Measuring productivity in African agriculture: A survey of applications of the superlative index numbers approach

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Abstract

This paper argues that partial productivity measures are inappropriate and at times misleading for assessing the performance of agricultural production technologies and systems. This is especially true where substantial changes in resource stock and flows accompany the production process. Superlative-index based total factor productivity measures are a more appropriate technique to compare production efficiency and sustainability of alternative systems. Mathematical formulations of intertemporal and interspatial total factor productivity measures with and without considering changes in resource stock and flows are shown. Then three case studies from sub-Saharan Africa in which this approach was applied are reviewed. These studies show that total factor productivity measures are biased if changes in resource stock and flows are not appropriately accounted for in intertemporal comparisons, and differences in input intensity are not accounted for in interspatial comparisons.

1   Introduction

A proper measurement of productivity of alternative farming practices is essential for understanding the efficiency and competitiveness of agriculture in sub-Saharan Africa (SSA) countries. Most productivity analyses are based on partial productivity measures such as yield per hectare (land productivity) or output per person (labour productivity). Such productivity measures can be misleading if considerable input substitution occurs. Although partial productivity measures provide insights into the efficiency of a single input in the production process, they mask many of the factors accounting for observed productivity differentials. A conceptually superior way to estimate productivity is total factor productivity (TFP). TFP is defined as the ratio of aggregate outputs to aggregate inputs used in the agricultural production process. There are two basic approaches to the measurement of productivity: the growth accounting approach, which is based on index numbers, and the parametric approach, which is based on an econometric estimation of production, cost or profit functions.

We advocate the use of the index number approach for four reasons. First, with the index number approach, detailed data with several input and output categories can be used regardless of the number of observations over time. There are therefore no problems of degrees of freedom or statistical reliability in working with small samples so long as they are live representatives. Second, indices may be constructed for individual components of total farm output, thus avoiding input–output separability assumptions. Third, under certain technical and market conditions, the econometric and index number approaches are equivalent. Recent advances in growth accounting theory have shown that non-parametric methods do impose an implicit structure on the aggregate production technology (Ohta 1974; Diewert 1976; Diewert 1981; Denny and Fuss 1983). Finally, the index based on TFP approach permits aggregation of different categories of outputs and inputs. In SSA it is normal practice for farmers to grow different types of crops in different years as well as in a mixture in a given year. They also use different types of input as required by the farming method.

There have been relatively few applications of TFP approach in the context of farming systems in SSA. In this paper we provide a survey of three applications where TFP approaches have been used. These applications concern two examples of measurements of TFP of alternative crop and livestock farming methods after adjusting for changes in soil quality in humid West Africa (Ehui and Spencer 1993; Ehui and Jabbar 1995; Ehui and Jabbar 2001), and one example of measurement of TFP of alternative tenure contracts in the highlands of East Africa (Gavian and Ehui 1999).

These applications present different but interesting results. First, adjusting for soil quality measurement has significant impact on TFP measures. The direction of the impact depends on whether quantity of soil nutrients decreases or increases. But this direction cannot be determined a priori if there are changes in composition of output and input types and levels over time or across systems. Second, livestock have significant and positive impact on TFP measures and therefore on total output growth. Ignoring the role of livestock in productivity measurement of a farming system will yield biased TFP

measures. Third, we show that relative to formal land contract, informal land tenure contracts have lower productivity significantly affecting relative output changes despite higher use of purchased inputs. This last empirical example on land tenure serves to demonstrate the wide applicability of the TFP approach to different circumstances.

The paper is organised as follows. The next section presents the concept of productivity with a graphical illustration. Section 3 presents the concept of interspatial and intertemporal total factor productivity. We show that total factor productivity indices may be constructed to compare performance of a given system at two points in time (intertemporal TFP indices) or to compare two systems at a given point in time (interspatial TFP indices). Section 4 demonstrates how changes in soil quality can be incorporated in conventional TFP measurement. Section 5 presents results of the empirical examples derived from the surveyed studies. The conclusions are presented in Section 6.

2 Concept of total factor productivity

Total factor productivity (TFP) is a ratio of total outputs (measured in an index form) to total inputs (also measured in an index form). If the ratio of total outputs to total inputs is increasing, then the ratio can be interpreted to measure that more outputs can be obtained for a given input level. Productivity, or TFP, captures the growth or changes in outputs not accounted for by the growth or changes in production inputs. Differences in TFP over time or across farming types can result from several factors (Ahearn et al. 1998), such as:

(i) differences in efficiency (less than the maximum output is produced from a given input bundle)

(ii) variation in scale or level of production over time, as the output per unit of input varies with the scale of production or

(iii) technical change. Technical change itself can result from quality improvement in input or quality improvements in the production process.

Figure 1 illustrates what a measure of productivity can capture. In the simplest case the production technology Y^s models the transformation of inputs X^s into outputs Y^s in period s or alternative system s. Y^{s+1} is the production frontier for period (or alternative
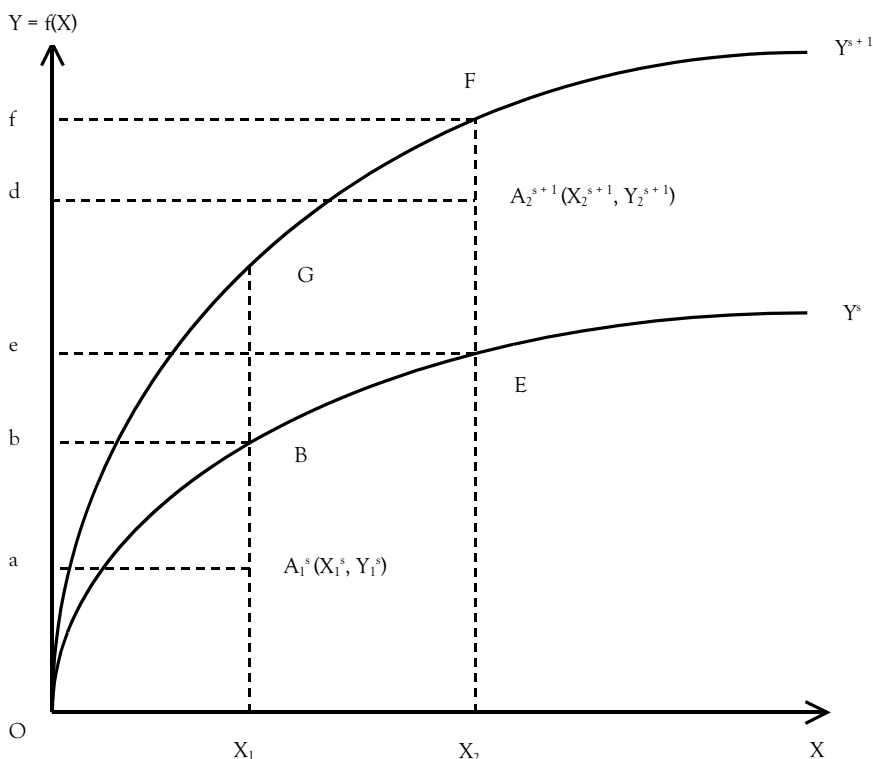

Figure 1. *Production relationships and productivity.*

system) s + 1. The frontier is the boundary of technology in each year or system. This is also defined as the maximum feasible output given input X. That is Y^s and Y^{s+1} indicate the maximum of output that can be obtained for a given level of X in each period, or in each farming system type at a point in time.

In Figure 1, any point along the curve, Y^s, indicates the maximum of Y^s that can be obtained for a given level of X. Any X, Y combination below the curve (e.g. point A_1^s) would represent 'technically inefficient' production since more of Y could be produced with the same level of X.

In Figure 1, the distance of production point $A_1^s(X_1^s, Y_1^s)$ to the frontier Y^s can be expressed as:

$$D_1^s(X_1^s, Y_1^s) = \frac{oa}{ob} \qquad (1)$$

That is a measure of how far the production point $A_1^s(X_1^s, Y_1^s)$ is from the frontier Y^s in period (or production system) s.

The curvature of the production function in Figure 1 depicts a production technology with decreasing returns scale. This means that at some point more X is required to produce a unit of Y than is required to produce each unit of Y than at point E. For example, at point B, less input is required to produce a unit of Y than at point E. If, over time, producers expand their production level, given the curvature of Y^s, they will realise lower leves of output per unit of input.

In period or system s + 1, the distance of production point $A_2^{s+1}(X_2^{s+1}, Y_2^{s+1})$ to the frontier is measured as:

$$D_2^{s+1}(X_2^{s+1}, Y_2^{s+1}) = \frac{od}{of} \qquad (2)$$

Additional units of Y can be produced for a given level of X through technical innovation (shift in the technology frontier), due to increase in resources (e.g. increased stock of soil nutrients). A change in production technology could be depicted in Figure 1 as a shift in productive surface from Y^s to Y^{s+1}. At each scale of production, more output is produced with the new technology represented in Y^{s+1} than with the original technology, Y^s.

For example, when the production technology is represented by Y^s, an input level of X_1 will result in the output of point B. However, after technical innovation or improvement in soil quality leading to the new production technology Y^{s+1}, the same input level of X_1 yields output at point G. This increased output would be captured by a measure of TFP over time or across farming types at a point in time.

The efficiency change in production between period s and s + 1 (or the efficiency change of a system s compared to an improved one, s + 1) is defined by the distance in s + 1 divided by the distance to the frontier in period (or system) s:

$$TEF(s, s+1) = \frac{D_2^{s+1}(.)}{D_i^s(.)} = \frac{od}{of} \bullet \frac{ob}{oa} \qquad (3)$$

Nin et al. (2000) defined equation (3) as a measure of catching up, or the rate at which a production unit is approaching or moving away from the frontier.

The technical change component of the total productivity change can be measured as the distance between the frontiers in periods s and s + 1. Technical change can be expressed as the ratio of the distance from the production point in s + 1 to the frontier in s, divided by the distance of the same point to the frontier in s + 1.

$$TCH(s, s + 1) = \frac{D_2^s(.)}{D_i^{s+1}(.)} = \frac{od \; / \; oe}{od \; / \; of} = \frac{of}{oe} \tag{4}$$

Total productivity change is the product of the efficiency change and the technical change indices.

$$M(s, s + 1) = \frac{D_2^{s+1}(.)}{D_1^s(.)} \bullet \frac{D_2^s(.)}{D_1^{s+1}(.)} = \frac{D_2^s(.)}{D_1^{s+1}(.)} = \left[\frac{od}{of} \bullet \frac{ob}{oa} \right] \left[\frac{of}{oe} \right] = \frac{od}{oe} \bullet \frac{ob}{oa} \tag{5}$$

M(s, s + 1) is also known as the Malmquist index that measures the total productivity change of a production unit between two periods or a production unit compared to another one over the same period. Caves et al. (1982) have shown that the geometric mean of two Malmquist indexes are equal to the Tornquist index based on the translog form (see below).

3 Interspatial and intertemporal total factor productivity measures

The major problem with the index number approach lies in deriving aggregate output and input measures that represents the numerous outputs and inputs involved in most production processes. Earlier approaches to TFP used a Laspeyres or Paasch weighting system where base period prices were used as aggregation weights. However, the Laspeyres or Paasch indexing procedure is inexact except when the production function is linear and all inputs are perfect substitutes (Christensen 1975; Diewert 1976). A better alternative is to use an index number that is exact for linear homogenous flexible functional forms (Christensen et al. 1971). The class of indices with this property has been termed 'superlative' by Diewert (1976). The most popular indexing procedure is the Divisia index, which is exact for the case of homogenous translog functions (Capalbo and Antle 1988). The translog function does not require inputs to be perfect substitutes, but rather permits all marginal productivities to adjust proportionally to changing prices.

Assume that the agricultural process in land held under farming system i at time t can be represented by the production function:

$$Q_{it} = F(X_{it}, T_{it}, D_i) \tag{6}$$

where Q_{it} is the output level, X_{it} is a vector of factor inputs, T_{it} is an index of technology, and D_i is a vector of dummy variables for every alternative system other than the reference base system. T_{it} and D_i denote also intertemporal and interspatial productivity difference indicators. Equation 6 assumes that the production function in each alternative system has common elements as well as differences resulting from the nature of the systems, which are maintained by the additional argument D. Suppose that we want to know the difference between the level of output on land held under farming system i at time s, and land held under farming system o at time t. Application of Diewert's (1976) quadratic lemma[1] to a logarithmic approximation of (6) gives:

$$\Delta LnQ = LnQ_{is} - LnQ_{0t} = \frac{1}{2}\sum_k \left[\frac{\partial LnF}{\partial LnX_k}\Big|X_k = X_{kis} + \frac{\partial LnF}{\partial LnX_k}\Big|X_k = X_{kot}\right]\left[LnX_{kis} - LnX_{kot}\right]$$

$$+\frac{1}{2}\left[\frac{\partial LnF}{\partial D_i}\Big|_i + \frac{\partial LnF}{\partial D_i}\Big|_o\right]\left[D_i - D_o\right] + \frac{1}{2}\left[\frac{\partial LnF}{\partial LnT}\Big|_{T=T_{is}} + \frac{\partial LnF}{\partial LnT}\Big|_{T=T_{ot}}\right]\left[LnT_{is} - LnT_{ot}\right] \tag{7}$$

---

1. Diewert's (1976) quadratic lemma basically states that if a function is quadratic, the difference between the function's values evaluated at two points is equal to the average of the gradient evaluated at both points multiplied by the difference between the points,

$$F(Z^1) - F(Z^0) = \frac{1}{2}[F(Z^1) + F(Z^0)]^T(Z^1 - Z^0),$$

where F(Z^r) is the gradient vector of F evaluated at Z^r, r = 0,1.

Let us define the interspatial effect as:

$$\theta_{io} = \frac{1}{2}\left[\frac{\partial LnF}{\partial D_i}\bigg|D = D_i + \frac{\partial LnF}{\partial D_i}\bigg|D = D_o\right][Di - Do] \tag{8}$$

and the intertemporal effect as:

$$\mu_{st} = \frac{1}{2}\left[\frac{\partial LnF}{\partial LnT}\bigg|_{T=T_{is}} + \frac{\partial LnF}{\partial LnT}\bigg|_{T=T_{ot}}\right][LnT_{is} - LnT_{ot}] \tag{9}$$

Constant returns to scale and perfect competition in input and output markets imply that $(\partial Ln\,F/\partial Ln\,X_k) = S_k$, where the term S_k represents the cost share for the kth input. Using these assumptions, we can rewrite equation (7) as:

$$\Delta LnQ = \frac{1}{2}\sum_k [S_{kis} + S_{kot}][LnX_{kis} - LnX_{kot}] + \theta_{io} + \mu_{st} \tag{10}$$

From equation (10) it can be observed that the output differential across alternative systems and time periods may be broken down into an input effect, a system (or spatial) effect and an intertemporal effect.

The first expression on the right hand side of equation 10 denotes the weighted sum of differences in factor intensities. Let us define this expression as $\rho_{io} = \frac{1}{2}\sum_k [S_{kis} + S_{kot}][Ln(X_{kis}) - Ln(X_{kot})]$, so that equation (10) becomes:

$$\Delta Ln\,Q = \rho_{io} + \theta_{io} + \mu_{st} \tag{11}$$

The difference in output levels can therefore be decomposed into three effects: (i) a factor intensity effect ρ_{io}; (ii) a system (or spatial) effect, θ_{io}, and (iii) an intertemporal effect, μ_{st}. If we want to measure the efficiency levels across alternative systems at a given point in time (where $t = s$), we rearrange the terms to isolate the system effect:

$$\theta_{io} = [Ln(Q)_i - Ln(Q)_0] - \frac{1}{2}\sum [S_{ki} + S_{k0}][Ln(X_{ki}) - Ln(X_{k0})] \tag{12}$$

The expression θ_{io} is the Tornqvist–Theil approximation (Tornqvist 1936; Capalbo and Antle 1988) to the change in productivity levels due to the type of production system at a particular point in time. The difference in the TFP of two systems is a function of the differences in output differential and factor intensities.

In the case of multiple outputs, the Tornqvist–Theil quantity index can also be used to aggregate the various outputs into a single index:

$$[Ln(Q)_i - Ln(Q)_0] = \frac{1}{2}\sum_j [r_{ji} + r_{j0}][Ln(Q_j)_i - Ln(Q_j)_0] \tag{13}$$

where r_{ij} and r_{io} denote the jth output revenue share in systems i and o, respectively. Q_j denotes the jth output level.

The general expression shown in equation (10) can be applied to provide a comparison of the rate of growth of productivity due to technical change for a particular farming system over time ($D_i = D_0$ and $s = t + 1$)

$$\mu_{t+1,t} = \left[Ln(Q)_{t+1} - Ln(Q)_t \right] - \frac{1}{2} \sum_{k \neq A} \left[S_{k,t+1} + S_{kt} \right] \left[Ln(X_{k,t+1}) - Ln(X_{kt}) \right] \quad (14)$$

where $\mu_{t+1,t}$ measures the intertemporal TFP of a production system over two periods. It is the Tornqvist–Theil approximation to the change in productivity levels due to technical change.

Equations (12) and (14) indicate that there are two components that contribute to any observed differences in TFP: differences in the level of output and differences in factor intensities. TFP is therefore the residual, or the portion of change in output levels not explicitly explained by changes in input levels. However, increases in factor intensities may occur without any increases in TFP. Changes in TFP levels and factor intensities are not independent but they are of different significance. Increases in TFP will occur if output increases proportionally more than increases in factor intensities. But increases in output that are due to increases in factor intensities are qualitatively (although not quantitatively) less significant than changes in TFP. Indeed output will increase if a farmer applies more purchased inputs. Unless there are improvements in the use of these inputs, this will be a change in factor intensity and not TFP. It is clear that with TFP changes, in contrast with factor intensity differentials, the farmer's capability to produce more with the same resources has improved. A good example is a recent data from the United States which show that output growth in the US manufacturing and non-farm business sectors were significantly higher than that in agriculture but growth in input use was negative in agriculture and very high in the other two sectors. Consequently productivity growth was a more important source of output growth in agriculture than it was for the rest of the economy (Ahearn et al. 1998).

4 Accounting for changes in soil quality in TFP measurement

Equations (12) and (14) do not account for changes in non-market environmental services (such as soil nutrients) that may take place during the production process. The agricultural sector uses common pool natural resources (e.g. air, water, soil nutrients etc.). The stock of these resources affects the production environment, but is, in many cases, beyond the control of the farmer. For example, soil nutrients are removed by crops, erosion or leaching beyond the crop root zone, or through other processes such as volatilisation of nitrogen. Agricultural production can also contribute to the stock of some of the nutrients, particularly of nitrogen, by leguminous plants. When the stock of resources is reduced, the farmer faces an implicit cost in terms of forgone productivity. Conversely, when the stock of resources is increased during the production process (e.g. via nitrogen fixation), the farmer derives an implicit benefit from the system. Squires (1992) showed that when common pool resources stocks are used, it is inappropriate for productivity measurement to treat the resource stock as a static conventional input. Rather, the resource stock is more appropriately specified as a technological constraint. This is because for a given input bundle, increases (decreases) in resource abundance shift the production function, increasing (decreasing) resource flows and output. In conventional TFP measures, changes in resource stock and flows are not accounted for, so the results are biased. In this section, the conventional TFP measures developed above are modified to take account of changes in resource stock and flows during production.

To derive the interspatial and intertemporal productivity measures in the presence of changes in soil quality, let us represent the agricultural process of farming systems in period t by the dual variable cost function.

$$G_{it} = G(Y_{it}, Z_{it}, W_{it}, B_{it}, T_t, D_i) \qquad (15)$$

Where G_{it} is the variable cost function for the optimal combination of variable inputs, W_t is a vector of variable input prices, B_{it} is a technology shift variable representing the level of resource abundance in period t and Z_{it} is a positive externality denoting the net resource flow $(B_{it+1} - B_{ot})$ from t to t + 1 or and from system i to o. As a positive externality, Z_{it} is treated as an output, thus contributing positively to aggregate profit, and Y_{it} as an index of outputs from the farming system. Note that $\partial G(.)/\partial B < 0$ and $\partial G(.)/\partial Z > 0$. This means, the higher the stock of nutrients, the lower the cost of production. However, as an output, increased production of Z will contribute to increased cost of production like any other output.

In case that the change in stock of nutrients is negative, we treat it as a cost to the production process. In this case the variable cost of production is rewritten as:

$$G'_{it} = G(Y_{it}, P_{zit}, W_{it}, B_{it}, T_t, D_i) \qquad (16)$$

Where P_{zit} represents the opportunity cost or replacement cost of externality Z_{it}. Assuming constant returns to scale and competitive output and factor markets, application of Diewert's (1976) quadratic logarthimic approximation to (15) and (16) gives:

$$\Delta LnG'' = \frac{1}{2}\left[R_{yis} + R_{yot}\right]\left[LnY_{is} - LnY_{ot}\right] + \frac{1}{2}\left[R_{zis} + R_{zot}\right]\left[LnZ_{is} - LnZ_{ot}\right]$$

$$+\frac{1}{2}\sum_{k}\left[S_{kis} + S_{kot}\right]\left[LnW_{kis} - LnW_{kot}\right] + \frac{1}{2}\left[\frac{\partial LnG}{\partial LnB}\bigg|_{B=B_{is}} + \frac{\partial LnG}{\partial LnB}\bigg|_{B=B_{ot}}\right] \quad (17)$$

$$\left[LnB_{is} - LnB_{ot}\right] + \theta'_{io} + \mu'_{st}$$

where

$$\theta'_{io} = \frac{1}{2}\left[\frac{\partial LnG}{\partial D}\bigg|_{D=D_{i}} + \frac{\partial LnG}{\partial D}\bigg|_{D=D_{o}}\right]\left[D_{i} - D_{o}\right] \qquad (18)$$

$$\mu'_{st} = \frac{1}{2}\left[\frac{\partial LnG}{\partial T}\bigg|_{T=T_{s}} + \frac{\partial LnG}{\partial T}\bigg|_{T=T_{z}} D = D_{o}\right]\left[T_{s} - T_{t}\right] \qquad (19)$$

According to Squires (1992), for a Schaefer type of technology shift variable (which behaves like common pool of resources, $\frac{\partial LnG}{\partial LnB} = G_{B}(G) = -1$. Thus equation (17) becomes:

$$\Delta LnG' = \frac{1}{2}\left[R_{yis} + R_{yot}\right]\left[LnY_{is} - LnY_{ot}\right] + \frac{1}{2}\left[R_{zis} + R_{zot}\right]\left[LnZ_{is} - LnZ_{ot}\right]$$

$$+\frac{1}{2}\sum_{k}\left[S_{kis} + S_{kot}\right]\left[LnW_{kis} - LnW_{kot}\right] - \left[LnB_{is} - LnB_{ot}\right] + \theta'_{io} + \mu'_{st} \qquad (20)$$

To measure intertemporal productivity in the dual space for two periods s and t, setting $D_{i} = D_{o} = 0$ yields:

$$\mu'_{is} = \left[LnG'_{s} - LnG'_{t}\right] - \frac{1}{2}\left[R_{ys} + R_{yt}\right]\left[LnY_{s} - LnY_{t}\right] - \frac{1}{2}\left[R_{zs} + R_{zt}\right]$$

$$\left[LnZ_{s} - LnZ_{t}\right] - \frac{1}{2}\sum_{k}\left[S_{kis} + S_{kt}\right]\left[L_{n}W_{ks} - LnW_{ks}\right] + \left[LnB_{s} + LnB_{t}\right] \qquad (21)$$

Similarly setting $T_{s} = T_{t} = 0$ gives a measure of interspatial TFP in the presence of soil quality changes.

$$\theta'_{io} = \left[LnG'_{i} - LnG'_{o}\right] - \frac{1}{2}\left[R_{yis} + R_{yo}\right]\left[LnY_{i} - LnY_{o}\right] - \frac{1}{2}\left[R_{zi} + R_{zo}\right]$$

$$\left[LnZ_{i} - LnZ_{o}\right] - \frac{1}{2}\sum_{k}\left[S_{ki} + S_{ko}\right]\left[LnW_{ki} - LnW_{ko}\right] + \left[LnB_{i} - LnB_{o}\right] \qquad (22)$$

To convert equations (21) and (22) in the primal space, we totally differentiate the log of cost equations $G = \sum_i W_i X_i$ with respect to time

$$\dot{G} = \sum S \dot{X}_i + \sum_i S_i \dot{W}_i \tag{23}$$

The Tornqvist approximation to equation (23) for periods s and t and systems i and o gives:

$$\Delta LnG' = \left[LnG'_{is} - LnG'_{ot} \right] = \frac{1}{2} \sum_k \left[S_{kis} + S_{kot} \right] \left[LnX_{is} - LnX_{kot} \right]$$
$$+ \frac{1}{2} \sum_k \left[S_{kis} + S_{kot} \right] \left[LnW_{kis} - LnW_{kot} \right] \tag{24}$$

Equating (24) and (20) gives measures of intertemporal and interspatial productivity in the primal space which are used in the empirical examples that follow:

$$\tau_{st} = -\mu'_{st} = \frac{1}{2} \left[R_{ys} + R_{yt} \right] \left[LnY_s - LnY_t \right] + \frac{1}{2} \left[R_{zs} + R_{zt} \right]$$
$$\left[LnZ_s - LnZ_t \right] - \frac{1}{2} \sum_k \left[S_{sk} + S_{kt} \right] \left[LnX_{ki} - LnX_{ko} \right] - \left[LnB_s - LnB_t \right] \tag{25}$$

$$\gamma_{io} = -\theta'_{io} = \frac{1}{2} \left[R_{yi} + R_{yo} \right] \left[LnY_i + LnY_o \right] + \frac{1}{2} \left[R_{zi} + R_{zo} \right]$$
$$\left[LnZ_i - LnZ_o \right] - \frac{1}{2} \sum_k \left[S_{ki} + S_{ko} \right] \left[LnX_{ki} - LnX_{ko} \right] - \left[LnB_i - LnB_o \right] \tag{26}$$

In the case of negative externality the same procedure is followed. But the cost structure is different (see appendix for the deviation). The interspatial and intertemporal productivity measures derived are:

$$\tau'_{st} = \left[LnY_s - LnY_t \right] - \frac{1}{2} \left[S_{zs} + S_{zt} \right] \left[LnZ_s - LnZ_t \right]$$
$$- \frac{1}{2} \sum_k \left[S_{ks} + S_{kt} \right] \left[LnX_{ks} - LnX_{kt} \right] - \left[LnB_s - LnB_t \right] \tag{27}$$

$$\gamma'_{io} = \left[LnY_i - LnY_o \right] - \frac{1}{2} \left[S_{zi} + S_{zo} \right] \left[LnZ_i - LnZ_o \right]$$
$$- \frac{1}{2} \sum_k \left[S_{ki} + S_{ko} \right] \left[LnX_{ki} - LnX_{ko} \right] - \left[LnB_i - LnB_o \right] \tag{28}$$

where s and t represent two distinct time periods; i and o represent two separate farming systems. R_y and R_z are the share in total revenues of output Y and resource flow Z. S_k and S_z are the cost share of variable input X_k and Z where the latter is treated as a negative resource flow.

5 Empirical examples

In this section we discuss three examples of applications of the superlative index numbers in the context of sub-Saharan Africa farming practices. Two of these are based on experimental data (Ehui and Spencer 1993; Ehui and Jabbar 1995; Ehui and Jabbar 2001) while the third is based on survey data (Gavian and Ehui 1999).

Since the farming systems have multiple and different crop outputs, Ehui and Spencer (1993), Ehui and Jabbar (1995), Gavian and Ehui (1999) and Ehui and Jabbar (2001) calculated an implicit output index by dividing the total value of all outputs by a price index obtained by weighing the individual output prices by the revenue share of each. They also calculated an implicit input quantity index as the ratio of total expenditures or inputs to the weighted input prices, the weight being the cost share of each input. Similarly to calculate the index of the soil nutrient stock, they share weighted the total quantity of main soil nutrients, i.e. nitrogen (N), phosphorus (P) and potassium (K) (in tonnes per hectare) available in the topsoil. In determining the cost share of the soil nutrients, they approximated the opportunity cost of each soil nutrient with its replacement cost, i.e. the market price of chemical fertiliser. Resource flows (the Z_s) are computed as the difference between the stock of nutrient between the years under study or between different systems.

5.1 Measuring the productivity of alternative cropping systems in presence of soil quality change

Ehui and Spencer (1993) developed intertemporal and interspatial total factor productivity indices using equations (25) and (26) to measure the intertemporal and interspatial productivity of four cropping systems denoted A, B, C and D between 1986 and 1988. In system A, land was cleared manually and cropped by a local farmer. Yam, melon and plantains were grown in 1986. In 1988, plantain, melon and cassava were grown. In all other systems, a tractor equipped with a shear blade cleared the land and cropped by the researchers. In system B, cassava, maize and cowpea were planted in 1986; only cassava was planted in 1988. All crops in system C were grown in alleys formed by hedgerows of nitrogen fixing trees or shrubs. In this system, known as alley cropping, the hedgerows were periodically pruned during the cropping season to prevent shading and reduce competition with food crops. In system D, plantain was grown during the 1986–88 period. No fertiliser was used in any of the cropping systems. The primary objective of this experiment was to assess the intertemporal and interspatial productivity of alley cropping (considered to be an improved system) compared to the traditional cropping systems that are generally considered to be not sustainable (Kang et al. 1990). Ehui and Spencer (1993) approximated the intertemporal productivity measure incorporating changes in soil quality to a measure of sustainability.

Estimated intertemporal and interspatial productivity indices for the four cropping systems are reported respectively in Tables 1 and 2. Two sets of indices were constructed with and without adjustment for changes in nutrient stock and flows. When no adjustment is made for changes in resource stock and flows, cropping systems A and C are highly unproductive as they respectively produce 20 and 2% as much output in 1988 as in 1986 using the 1986 input bundle (Table 1). Cropping systems B and D are highly productive as they respectively produce 6.38 and 3.27 times more output in 1988 as in 1986 using the 1986 input bundle. When adjustment is made for changes in resource stock and flows, system A remains non-productive at the same level as before, system B remains productive at about the same level as before but the productivity of systems C and D dramatically change. Because of net positive change in resource stock due to improved soil quality through fixation of nitrogen by leguminous trees, system C produces 12.23 times more output in 1988 as in 1986 using the 1986 input bundle. On the other hand, because of net negative resource stock due to nutrient mining by plantain, system D produces only 88% of output in 1988 as in 1986 using the 1986 input bundle.

Table 1. *Intertemporal total factor productivity (sustainability) indices for four cropping systems under experimental conditions, in south-western Nigeria, 1986–88.*

Cropping system	Resource stock and flows not adjusted	Resource stock and flows adjusted
System A	0.20	0.22*
System B	6.38	6.25*
System C	0.02	12.23*
System D	3.27	0.88**

* = Net positive change in resource stock.
** = Net negative change in resource stock levels.
Source: Ehui and Spencer (1993).

Table 2. *Interspatial total factor productivity (economic viability) indices for four cropping systems under experimental conditions in south-western Nigeria for 1986 and 1988.*

Cropping system	Resource stock and flows not adjusted (1986)	Resource stock and flows adjusted (1986)	Resource stock and flows not adjusted (1988)	Resource stock and flows adjusted (1988)
System A	1.00	1.00	1.00	1.00
System B	1.73	0.73**	68.50	9.26**
System C	5.37	0.76**	0.37	1.12*
System D	0.06	2.40*	1.04	0.14**

* = Net positive change in resource stock.
** = Net negative change in resource stock levels.
Source: Ehui and Spencer (1993).

Clearly the results of Ehui and Spencer (1993) showed that if we do not account for the positive resource quality change due to trees in system C, the intertemporal productivity index will lead to the erroneous conclusion that the system is not productive. Soil nutrients increased by 31%, representing nearly 30% of the net revenue in 1988. This is important to the value of output which explains the high intertemporal

productivity index in system C. Similarly, if we do not account for the depleted resources in system D, the erroneous conclusion would be reached that the system is productive, when in fact it is not.

In Table 2, Ehui and Spencer (1993) compared the productivity (interspatial TFP) of all cropping systems relative to the traditional farming system (system A). In 1986 (relative to system A) systems B and C are more productive and system D is less productive when changes in resource flows are not accounted for. After accounting for changes in resource flows, systems B and C are less productive and system D more productive than the reference base system.

In 1988, productivity indices for all the systems show a different pattern. Without adjustment in resource flow, system B is many times more productive and system C is less productive than system A while system D is equally productive as system A. With adjustment in nutrient stocks and flows, systems B and C are more productive than the reference bases system while system D is less productive. The changes in productivity measures in 1988 compared to 1986 are attributable to the changes in soil nutrient status over the two-year period. For example, system C (where crops are grown in association with leguminous trees), soil nutrients increased by 2.3% in 1988 compared to system A, with a revenue share of about 6%. In system D, where only plantain is grown, soil nutrients decreased by 21% compared to system A representing about 7% of the full cost faced by the farmer in 1988. Soil nutrients decreased by 16% for system B in 1988 representing about 10% of the total cost. As shown in Table 1, when variations in resource stock levels and the flows are not accounted for, productivity measures are biased. The direction of biases depend on the magnitude of changes in resource stock and flow levels.

5.2 Assessing the productivity of crop and livestock systems after accounts for soil quality change

Ehui and Jabbar (1995, 2001) calculated intertemporal and interspatial total factor productivity indices using experimental data over a 7-year period (1983–90) for three production systems: traditional farming (non-alley) with a two year fallow; alley farming with a two year fallow; and continuous alley farming. In alley farming, crops are grown between alleys formed by leguminous trees, which are pruned periodically to use as mulch. In alley systems, part of the tree foliage in non-crop season is used as protein rich feed supplement for small ruminants. The objective of this experiment was to compare the productivity of the alternative farming methods with and without accounting nutrient stock and flows and with and without livestock.

Three scenarios were considered in the analysis: (a) taking into account only direct inputs and outputs for crop and not accounting for nutrient stock and flows, (b) taking into account crop production and nutrient stock and flows, and (c) taking into account crop and small ruminant livestock production, and nutrient stock and flows.

The results for intertemporal TFP indices are shown in Table 3. When only crops are produced and changes in the stock and flows of nutrients are not accounted for, continuous alley farming is a highly productive system while the other two systems are not. In continuous alley farming, about 1.5 times as much output was produced in 1990 as in 1983 using the 1983 input bundle. The annual average rate of growth in productivity for the continuous alley farming system was 5.4%. Non-alley farming with fallow and alley farming with fallow had lower annual average rate of growth of –1.19% and –7.3%, respectively. When changes in nutrient stock and flows over this period are accounted for, none of these systems is productive and continuous alley farming is even worse than non-alley farming. This is because continuous alley farming gives a higher flow of crop output but at the same time mines the soil in the long run. Soil mining in continuous alley farming is, however, lower than in non-alley farming. On the other hand, intermittent short fallow in alley farming allows resource stock to be maintained but because of absence of output in fallow years, the system is not sustainable.

Table 3. *Intertemporal total factor productivity indices for three production systems under experimental conditions in south-western Nigeria, 1983–90.*

Production systems	Crops, not accounting for nutrient stock and flows	Crops and accounting for nutrient stock and flows	Crops and livestock and accounting for nutrient stock and flows
Non-alley farming with fallow	0.92	0.69**	0.78**
Continuous alley farming	1.46	0.64**	1.28**
Alley farming with fallow	0.60	0.56*	0.60

* = Positive resource flow;   ** = negative resource flow.
Source: Ehui and Jabbar (1995, 2001).

The performance of all the systems improves when livestock is mixed with crop production. However, only continuous alley farming with livestock is productive, the other two are not. Continuous alley farming produces 1.3 times more output in 1990 as in 1983 with the 1983 input bundle. The annual average rate of growth in productivity for the continuous alley farming over the seven-year period is estimated at 3.5% as opposed to –6.4% when livestock are not included in the system. In non-alley system, there is no improved feed for livestock while in alley with fallow, flow of output is smaller than in continuous alley system. The implication of this result is that livestock contribute significantly to the productivity of farming systems than is known or is acknowledged in the literature. In this case the productivity level is twice the level obtained when crops alone are considered.

Results of interspatial TFP indices for 1983 and 1990 are shown in Table 4. The non-alley system is the base reference (i.e. its productivity level is set equal to 1). When only crops are considered without accounting for changes in nutrient stock and flows, the alley systems were more efficient in both 1983 and 1990. They produce 1.4 to 2.1 times more output as the non-alley system, using the input bundle of the non-alley system. When changes in the nutrient stock and flows are accounted for, TFP levels are lower than when nutrient stock and flows are not accounted for. When small stock animals are combined with crop production and changes in nutrient stock and flows are

also accounted for, the productivity of the alley systems in the base year increases significantly demonstrating the contribution of smallstock to the productivity of the alley systems. In the terminal year, the competitive advantage of the alley systems reduces but they still remain significantly more efficient than the non-alley systems. Extrapolation of the 1990 TFP levels indicates that the continuous alley will continue to be more efficient in the future. The alley farming system with fallow will probably continue to be more efficient than the non-alley system but the gap in efficiency levels will be small.

Table 4. *Interspatial total factor productivity indices for three production systems under experimental conditions in south-western Nigeria, 1983 and 1990.*

Production systems	Crops, not accounting for nutrient stock and flows		Crops and accounting for nutrient stock and flows		Crops and livestock accounting for nutrient stock and flows	
	1983	1990	1983	1990	1983	1990
Non-alley fallow	1.00	1.00	1.00	1.00	1.00	1.00
Continuous alley	1.91	1.36	1.04*	0.95*	1.87*	1.26*
Alley farming with fallow	2.11	1.45	1.04*	1.03*	2.06*	1.18

* = positive nutrient or resource flow.
Source: Ehui and Jabbar (1995, 2001).

There are two practical implications of these results. First, TFP estimates change when we account for changes in soil quality as measured by nutrient stock and flow. The direction of change depends on the magnitude of resource stocks and flows. But the absolute level of TFP depends on the conventional output and input levels which depend on climatic and other stochastic processes. The levels of output and inputs vary also system by system. Second, incorporation of livestock in the alley systems nearly fully compensate for the nutrient losses that take place slowly over time due to continuous crop production but may not become visible at points in time. Livestock, therefore, has the potential for maintaining internal stability and long-term viability of the alley system.

5.3 The relative efficiency of alternative land tenure contracts

This example is different from the previous ones. But it serves to illustrate the wide applicability of superlative index numbers in the context of farming systems. Gavian and Ehui (1999) used the interspatial measures of TFP in equation (12) based on Divisia index, to test the relative efficiency of three informal and less secure land contracts in Ethiopia (rented, share-cropped and borrowed) relative to lands held under formal contract with the Ethiopian government. In Ethiopia, land is publicly owned but individual families are allocated usufruct right by local Peasant Associations (PA) on specific amounts of land. Previously subletting was illegal but more recently informal land market has emerged in which new families or families with surplus labour may obtain land under different arrangements (renting, sharecropping or borrowing) from

families having formal allocation. This study was based on data collected through a multiple visit sample survey.

Table 5 shows the average total factor productivity levels for each of the three informal contracts relative to the PA allocated land. Land and total factor productivity levels are lower for these contracts relative to the formal contract. Borrowed lands have the lowest TFP levels producing 16% less output than the PA allocated lands using the same input bundle. The shared lands are 13% less efficient than the PA allocated lands, whereas rented lands are 10% less efficient.

Table 5. *Total factor productivity, land productivity and factor intensities by tenure arrangements in central Ethiopia.*

		Informally-contracted fields		
Indicators	PA allocated fields*	Rented	Shared	Borrowed
Total factor productivity	1.00	0.93	0.89	0.84
Overall land productivity	1.00	0.99	0.93	0.92
Wheat	1.00	1.15	1.24	0.95
Barley	1.00	0.88	0.78	0.95
Legumes	1.00	0.96	0.98	1.03
Residues	1.00	1.01	0.99	0.99
Overall factor intensity	1.00	1.06	1.04	1.10
Labour	1.00	1.00	0.99	0.98
Power	1.00	1.01	0.98	1.01
Chemicals	1.00	1.04	1.06	1.10
Seeds	1.00	1.01	1.01	1.01

* Peasant Association (PA) allocated fields provide better security than informally contracted fields.
Source: Gavian and Ehui (1999).

The overall land productivity levels for informally contracted fields are also lower than for PA allocated fields. However, the gap is smaller than the gap in TFP levels due to the relatively high levels of factor intensity on informally contracted fields. The higher level of total inputs applied to informally contracted fields increases the level of land productivity but not the level of TFP. For example, the factor intensity level on borrowed land is 10% higher than the PA allocated lands but the TFP level is 16% lower. It can be seen from Table 5 that the lower output levels of informally contracted lands relative to the PA allocated lands are entirely the results of lower total productivity on these lands.

Chemical inputs (fertilisers and herbicides) were the major contributor to higher levels of inputs for all the informal contracts, whereas the contribution of animal power, human labour and seeds remain roughly the same. The increase in the level of chemicals was inversely proportional to the degree of land tenure security as defined above. The more insecure the land, the more farmers applied chemical inputs. The largest increase (10%) was for borrowed lands.

The high input intensities, combined with low land productivity ratios and thus low TFP, indicate that the capacity of rented, shared and borrowed lands to produce more output is not hampered by under-investment due to land insecurity. Rather than applying less input, as theory would suggest, farmers applied more inputs, in particular more chemical fertilisers, on informally contracted fields.

Gavian and Ehui (1999) gave several reasons for this high input/low output combination on informally contracted fields. First, informally contracted fields may have poor soil quality. This would be true if farmers with formal contract decide to keep their best fields to themselves, offering only inferior fields to other farmers under informal contracts. Second, land-importing farmers may use labour inefficiently. As young adults, borrowers usually have strong obligations to contribute labour to the family farm. Additionally, they tend not to own the oxen needed to plough their borrowed fields. Although they use the same amount of total human and animal days per hectare as PA farmers, they do so by relying on labour and oxen exchanges, after tending to family fields. This would imply that borrowers were not planting and harvesting at the optimal time. Thus it appears likely that the TFP efficiency gap is due to less experience, poor soil quality and timing of farm operations rather than tenure insecurity.

6 Conclusions

In this paper, we showed that superlative index numbers based on the Divisia indices can be used to measure the total productivity of alternative farming practices. The superlative index numbers are conceptually superior to the partial productivity measures such as land and labour productivity because they are exact for homogenous flexible functional forms, i.e functions capable of providing a second order approximation to any arbitrary functional form. For example, the Divisia index is exact for the case of homogenous translog functions, which does not require inputs to be perfect substitutes, but rather permits all marginal productivities to adjust proportionally to changing prices.

This paper reviewed three applications of the Tornqvist–Theil approximation to the Divisia index in farming systems of sub-Saharan Africa. To account for soil quality changes the conventional approach to TFP measurement was modified to account for soil nutrient stocks and flows. The empirical results show that the Divisia indices are sensitive to changes in the stock and flow of soil nutrients. In farming systems where nutrient status of soil changes significantly during the production process, the indices provide markedly different results from conventional TFP measurements. Also the review showed that the performance of tropical farming systems improves and systems become more sustainable when small stock livestock is mixed with crop production. Finally the Divisia index approach was used to compare the efficiency of informal land tenure contracts in sub-Saharan Africa. Although the informally contracted lands are farmed 7 and 10% less efficiently, the decomposition of the TFP indices indicated that farmers actually apply inputs more intensely. The gap in TFP thus resulted from inferior quality of inputs rather than a lack of incentive to allocate inputs to a mixed crop–livestock farming. The challenge facing research is to collect data on appropriate prices and quantity to make the use of these tools meaningful. The bigger challenge is on the proper and accurate measurement of soil nutrients in order to properly account for soil quality changes.

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Appendix

Derivation of interspatial and intertemporal total factor productivity (TFP) measures in the presence of negative externality.

Applying quadratic comma to equation (16) gives:

$$\Delta LnG' = \left[LnY_{is} - LnY_{ot} \right] + \frac{1}{2} \left[S_{zis} + S_{aot} \right] \left[LnP_{zis} - LnP_{zot} \right]$$

$$+ \frac{1}{2} \sum \left[S_{kis} + S_{kot} \right] \left[LnW_{kis} - LnW_{kot} \right] - \left[LnB_{is} - LnB_{ot} \right] + \theta'_{io} + \mu'_{st} \tag{A1}$$

noting $\partial LnG / \partial LnB = -1$ and $\theta'_{io} + \mu'_{st}$ are defined as in equations (18) and (19).

Cost G' is now defined as $G' = \sum W_i X_i + P_z Z$ since Z is treated as a cost. Totally differentiating log of G' with respect to time yields:

$$G' = \sum_i S_i \dot{X}_i + \sum_i S_i \dot{W}_i + S_z \dot{Z} + S_z \dot{P}_z \tag{A2}$$

The Tornqvist approximation of (A2) gives:

$$\Delta LnG' = \left[LnG'_{is} - LnG'_{ot} \right] = \frac{1}{2} \sum_k \left[S_{kis} + S_{kot} \right] \left[LnX_{kis} - LnX_{kot} \right]$$

$$+ \frac{1}{2} \left[S_{zis} + S_{zot} \right] \left[LnP_{zis} - LnP_{zot} \right] + \frac{1}{2} \left[S_{is} + S_{zot} \right] \left[LnZ_{is} - LnZ_{ot} \right] \tag{A3}$$

Equating (A3) with (A1) and solving for $-\mu'_{st}$ (when setting $\theta'_{io} = 0$) and for $-\theta'_{io}$ (when $\mu'_{st} = 0$) yields the intertemporal productivity measure:

$$\tau'_{st} = -\mu'_{st} = \left[R_{ys} + R_{yt} \right] \left[LnY_s - LnY_t \right] - \frac{1}{2} \sum_k \left[S_{ks} + S_{kt} \right]$$

$$\left[LnX_{ks} - Ln_{kt} \right] - \frac{1}{2} \left[S_{zs} + S_{zt} \right] \left[LnZ_s - LnZ_t \right] - \left[LnB_s - LnB_t \right]$$

and the interspatial measure:

$$\tau'_{io} = -\vartheta'_{io} = \left[LnY_{io} - LnY_o \right] - \frac{1}{2} \left[S_{zi} + S_{zo} \right] \left[LnZ_i - LnZ_o \right]$$

$$- \frac{1}{2} \sum_k \left[S_{ki} + S_{ko} \right] \left[LnX_{ki} - LnX_{kio} \right] - \left[LnB_i - LnB_o \right]$$