E[Q(Y)Y]\}$. *Then an actuarially fair premium for revenue*

insurance is smaller than that for yield insurance when $F_L \prec_c^A F$ *where* $F_L(p,y)$ $= \max[0, F_P(p) + F_Y(y) - 1]$ *and* $A = \{(p,y) : py \le \beta \int pydF\}$.

Proof. Part (a): Because the function max[.,0] is convex and using Jensen's inequality, we have $\Pi_Y(F_P, F_Y) = E[P]\int \max[\beta E[Y] - y, 0]dF_Y \le \int \max[\beta E[P]E[Y] - py, 0]dF_P dF_Y$ $= \Pi_R(F_P F_Y)$. By Proposition 1, $\Pi_R(F) \ge \Pi_R(F')$ if $F \prec_c^A F'$ where $F' = F_P F_Y$ and $A = \{(p,y) : py \le \beta E[P]E[Y]\}$.

Part (b): There are two steps. First, we show that conditions (i) and (ii) are sufficient to assure that the premium rate for yield insurance is higher than that for revenue insurance when the price and yield are countermonotonic.[11] Second, we apply Proposition 1.

Step 1. The Frechet lower bound $F_L(p,y) = \max[0, F_P(p) + F_Y(y) - 1]$ is the probability distribution of $(F_P^{-1}(1-U), F_Y^{-1}(U))$ where U is uniformly distributed on $[0,1]$, or $(Q(Y), Y)$, so that P and Y are functionally dependent. Now we show that conditions (i) and (ii) imply that $\Pi_R(F_L) \le \Pi_Y$. This inequality holds assuredly if for any y

$$\beta E[Q(Y)Y] \ge Q(y)y \text{ implies } y < \beta EY \text{ and } \beta E[Q(Y)Y] - Q(y)y \le EP(\beta EY - y). \quad (6)$$

Because $E[Q(Y)Y] < EQ(Y)EY = EPEY$, the last inequality in (6) is satisfied if $Q(y) \ge EP$, or $y \le Q^{-1}(EP)$. And so, relationship (6) adheres if $Q(y)y$ is increasing in y, which is assured by condition (i), and $\hat{y} \le \min[\beta EY, Q^{-1}(EP)]$ where $\hat{y}$ $= \sup\{y : Q(\hat{y})\hat{y} = \beta E[Q(Y)Y]\}$, which is precisely condition (ii).

Step 2. The proof is completed by observing that, by Proposition 1, $\Pi_R(F_L)$ $\ge \Pi_R(F)$ if $F_L \prec_c^A F$ where $A = \{(p,y) : py \le \int pydF\}$.

Part (a) follows because, by Jensen's inequality, the premium for yield insurance is always smaller than the revenue counterpart when price and yield risks are independent. The condition in part (b) can be stated in terms of the price elasticity with the inverse demand given by $Q(y)$. Then condition (i) is equivalent to the requirement that the price

elasticity is sufficiently small (in absolute value): $\eta(y) \geq -1$ where $\eta(y) = [y\partial Q(y)/\partial y]$ $/Q(y)$. Using Jensen's inequality and the functional dependence between $P = Q(Y)$ and Y (implied by countermonotonicity), it is easy to check that condition (ii) is satisfied when the inverse demand function $Q(y)$ is concave, given that condition (i) holds.

EXAMPLE 2. Consider the binary (bivariate Bernoulli) distribution of price and yield in Figure 3. The Frechet lower bound is obtained by setting $\varepsilon = -1/4$. Then the premium for revenue insurance is $\Pi_R(F_L) = 0.5(0.5\beta(\overline{p}\,\underline{y} + \underline{p}\,\overline{y}) - \min[\overline{p}\,\underline{y}, \underline{p}\,\overline{y}])$, while the premium for yield insurance is $\Pi_Y = 0.25(\underline{p} + \overline{p})(0.5\beta(\underline{y} + \overline{y}) - \underline{y})$. Conditions (i) and (ii) in part (b) of Proposition 4 are satisfied (for any $\beta > 0$) if $\overline{p}\,\underline{y} \leq \underline{p}\,\overline{y}$, or $a = \overline{p}/\underline{p} \leq \overline{y}/\underline{y} = b$. And so, if the price "dispersion" parameter, a, is smaller than the yield "dispersion" parameter, b, the premium for revenue insurance is smaller than that for yield insurance when the strength of the negative price-yield correlation is sufficiently high: $\varepsilon = -1/4$.

In contrast, if the price "dispersion" is sufficiently greater than the yield "dispersion," $a > 2b - 1$, the relationship between the premiums is reversed if the feasible level of coverage is sufficiently small $\beta < 2(a + 1 - 2b)/(a(b-1) - b + 1)$. When the strength of the negative price-yield correlation is weaker, the effect of the level of coverage, β, on the difference between the premium rates for yield and revenue insurances, $\hat{\Pi}_Y$ and $\hat{\Pi}_R$, is, generally, non-monotone as well. This is illustrated in Figure 4 where $\varepsilon = -0.025$, $\underline{p} = \underline{y} = 1$, and $\overline{p} = \overline{y} = 2$.

$Y \setminus P$	$\underline{p}$	$\overline{p}$
$\underline{y}$	$1/4 + \varepsilon$	$1/4 - \varepsilon$
$\overline{y}$	$1/4 - \varepsilon$	$1/4 + \varepsilon$

FIGURE 3. Bivariate Bernoulli distribution of price and yield

5. Concluding Remarks

This paper analyzes the effect of an increase in dependence among multiplicative risks, random price and yield, on the actuarially fair premium for revenue insurance policy. The indemnity payment under revenue insurance is equivalent to the payoff of a put option on revenue with the strike price equal to the revenue guarantee. Because the indemnity function is not supermodular in the two risks over the entire domain of possible realizations, the approach taken is to localize the concept of greater dependence to assure monotonicity. When an increase in dependence is restricted to a certain subset of the support of the joint probability distribution of price and yield, the effect on the actuarially fair premium for revenue insurance can be easily signed. Furthermore, circumstances under which the premiums for revenue and the corresponding yield insurance can be unambiguously compared become more transparent.

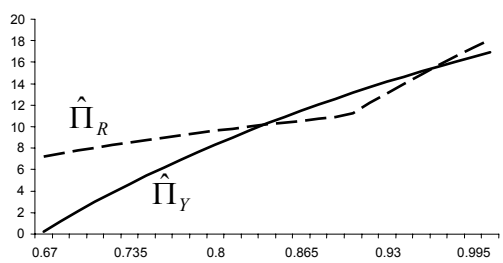

FIGURE 4. Premium rates for revenue and yield insurances as functions of β

Endnotes

1. The contract details and the description of various revenue insurance policies available to agricultural producers are provided in Harwood et al. 1999. In one of the more popular contracts, Crop Revenue Coverage, the revenue guarantee is not fixed at the time when the insurance contract is purchased but depends on the realized harvest-time price. Other revenue insurance policies such as Revenue Assurance and Income Protection conform to the standard insurance contract with the revenue guarantee fixed when the contract is signed.
2. The strength of price-yield correlation is likely to differ across growing regions because of the variability in the geographical concentration of production as well as across different crops within a region (e.g., Harwood et al. 1999).
3. Hennessy, Babcock, and Hayes (1997) show that revenue insurance is always cheaper than the combination of separate price and yield insurances. The issue of comparing the premiums for revenue and yield insurances for a given coverage level is of practical significance (e.g., see Miller, Kahl, and Rathwell 2000, Turvey and Amanor-Boadu 1989, Turvey 1992a,b; Skees et al. 1998). Most of the agricultural economics literature in this area focuses on the empirically sound estimates of insurance premiums (e.g., Stokes 2000; Buschena and Lee 1999; Richards and Manfredo 2003). In addition to option pricing techniques, empirical estimates of revenue insurance premiums can be obtained using a simulation approach based on imposing correlations through reordering of the draws (e.g., Hart, Hayes, Babcock 2003). The importance of accounting for the negative price-yield dependence as a determinant of premium subsidy levels is recently emphasized in Wang, Hanson, and Black 2003.
4. The concordance order studied by these authors places no restrictions on set A; that is, $A = supp(F(x,\infty)) \times supp(F(\infty, y))$.

5. It is assumed throughout this paper that the EICs always act on points such that all four points (x, y), (x', y'), (x', y), (x, y') belong to set A as implied by the definition. This, of course, depending on the nature of set A, may significantly restrict the set of dependence structures under scrutiny. As demonstrated here, such restrictiveness is useful when dealing with functions that possess supermodularity or submodularity properties on a subset of the distribution support.
6. For most insurance products, the coverage level varies between 50 and 85 percent of the expected revenue (or 50 to 90 percent of the expected yield in the case of yield insurance in Section 4). Also, it appears that in actuality the revenue guarantee is set without an explicit regard for price-yield correlation, i.e., it is calculated as $\beta P_g Y_g$ instead of $\beta E[PY]$, where it is implicit that $P_g = E[P], Y_g = E[Y]$ (USDA 2001b).
7. This can be shown by integrating the first inequality with respect to F and the second inequality with respect to F' over set $C = \{(p, y) : \beta \int tsdF \le py \le \beta \int tsdF'\}$ and using the definition of $F \prec_c^C F'$ to obtain a contradiction.
8. Recall that $F \prec_c^A F'$ means that there exists a sequence of probability distributions $F = F_1, ..., F_n = F'$ such that F_i is obtained from F_{i-1} through an EIC on set A. By Proposition 2, it follows that the expected indemnity decreases (increases) under each EIC transformation depending on the subset of the distribution support in which the EIC takes place.
9. This indemnity corresponds to the Crop Revenue Coverage insurance plan (hence, the subscript in [4]) and provides a so-called replacement cost protection in addition to a revenue guarantee (e.g., Makki and Somwaru 2001).
10. Note that sets A_1 and A_2 do not overlap.
11. Random variables that reach the Frechet lower bound are said to be counter-monotonic or mutually exclusive in economics, finance, and actuarial sciences (Dhaene and Denuit 1999).

References

Buschena, D., and Z. Lee. 1999. "Reliability of Options Markets for Crop Revenue Insurance Rating." *Journal of Agricultural and Applied Economics* 24(2) (December): 398-423.

Denuit, M., J. Dhaene, and C. Ribas. 2001. "Does Positive Dependence between Individual Risks Increase Stop-Loss Premiums?" *Insurance: Mathematics and Economics* 28(3) (June): 305-08.

Dhaene, J., and M. Denuit. 1999. "The Safest Dependence Structure among Risks." *Insurance: Mathematics and Economics* 25(1) (September): 11-21.

Epstein, L.G., and S.M. Tanny. 1980. "Increasing Generalized Correlation: A Definition and Some Economic Consequences." *Canadian Journal of Economics* 13(1) (February): 16–34.

Hart, C.E., D.J. Hayes, and B.A. Babcock. 2003. "Insuring Eggs in Baskets." CARD Working Paper 03-WP 339, Center for Agricultural and Rural Development, Iowa State University. July.

Harwood, J., R. Heifner, K. Coble, and A. Somwaru. 1999. "Managing Risk in Farming: Concepts, Research, and Analysis." Agricultural Economic Report No. 774. Economic Research Service, U.S. Department of Agriculture. March.

Hennessy, D.A., B.A. Babcock, and D.J. Hayes. 1997. "Budgetary and Producer Welfare Effects of Revenue Insurance." *American Journal of Agricultural Economics* 79(3) (August): 1024-34.

Hennessy, D.A., A.E. Saak, and B.A. Babcock. 2003. "Fair Value of Whole-Farm and Crop-Specific Revenue Insurance." Paper presented at American Association of Agricultural Economics annual meeting, Montreal, Quebec, July 27-29.

Joe, H. 1997. *Multivariate Models and Dependence Concepts*. London: Chapman and Hill.

Makki, S.S., and A. Somwaru. 2001. "Asymmetric Information in the Market for Yield and Revenue Insurance Products." Technical Bulletin No. 1892, Economic Research Service, U.S. Department of Agriculture. April.

Miller, S.E., K.H. Kahl, and P.J. Rathwell. 2000. "Revenue Insurance for Georgia and South Carolina Peaches." *Journal of Agricultural and Applied Economics* 32(1) (April): 123-32.

Richards, T.J., and M.R. Manfredo. 2003. "Infrequent Shocks and Rating Revenue Insurance: A Contingent Claims Approach." *Journal of Agricultural and Resource Economics* 28(2) (August): 233-51.

Skees, J.R., J. Harwood, A. Somwaru, and J. Perry. 1998. "The Potential for Revenue Insurance Policies in the South." *Journal of Agricultural and Applied Economics* 30(1) (July): 47-67.

Stokes, J.R. 2000. "A Derivative Security Approach to Setting Crop Revenue Coverage Insurance Premiums." *Journal of Agricultural and Resource Economics* 25(1) (July): 159-76.

Tchen, A.H. 1980. "Inequalities for Distributions with Given Marginals." *Annals of Probability* 8(4) (August): 814-27.

Turvey, C.G. 1992a. "An Economic Analysis of Alternative Farm Revenue Insurance Policies." *Canadian Journal of Agricultural Economics* 40(3) (November): 403-26.

———. 1992b. "Contingent Claim Pricing Models Implied by Agricultural Stabilization and Insurance Policies." *Canadian Journal of Agricultural Economics* 40(2) (July): 183-98.

Turvey, C.G., and V. Amanor-Boadu. 1989. "Evaluating Premiums for a Farm Income Insurance Policy." *Canadian Journal of Agricultural Economics* 37(2) (July): 233-47

U.S. Department of Agriculture (USDA). 2001a. "Revenue Insurance, 70/75 Sales Skyrocket." http://www.rma.usda.gov/news/2001/03/010328_crc.html. (accessed July 2004) Risk Management Agency, April.

———. 2001b. *Crop Insurance Handbook (CIH) 2002 and Succeeding Crop Years.* Washington, DC: Risk Management Agency, FCIC-18010(6-01).

Wang, H.H., S.D. Hanson, and J.R. Black. 2003. "Efficiency Costs of Subsidy Rules for Crop Insurance." *Journal of Agricultural and Resource Economics* 28(1) (April): 116-37.

Yin, S., and Turvey, C.G. 2003. "The Pricing of Revenue Assurance: Comment." *American Journal of Agricultural Economics* 85(4) (November): 1062-65.