# Disproportionate Joint Cost Allocation

# at Individual-Farm Level Using Maximum Entropy

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**Abstract**

This paper addresses the allocation of joint cost among enterprises – also called 'production branches' or 'activities' – and presents an approach based on maximum entropy and standard costs from farm-management literature as allocation factors. The approach allows us to discard the widely applied assumption of a proportional joint-cost allocation. Since it provides a disproportionate joint cost allocation, the distinctive feature of the approach is that it favours the adjustment of large standard costs rather than of small ones.

**Key words**: joint cost allocation, maximum entropy

# 1 Introduction

When allocating joint costs among enterprises (also referred to in the literature as 'production branches' or 'activities'), which signifies an important challenge in the analysis of full cost, we normally struggle with a data gap or an under-determined (cost) model, since a scarcity of available resources such as time and money means that the 'true' allocation is not available. As a method for overcome data gaps and allowing information recovery, maximum entropy represents a promising tool for addressing the joint-cost allocation problem.

This paper aims to present an approach that makes the virtue of maximum entropy available for an empirical application of joint-cost allocation at individual-farm level. To our knowledge, there is no farm- specific joint-cost allocation approach based on maximum entropy, though there are several maximum-entropy-based analyses that address joint-cost allocation on a regional or country level (e.g. Lence and Miller, 1998; Léon et al., 1999; Garvey and Britz, 2002; Peeters and Surry, 2005; Fragoso and da Silva Carvalho, 2012).

There are two major differences between a cost allocation at regional level and at individual-farm level. Firstly, a common production technology is assumed in all regional analyses when performing the cost allocation. As regards the diversity of Swiss farms in general and crop farms in particular, a common production technology cannot be assumed. Secondly, the main objective differs. While a regional analysis focuses on input coefficients representing a farm type, a farm-level cost analysis aims to allocate the joint costs entirely among the enterprises of a particular farm.

# 2 Method

## 2.1 Proportional allocation

For simplicity's sake, the following cost allocation is presented for one joint-cost item and one farm only, i.e., machinery costs. The farm produces $i$ arable crops ($i=1,2,..,I$). An arable crop is considered an enterprise (e.g. potatoes). Each crop $i$ is grown on an area $x_i$ measured in hectares. From the accountancy figures, we know the total joint costs $y$, i.e. the total machinery costs at farm level in Swiss Francs (CHF). As a result of the joint-cost allocation, we are looking for $\beta_i$, the (machinery) cost in CHF per hectare of crop $i$ where $\mu_i$ serves as an allocation factor. For the latter standard costs per hectare, also referred to as 'budgeted costs' or 'forecast costs', are used.

$$\beta_i = \frac{\mu_i}{\sum_{i=1}^{I} x_i \mu_i} y \qquad (1)$$

Together, the numerator and denominator on the right-hand side of Equation 1 represent the share of one hectare of enterprise *i* out of the farm-wide joint costs. Taken as a whole, all shares form the so-called 'apportionment formula' or 'allocation key'.
To perform a *Proportional* cost allocation, we reformulate Equation 1 to:

$$\beta_i = \alpha \mu_i \qquad (2)$$

The factor alpha ($\alpha$) is defined as follows:

$$\alpha = \frac{y}{\sum_{i=1}^{I} x_i \mu_i} \qquad (3)$$

Figure 1 graphically illustrates joint-cost allocation (i.e. for machinery costs), and underscores the adjustment of allocation factors, which is the core element of the allocation procedure. Specifically, the costs of two crops – grassland (*G*) and potatoes (*P*) with potatoes representing a crop with markedly higher standard costs – are adjusted. If we assume that the farm as a whole has lower costs than suggested by the farm management literature ($\alpha < 1$), the proportional line (*Prop*) whose slope is equal to $\alpha$ lies below the angle bisector (45°). Applying the standard costs $\mu_{Grassland}$ and $\mu_{Potatoes,}$ the proportional joint-cost allocation leads to $\beta_{G\_prop}$ and $\beta_{P\_prop}$, respectively.

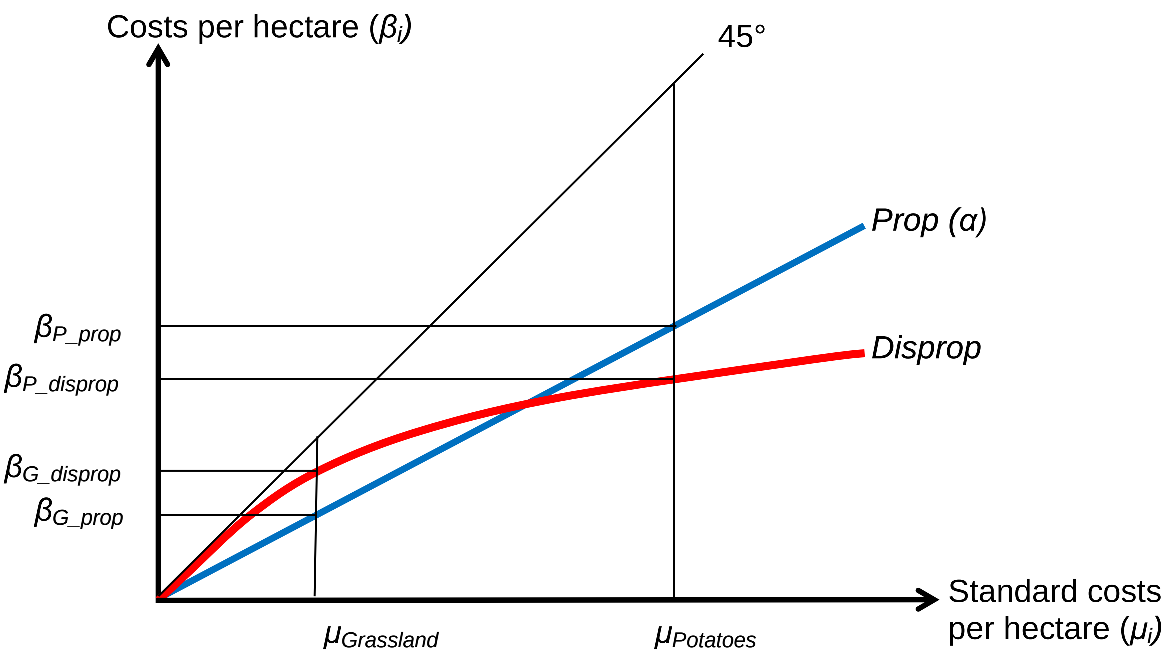


**Figure 1. Proportional and disproportionate joint-cost allocation**

G = Grassland; P = Potatoes; Prop = proportional; Disprop = disproportionate

## 2.2 *Core maximum entropy model*

The following outline of a joint-cost allocation model using maximum entropy is based on Golan et al. (1996: Chapter 3) and aims to derive the joint cost (i.e. machinery) of enterprise $i$ per hectare ($\beta_i$). Since we use standard costs per hectare ($\mu_i$) in the model, we must focus on the single-hectare level. For the model specification, each individual hectare of crop $i$ is treated as an independent activity. Accordingly, the number of hectares is crop-specific and denoted as $N(i)$, while $j$ refers to the individual hectare's number [$j = 1,2,\ldots,N(i)$]. Assuming that $N(i)$ is an integer, the individual hectares of crop $i$ are denoted as $x_{i,j}$:

$$x_i = \sum_{j=1}^{N(i)} x_{i,j} \tag{4}$$

$\beta_i$, the cost per hectare as already mentioned above, is provisionally defined for each hectare individually as $\beta_{i,j}$. The allocation of joint cost $y$ can be formulated as follows:

$$y = \sum_{i=1}^{I} \sum_{j=1}^{N(i)} \beta_{i,j} \tag{5}$$

We assume that $\beta_{i,j}$ lies in a range characterised by $K$ support points ($z_{i,k}$). With regard to the diversity within crop farming, we assume that the 'true' value for $\beta_{i,j}$ lies within a range of $\mu_i \pm \mu_i$. Following Howitt and Reynaud (2003), three support points are defined ($K$ = 3). The three support points are $0$, $\mu_i$ and $2\mu_i$, respectively. Since support points ($z_{i,k}$) refer to crops and do not differ among the different hectares of crops, the hectare-wise distinction need not be considered for support points. Each $\beta_{i,j}$ is defined as a weighted sum of its support points:

$$\beta_{i,j} = \sum_{k=1}^{K} p_{i,j,k} z_{i,k} \tag{6}$$

$p_{i,j,k}$ represents the probability of support point $k$ of hectare $j$ of enterprise $i$ being applied. For each hectare $j$ of all enterprises, the probabilities must add up to 1:

$$\sum_{k=1}^{K} p_{i,j,k} = 1 \qquad \forall\, i,j \tag{7}$$

Maximising the Shannon Entropy measure $H$ allows us to determine the probabilities $p_{i,j,k}$:

$$\max H = \left[ -\sum_{i=1}^{I} \sum_{j=1}^{N(i)} \sum_{k=1}^{K} p_{i,j,k} \ln p_{i,j,k} \right] \tag{8}$$

Given that support points within the different hectares of a specific crop are identical, the resultant probabilities must also be identical. Since there are no differences among the several hectares of crop $i$, it holds that $p_{i,j,k} = p_{i,k} \;\; \forall\, j$

Consequently, Equation 7 can be reformulated, which facilitates the subsequent model formulation:

$$\sum_{k=1}^{K} p_{i,k} = 1 \qquad \forall\, i \tag{9}$$

The fact that probabilities of all hectares of a particular crop are equal leads to a further simplification: $\beta_{i,j} = \beta_i \;\; \forall\, j$

Thus, Equation 6 is reformulated as follows:

$$\beta_i = \sum_{k=1}^{K} p_{i,k} z_{i,k} \tag{10}$$

Using Equations 4 and 10, Equation 5 can also be formulated differently:

$$y = \sum_{i=1}^{I} x_i \beta_i \tag{11}$$

Finally, the Shannon Entropy equation (8) is reformulated, making use of Equation 4:

$$\max H = \left[ -\sum_{i=1}^{I} x_i \sum_{k=1}^{K} p_{i,k} \ln p_{i,k} \right] \tag{12}$$

The variable $x_i$ serves as a weighting factor that takes account of the differing number of hectares of the farm's enterprises.

Together, Equations 9 to 12 form the *CoreModel*, even allowing the use of non-integer values for crop areas. Unlike the above description, which refers to one cost item and a single farm, the model is solved for all joint-cost items and all farms.

Returning to the joint-cost allocation issue, the question arises as to how the *CoreModel* differs from a *Proportional* cost allocation. The Shannon measure of entropy (Equation 12) reaches its maximum value when the distribution of all probabilities $p_{i,k}$ is uniform. Thus, the maximum is attained when each of the support points of all crops is assigned the probability of *1/K*, i.e. if $\beta_i$ is equal to the standard costs $\mu_i$. The approach therefore minimises the deviation from the standard costs.

Bearing in mind that the adjustment is performed at the one-hectare level, the absolute differences between support points are of importance. For instance, if total costs $y$ are smaller than suggested by the standard costs from farm management literature, the model must cause a reduction of the allocation factors. In absolute terms, a 1 % probability shift has a stronger impact on a crop with high standard costs (such as potatoes) than on one with low costs (grassland). Figure 1, which also includes the adjustment of allocation factors via maximum entropy cost with *Disprop*, produces the results $\beta_{G\_disprop}$ and $\beta_{P\_disprop}$ for grassland and potatoes, respectively. The probability distribution of the maximum entropy approach therefore leads to a disproportionate adjustment of standard costs. It is important to note that *Disprop* never intersects the angle bisector (45°), because all standard costs are adjusted in either a diminishing or increasing direction.

From a production technology perspective, a disproportionate adjustment better addresses the adjustment of costs of the production processes than does a proportional adjustment. Potatoes, for instance, incur much higher machinery costs than grassland. If farm-wide machinery costs differ greatly from the expected values in the farm-management literature, there are more possibilities in practice for adjusting machinery costs for potatoes, since more operational steps are applied (e.g. for plant protection). Generally speaking, the higher the standard costs $\mu_i$, the greater the possibilities for adjusting costs.

### 2.3 Inequality restrictions

The disproportionate adjustment of the *CoreModel* can lead to a situation in which crops with high standard costs are so strongly reduced that they even undercut crops with low standard costs. Such a result is only possible if the deviation factor alpha has an extremely low value (e.g. $\alpha < 0.5$). Potatoes, for instance, would have lower absolute machinery costs than grassland, which is useless from a production technology point of view. To ensure a plausible rank order among production branches we impose inequality restrictions as suggested by Campbell and Hill (2006). Although it would be possible to apply an inequality restriction for each crop, we define groups including similar crops. Accordingly, the *Inequality* application allows us to maintain the rank order between groups, while the rank order within groups may change.

## 3 Data

Data from 36 crop farm observations of the Swiss Farm Accountancy Data Network (FADN) is used to apply the different joint allocations. 12 different enterprises are considered while we focus on the allocation of two different joint cost items, labour (measured in normal working days) and machinery costs (in CHF), respectively. The allocation factors ($\mu_i$) are taken from farm management literature.
Based on $\mu_i$'s the factor alpha is calculated for all farm observations. The mean value of labour input (alpha = 2.5) indicates that far more labour is used than suggested by farm management literature. As regards machinery costs, the mean value of alpha is 0.8.

## 4 Results

Table 1 presents the results of *Proportional* joint-cost allocation and the two maximum-entropy applications *CoreModel* and *Inequality*, respectively. The results refer to the average of the enterprise cases involved, the number of which is indicated in the second column.

For labour, the results for *Proportional* on the one hand and both maximum-entropy applications on the other differ significantly. Bearing in mind that the farms in question use much more labour than suggested by the farm-management literature, the difference due to the disproportionate allocation under maximum entropy becomes obvious. For crops with low standard labour costs such as forest and fallow land, the results are lower for maximum-entropy applications than for *Proportional*. Conversely, potatoes and other activities exhibit higher results under the maximum-entropy applications. The *CoreModel* exhibits a slightly stronger deviation from *Proportional* than does *Inequality*, with *CoreModel* deviations falling within a range of -22 % (forest) to +19 % (potatoes), whilst the *Inequality* application shows a deviation of between -15 % (forest) and +18 % (other activities).

Given a mean value for factor alpha of 0.8, the allocation factors must in general be reduced for machinery. For crops with low standard machinery costs (e.g., forest and fallow land), the *Proportional* allocation leads to a stronger reduction and hence lower results than both maximum-entropy applications. Conversely, maximum-entropy applications show a more substantial reduction for crops with high standard costs (e.g., potatoes and other activities), leading to lower results. A substantial difference between the *CoreModel* and *Inequality* applications can be observed for these two crops. The absolute results of *CoreModel* are much lower. For *CoreModel*, the deviations from the *Proportional* allocation range between -29 % (other activities) and +19 % (fallow land), while those of *Inequality* range between -16 % (other activities) and +11 % (forest).

**Table 1. Mean values of labour and machinery-cost results for all enterprises**

| Enterprise | No. of Cases | *Labour in NWD per ha* | | | *Machinery in CHF per ha* | | |
|---|---|---|---|---|---|---|---|
| | | *Propor-tional* | *Core Model* | *Inequa-lity* | *Propor-tional* | *Core-Model* | *Inequa-lity* |
| Wheat | 33 | 8.7 | 8.2 | 8.3 | 1275 | 1339 | 1286 |
| Barley | 22 | 7.7 | 7.4 | 7.5 | 1367 | 1410 | 1383 |
| Maize | 15 | 9.3 | 8.7 | 8.8 | 1266 | 1310 | 1294 |
| Silage Maize | 15 | 8.8 | 8.6 | 8.7 | 2217 | 2132 | 2200 |
| Potatoes | 7 | 37.5 | 44.6 | 43.2 | 3345 | 2582 | 3002 |
| Sugar Beet | 23 | 17.0 | 18.2 | 17.2 | 2376 | 2224 | 2311 |
| Oilseeds | 31 | 7.4 | 6.7 | 6.8 | 1124 | 1201 | 1169 |
| Peas | 13 | 8.2 | 7.7 | 8.0 | 1080 | 1196 | 1088 |
| Grassland | 36 | 9.4 | 9.1 | 9.2 | 1884 | 1851 | 1916 |
| Fallow Land | 13 | 6.8 | 6.4 | 6.5 | 449 | 536 | 488 |
| Forest | 20 | 2.6 | 2.0 | 2.2 | 312 | 361 | 345 |
| Other Activities | 7 | 117.1 | 138.2 | 137.7 | 3508 | 2478 | 2942 |

NWD = normal working days

## 5 Conclusions

This paper presents an approach for the allocation of joint cost among farm enterprises at individual-farm level, based on maximum entropy and standard costs from farm-management literature as allocation factors. Adding *Inequality* restrictions ensures that a rough rank order between enterprises is maintained. Accordingly, this application is suited to farm management analyses at individual-farm level, and in the end allows the potential of maximum entropy to be used on behalf of joint-cost allocation. As a normative approach, the optimal solution is provided for each farm separately, bearing in mind the farm-specific joint-cost situation.

Compared to a *Proportional* joint-cost allocation, which is usually applied in the literature for full cost analyses, the application of maximum entropy for joint-cost allocation leads to a disproportionate adjustment of standard costs, reflecting a probability distribution in which the adjustment of high standard costs is more likely than the adjustment of low ones. As a result, the maximum-entropy applications bring joint-cost allocation more in line with reality, as well as allowing the strong assumption of proportional allocation to be discarded.

Based on crop-farm observations from the Swiss Farm Accountancy Data Network (FADN), the average allocated joint costs of the enterprise are compared for a *Proportional* cost allocation and for the two maximum-entropy applications *CoreModel* and *Inequality*, respectively. The shown differences highlight that the choice of allocation methods is of empirical importance.

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