Incorporating Risk in a Positive Mathematical Programming Framework: a New Methodological Approach

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Abstract

In this paper we develop a new methodological proposal to incorporate risk into a farm level Positive Mathematical Programming (PMP) model. Our model presents some innovations with respect to the previous literature and estimates simultaneously the resource shadow prices, the farm non-linear cost function and a farm-specific coefficient of absolute risk aversion. The proposed model has been applied to three farm samples and the estimation results confirm the calibration ability of the model and show values for risk aversion coefficients consistent with the literature. Finally we simulate different scenarios of crop price volatility to test the model reactions as well as the potential role of an agri-environmental scheme as risk management tool.

Keywords: risk aversion, positive mathematical programming, farm behaviour

1. Introduction

Risk is an important component of agricultural production and it affects farmers' production choices. Hardaker *et al*. (1997) classified the risk in agriculture as production risk, market risk, institutional risk, personal risk and financial risk. Under a risky environment, the decision-maker makes the choices based on his expectation of uncertain outcomes and these expectations are often based on past experiences. Most empirical studies showed that farmer is a risk averse agent as he is willing to sacrifice some income to ensure against the risky consequences (Feder, 1980). Given the risk averse attitude of the farmers, the recent years increase in price volatility on world and EU markets is negative as it makes farmer income uncertain. These unpredictable price variations may lead to non optimal production decisions in the short run and may discourage farm investments leading to a decrease in farm profitability and competitiveness in the medium-long run. Since risk is a structural component of agriculture and the farmer is not a risk neutral agent, ignoring risk in modelling farmer behaviour is likely to lead to biased results. In a mathematical programming model, risk faced by farmers can be introduced either by randomising the behaviour of input and output prices or by introducing uncertainty in the supply of limiting inputs as well as in the technical coefficients specification. There are different techniques to accommodate risk in a mathematical programming framework, such as the mean-variance approach (Freund, 1956; Paris, 1979; Coyle, 1992, 1999), the Minimisation of the Total Absolute Deviations (MOTAD) (Hazel, 1971; McCarl and Onal, 1989), the target MOTAD (Tauer, 1983), the chance constrained programming (Charnes and Cooper, 1959) and the discrete stochastic sequential programming (Kaiser and Messer, 2011).

Besides the inclusion of risk another important issue in farmer behaviour analyses is the ability of the model to calibrate to the observed base year situation. Although normative mathematical programming models, which lack of any calibration, dominated the efforts in agricultural economics modelling for decades, nowadays a wide divergence between the modelled outcome and the observed outcome is unacceptable in policy analysis. Although the addition of a risk term in a normative mathematical programming model may improve the model performance and may overcome the overspecialisation problem, typical in linear programming specifications, it is often not enough to reproduce the observed farmer's production decision. Positive Mathematical Programming (PMP) is a powerful calibration method introduced in the 90s with the aim of overcoming the drawbacks of normative models. PMP method is able to recover a non linear cost function that allows to exactly reproduce the observed activity levels and to provide information about the effect of parameter changes on the farm input allocation (Howitt, 1995).

In this paper we develop a new methodological proposal which incorporates farm risk in a farm level PMP model. Given the importance of accounting for risk in farm level analyses and the powerful calibration ability of PMP, we think that the incorporation of the risk component in a PMP framework will be one of the new research frontier in farmers' behaviour analyses. So far, there have been a few attempts in the literature to introduce risk modelling in a PMP framework (Paris and Arfini, 2000; Severini and Cortignani, 2012; Petsakos and Rozakis, 2011). This may be explained by the difficulty in estimating two different non-linear terms in the objective function, the cost function and the risk component. The idea of combining risk modelling with PMP relies on the information contained in the farm non-linear cost function estimated in the PMP procedure. As this cost function incorporates any type of model misspecification, data errors, aggregate bias, price expectation and risk behaviour (de Frahan *et al.*, 2007), it should be possible to isolate the risk component from the farm non linear cost function such that the impact of risk on farmer's choices may be studied. Our proposal presents some innovation compared to the previous literature, since it merges the first linear phase with the second non-linear phase of the PMP of a farmer expected utility maximisation problem. This allows to estimate simultaneously the farmer's risk aversion coefficient, the farm non-linear cost function as well as the shadow prices of limiting resources (e.g. land) by imposing the dual conditions of optimality.

The proposed model has been applied to three representative farm samples in order to check the ability of the model to calibrate to the base year observed activity levels and to estimate a farmer specific absolute risk aversion coefficient. Then, the calibrated model has been used to perform some simulations of different crop price volatility scenarios aiming at testing the ability of the model to represent the farmer's reaction to changes in economic conditions. This exercise has explored the potential role of an agri-environmental scheme (AES), the option to convert a share of cropland to grassland, as risk management tool. The idea is that, since the adoption of AESs guarantees a fixed payment to the farmer independent of market conditions and crop yields, these measures may act as an insurance against price and yield risk. The mathematical programming framework is suitable for our purpose as it allows to model easily the grassland program which competes for farmland with the other crops.

The paper is organised as follows: section 2 presents the existing literature on PMP and the attempts to incorporate risk in PMP models; section 3 describes our methodological proposal to integrate risk in a PMP model and its innovation compared to the extant attempts; section 4 details the empirical model and data, while in section 5 we analyse the results of the calibration and of simulated scenarios; section 6 draws discussion and conclusions.

2. Risk in PMP Models

The standard PMP approach is a three step procedure which uses the dual information provided by the calibration constraints of the first step to recover a farm non linear cost function which calibrates the model to the observed activity levels. Although the PMP methodology was already applied in the '80s in agricultural economic analyses, it was formalised and published for the first time by Howitt in 1995. Following the seminal papers by Howitt (1995) and Paris and Howitt (1998), there have been many methodological developments in the area of PMP, aiming to improve the standard approach. Paris and Arfini

(2000) dealt with the problem of zero activity levels in some farms of an homogenous sample proposing the self-selection approach. Paris (2001) proposed the Symmetric Positive Equilibrium Problem (SPEP) as a way to avoid a linear representation of the technology and to make the demand and supply of fixed inputs responsive to output levels and input price changes. One of the most important modifications to the original PMP approach was proposed by Heckelei and Wolff (2003), who proposed to skip the first step of PMP, which may lead to biased results, and to employ directly the first order conditions of the desired programming model to estimate simultaneously the non-linear cost function and the dual values. The work of Heckelei and Wolff represents a remarkable attempt to join mathematical programming model with econometric techniques within the new framework of econometric mathematical programming (EMP) and empirical application of their proposal can be found in de Frahan *et al.* (2007) and in Buysse *et al.* (2007). A recent extension of the PMP approach is represented by the model proposed by Arfini and Donati (2011) and discussed in the Paris' book 'Economic Foundations of Symmetric Programming' (2011: 397-404). Arfini and Donati merged the first linear phase with the second non-linear phase of the PMP and they estimated simultaneously the parameters of the non-linear cost function, the shadow price of resources and the differential marginal costs. The authors (2011) provided also an empirical application to the analyses of some Health Check CAP reform proposals.

As stated above, a relatively new research frontier in the area of mathematical programming concerns the integration of risk modelling in a PMP framework. The idea raises by the information contained in the dual values of the calibration constraints of the standard PMP. As this information captures also the risk behaviour, it should be possible to make the risk component explicit and separate it from the other non-linear cost components. This would allows to identify the farmer attitude towards risk and the role of risk in farmer's choices as well as to perform simulations under different risk scenarios. So far there have been a few studies in this direction.

The first attempt was made by Paris and Arfini (2000), who introduced risk in a PMP model relying upon the mean-variance approach proposed by Freund (1956). Although their study drove the attention of agricultural economists on the new challenging problem of incorporating risk into PMP, they applied an exogenous absolute risk aversion coefficient and their model still relied upon the standard three PMP steps.

A more recent attempt has been carried out by Severini and Cortignani (2012). Their work extended the work of Heckelei and Wolff (2003) by the inclusion of a "gross margin risk" modelled by the mean-variance approach. The gross margin risk is determined by the random behaviour of prices and yields and it is not possible to isolate the price risk from the yield risk. The authors estimated endogenously the farmer's absolute risk aversion coefficient by skipping the first phase of PMP and applying directly the Generalised Maximum Entropy estimation on the first order conditions of the desired model. The model estimates simultaneously the non-linear cost function, a farmer specific absolute risk aversion coefficient and the shadow price of land. The paper presents an illustrative empirical application to a small sample of farms located in the centre of Italy with the aim of evaluating the effect of a revenue insurance schemes on farm production choices and on farm gross margins. In their work, Severini and Cortignani (2012) did not consider any structural foundation behind the model.

Petsakos and Rozakis (2011) proposed an innovative framework for integrating risk into PMP. By applying a second order Taylor expansion to a logarithmic utility function they got an expected utility function, which has been used in a three step procedure to calibrate the variance-covariance matrix of the gross margin per unit of activity. Although the risk aversion coefficient is not explicit in the model, it can be derived applying the Arrow-Pratt rule on the expected utility and it exhibits DARA preferences. Although the work of Petsakos and Rozakis represents a remarkable proposal to include risk into a PMP framework, it presents some weaknesses. First, they consider the misspecification of the initial variance-covariance matrix of the unitary gross margin as the only reason why the starting model does not reproduce the observed activity levels and they did not calibrate any cost function. Second, their methodological proposal estimates some negative values for the resources shadow values. Finally, their PMP procedure still relies upon three steps.

3. Theoretical Model

Given the few attempts found in the literature to integrate risk into a PMP framework, and lacking an established consensus on the most suitable one, we have elaborated a new proposal for the integration of agricultural risk in a farm level PMP model which consists in two phases: an estimation and a simulation phase. This new methodological proposal merges the first linear phase of PMP with the second non-linear phase by using the dual relationships of a farmer's expected utility maximisation problem. The problem incorporates the risk term according to the mean-variance approach. The model estimates simultaneously the differential marginal cost and the shadow price of resources which usually belong to the first PMP phase, as well as the farm non-linear cost function and the farmer specific coefficient of absolute risk aversion. In addition no calibration constraints are made explicit in the model. The model specification is the following:

$$\min_{\mathbf{u},\mathbf{y},\lambda,\alpha} \frac{1}{2}\mathbf{u}'\mathbf{u} + \mathbf{y}'\mathbf{b} + \mathbf{c}'\overline{\mathbf{x}} + \boldsymbol{\lambda}'(\overline{\mathbf{x}} + \boldsymbol{\varepsilon}) + \alpha\overline{\mathbf{x}}'\mathbf{V}\overline{\mathbf{x}} - E(\tilde{\mathbf{p}})'\overline{\mathbf{x}} \tag{1}$$

subject to

$$\mathbf{c} + \alpha\mathbf{V}\overline{\mathbf{x}} + \mathbf{A}'\mathbf{y} + \boldsymbol{\lambda} \geq E(\tilde{\mathbf{p}}) \tag{2}$$

$$\mathbf{c} + \boldsymbol{\lambda} = \mathbf{Q}\overline{\mathbf{x}} + \mathbf{u} \tag{3}$$

$$\mathbf{y} \geq 0, \boldsymbol{\lambda} \geq 0, \alpha \geq 0 \tag{4}$$

where, $\overline{\mathbf{x}}$ is the vector of observed activity levels, $\mathbf{c}$ is the vector of accounting costs per unit of activity and $E(\tilde{\mathbf{p}})$ is the vector of expected prices, $\mathbf{Q}$ is the quadratic matrix of the non linear cost function which is common to all farms of the same sample while $\mathbf{u}$ is the specific farm deviation from the common cost function; $\mathbf{b}$ represents the vector of resource availability and $\mathbf{A}$ is the matrix of technical coefficients, $\mathbf{V}$ is the variance-covariance matrix of activity prices, while α is the farmer's absolute risk aversion coefficient. $\mathbf{y}$ and $\boldsymbol{\lambda}$ are the vectors of resource shadow values and of the shadow values of the calibration constraints respectively and $\boldsymbol{\varepsilon}$ is the perturbance term vector which prevents linear dependency between the constraints (Paris and Howitt, 1998).

The objective function (1) minimises the square of the individual farm deviation, $\frac{1}{2}\mathbf{u}'\mathbf{u}$, from the common cost function and the difference between the primal and the dual objective function of the farmer's expected utility maximisation problem. The constraint (2) represents the economic equilibrium condition stating that the marginal cost must be larger or equal to the marginal revenue, while the constraint (3) indicates the relationship between the marginal cost of the first phase of the standard PMP and the marginal cost of the farm non linear cost function. This last constraint allows to estimate the implicit cost $\boldsymbol{\lambda}$ for each activity. The use of the dual relationships (constraint 2) allows to merge the first two phases of PMP which has some advantages. First, the calibration constraints are not introduced in the model avoiding critiques raised against this kind of constraint. Second, the model allows the simultaneous estimation of the shadow prices of resources, $\mathbf{y}$, the shadow prices of activities, $\boldsymbol{\lambda}$, the quadratic matrix of the cost function, $\mathbf{Q}$, the individual farm deviations from the cost function, $\mathbf{u}$, and the farmer’s absolute risk aversion coefficient α. The coefficient of absolute risk aversion is farm specific and it exhibits constant absolute risk aversion (CARA) preferences. We impose neutral or risk averse behaviour forcing the absolute risk aversion coefficients be non-negative.

Despite the proposals by Cortignani and Severini (2012) and Heckelei and Wolff (2003), the model (1)-(4) does not include the primal resource constraint that links the input demand with the input supply. The inclusion of this constraint in the estimation phase of the models of Cortignani and Severini and Heckelei and Wolff seems to introduce a tautology as all the information about this primal resource constraint is perfectly known.

Since the model (1) - (4) is a mathematical programming model with inequality constraints and sign restricted variables, a set of Karush-Kuhn-Tucker (KKT) conditions provides the solution of the model. In order to derive the KKT conditions, we write the Lagrange function of the model:

$$\begin{aligned} L = {} & \frac{1}{2}\mathbf{u}'\mathbf{u} + \alpha\bar{\mathbf{x}}'\mathbf{V}\bar{\mathbf{x}} + \mathbf{y}'\mathbf{b} + \boldsymbol{\lambda}'(\bar{\mathbf{x}} + \boldsymbol{\varepsilon}) + \mathbf{c}'\bar{\mathbf{x}} \\ & - E(\tilde{\mathbf{p}})'\bar{\mathbf{x}} + \mathbf{w}'(E(\tilde{\mathbf{p}}) - \mathbf{c} - \alpha\mathbf{V}\bar{\mathbf{x}} - \mathbf{A}'\mathbf{y} - \boldsymbol{\lambda}) + \mathbf{v}'(\mathbf{c} + \boldsymbol{\lambda} - \mathbf{Q}\bar{\mathbf{x}} - \mathbf{u}) \end{aligned} \tag{5}$$

where, $\mathbf{w}'$and $\mathbf{v}'$represent the Lagrange multipliers associated to each constraint. From the Lagrange function we can derive the KKT conditions, where the sign is dictated by the direction of the optimisation and by the sign of the variables, and their associated complementary slackness conditions:

$$\frac{dL}{d\mathbf{u}} = \mathbf{u} \text{ - } \mathbf{v} = 0 \tag{6a}$$

$$\mathbf{u}'\frac{dL}{d\mathbf{u}} = \mathbf{u}'(\mathbf{u} - \mathbf{v}) = 0 \tag{6b}$$

$$\frac{dL}{d\mathbf{y}} = \mathbf{b} - \mathbf{A}\mathbf{w} \geq 0 \tag{7a}$$

$$\mathbf{y}'\frac{dL}{d\mathbf{y}} = \mathbf{y}'(\mathbf{b} - \mathbf{A}\mathbf{w}) = 0 \tag{7b}$$

$$\frac{dL}{d\boldsymbol{\lambda}} = \bar{\mathbf{x}} + \boldsymbol{\varepsilon} - \mathbf{w} + \mathbf{v} \geq 0 \tag{8a}$$

$$\boldsymbol{\lambda}'\frac{dL}{d\boldsymbol{\lambda}} = \boldsymbol{\lambda}'(\bar{\mathbf{x}} + \boldsymbol{\varepsilon} - \mathbf{w} + \mathbf{v}) = 0 \tag{8b}$$

$$\frac{dL}{d\alpha} = \bar{\mathbf{x}}'\mathbf{V}\bar{\mathbf{x}} - \mathbf{w}'\mathbf{V}\bar{\mathbf{x}} \geq 0 \tag{9a}$$

$$\alpha\frac{dL}{d\alpha} = \alpha(\bar{\mathbf{x}}'\mathbf{V}\bar{\mathbf{x}} - \mathbf{w}'\mathbf{V}\bar{\mathbf{x}}) = 0 \tag{9b}$$

$$\frac{dL}{d\mathbf{w}} = E(\tilde{\mathbf{p}}) - \mathbf{c} - \alpha\mathbf{V}\bar{\mathbf{x}} - \mathbf{A}'\mathbf{y} - \boldsymbol{\lambda} \leq 0 \tag{10a}$$

$$\mathbf{w}'\frac{dL}{d\mathbf{w}} = \mathbf{w}'(E(\tilde{\mathbf{p}}) - \mathbf{c} - \alpha\mathbf{V}\bar{\mathbf{x}} - \mathbf{A}'\mathbf{y} - \boldsymbol{\lambda}) = 0 \tag{10b}$$

$$\frac{dL}{d\mathbf{v}} = \mathbf{c} + \boldsymbol{\lambda} - \mathbf{Q}\bar{\mathbf{x}} - \mathbf{u} = 0 \tag{11a}$$

$$\mathbf{v}'\frac{dL}{d\mathbf{v}} = \mathbf{v}'(\mathbf{c} + \boldsymbol{\lambda} - \mathbf{Q}\bar{\mathbf{x}} - \mathbf{u}) = 0 \tag{11b}$$

KKT condition (6a) indicates that the dual value, $\mathbf{v}$, associated to the marginal cost function equation is equal to the farm deviation from the cost function, $\mathbf{u}$; since the model tries to keep $\mathbf{u}$ as small as possible, $\mathbf{v}$ should result in a small positive or negative number too. $\mathbf{w}$ is the dual value of the economic equilibrium constraint (2) and it can be interpreted as the shadow output quantity, thus $\mathbf{w} = \mathbf{x}$. Substituting $\mathbf{v} = \mathbf{u}$ and $\mathbf{w} = \mathbf{x}$ in (7a) and (8a), we can recognize in these two conditions the resource constraints and the calibration constraints respectively. Hence, the model (1)-(4) implicitly represents the constraints of a first phase model of the standard PMP and as a consequence the estimated model calibrates to the base year activity level without making the first phase explicit; this prevents from the critiques raised against the standard PMP approach (Heckelei and Wolff, 2003). The other KKT conditions represent a tautology (condition 9a) and the constraints of the model (conditions 10a and 11a). We estimated the model by Ordinary Least Squares where $\mathbf{u}$ is treated as the error term of the estimation procedure.

The estimated variables of the model (1) - (4) are then used to construct a non-linear model which includes both the estimated farm quadratic cost function and the estimated risk term (equations 12 -14). The model calibrates the endogenous variable levels to the base year without the calibration constraints and it can been used in simulation analysis:

$$\max EU(\tilde{\pi}) = E(\tilde{\mathbf{p}})'\mathbf{x} - \frac{1}{2}\mathbf{x}'\hat{\mathbf{Q}}\mathbf{x} - \hat{\mathbf{u}}'\mathbf{x} - \frac{1}{2}\hat{\alpha}\mathbf{x}'\mathbf{V}\mathbf{x} \tag{12}$$

subject to

$$\mathbf{A}\mathbf{x} \le \mathbf{b} \tag{13}$$

$$\mathbf{x} \ge 0 \tag{14}$$

where, $\mathbf{x}$ is the vector of endogenous activity levels, $\hat{\mathbf{Q}}$, $\hat{\mathbf{u}}$ and $\hat{\alpha}$ have been estimated previously and $E(\tilde{\mathbf{p}})'$, $\mathbf{V}$, $\mathbf{A}$ and $\mathbf{b}$ are exogenous parameters. Equation (12) is the farmer expected utility to be maximised which is equal to the expected revenue minus the estimated farm non linear cost function and the risk term; equation (13) is the resource constraint.

Our methodological proposal for the incorporation of risk in a PMP framework represents an innovative approach compared to the previous studies in this challenging research area. Our model differs from the work of Paris and Arfini (2000) as we estimate endogenously the farmer's coefficient of absolute risk aversion and we do not rely upon the standard three-step PMP. Although the endogenous variables in our estimation approach are the same variables estimated in the estimation model of Severini and Cortignani (2012), our proposal presents some differences from their approach. While Severini and Cortignani skipped the first phase of the PMP and they estimated directly the optimality conditions of the desired model, we merged the first linear phase with the second non-linear phase by using the dual relationships of an expected utility maximisation problem. By this approach, we could use additional information such as the variable accounting cost per unit of activity available in our dataset and the difference between the primal and the dual objective function of a farmer expected utility maximisation problem. In addition, our model specifies only the dual constraint on marginal costs while the estimation of the optimality conditions requires the specification of both the dual constraint and the resource constraints. Therefore, when the set of resource constraints is large the estimation model of Severini and Cortignani may be highly constrained, while our model not. Finally, the estimation carried out by Severini and Cortignani, according to the procedure proposed by Heckelei and Wolff (2003), introduces deviations in the output level, while our deviations concern the individual deviation from the common cost function. It is reasonable to think that homogenous farms share similar technology, which differs by small changes from one farm to another. Finally, our approach differs from the proposal of Petsakos and Rozakis (2011) as we applied the E-V approach, estimating directly both the non-linear cost function and the non-linear risk term.

4. Empirical Model and Data

In this section an empirical application of the theoretical model presented in section 3 will be provided considering crop price risk only. First the model (1)–(4) has been estimated and the ability of the model to calibrate to the base year activity levels and to estimate farmer specific absolute risk aversion coefficients has been checked. Then, the calibrated model (12) – (14) has been applied in simulation scenarios to analyse the farmer's reactions to the introduction of an agri-environmental program, the option to convert a share of cropland to grassland, under different crop price volatility scenarios. The ability of the model to represent the farmer's response to different risk-level scenarios has been investigated as well as the potential role of the grassland program as a strategy to cope with price risk. This has been investigated by detecting the change in land allocation among crops.

The model has been applied to three farm samples of the Emilia-Romagna region in Italy, differentiated by size (small, medium and large). Each sample is composed by fourteen farms and the small sample size is justified by the illustrative role of the empirical application. The estimation of the model and the simulations have been applied to each farm sample separately. Five crops have been included in the empirical model: sugar beet, common wheat, corn, barley, grassland under environmental commitments. The first four crops are the most widely grown crops in the area under study, while the committed grassland is the AES considered in our model. For the farms where the AES is not in place in the baseline, we have set a very small allocation to take into account this scheme as an option; in these farms, AES becomes a latent activity, that is an option in the farmer's production plan discarded in the base year (Arfini and Donati, 2013; Severini and Cortignani, 2008; Rohm and Dabbert, 2003). According to most PMP theoretical papers, we have introduced the land constraint as the only resource constraint and we adopt a quadratic cost function, the most frequently used functional form in PMP works.

Expected output prices, accounting variable cost per unit of output, observed activity levels, amount of farmland and the matrix of technical coefficients are all farm specific exogenous variables, while the variance-covariance matrix is exogenous and common to all farms. The solution of the model results in the simultaneous estimation of the endogenous variables: the quadratic matrix of the cost function, which is common to all farms of the same sample, and the farm specific variables, which are the shadow values of the calibration constraints and of land, the farmer's absolute risk aversion coefficient, and the farm deviations from the common cost function.

Different price-risk scenarios have been constructed by manipulating on a yearly base the monthly crop price time series which have been used to build the variance-covariance matrix. Hence, there is a different variance-covariance matrix in each of the four simulated scenario corresponding to a different degree of crop price volatility. Price volatility[1] in scenario 1 is set equal to half of the volatility in the baseline; scenario 2 and scenario 3 show price volatility smaller and higher by 10% than the baseline respectively; volatility of scenario 4 is 50% higher than the baseline. It has been assumed that the grassland under agri-environmental commitments is not sold on the market, thus this activity is a no-risk activity, since it affects farm income just by its fixed agri-environmental payment.

The three farm samples employed in the empirical model have been constructed from the 2009 data on territorial aggregates of AGREA (the Regional Agency for the payments to farmers) and of RICA (the Italian FADN database) considering only crop farms in plain areas in Emilia-Romagna. From the data on territorial aggregates we have constructed the

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[1] Price volatility of a crop has been calculated as the monthly price variation of that crop on a yearly base.

representative arable crop farms[2] in the plain area of seven Emilia Romagna provinces (Piacenza, Parma, Reggio-Emilia, Modena, Bologna, Ferrara, Ravenna). The representative farms have been grouped into three samples according to their size: small farms (0-20 hectares), medium farms (30-100 hectares) and large farms (more than 100 hectares). The payment for committed grassland is set at 240 euro/ha according to the Rural Development Program (RDP) of Emilia-Romagna (Regione Emilia-Romagna, 2005).

5. Results

In this section the results of the model estimation and of each simulated scenario will be presented for the three farm samples.

5.1 Estimation Results

The model is able to reproduce the base year observed variable levels in each farm sample. The percentage deviations between the observed activity levels and the level reproduced by the model is lower than 1% for almost all farms and most of them show a deviation even lower than 0.5%. The model estimation of the quadratic matrix of the non linear cost function shows substitution relationships between crops; the only exceptions are represented by the estimated parameters for sugar beet that show complementary relationships with the other crops in the small farm sample and by the grassland under environmental commitment which seems to be complementary to some crops[3].

The model is able to estimate a farm specific absolute risk aversion coefficient; six farmers from the small farm sample, one farmer from the medium farm sample and three farmers from the large farm sample show a neutral attitude towards risk, while the others exhibit a risk averse behaviour (table 1). We have calculated the farmers' relative risk aversion coefficients by applying the Arrow-Pratt rule and the results are consistent with the range of 0-7.5 indicated in the literature for that coefficient (Chavas and Holt, 1996); only three farmers from the medium sample and three farmers from the large sample show values just above that range.

5.2 Simulation results

The simulation results show the ability of the model to represent the farmers' reactions to changes in price volatility. The analyses has been focused on the changes in the share of land committed to grassland under different scenarios of crop price volatility as well as on the changes in land allocation among the other crops. The variation in the crop allocation confirms the importance of accounting for risk in the analyses of farmers' behaviour, while the increasing share of farmland contracted to AES supports the potential role of such a

scheme as an income stabiliser tool. While risk neutral farmers do not change their crop mix under different scenarios, variations in crop price volatility leads to changes even strong in land allocation in the case of risk averse farmers. If the volatility was lower compared to the baseline situation, all risk averse farmers would not adopt the grassland program. This

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[2] The representative farm of a territorial aggregate has been constructed from the aggregate data. The allocation of land among crops in the representative farm has been determined according to the crop proportion in the aggregates. The variable costs and the crop prices of the representative farm belong to the average costs and average crop prices of the territorial aggregates.

[3] For space reasons we do not report these results, but they are available from the authors upon request.

Table 1 Estimates of farmer's absolute and relative risk aversion coefficients.

	Small farm sample		Medium farm sample		Large farm sample	
	Absolute risk aversion coefficient	Relative risk aversion coefficient	Absolute risk aversion coefficient	Relative risk aversion coefficient	Absolute risk aversion coefficient	Relative risk aversion coefficient
1	0.00190	5.43	0.00040	11.39	0.00010	7.90
2	0.00000	0.00	0.00002	1.04	0.00004	9.61
3	0.00080	4.44	0.00040	8.42	0.00002	2.49
4	0.00000	0.00	0.00004	2.24	0.00000	0.16
5	0.00230	5.42	0.00010	3.48	0.00003	2.90
6	0.00000	0.00	0.00008	4.02	0.00000	0.00
7	0.00070	3.81	0.00020	7.42	0.00002	3.13
8	0.00000	0.00	0.00000	0.00	0.00000	0.00
9	0.00120	5.42	0.00050	12.22	0.00008	8.37
10	0.00000	0.00	0.00009	4.56	0.00001	2.42
11	0.00140	3.53	0.00030	6.32	0.00002	4.14
12	0.00030	2.70	0.00003	1.90	0.00001	2.31
13	0.00070	3.10	0.00009	3.40	0.00003	3.46
14	0.00000	0.00	0.00006	2.90	0.00000	0.00

may be explained by the small share of land allocated to the AES in the baseline which would easily become zero with a small decrease in crop price volatility. On the other hand, an increase in crop price volatility would promote the adoption of the AES. While in the baseline only one farm contracts more than 1% of its land to grassland, an increase in crop price volatility by just 10% (scenario 3) leads to a share contracted which varies from 6.1% to 12.1% in the small farm sample, from 0.7% to 1.9% in the medium farm sample and from 1.7% to 7.8% in the large farm sample (table 2). In scenario 4, where the volatility is set 50% higher than the baseline volatility, this share would be larger than 12% for the small farms, and some of them would commit to grassland even more than one fourth of their farmland. Under this scenario most of the large farms would adopt grassland program on more than 10% of their land and one of them would contract 19.6%, while medium farms would not commit to grassland more than 6.3% of farmland. The growth of the AES farmland share under scenarios of increasing crop price volatility show the potential role for such a scheme as income stabiliser. Indeed, when the crop price fluctuations rise, risk averse farmers are willing to convert some share of high income and high risk crops to a free risk activity.

In order to account for the variation in the crop mix caused by changes in the crop price volatility, we will consider the aggregate data of the hectares allocated to each crop within each sample (figures 1-3). This variation is the result of a direct and indirect effect. The direct effect consists in the changes of the crop price variance leading to a decrease of the highest risky crops when the volatility rises. The indirect effect is due to the cross terms of the quadratic matrix of the non linear cost function and of the variance-covariance matrix. The cross terms indicate the relationship between the hectares allocated to a crop and the marginal cost of production of another crops. A positive cross term leads to a rise in the marginal cost of production of a crop as a consequence of a rise in the hectares allocated to another crop.

When the crop price volatility rises, the share of farmland allocated to sugar beet increases in all the three samples. This is consistent with what we may expect, as sugar beet is the lower risky crop, besides the grassland under commitment; in addition, its cross terms in the variance-covariance matrix are negative, thus an increase in its share leads to a decrease in the

Table 2. Share of farmland allocated to grassland under AES.

	Small farm sample					Medium farm sample					Large farm sample				
	base	scenario 1	scenario 2	scenario 3	scenario 4	base	scenario 1	scenario 2	scenario 3	scenario 4	base	scenario 1	scenario 2	scenario 3	scenario 4
1	1.96	0.00	0.00	9.42	26.04	0.25	0.00	0.00	1.20	3.35	0.08	0.00	0.00	5.61	19.59
2	0.66	0.66	0.66	0.66	0.66	0.31	0.00	0.00	1.15	3.58	0.03	0.00	0.00	4.26	15.85
3	1.32	0.00	0.00	6.13	14.17	0.31	0.00	0.00	1.25	2.83	0.05	0.00	0.00	1.69	7.21
4	0.76	0.76	0.76	0.76	0.76	0.15	0.00	0.00	1.15	2.84	0.25	0.00	0.00	2.05	2.24
5	2.17	0.00	0.00	9.37	26.15	0.31	0.00	0.00	1.30	3.64	0.05	0.00	0.00	3.38	12.18
6	0.59	0.59	0.59	0.59	0.59	0.21	0.00	0.00	0.94	1.98	0.08	0.08	0.08	0.08	0.08
7	1.10	0.00	0.00	7.47	12.36	0.21	0.00	0.00	1.15	3.86	0.05	0.00	0.00	4.36	10.39
8	0.83	0.83	0.83	0.83	0.83	0.11	0.11	0.11	0.11	0.11	0.03	0.03	0.03	0.03	0.03
9	1.16	0.00	0.00	6.80	13.44	0.26	0.00	0.00	1.89	6.31	0.06	0.00	0.00	7.79	17.35
10	0.55	0.55	0.55	0.55	0.55	0.17	0.00	0.00	1.06	3.66	0.03	0.00	0.00	4.08	8.19
11	1.64	0.00	0.00	12.12	35.04	0.24	0.41	0.00	1.04	1.76	0.39	0.00	0.00	4.86	11.41
12	0.71	0.00	0.00	6.52	17.91	0.13	0.00	0.00	0.71	2.32	0.63	0.00	0.00	4.90	8.83
13	1.24	0.00	0.00	7.33	15.77	0.41	0.00	0.00	1.39	3.34	0.05	0.00	0.00	6.30	16.59
14	0.53	0.53	0.53	0.53	0.53	0.20	0.00	0.00	0.99	2.50	0.03	0.03	0.03	0.03	0.03

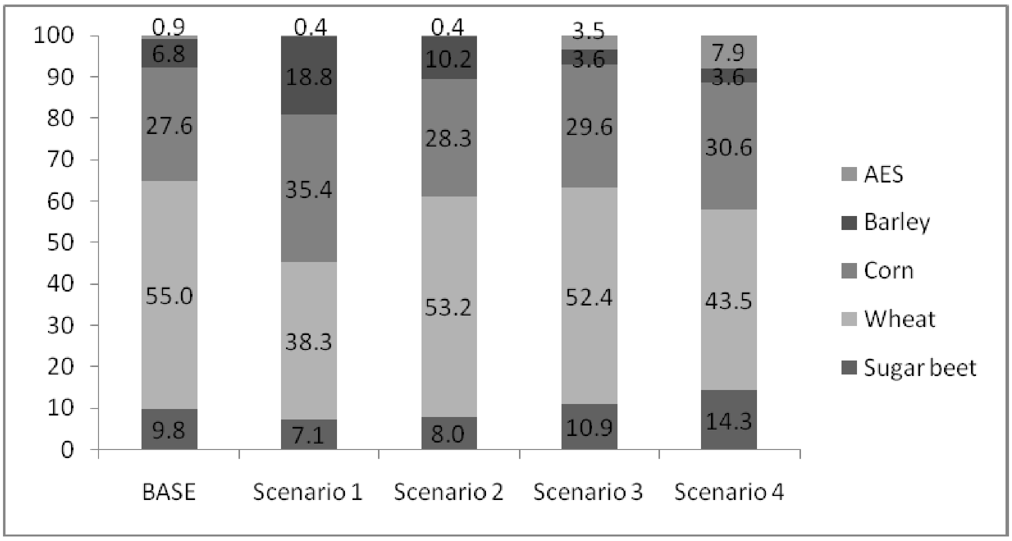

Figure 1. Share of farmland allocated to crops in the small farm sample.

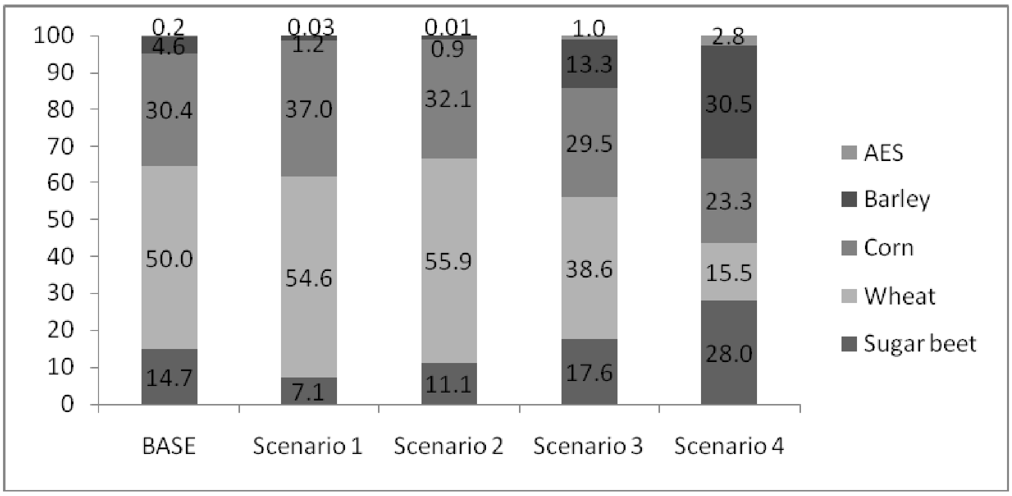

Figure 2. Share of farmland allocated to crops in the medium farm sample.

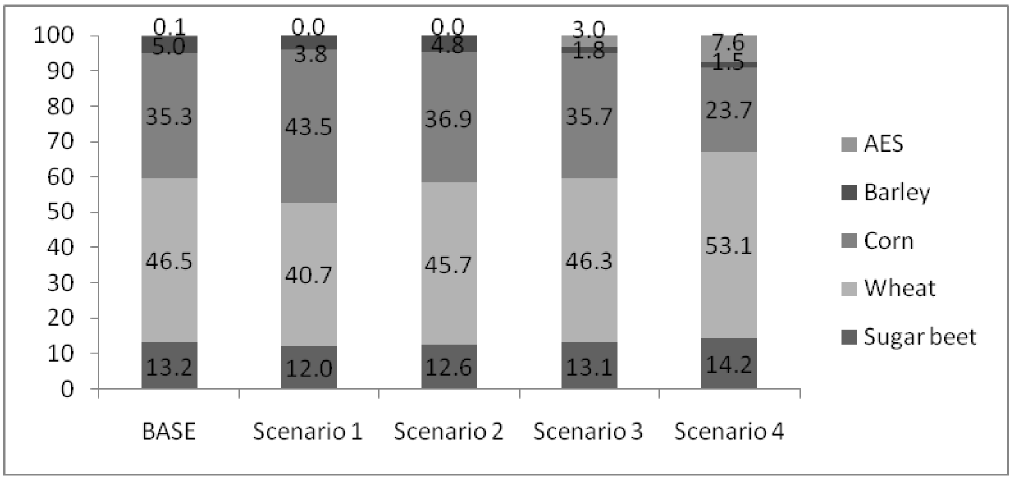

Figure 3. Share of farmland allocated to crops in the large farm sample.

risk cost of the other crops. In scenario 4, where the price volatility is set 50% higher than the baseline, the share of sugar beet reaches 28% in the medium farm sample, while it is around 14% in the small and large farm samples. This may explain the lower share of farmland committed to grassland program in the medium farm sample under this scenario, which compensates the lower adoption of the free risk AES with an higher adoption of the low risk sugar beet compared to the other two samples.

Wheat is the higher risky crops and its share changes depend on the samples. Given the high risk feature of wheat price we may expect a decrease in the hectares allocated to wheat when the price fluctuations of all crops rise. This is verified in the medium farm sample, while in the large farm sample the share of farmland allocated to wheat exhibits an increasing behaviour. The latter may be explained by the complementary relationship between wheat and the committed grassland as shown by the negative cross term of the quadratic matrix of the cost function of the large farm sample: as the increase in volatility leads to a larger grassland adoption, this contributes to reduce the wheat costs and promote a larger share of wheat grown. The share of farmland allocated to corn decreases with a rise in the crop price volatility in the medium and large farm sample. In the small farm sample the hectares devoted to corn do not show any relationships with volatility, likely because the negative cross term between sugar beet and corn and committed grassland and corn in the quadratic matrix of the cost function drops the effect of corn price variations.

Besides wheat, the second higher risky crops among the five considered is barley. In the small farm sample, the increase in crop price volatility leads to a reduction in the share of farmland allocated to barley. For this crop the effect of volatility is stronger than the complementary relationships with sugar beet. In the medium farm sample the share of farmland devoted to barley increases with volatility growth, while the large farm sample shows an irregular path: a rise in the barley share from scenario 1 to 2 and a decline from scenario 3 to 4.

6. Discussion and Conclusions

In this paper we have developed a new methodological approach which accommodates risk in a PMP framework. Risk modelling and calibration of mathematical programming models are two relevant issues in the analysis of farmers' behaviour. The literature is still rather scarce in this area and our proposal represents an innovative approach compared to previous studies. The farmers' absolute risk aversion coefficient is estimated endogenously in the model together with the farm non-linear cost function, the shadow prices of resources and the shadow prices of activities. The simultaneous estimation prevents from the critiques raised against the standard PMP approach. By the KKT conditions we have shown that the model is consistent with the PMP theory and calibrates to the base year observed activity levels without making any calibration constraint explicit.

We have provided an empirical application of our model on three representative farm samples differing by farm size. We showed that the model calibrates to the base year observed activity levels for all farms. Moreover, the values of the estimated coefficients of farmers' risk aversion are consistent with the range provided in the literature.

We have performed some simulations to test the model's ability to represent the farmers' responses to changes in economic conditions (crop price volatility) as well as to investigate the role of agri-environmental grassland as a farmer's strategy to cope with risk. The idea is that the grassland under agri-environmental contract may represent an income risk management tool for the farmer, since it guarantees a fixed payment independent of the market conditions.

The simulation results confirm the potential role of the grassland program as an insurance against farmer's income risk as the share of farmland contracted rises for risk averse farmers under scenarios of increasing crop price volatility. In the baseline the share of land under agri-environmental grassland is lower than 1% for all farms but one: if the volatility is set 50% higher than the baseline volatility all risk averse small farms would contract more than 12% of their farmland and for some of them this share would reach one fourth; under this scenario large farms would devote more than 10% of their land to agri-environmental grassland, while the adoption in medium farms would be smaller. The changes in the farmland allocation among the other crops under simulated scenarios depends not only on the crop price risk level but also on the cross relationships between crops expressed in the cross terms of the variance-covariance matrix and of the quadratic matrix of the cost function.

Despite the innovations of our model compared to the previous proposals, our model still makes some assumptions. For example, we assume that farmers exhibit CARA risk preferences and that income volatility is due only to the price changes, while yields are kept constant over time. It would be interesting to further develop the model by assuming DARA preferences and by introducing variable crop yields over time. In the latter it would be challenging to separate the effects of yield variation and price variation on farmer's behaviour. A further extension of our model would consist in modelling and calibrating activities whose base year level is equal to zero in some farms according to the proposal of Paris and Arfini (2000).

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