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Ignoring seasonality, a type of specification error, may result in biased estimates of parameters. To avoid this problem, one of three approaches is generally taken: (1) deseasonalize the data, (2) introduce an additional variable, such as weather, to explicitly account for the seasonal variation, or (3) use dummy variables to capture the effects of seasonality.

The first approach, using seasonally adjusted data, is undesirable for a number of reasons. First, it implicitly assumes that seasonality is separable. Gersovitz and MacKinnon (1978) argue that economic theory provides no support for this assumption. Second, using seasonally adjusted data imposes a certain pattern of variation on the regression parameters which may be inappropriate (Ladd, 1964). Finally, if seasonality in the dependent variable is to be explained, then seasonally unadjusted data should be used (Thomas and Wallace, 1971).

The second approach, accounting for seasonality by including an additional variable, can be difficult to implement due to data availability, collinearity problems with included variables, and specification problems. In addition, it is doubtful that this approach offers any improvement over the dummy variable approach (see Gersovitz and MacKinnon, p. 270).

The third approach, using a variable which assumes the value of one in the jth season and zero otherwise, has the advantage of simplicity

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and flexibility in its implementation. Further, if all parameters are allowed to vary seasonally, then the estimated relation can approximate the true functional form in which seasonality is inseparable (Gersovitz and MacKinnon, p. 265). Finally, seasonal effects can be directly estimated. Multicollinearity, however, is a potential disadvantage; OLS estimates can be very imprecise when both intercept and slope coefficients are allowed to vary simultaneously. One way to overcome this problem is to impose some additional restrictions on the model. One such restriction is to require the regression coefficients pertaining to adjacent seasons not to differ greatly. Gersovitz and MacKinnon applied this "smooth seasonality" restriction to estimating the demand for soft drinks and found that it compared well to the explicit use of weather variables. In addition, their sampling experiments indicate that the smoothness seasonality restriction can result in considerable gains in efficiency over OLS while yielding seasonal coefficient estimates which vary in a simple regular fashion.

The purpose of this paper is to explore further the performance of the Smoothness estimator. The procedure is applied to estimating the monthly variation in the long-run milk sales response to generic advertising expenditures in the New York City market. The first section reviews the smoothness methodology. Empirical results are then presented, and these results are evaluated and discussed.

The Smoothness Methodology

Suppose, for simplicity, that the model to be estimated is

$$Y_t = \sum_{j=1}^{\lambda} \beta_j D_{jt} X_{jt} + \varepsilon_t \qquad \varepsilon_t \sim (0, \sigma^2) \tag{1}$$

where Y_t and X_t are scaler time series at time t, and $D_{jt} = 1$ if t is in the jth season and $D_{jt} = 0$ otherwise. The smoothness restriction is imposed formally on (1) by the expression

$$\Delta^{d+1} \beta_j = \mu_j \qquad \mu_j \sim (0, \zeta^2) \tag{2}$$

where Δ is the difference operator and d is the degree of smoothness. Thus, for example, first-degree smoothness implies that second differences in the β_j are small, i.e.

$$\Delta^2 \beta_j = (\beta_j - \beta_{j-1}) - (\beta_{j-1} - \beta_{j-2}) = \beta_j - 2 \beta_{j-1} + \beta_{j-2} \tag{3}$$

is small for all seasons.

Estimating (1) subject to (2) is a Bayesian procedure that was first proposed by Shiller (1973) to mitigate the effects of multicollinearity in the estimation of distributed lags. Gersovitz and MacKinnon modified the Shiller methodology to make it applicable to seasonality estimation. In particular, the periodicity of the seasons implies that the first coefficient is linked to the last, as well as the second: e.g., if $\lambda = 4$ (for quarterly data) and $d = 1$ then

$$\begin{aligned} (\beta_3 - \beta_2) - (\beta_2 - \beta_1) &= \mu_1 \\ (\beta_4 - \beta_3) - (\beta_3 - \beta_2) &= \mu_2 \\ (\beta_1 - \beta_4) - (\beta_4 - \beta_3) &= \mu_3 \\ (\beta_2 - \beta_1) - (\beta_1 - \beta_4) &= \mu_4 \end{aligned} \tag{4}$$

or in matrix notation

$$R_1 \beta = \mu \tag{5}$$

where

$$R_1 = \begin{bmatrix} 1 & -2 & 1 & 0 \\ 0 & 1 & -2 & 1 \\ 1 & 0 & 1 & -2 \\ -2 & 1 & 0 & 1 \end{bmatrix} . \tag{6}$$

The seasonal linkage of the coefficients results in linear dependence in the R_d matrix (the first $\lambda - 1$ rows sum to minus the λ row). For example, in (6), the first three rows sum to minus the fourth row. This makes the (Shiller) assumption that μ comes from a spherical normal distribution with covariance matrix $\zeta^2 I$ inadmissable, but this problem is circumvented by replacing the $k = \sigma/\zeta$ parameter in the smoothness estimator

$$\hat{\beta}_s = (\tilde{X}' \tilde{X})^{-1} (\tilde{X}' \tilde{Y}) \tag{7}$$

where
$$\tilde{X} = \begin{bmatrix} D_1 X & D_2 X & \ldots & D_\lambda X \\ & kR_1 & & \end{bmatrix} \tag{8}$$

and
$$\tilde{Y} = \begin{bmatrix} Y \\ 0 \end{bmatrix} \tag{9}$$

with
$$k' = \left((\lambda - 1)/\lambda\right)^{\frac{1}{2}} k . \tag{10}$$

The Smoothness estimator (7) can be computed using an OLS regression package by adding λ zeroes to the Y vector and λ rows of R_1, multiplied by k', to the X matrix. The value of k' determines the weight that the dummy observations carry in estimating the parameters. If $k = 0$, the Smoothness estimates reduce to OLS estimates. The OLS estimates are modified to a greater extent by the smoothness restriction as k' increases in magnitude. The choice of k, which depends on σ and ζ, is discussed in the next section.

III. Empirical Results

An economic model designed to provide dairy farmers with an estimate of their return on investment in generic advertising indicated that estimated returns are highest, ceteris paribus, when the price differential between Class I and Class II milk is the greatest (Thompson, Eiler, Forker 1976). This suggests that the advertising should be concentrated in the months containing the largest price differential (usually January through June). However, the in the context of the TEF model, a necessary condition for this conclusion to be valid is that the (long-run) milk sales response to advertising remain the same throughout the year. To gain some insights into the seasonal behavior of this response, the smoothness methodology is applied to the milk sales response function used in Thompson (1978). This function, modified to permit seasonal variation in the long-run advertising elasticity[1/], is written as

$$\ln q_t = \sum_{j=1}^{12} \phi_j D_{j,t} + \theta \ln I_{t-1} + \delta \ln PC_{t-1} + \gamma \ln PM_{t-1} +$$

$$\sum_{i=0}^{4} \sum_{j=1}^{12} \beta_{ij} D_{j,t-i} \ln A_{t-i} + u_t \qquad (11)$$

where q is per capita fluid milk sales, D_j are monthly dummy variables, I is

[1/] The "long-run" advertising elasticity is the sum of the initial plus carry-over effects of the advertising expenditure. The idea underlying (11) is that the initial and secondary effects of an advertising expenditure made in, say, January will differ from the corresponding effects of an expenditure made on, say, June, i.e., the month of the expenditure determines the structure of the initial and lagged response.

real per capita personal income, PC is the real price of cola, PM is the real price of milk, and A is the real per capita expenditure on generic milk advertising[2/]. The appropriate length for the distributed lag relationship between milk sales and advertising expenditures was set at four months based on previous analysis (Thompson 1978, Kinnucan 1981).

Monthly data covering 1975:1 to 1978:6 for New York City market are used. To accomodate the double-log specification, an arbitrarily small value of .0001 was used for months with zero advertising expenditures. Due to the four-period lag, the effective data period is May 1975 through June 1978, for a total of 38 observations.

The model as specified in (11) cannot be estimated with the data set described above due to insufficient degrees of freedom. This equation allows all components of the long-run advertising elasticity to vary simultaneously with the seasons and thus consumes 60 degrees of freedom which, by itself, exceeds the 38 observations available. To overcome this problem, a more restricted form of (11) was estimated, namely

$$\ln q_t = \sum_{j=1}^{12} \phi_j D_{j,t} + \theta \ln I_{t-1} + \delta \ln PC_{t-1} + \gamma \ln PM_{t-1} + \beta_0 \ln A_t + \beta_1 \ln A_{t-1} + \beta_2 \ln A_{t-2} + \sum_{j=1}^{12} \beta_{3,j} D_{j,t-3} \ln A_{t-3} + \beta_4 \ln A_{t-4} + u'_t \qquad (12)$$

[2/] A more precise description of the data used, along with sources, is provided in the footnotes of the appendix table containing the data.

Equation (12) allows only the effect of the third lagged month to vary monthly; the effect of the other components of the long-run advertising elasticity are assumed to be constant throughout the year. The third lagged month was selected to represent the seasonal change in the long-run elasticity for the following reasons: t-tests based on OLS estimates indicate that the coefficients of A_t and A_{t-1} are not statistically different from zero at the usual levels of statistical significance, and the coefficient of A_{t-3} accounts for the largest part of the total sales response to advertising (37 percent) and is statistically significant at the .01 level. The data permits seven degrees of freedom for the estimation of (12).

OLS estimates of the long-run advertising elasticity based on (12) are presented in the last column of table 1. These estimates are quite erratic. A look at the simple correlation coefficients between intercept and slope dummies revealed numerous correlations in excess of .95. To reduce this source of collinearity (12) was re-estimated (without the smoothness restriction) first omitting intercept dummies and then using quarterly intercept dummies[3/]. The estimated long-run advertising elasticity based on the no-intercept dummy model is quite constant throughout the year, becoming slightly more elastic in the spring months (see column one of table 1). However, an F-test comparing this model to the quarterly-intercept dummy model rejected the former at the .05 significance level ($F = 4.323$ compared to $F_{.05}(3, 15) = 3.29$).

[3/] The "quarters" cover December - February, March - May, and June - August with the September - November period as the base. This classification more nearly represents actual seasonal changes in milk consumption than do calendar quarters.

Table 1. SEASONAL LONG-RUN GENERIC ADVERTISING ELASTICITIES FOR FLUID MILK[a], New York City Market, May 1975 to June 1978 Data

Month	OLS estimates obtained when the model contains:		
	No Intercept Dummies	Quarterly Intercept Dummies	Monthly Intercept Dummies
Jan.	.0329	.0478	.0626
Feb.	.0362	.0552	.0946
Mar.	.0436	.0184	-.0279
Apr.	.0462	.0259	.0386
May	.0492	.0221	-.0427
June	.0340	.0380	.0346
July	.0327	.0382	.1243
Aug.	.0324	.0373	.0206
Sept.	.0277	.1084	.0873
Oct.	.0286	.1233	.0234
Nov.	.0290	.1217	.1404
Dec.	.0282	.0430	.0357

[a] Based on letting the carryover effects from the A_{t-3} expenditure to vary monthly.

The seasonal variation in the long-run advertising elasticities based on the quarterly-intercept dummy model is more regular than those from the monthly intercept dummy model (column two of table 1). These estimates suggest that the long-run milk sales response to advertising peaks in the September - November period and troughs in the March - May period. An F-test comparing the quarterly and monthly dummy intercept models indicates that the latter does not represent a significant improvement over the former ($F = 1.671$ compared to $F_{.05}\ (7, 8) = 3.50$). Therefore, subsequent analysis is based upon specifying intercept changes to occur quarterly rather than monthly.

As discussed above, more precise estimates of seasonality parameters can be obtained by imposing a smoothness restriction on the model. To investigate the extent to which this restriction has the potential for improving upon the OLS, estimates of the long-run advertising response (12) (using quarterly intercept dummies) was re-estimated using the smoothness methodology. An assumption of first-degree smoothness ($d = 1$) was imposed[4/]. A procedure, employed by Gersovitz and MacKinnon, was used to estimate the k parameter. First, the standard error of the OLS regression for (12) was used as an estimate of σ. To estimate ζ it was assumed that the β_{3j} coefficients had a mean equal to the mean of the OLS coefficients and followed a square wave[5/] which varied from +100 percent to -100 percent

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4/ While other degrees of smoothness are potentially relevant, Shiller (p. 779) states that $d = 1$ will probably do for most applications.

5/ The square wave assumption requires the coefficients to lie on a square wave with given amplitude, so that they are equal to their largest value for half the year and their smallest for the other half. Gersovitz and MacKinnon obtained better results with this assumption than the triangular wave assumption, which assumes seasonal coefficients follow a triangular pattern with the true amplitude, rising linearly for half the year, and falling linearly for the other half. The square wave assumption results on a smaller k-value than the triangular wave assumption; hence, is less restrictive.

of that mean. $\hat{\zeta}$ was then computed from the formula

$$\hat{\zeta} = \sqrt{\hat{\beta}_3{}'R_1{}'R_1 \;\; \hat{\beta}_3/12}$$

where $\hat{\beta}_3$ is the vector containing $2 \sum_{j=1}^{12} \hat{\beta}_{3j}/12 = .03783$ as the first six elements and zeroes as the remaining six elements. This resulted in a value for $\hat{k} = \hat{\sigma}/\hat{\zeta} = 1.19949$.

To determine the sensitivity of the estimates to the magnitude of k and, by implication, the square wave assumption upon which $\hat{k}$ is based, additional estimates based on ½ $\hat{k}$ and 2 $\hat{k}$ are generated. To account for the linear dependence in the R_1 matrix, $\hat{k}$ is multiplied by $(11/12)^{\frac{1}{2}} = .9574$, according to (10), to obtain $\hat{k}' = 1.14839$ for the actual weighting of the dummy observations in the augmented data matrix $\tilde{X}$.

OLS and Smoothness estimates for the β_{3j} parameters, along with the corresponding standard errors, are presented in table 2. According to the OLS estimates, advertising placed in October has the largest third round effect (with an impact elasticity of .086) while advertising placed in March has the smallest third round effect (with an impact elasticity of -.019). This means that a ten percent increase in October advertising will increase January milk sales by .86 percent, whereas the same increase in March would reduce June sales by .19 percent, ceteris paribus.

As can be seen, monthly differences in the parameter estimates are more gradual for the Smoothness estimates. Even a mild imposition of the smoothness restriction results in a considerable reduction in the magnitude of the September - November third month lag response (the ½ $\hat{k}'$ estimates in table 2).

Table 2. SEASONAL IMPACT GENERIC ADVERTISING ELASTICITIES FOR FLUID MILK[a], New York City Market, May 1975 to June 1978 Data

Month in Which the Expenditure Occured	The Measured Impact on Sales Three Months Later:			
	OLS Estimates	Smoothness Estimates		
		($\frac{1}{2}\hat{k}'$)	($\hat{k}'$ = 1.14839)	($2\hat{k}'$)
Jan.	.01013 (.00688)	.00234 (.00516)	.00878 (.00829)	.00617 (.00756)
Feb.	.01753 (.00950)	.00661 (.00872)	.01301 (.01130)	.00871 (.00982)
Mar.	-.01927 (.01275)	-.01272 (.01528)	-.00179 (.01285)	.00382 (.01055)
Apr.	-.01176 (.01092)	-.00665 (.01337)	.00258 (.01170)	.00679 (.01016)
May	-.01559 (.01336)	-.00977 (.01607)	.00228 (.01387)	.00715 (.01159)
June	.00028 (.02044)	.00106 (.02224)	.00410 (.01708)	.00595 (.01299)
July	.00048 (.01932)	.00100 (.02144)	.00325 (.01681)	.00485 (.01301)
Aug.	-.00039 (.01819)	.00085 (.01990)	.00322 (.01531)	.00452 (.01163)
Sept.	.07067* (.02875)	.02343 (.02304)	.00963 (.01531)	.00447 (.01087)
Oct.	.08560* (.03481)	.02924 (.02801)	.01189 (.01847)	.00494 (.01234)
Nov.	.08400* (.03450)	.03000 (.02711)	.01175 (.01732)	.00507 (.01121)
Dec.	.00532 (.00652)	.00147 (.00735)	.00479 (.00796)	.00298 (.00727)
RSS	.01027	.01499	.01555	.01704
X^2 [b]	------	4.934	7.867	9.977
ν	------	.543	.608	.779

*statistically significant at $p \leq .05$.

[a] Based on letting carryover effects from the A_{t-3} expenditure to vary monthly.

[b] Computed using the ζ^2 values .001908, .000447, and .000119 for $k = \frac{1}{2}\hat{k}$, $\hat{k}$, and $2\hat{k}$, respectively.

For all estimators, the December - August sales response to advertising lagged three periods is not statistically different from zero at even the 10% level. While the Smoothness estimates did result in efficiency gains (smaller standard errors) in most of the months, the concomitant decrease in the magnitude of the corresponding coefficients resulted in a reduction in statistical significance in most cases when compared to the OLS estimates.

The appropriateness of the smoothness restriction can be tested in several ways. One is to see whether our prior ideas embodied in the restriction are contradicted by the sample. To determine this, a test statistic derived by Theil (1971, pp. 350-51) can be used. Under the null hypothesis that sample and prior information are not in conflict with each other, the test statistic is distributed as chi-squared distribution with 11 degrees of freedom. The value of the X^2 test statistic for the alternative Smoothness estimates is presented in the second to last row of table 2. The data fails to reject the null hypothesis for all k-values at the 10% level ($X^2_{.10}(11) = 17.272$).

Another evaluation of the smoothness restriction is to see if smoothness estimates are Mean Square Error (MSE) superior to OLS estimates[6/]. A test-statistic, developed by Fomby (1979) in a different context, can be used for this purpose. Under the null hypothesis that the MSE of the Smoothness estimates is less than the OLS estimates, this ν-statistic has a non-central F-distribution. The values of this statistic for the Smoothness estimates are presented in the bottom row of table 2. Comparing these values with the corresponding critical $F_{\frac{1}{2}}$ .25 (11,13) = 1.609[7/] indicates

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6/ Mean square error refers to variance plus bias squared.

7/ A table of critical values for the non-central F-distribution is provided in Wallace and Toro-Vizcarrondo (1969).

that the null hypothesis of MSE superiority of the Smoothness estimates cannot be rejected even at the .25 level. This result, combined with that of the previous test, suggests that the Smoothness estimates do offer an acceptable alternative to the OLS estimates, particularly when the smoothness restriction is weakly imposed.

The long-run elasticities corresponding to the A_{t-3} impact elasticities are presented in table 3 for the OLS and Smoothness estimates. The smoothness restriction lowers the peak and raises the rough of the estimated seasonal response when compared to the unrestricted (OLS) estimates.

Table 3. SEASONAL LONG-RUN[a/] GENERIC ADVERTISING ELASTICITIES FOR FLUID MILK, New York City Market, May 1975 to June 1978 Data

		Smoothness Estimates		
Month	OLS Estimates	$\frac{1}{2}\hat{k}$	$\hat{k} = 1.14839$	$2\hat{k}$
Jan.	.04781	.03838	.04044	.03832
Feb.	.05521	.04265	.04467	.04086
Mar.	.01841	.02332	.02987	.03597
Apr.	.02592	.02939	.03423	.03894
May	.02209	.02627	.03394	.03930
June	.03796	.03710	.03576	.03810
July	.03816	.03704	.03491	.03700
Aug.	.03729	.03689	.03488	.03667
Sept.	.10835	.05947	.04129	.03662
Oct.	.12328	.06528	.04355	.03709
Nov.	.12168	.06604	.04341	.03722
Dec.	.04300	.03761	.03645	.03513

a/ Based upon letting the carryover effects from the A_{t-3} expenditure to vary monthly.

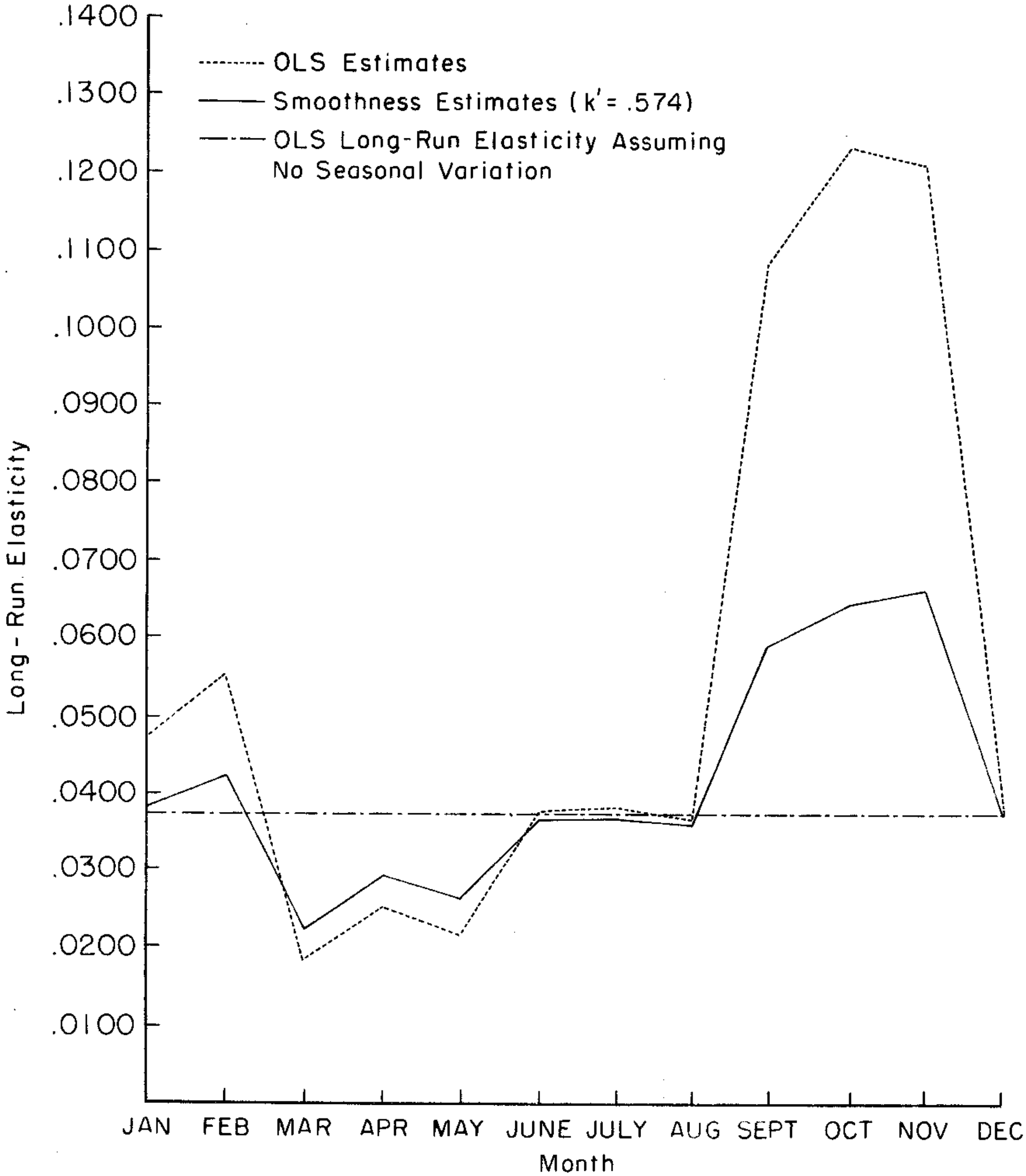
The estimated elasticities are quite sensitive to the magnitude of the k parameter: increasing it by a factor of two results in a nearly constant estimated seasonal response while halving it accentuates the peak response which occurs in the months of September, October and November. For these data, then, the underlying assumption upon which the k parameter is computed is quite important. The oversmoothing which occurs when $k = 2\hat{k}$ suggests that $2\hat{k}$ may serve as an upper bound for k. Neither the MSE test nor the X^2 test was capable of discriminating between the $\hat{k}'$ and $\frac{1}{2}\hat{k}'$ estimates. More powerful tests need to be developed if objective selection of the appropriate k-value Smoothness estimates is to be achieved.

Conclusions and Implications

The evidence presented in this paper shows a distinct monthly variation in the long-run milk sales response to generic advertising expenditures in the New York City market. Data limitations prevented analyses of the seasonal variation in all the components of the long-run response so that only that component accounting for the largest share of the total response was examined. This shortcoming can be overcome as additional data becomes available. In addition, the potential problem of twelfth-order serial correlation in the residuals was ignored. A more refined analysis would include consideration of this problem as well.

The Smoothness estimates of the seasonality in the long-run advertising elasticities can be considered mean square error superior to OLS estimates. As illustrated in figure 1, the Smoothness estimates based on the weakly imposed square wave assumption track the OLS estimates, but with reduced amplitude.

FIGURE I. SEASONAL LONG-RUN GENERIC ADVERTISING ELASTICITIES FOR FLUID MILK,[1] New York City Market, May 1975 to June 1978 Data



[1] Based on letting the carry-over effects from the A_{t-3} expenditure to vary monthly

Unfortunately, existing statistical tests are not powerful enough to discriminate between Smoothness estimates for small deviations in the k-parameter. Where Smoothness estimates are highly sensitive to small changes in this parameter, as is the case for the data used in this study, the lack of an objective means of selecting the "best" estimates is a serious problem. Thus, while the Smoothness methodology offers a highly flexible alternative to OLS for estimating seasonal variation in regression parameters, more precise guidelines for selecting the appropriate k-value need to be established.

Smoothness vis-a-vis OLS estimates of the seasonally varying advertising parameters have important implications for optimal advertising policy. For example, the optimal level of generic milk advertising in the New York City Market for the period July 1977 - June 1978 is $2,038,689 (in 1971 dollars) when the OLS estimates are used (table 4)[8/]. This figure would be reduced by at least 18 percent (to $1,676,339) if the Smoothness estimates presented in table 3 are used. Further, while the OLS estimates result in a seasonal allocation of 45 percent of the advertising budget for the months of September, October, and November, the corresponding allocation based on the Smoothness estimates would be no more than 36 percent for these months. Thus, in the absence of objective selection criterea, widely differing policy recommendations can emerge from the same data and model.

8/ The optimum advertising expenditure is defined as that expenditure which maximizes producer surplus less cost of advertising and was computed on the basis of a model developed by Thompson et al. (1976) where alternative uses of the advertising budget are assumed to earn at least a 20 percent marginal rate of return.

Table 4. OPTIMAL SEASONAL ALLOCATION OF THE GENERIC MILK ADVERTISING BUDGET BASED ON ALTERNATIVE ESTIMATES OF THE MONTHLY LONG-RUN ADVERTISING ELASTICITIES, New York City Market, July 1977-June 1978.

	The Optimal[1/] Monthly Expenditure Based on:		
	OLS Estimates	Smoothness Estimates of β	
Yr./Month	of β	$\frac{1}{2} = \hat{k}$	$\hat{k} = 1.14839$
1977 July	$ 134,390	$ 130,984	$ 124,526
Aug	131,772	130,460	124,352
Sept	288,320	187,355	139,012
Oct	323,574	208,298	150,181
Nov	309,787	203,236	144,945
Dec	148,701	132,729	129,238
1978 Jan	162,314	134,998	141,106
Feb	164,583	133,427	138,663
Mar	70,165	87,178	108,644
Apr	92,679	103,758	130,809
May	82,906	96,951	121,559
June	129,503	126,969	122,955
Annual	$2,038,689	$1,676,339	$1,575,988

[1/] Assumes a 20 percent marginal rate of return for alternative uses of the advertising budget. All figures are expressed in terms of 1971 dollars.

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APPENDIX

Appendix Table 1. Milk Sales, Generic Advertising Expenditures, and Other Data for New York City SMSA (January 1975 to June 1978)

		Adjusted Per Capita Sales [a/] (Ounces)	Per Capita Monthly Advertising [b/] (Dollars)	Per Capita Personal Income [c/] (Dollars)	Retail Milk Price ½ gallon [d/] (Dollars)	Population SMSA [e/] (000)	Population MCA [f/] (000)	Consumer Price Index [g/]	Cost of Advertising Index [h/]	Retail Cola Price 72 oz. Carton [i/] (Dollars)
1975	Jan.	9.15	---	6,929.8	.85	12,082.8	18,595.5	161.7	140.0	1.985
	Feb.	9.30	.01742	6,930.5	.83	12,152.6	18,574.8	163.2	140.0	1.970
	Mar.	9.18	.00430	6,931.3	.82	12,222.4	18,554.1	163.4	140.0	1.995
	Apr.	9.03	.00404	6,932.0	.78	12,292.2	18,533.4	163.7	154.0	1.999
	May	8.86	.00610	6,932.8	.80	12,362.0	18,512.7	164.3	154.0	1.921
	Jun.	8.55	.01686	6,933.5	.79	12,432.8	18,492.0	165.2	154.0	1,894
	Jul.	7.95	.00898	6,934.3	.76	12,501.5	18,471.3	166.6	139.0	1.894
	Aug.	8.19	.00427	6,979.2	.78	12,502.5	18,472.0	167.5	139.0	1.821
	Sep.	8.73	.00925	7,024.0	.77	12,503.4	18,472.7	169.3	139.0	1.811
	Oct.	8.93	.00958	7,068.9	.78	12,504.4	18,473.4	170.0	159.0	1.805
	Nov.	8.85	.01288	7,113.7	.83	12,505.3	18,474.1	171.4	159.0	1,805
	Dec.	8.96	---	7,158.6	.82	12,506.2	18,474.8	172.4	159.0	1.804
1976	Jan.	8.82	.00128	7,203.5	.83	12,507.2	18,475.5	172.7	175.0	1.801
	Feb.	8.46	.00700	7,248.3	.81	12,508.1	18,476.2	173.5	175.0	1.811
	Mar.	8.94	.00702	7,293.1	.84	12,509.0	18,476.9	173.9	175.0	1.810
	Apr.	8.53	.01128	7,337.9	.83	12,510.0	18,477.6	174.3	191.0	1.810
	May	8.57	.01394	7,382.8	.84	12,510.9	18,478.3	179.9	191.0	1.813
	Jun.	8.31	.01059	7,427.5	.84	12,511.8	18,479.0	176.0	191.0	1.815
	Jul.	7.95	.00579	7,472.4	.84	12,512.8	18,479.6	176.7	175.0	1.815
	Aug.	7.93	.01451	7,536.4	.84	12,481.9	18,448.2	177.6	175.0	1.811
	Sep.	8.46	.00475	7,600.6	.84	12,451.0	18,416.8	178.6	175.0	1.812
	Oct.	8.73	.01592	7,665.3	.86	12,420.0	18,385.4	179.0	205.0	1.830
	Nov.	8.68	.01041	7,730.2	.86	12,389.1	18,354.0	179.0	205.0	1.851
	Dec.	8.85	---	7,795.5	.86	12,358.2	18,322.5	179.7	205.0	1.863

Appendix Table 1 (continued).

		Adjusted Per Capita Sales[a/] (Ounces)	Per Capita Monthly Advertising[b/] (Dollars)	Per Capita Personal Income[c/] (Dollars)	Retail Milk Price ½ gallon[d/] (Dollars)	Population SMSA[e/] (000)	Population MCA[f/] (000)	Consumer Price Index[g/]	Cost of Advertising Index[h/]	Retail Cola Price 72 oz. Carton[i/] (Dollars)
1977	Jan.	8.75	—	7,861.0	.86	12,327.3	18,291.1	180.5	202.0	1.872
	Feb.	8.68	.00391	7,926.9	.87	12,296.4	18,259.7	182.1	202.0	1.863
	Mar.	8.70	.00840	7,993.1	.86	12,265.5	18,228.3	182.9	202.0	1.864
	Apr.	8.06	.00085	8,059.6	.86	12,234.6	18,196.8	183.7	220.0	1.863
	May	8.40	.00779	8,126.5	.86	12,203.7	18,165.4	184.6	220.0	1.869
	June	8.30	.01275	8,193.7	.86	12,172.8	18,134.0	186.2	220.0	1.883
	July	8.578	.01398	8,250.8	.87	11,113.5	17,577.9	186.4	202.0	1.868
	Aug.	8.547	.00515	8,308.0	.86	11,100.6	17,560.7	187.2	202.0	1.883
	Sep.	9.719	.00383	8,365.1	.86	11,087.7	17,543.4	187.3	202.0	1.937
	Oct.	9.712	.01527	8,422.2	.86	11,074.8	17,526.2	187.6	231.0	1.969
	Nov.	9.756	.01165	8,479.3	.86	11,062.0	17,509.0	188.5	231.0	1.992
	Dec.	9.682	.00476	8,536.5	.87	11,049.1	17,491.7	188.8	231.0	2.003
1978	Jan.	9.606	.00137	8,593.6	.87	11,036.2	17,474.5	189.3	218.0	1.996
	Feb.	9.690	.00575	8,650.7	.87	11,023.3	17,457.3	190.7	218.0	2.062
	Mar.	9.665	.00805	8,707.9	.87	11,010.4	17,440.0	191.9	218.0	2.060
	Apr.	9.822	.00858	8,765.0	.88	10,997.6	17,422.8	192.9	242.0	2.086
	May	9.392	.00646	8,822.1	.88	10,984.7	17,405.6	194.1	242.0	2.088
	Jun.	9.063	.00825	8,879.3	.88	10,971.8	17,388.3	196.1	242.0	2.109

FOOTNOTES FOR APPENDIX TABLES

a/ The net sales within the Standard Metropolitan Statistical Area (SMSA) were adjusted for the type of days in the month, i.e., number of Sundays, Mondays, etc. The sales were also placed on a per capita basis according to the population in the SMSA. The SMSA for the July 1977-June 1978 data was defined to exclude Bergen County of New Jersey. Source for adjusting data for calendar composition: John P. Rourke, Adjusting In-Area Sales Data for Calendar Composition, USDA, Agr. Mktg. Ser. Fed. Milk Order Mktg. Stat., MOMS, No. 196, April 1976, FMOMS No. 210, June 1977, and FMOS No. 233, August 1979.

b/ Includes media advertising expenditures for television, radio and newspaper. Advertising expenditures were placed on a per capita basis according to the population in the media coverage area (MCA). The July 1977-June 1978 data excludes expenditures for network T.V. advertising. Source: Advertising invoices of American Dairy Association and Dairy Council of Syracuse, New York.

c/ Personal income within SMSA before taxes. Personal income was placed on a per capita basis according to the population of the SMSA. Source: New York State Department of Commerce, Personal Income, New York State By County, 1974 and 1975, July 11, 1977. Historical growth rates were used to estimate January 1976 to June 1978 data.

d/ Prevailing food store Metro Area fluid whole milk price in dollars per half gallon. Source: Survey of Prices Charged for Milk on Retail Routes, Food Stores and Dairy Stores 25 Upstate Markets, various monthly issues.

e/ SMSA counties for NYC Metro are: Nassau, New York City--five boroughs, Rockland, Suffolk, Westchester, and Bergen, New Jersey. The population figures for July 1977-June 1978 exclude Bergen County, New Jersey. Monthly population estimates are a linear interpolation of annual estimates. Population source: New York State Statistical Yearbook, various issues. The 1977-78 annual figures were provided by Bob Scardamalio of the NYS Dept. of Commerce, Division of Economics and Statistics.

f/ Media Coverage Area (MCA) population. Estimated population viewing television stations of a given market. Source: New York State Statistical Yearbook and Federal Population Series, P-26, various issues. Nonlinear population estimates were made for 1976 and 1977. The July 1977 to June 1978 figures are linear interpolations of annual estimates obtained from Broadcasting Yearbook, Storer Broadcasting Co., 1978, 1979 issues.

FOOTNOTES (Continued)

g/ Consumer Price Index (CPI) for all items in the New York - Northeastern New Jersey area, 1967=100. Source: NYS Bureau of Business Research, Quarterly Summary of Business Statistics, 1975-1979 issues.

h/ Cost of Advertising Index (composite of all time periods) where first quarter 1971=100. This index reflects variations in the cost of prime-time spot television. Source: United Dairy Industry Association, correspondence, Barbara J. Deering, January 7, 1976. Estimates for 1976 and 1977 were made in consultation with personnel from D'Aray-MacManus & Masius, Inc. July 1977-June 1978 figures are based on extrapolation from past trends.

i Retail price of cola drink (throwaway, 72 oz. carton) in the New York-Northeastern New Jersey area. Source: United States Department of Labor, Bureau of Labor Statistics, Estimated Retail Food Prices by City, various monthly issues.