The CRESH revival - or how to model agriculture in AGE models more realistically

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Abstract

Applied general equilibrium (AGE) modeling is a versatile tool for guiding public policies. However, the standard treatment of production function in AGE models is not completely satisfactory. It is commonly based on nested CES functions that are rigid and empirically hard to estimate. In this paper, I will show that by applying a more general functional form - CRESH - the agricultural sector can be presented more realistically in AGE models.

Key words: AGE modeling, production function

1 Introduction

At present a considerable share of applied policy analysis relies on information generated by applied general equilibrium (AGE) models[1]. Agricultural policy issues that are typically investigated with AGE models are numerous: domestic agricultural policies, trade policies, environmental regulation in agriculture, climate change mitigation and adaptation, etc. In contrast to partial equilibrium models, the AGE models naturally take into account both supply and demand side effects and, consequently, the effects on the economy as a whole. The most fundamental part of the supply side analysis is the production function itself. Thus it is surprising to note how much of agricultural AGE modeling relies on standard treatment of production that is by many ways unsatisfactory. The standard treatment is to use constant elasticity of substitution (CES) function for the primary factors and nest that as fixed proportions, Leontief function with intermediate inputs. It is also common that the parameters in these models are not based on empirical evidence. And even if the parameters would be based on real estimates, it is common that the estimation is conducted with some other functional form than CES and therefore the concordance between the estimated parameters and the model's functional form comes into question.

CES function has become popular as it originally increased realism in AGE models by allowing for substitution between primary factors. In the very fundamental settings the substitutability is restricted to two main primary factors, capital and labor, and for such cases the CES approach is evidently able to deliver important improvement in realism. However, the CES function can only handle two inputs at maximum and attempts to widen the concept to more than two inputs have been compromises with realism. The basic solution is to assume that all the inputs are substitutable in the same nest with constant elasticity. This is evidently restricting assumption and hardly realistic. More advanced approach is to place inputs at hierarchically differing nest levels. The nesting of labor, capital and energy as shown by van der Werf (2008) at different nest structures has been relatively common feature of AGE models dealing with energy and climate policies. However, the choice of the nest structure has been largely arbitrary. Furthermore, it has been based on little empirical evidence[2].

[1] Computable general equilibrium (CGE) is equally commonly used name for this family of models

[2] van der Werf (2008) actually estimated the nested CES-functions for OECD countries EU-KLEMS data and found that KL(E)-nest fit the data best. Aside of that, no comprehensive studies exist.

The substitutability of more than two factors is substantially more complicated affair. Firstly, CES function is symmetric - substitutability is necessarily same regardless of direction of change. Secondly, it is a measure for substitutability but not a measure for complementarity - in CES world the inputs are either complements or they are not, without degrees. More general functional forms would be more desirable and realistic as they allow for asymmetric substitutability and degrees of complementarity as well. Although incorporating flexible functional form functions in AGE models has proved difficult, there exists one exception that works smoothly in AGE models - so called CRESH function[3] first suggested by Hanoch (1971). One convenience in CRESH function is that it can be easily linearized in order to apply it in widely used MONASH-type AGE models. Despite its overall convenience and generality, the use of the CRESH functional form in AGE modeling has thus far remained marginal at best.

Agriculture is an industry that markedly uses more than two primary factors as it is an important user of land. Moreover there are few intermediate inputs used in agriculture that might be seen as substitutable with the primary factors. E.g. substitutability between land and fertilizer is interesting to investigate as it reflects the ease of shifting between intensive and extensive margin of production. Therefore, agriculture presents a good case for testing the properties of CRESH production function. In my paper I will show that by implementing agricultural production in AGE models as CRESH rather than a nested CES we could achieve increased realism with more convenient empirical estimation.

The paper is structured as follows. The chapter 2 is the empirical part of the work. I present the econometric models and the estimation results. The chapter 3 presents the results from the AGE modeling work where I compare performances of various production function types. In chapter 4 I discuss the implications of the results and chapter 5 concludes the paper.

2 Empirical evidence

In my analysis I have used a panel dataset of an EU country farm enterprises that includes several variables of inputs, outputs and production capital. The dataset is an unbalanced panel for years 2004-2009 and has farms identified by their lines of production lines.[4] By complementing this data with price information I could estimate all the nested CES functions for capital, labor and land inputs, and CRESH functions with aforementioned three primary factors with and without fertilizer.

2.1 Econometric models

I estimated the agricultural production function with three primary inputs: capital (K), labor (L) and land (M). First I estimate the nested CES functions: (KL)M, (KM)L and (LM)K. The general functional form for three input nested CES function is:

$$Y = A_i(ax_1^{\rho_u} + (bx_2^{\rho_l} + (1-a-b)x_3^{\rho_l})^{\frac{1}{\rho_l}})^{\frac{1}{\rho_u}}, \quad (1)$$

where upper level elasticity of substitution ρ_u measures the substitutability between input x_1 and the composite of the inputs x_2 and x_3. The lower level elasticity of substitution ρ_l measures the substitutability between the inputs x_1 and x_2. I can also test whether the basic non-nested (K,L,M)-structure can be safely rejected, i.e. $\sigma_{(K,L),LAND} = \sigma_{K,L}$. For the empirical estimation I apply same linearization to a system of equations as was used van der Werf (2008) that assumes cost minimization. It should be noted that increasing the nesting levels greatly increases the complexity of estimated system. In order to estimate systems with more than two

[3]CRESH stands for Constant Ratios of Elasticity of Substitution, Homothetic

[4]The lines of production that I included in the analysis are dairy, pig production, poultry, other animal production, grains and oilseeds, and other plant production.

levels of inputs would additionally require assumptions about unlikely input price indices. In absence of such indices I limit my CES function analysis to the three input case only.

By following original presentation by Hanoch (1971), the CRESH function is defined as an implicit function

$$F(Y,\underline{x}) = \sum_{i=1}^{n} D_i[x_i/h(Y)]^{d_i} - 1 \equiv 0, \qquad (2)$$

where $\underline{x}$ is the vector of the factors of production and Y is the output. D_i and d_i are the parameters of the function. $h(Y)$ is function of output Y that can take various forms. Cost minimization assumption yields a system of equations that can be estimated. Estimation is not restricted by the number of inputs as in CES case and actually requires less data when price information is required only for the inputs but not for the outputs. Additionally, the CES estimation requires data of yearly changes, so only the firms with more than two observations could be included in estimation. CRESH function is thus more conveniently estimated than the any form of CES function.

I carried out the analysis with multilevel structural equation statistical models by using R package lavaan.[5] Structural equation modeling is general statistics counterpart to the simultaneous equation models in econometrics. For both CES and CRESH estimation I needed to estimate several equations simultaneously. Multiple groups modeling framework enabled me to take into account evident differences between lines of production while at the same time taking advantage of deriving the estimation from the complete data set. I could thus draw comparison between functional forms and derive elasticities of substitution for each lines of production.

2.2 *Empirical results*

It should be noted that only the CES models are nested within each other so their comparison by goodness of fit values is thus meaningful. However, as we will see, the differences between goodness of fit values between CES and CRESH functions are so significant that it is quite safe to conclude that CRESH much better corresponds to real world than what CES does.

We start with CES function. Table 1 presents the goodness of fit values of nested CES functions. It shows that the structure where land and labor are first nested with each other and then as composite with capital fits the data best. Table 2 reports the p-values of two-sided t-test for testing $\sigma_{upper} = \sigma_{lower}$ of each line of production. It shows that the (L,M)K-nest has best fit, it is not statistically distinguishable from non-nested general case.

Table 1 Goodness of fit values for nested primary factors CES functions.

(K,L)M	(K,M)L	(L,M)K
0.27	0.38	0.53

Furthermore, closer examination of the elasticity of substitution values themselves reveal that the best nested function is very close to unit elastic Cobb-Douglas form (see table 3).

I estimated the CRESH function for two input combinations: the three primary factors alone and three primary factors plus fertilizer. Additionally I conducted the estimations for three different $h(Y)$ specifications: general homogeneous, constant returns to scale and S-shaped cost curve. Table 4 summarizes the results. It turns out that $h(Y)$ specification does not alter the results significantly and including fertilizer somewhat decreases goodness of fit.

[5] CRESH system has an additional complication when compared to CES system - it contains non-linear restrictions. The standard R package for estimating the systems of equations systemfit does not allow non-linear restrictions. Fortunately, package lavaan that is primarily aimed at latent variable analysis does not suffer from that limitation.

Table 2 p-values for non-nested function.

	(K,L)M	(K,M)L	(L,M)K
DAIRY	0.00	0.00	0.13
BEEF	0.37	0.00	0.00
PORK	0.21	0.00	0.00
POULTRY	0.00	0.00	0.00
GR_OILS	0.05	0.15	0.00
OTH_PLNT	0.00	0.00	0.00

Table 3 Estimated elasticities of substitution - CES function.

	(K,L)M $\sigma_{(K,L)M}$	(K,L)M $\sigma_{K,L}$	(K,M)L $\sigma_{(K,M)L}$	(K,M)L $\sigma_{K,M}$	(L,M)K $\sigma_{(L,M)K}$	(L,M)K $\sigma_{L,M}$
DAIRY	0.81	0.93	0.85	0.99	0.99	1.00
BEEF	0.96	0.99	1.53	1.00	1.12	1.00
PORK	0.96	0.99	1.43	0.99	1.07	1.00
POULTRY	0.29	0.86	0.20	0.98	0.81	1.00
GR_OILS	0.90	0.97	0.93	1.00	0.90	1.00
OTH_PLNT	0.75	0.88	0.63	0.99	0.79	0.99

However we see that CRESH function seems to correspond to real world better that any of the CES nests do.

Table 4 Goodness of fit values for nested CRESH functions.

	General homogeneous	CRS	S-shaped cost curve
Primary factors	0.64	0.71	0.64
Primary factors + fertilizer	0.58	0.70	0.58

For brevity, I present here CRESH results only for dairy and grains and oilseeds production lines. The elasticities are in the Morishima form [6]. Tables 5 and 6 summarize the elasticities of substitution for dairy production with primary factors with and without fertilizer input, respectively. Tables 7 and 8 do the same for grains and oilseed production.

Table 5 CRESH elasticities of substitution for dairy production, CRS, primary factors

	Capital	Labor	Land
Capital	0.00	0.96	1.93
Labor	-0.12	0.00	-0.19
Land	0.57	0.28	0.00

Table 6 CRESH elasticities of substitution for dairy production, CRS, primary factors and fertilizer

	Capital	Labor	Land	Fertilizer
Capital	0.00	0.96	1.93	0.97
Labor	-0.12	0.00	-0.19	-0.10
Land	0.58	0.29	0.00	0.30
Fertilizer	0.37	0.31	0.57	0.00

[6] Elasticities for other production lines are available upon request. The same goes to Allen-Uzawa form elasticities

Table 7 CRESH elasticities of substitution for grains and oilseed production, CRS, primary factors

	Capital	Labor	Land
Capital	0.00	0.99	1.99
Labor	-0.10	0.00	-0.19
Land	0.22	0.11	0.00

Table 8 CRESH elasticities of substitution for grains and oilseed production, CRS, primary factors and fertilizer

	Capital	Labor	Land	Fertilizer
Capital	0.00	0.99	1.99	1.00
Labor	-0.10	0.00	-0.19	-0.10
Land	0.22	0.11	0.00	0.12
Fertilizer	0.31	0.29	0.57	0.00

We see from tables 5 and 7 that elasticity between land and labor is lowest and they are thus least substitutable. This result is well in line with CES results, which indicated that land and labor should be modeled at lowest nest in CES function. However, the CRESH specification indicates much less substitutability for these factors. Indeed, labor and land are actually weak substitutes or complements, depending on the direction. We can also note that when we include fertilizer in the production function, it turns out to be relatively little substitutable input. However, its substitutability varies among other inputs and thus modeling it in Leontief function along with other intermediate inputs seems to have no empirical backing.

3 Modeling results

In order to compare the functioning of different production function specifications in an AGE model, I apply the static ORANI-G model. The closure for the model is the standard short run closure described in Horridge (2003). Land use is modeled so that new land is not created but it could shift between the land using industries. The database of this exercise is for the same country that coincides with the parameter estimations.

In order to demonstrate the relative merits of different production function specifications, I ran an imaginary simulation which decreases the agricultural subsidy payments by 20% at each production line. I ran four separate cases:

1. Non-nested CES function (KLM)
2. Nested CES function ((LM)K)
3. CRESH function with primary factors
4. CRESH function with primary factors plus fertilizer

Figure 1 displays some macro level results for each of the four cases. The red bar is the change in expenditure side real GDP and green is the change in employment. [7]. The changes are qualitatively similar in each case. Nested CES displays larger changes than non-nested case. With CRESH, the fertilizer inclusions generates bigger overall effects. As CRESH is the most general form of our functions, it is not surprising to see that it produces less overall

[7]The unexpected direction of change can be attributed to the short-run closure of the model; the capital is fixed in the short-run and production cannot adjust to optimal input use. Indeed, long-run closure yields real GDP improvement due to subsidy reduction. However, there are no visible differences between functional specifications in long-run closure as the economyis set to adjust fully.

economic changes than the more rigid CES case - there is more possibilities to adjust in CRESH economy. Labor decreases more than the real GDP, but relatively less in CRESH case because labor is more complement in CRESH and its use cannot adjust as much as it does in CES. The terms of trade, which is displayed by the yellow bars improves in all of the cases but more modestly with CRESH specification. Improvement in terms of trade indicates that production shifts from agriculture to more highly processed goods and services. By including fertilizer in the CRESH function, we can see smaller overall effects as there is now even more room for adjustment. The difference is modest.[8]

Figure 2 displays the percentage changes of the primary factor and feritilizer price levels. Again, the qualitative changes are similar in all of the cases. A marked difference between CES and CRESH is that land prices decrease significantly more with CRESH than with CES. For the labor, the situation is opposite. The CRESH therefore portrays a shift to production structure that is less land intensive than with CES function. The inclusion of fertilizer causes smaller change in land use but larger changes in fertilizer use as the intensive margin of land use is more accurately presented.

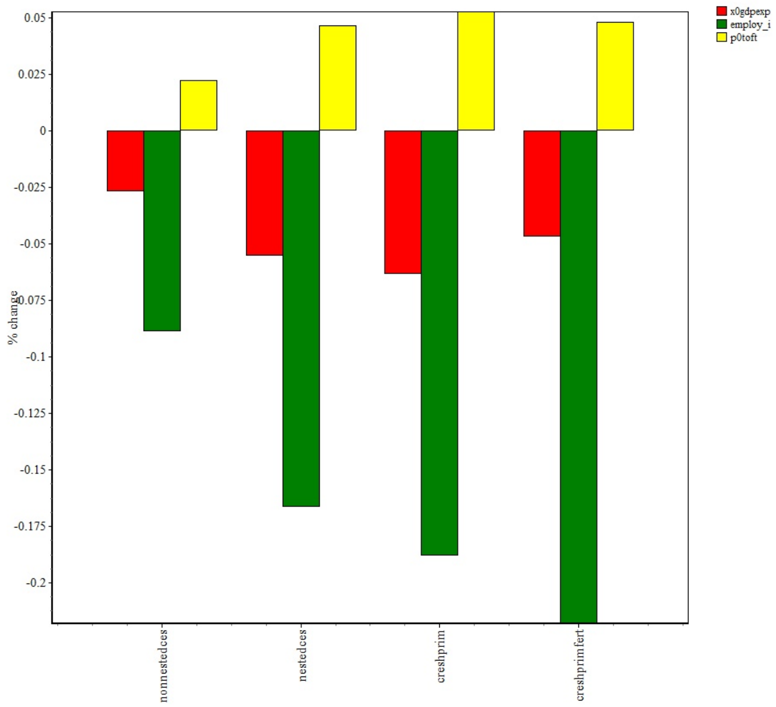

Figure 1 Changes in macro variables by different production function specifications.

Figure 3 displays percentage changes in output for the main agricultural lines of production. Notable difference between CES and CRESH specifications is that variation by the lines of production is much higher for the CRESH functions. The econometric estimation already showed much more variation by production line when estimated with CRESH function and variation is carried to the simulation results as well. The production line that is most land intensive, the grains and oilseed production decreases its output most compared to other production lines in CRESH cases. In overall, the changes are also more moderate with CRESH - output in agriculture decreases less with CRESH than with CES. It is curious to note that by including fertilizer use in CRESH function, the output actually decreases more.

[8]Including fertilizer in production function could be sensible if the environmental effects is of interest.

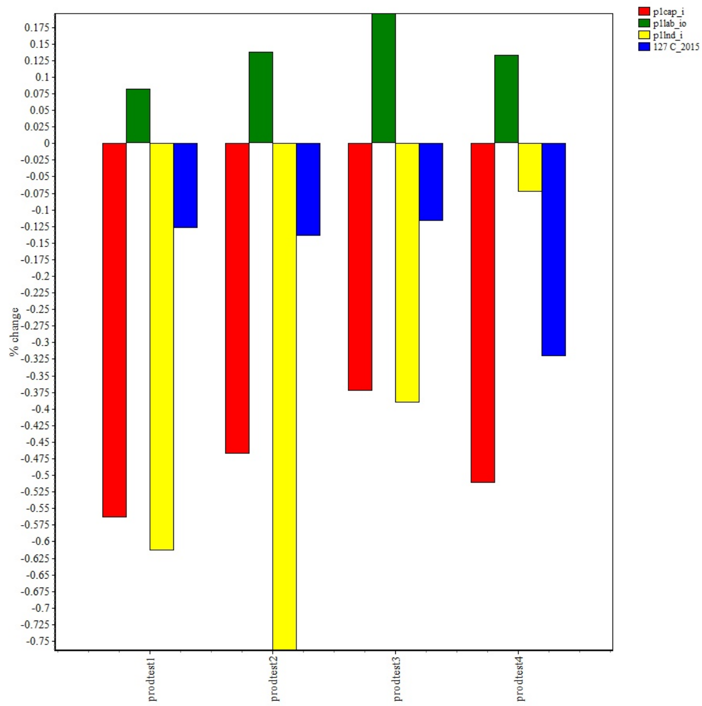

Figure 2 Changes of primary factor and fertilizer prices. Red = capital, green = labor, yellow = land and blue = fertilizer.

Figure 4 displays percentage changes in land use by line of production. Land use changes much more with CRESH function than with CES. In all the cases the grains and oilseeds shows largest changes. Fertilizer inclusion has quite moderate impact on overall land use by allowing little bit of intensive margin adjustment. The results reflect well the empirical estimates for land substitutability. The changes appear large, but on the other hand, the subsidy reduction was quite large as well. The CRESH specification suggest stronger adjustment in factor input use resulting to less land intensive production.

4 Discussion

Although my results apply to one particular dataset and one particular sector, I think that it might be of interest to anyone who aims at improving the realism of an AGE model by empirical analysis. Based on the results above it is easy to see that the functional form makes most of the difference for modeling agricultural production function. This is due to both 1) different parameter values achieved in estimation and 2) different functional forms in the model as well. If I had set all the elasticity values to unity (Cobb-Douglas case) I would have naturally found same result for each simulation. But as the same data set yields considerably different results, it does not seem meaningful to separate estimates and the functional form: consistent parameterization of AGE model production function is ideally done by implementing the same functional form that is used for the estimation.

The CRESH function is more straightforward to estimate and nested CES function on the other hand has severe limitations on that front. I suggest that if AGE modelers desire to improve the realism of their models by conducting empirical econometric work, they should seriously consider using CRESH as the estimated functional form and apply it in their models as well. Starting again from practicalities, it is easier to build the production function with CRESH than corresponding nested CES, at least with ORANI- and MONASH-type of models. Furthermore,

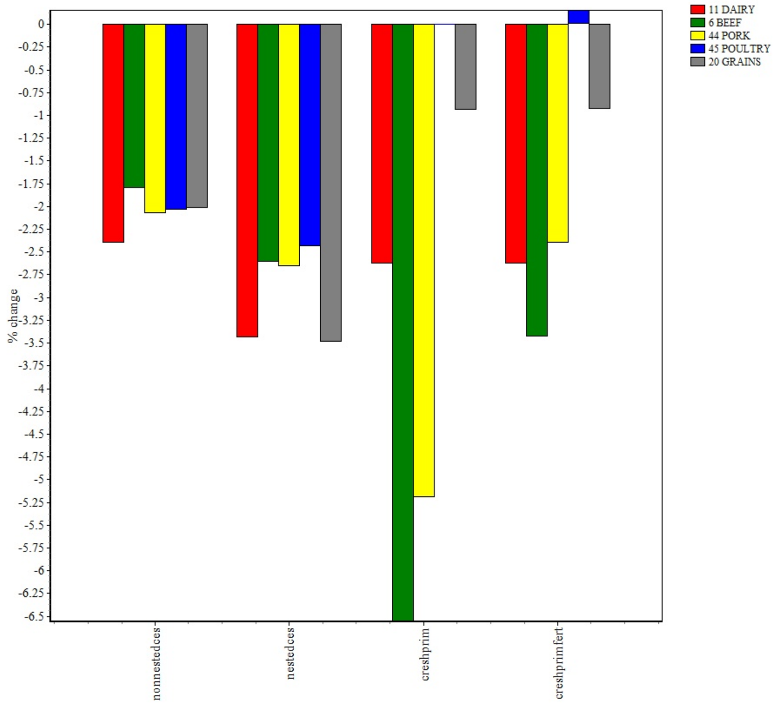

Figure 3 Changes in output by production line.

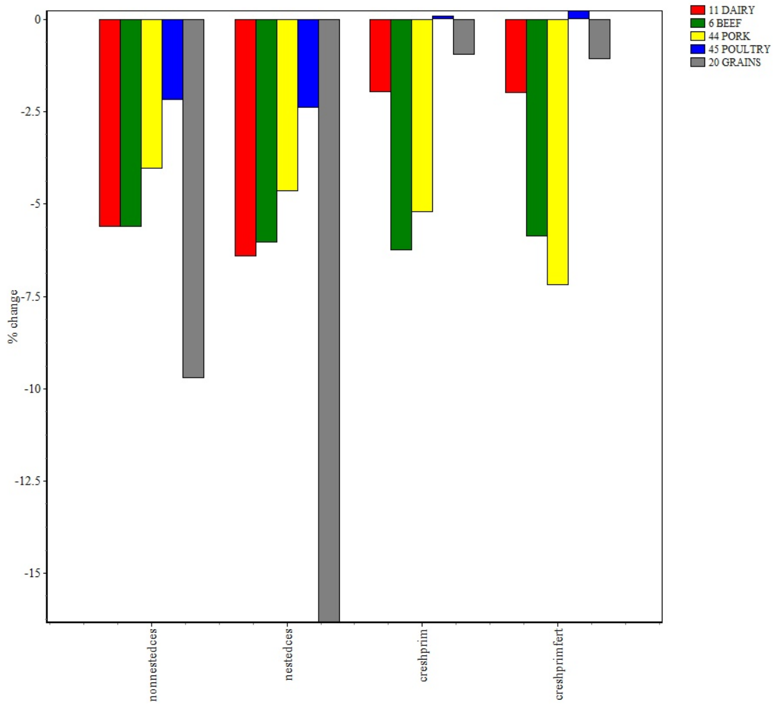

Figure 4 Changes in landuse by production line.

the CRESH is more likely to generate the empirically found variation in the model results as well. As CES is inherently incapable of producing pairwise substitutability and only one kind of complementarity, it might miss some features present in real life sectoral production.

The a priori expectations of agricultural sector's behavior lends support to choosing CRESH as well. The CES functional form shows very uniform changes in output per production line while the changes in factor use are not entirely plausible. The CRESH at the same time reveals more variation between the production lines of agriculture that CES largely misses. My analysis gives some support for treating fertilizer use as weak substitute for other factors. The results seem realistic, but on the other hand the effects are moderate.

The consequences for the policy analysis could be speculated as well. In this simple subsidy reduction case we could see that CRESH specification based on empirical evidence showed less reduction in agricultural output but stronger adjustment in land use. Could it be that agriculture as a sector has higher ability to adjust to policy changes than is currently portrayed in AGE analysis?

More thorough analysis is naturally required for firmer conclusions. The closure setting in this analysis is of course quite limited and long-run and dynamic model investigations would shed more light to significance of production function form in agricultural AGE modeling. Additionally, historical simulations of agricultural sector is required to justify definitive claims for improved realism. Before that, my conclusions are best seen as still tentative.

5 Conclusion

In this paper I have envisioned that realism of agricultural AGE models could be considerably improved by using CRESH functional form in production. Empirical evidence already supports the CRESH form over the less general and more rigid CES specifications. Nesting CES functions changes modeling results only a little while shifting to CRESH functional form produces significantly different and arguably more realistic results. Variation among production lines is much more evident in results generated by CRESH functional form. Including fertilizer in CRESH seems prominent way to include intensive margin adjustment in AGE models with land use.

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